Electromagnetic isospin breaking in hadronic vacuum polarization within a VMD model

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Abstract. One of the challenging components of modern lattice-QCD computations of the leading hadronic vacuum polarization contribution to the muon $(g-2)_\mu$ is the isospin-breaking correction due to electromagnetism. Dominated by long distances, this correction suffers from severe signal-to-noise issues and finite-volume effects. To provide infinite-volume benchmarks for various quark-level Wick contractions, we estimate their sizes phenomenologically at the physical point, using a previously developed vector-meson-dominance model. The pion contributions in this model sum to around -7.1×10^{-10}, dominated by the electromagnetic pion mass shift. The ultraviolet-finite (2 + 2)a ("dumbbell") contraction accounts for approximately two thirds of this model sum; for both, about 30% comes from Euclidean times beyond 2.8 fm.

1 Introduction

The muon anomalous magnetic moment $a_\mu \equiv (g-2)_\mu/2$ is measured with a precision of 124 ppb, $\Delta a_\mu^{\rm exp} \simeq 1.5 \times 10^{-10}$ [1, 2]. The leading hadronic vacuum polarization (HVP) contribution dominates the uncertainty of the Standard-Model prediction in the 2025 Theory Initiative update [3].[1] In view of tensions among data-driven evaluations based on e^+e^- data [7, 8], that update takes the HVP from lattice QCD [9–14]; for recent BMW/DMZ results, see Refs. [15, 16]. Hadronic τ decays offer an alternative input, once corrected for isospin breaking [17–19].

Lattice calculations commonly incorporate isospin-breaking effects through a perturbative expansion about the isospin-symmetric point [20–22],

$$a_\mu^{\rm HVP} = a_\mu^{\rm HVP}\big|_{\rm iso} + \delta m \left.\frac{\partial a_\mu^{\rm HVP}}{\partial\, \delta m}\right|_{\rm iso} + \alpha_{\rm em} \left.\frac{\partial a_\mu^{\rm HVP}}{\partial \alpha_{\rm em}}\right|_{\rm iso} + \ldots, \qquad \delta m = m_d - m_u, \tag{1}$$

where $\alpha_{\rm em}$ is the fine-structure constant, m_d and m_u are the down- and up-quark masses. The electromagnetic (QED) correction to the HVP, a few units of 10^{-10} [9, 11, 23, 24], must be determined with an accuracy of a few tens of percent. Achieving this precision is challenging because most of the correction comes from long distances, where the signal deteriorates. This requires reconstructing the tail [11, 23, 25, 26] or using e^+e^- data for the physical HVP tail, including isospin breaking [15, 23]. The correction also involves a number of Wick

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[1]HVP also enters precision atomic spectroscopy [4] and lattice determinations of hadronic contributions to the running of the electroweak gauge couplings [5, 6].

contractions, which are costly to evaluate [27] and noisy when quark-disconnected [28]. Furthermore, only some of these contractions are ultraviolet (UV) finite [25].

Here we estimate the QED correction to HVP using a vector-meson-dominance (VMD) model [29].[2] To connect this hadronic description to lattice Wick contractions, we use the Cottingham-like formula [32], which joins two currents of the forward light-by-light four-point function with a photon propagator. This relation underlies lattice studies of the UV-finite dumbbell [25] and of isospin-violating HVP at the SU(3)-symmetric point [33]. Leading-order (LO) partially quenched chiral perturbation theory (PQChPT) then supplies the charge factors for individual contractions [25, 34, 35].

The model uses an effective VMD Lagrangian of pions, ρ and ω mesons, photons and nucleons [36–38], with parameters fixed by experiment. In the heavy-nucleon limit, nucleon loops generate the anomalous vertices and fix the isospin structure of the form factors.

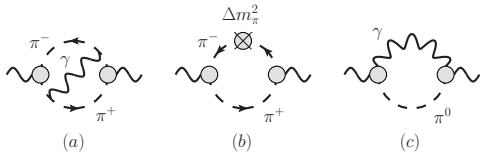

Figure 1. The three pion contributions retained in the model [29]: (a) photon correction to the $\pi^+\pi^-$ loop, (b) conversion term due to the charged-pion mass shift Δm_π^2, (c) $\pi^0\gamma$ loop. Shaded blobs: couplings to the external photon, dressed by the ρ and, for (c), also by ω mesons.

Contribution (a) in Fig. 1 is the photon correction to the $\pi^+\pi^-$ loop, with the pion mass fixed at its isospin-symmetric value. We adopt the Edinburgh consensus (EC) scheme [39]: in the isospin-symmetric theory ($\alpha_{\rm em} = 0$, $m_u = m_d$) all pions have $m_\pi^{\rm iso} = 135.0$ MeV, and QED, switched on with $m_u = m_d$ and the hadronic scale (here the ρ, ω and f_π parameters) held fixed, raises $m_{\pi^\pm}$ to its physical value but leaves the neutral pion essentially unaffected. The conversion term (b) accounts for this shift of the charged-pion mass and hence of the two-pion threshold. Assuming that the leading pion-mass dependence comes from the two-pion states, we evaluate it as

$$\Delta\overline{\Pi}^{(\rm ct)}(Q^2) \simeq \overline{\Pi}^{(\pi\pi)}(Q^2)\Big|_{m_\pi = m_{\pi^\pm}} - \overline{\Pi}^{(\pi\pi)}(Q^2)\Big|_{m_\pi = m_\pi^{\rm iso}}, \tag{2}$$

where $\overline{\Pi}^{(\pi\pi)}$ denotes the isospin-symmetric two-pion component of the scalar hadronic vacuum polarization function $\overline{\Pi}$, which includes the electromagnetic coupling and is renormalized to vanish at zero momentum, and Q^2 is the Euclidean momentum squared. Contribution (c) comes from the $\pi^0\gamma$ intermediate state and is governed by the anomalous pion transition form factor.

2 Lattice matching and results

In the time-momentum representation [40] the HVP contribution is the Euclidean correlator of the current $J_\mu = \sum_f Q_f\, \bar{q}_f \gamma_\mu q_f$ (Q_f in units of e), folded with a known QED kernel [41],

$$a_\mu^{\rm HVP} = \left(\frac{\alpha_{\rm em}}{\pi}\right)^2 \int_0^\infty {\rm d}t\, f(t)\, C(t), \qquad C(t) = -\frac{1}{3}\sum_{k=1}^{3}\int {\rm d}^3x\, \langle J_k(t,\vec{x}) J_k(0)\rangle, \tag{3}$$

[2] Two-pion states, including the ρ resonance, dominate the HVP beyond about 1 fm in Euclidean time. Chiral perturbation theory describes the low-energy region, with HVP calculations now reaching three loops [30, 31].

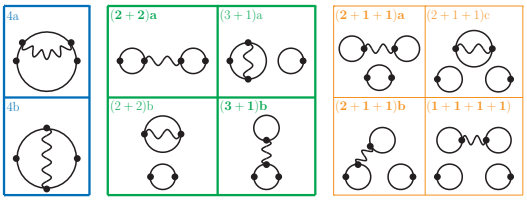

Figure 2. Wick contractions of the $O(\alpha_{\rm em})$ correction to the HVP. Solid lines: quark propagators; wavy lines: photon; dots: insertions of the electromagnetic current. The contractions labeled in bold are UV finite [25].

Table 1. LO PQChPT coefficients of Eq. (6) for the light-quark contributions. Each row sums to one; the (2 + 1 + 1) and (1 + 1 + 1 + 1) contractions vanish at this order.

(i)	4a+4b	(2+2)a	(2+2)b	(3+1)a	(3+1)b
$C_{(i)}^{(\pi\pi\gamma)}$	34/81	50/81	25/81	−14/81	−14/81
$C_{(i)}^{(\pi\gamma)}$	34/9	−25/9	0	0	0

Table 2. Model predictions (10^{-10}; infinite volume, EC scheme) for (a)–(c) and, via Eq. (6), the Wick contractions (UV-finite in bold); the negligible SD window (< 0.02) is not shown.

window [fm]	0.4–1.0	1.0–1.5	1.5–2.0	2.0–2.8	2.8–∞	total
(a) $\pi^+\pi^-\gamma$	−0.1	+0.1	+0.2	+0.3	+0.3	+0.7
(b) conversion	−0.4	−1.2	−1.6	−2.3	−2.5	−7.9
(c) $\pi^0\gamma$	0.0	0.0	0.0	0.0	0.0	+0.1
4a+4b	−0.2	−0.4	−0.5	−0.7	−0.8	−2.6
(2+2)a	−0.3	−0.7	−1.0	−1.3	−1.4	−4.7
(2+2)b	−0.1	−0.3	−0.4	−0.6	−0.7	−2.2
(3+1)a	+0.1	+0.2	+0.2	+0.3	+0.4	+1.2
(3+1)b	+0.1	+0.2	+0.2	+0.3	+0.4	+1.2
total	−0.5	−1.1	−1.4	−1.9	−2.2	−7.1

where $f(t) \simeq \pi^2 m_\mu^2 t^4/9$ at small t (this leading term cancels in the recently proposed mass-rescaled muon-electron $(g-2)$ difference [42]). The model enters through the dispersion relation: the correlator follows from the imaginary part of the vacuum polarization,

$$C(t) = \frac{1}{24\pi^2}\int_{s_{\rm thr}}^{\infty} {\rm d}s\ \sqrt{s}\,R(s)\,e^{-\sqrt{s}\,t}, \qquad R(s) = -\frac{3}{\alpha_{\rm em}}\ {\rm Im}\,\overline{\Pi}(s). \tag{4}$$

Here $s_{\rm thr}$ is the lowest threshold. To separate distance scales we use the windows

$$a_\mu^{\rm win} = \Big(\frac{\alpha_{\rm em}}{\pi}\Big)^2 \int_0^\infty {\rm d}t\, f(t)[\Theta(t-t_1)-\Theta(t-t_2)]C(t), \tag{5}$$

with $\Theta(t) = \frac{1}{2}[1+\tanh(t/\Delta)]$ and $\Delta = 0.15$ fm. The standard short-distance (SD), intermediate (ID) and long-distance (LD) windows are separated at $t = 0.4$ and 1.0 fm; for SD (LD), the lower (upper) term is replaced by 1 (0).

On the lattice the $O(\alpha_{\rm em})$ correction to $C(t)$ splits into the Wick contractions of Fig. 2. Those in which the photon joins two different quark loops are UV finite [25]; the largest, the

"dumbbell" (2 + 2)a, is the natural target of a scheme-independent lattice determination. In the others, 4a, 4b, (2 + 2)b, (3 + 1)a and (2 + 1 + 1)c, the photon is attached twice to the same quark loop, so their bare contributions are UV divergent and require counterterms.

For properly renormalized currents, counterterms cancel these UV divergences [32]. For a lattice comparison, their finite parts are fixed by matching pure QCD at the EC inputs to physical QCD+QED, with strong isospin breaking treated separately. In this convention, 4a+4b includes the connected quark-mass insertions, (3 + 1)a the valence-disconnected ones, and (2 + 2)b the sea-quark-mass and strong-coupling insertions into the connected HVP. The one-photon-reducible part of (2 + 2)a is assigned to the NLO HVP [43].

LO PQChPT charge factors for π^0 exchange and charged-pion loops carry over from light-by-light scattering once the photon-connected currents are specified, as shown for (2 + 2)a in Ref. [25]. The model estimate for quark contraction (i) reads

$$\overline{\Pi}_{(i)} = C_{(i)}^{(\pi\pi\gamma)}[\overline{\Pi}^{(\pi\pi\gamma)} + \Delta\overline{\Pi}^{(\mathrm{ct})}] + C_{(i)}^{(\pi\gamma)}\,\overline{\Pi}^{(\pi^0\gamma)}, \tag{6}$$

where $\overline{\Pi}^{(\pi\pi\gamma)}$ and $\overline{\Pi}^{(\pi^0\gamma)}$ denote contributions (a) and (c), with the coefficients of Table 1. The conversion term shares the $\pi^+\pi^-\gamma$ coefficients, as the pion-mass shift has the flavor structure of a photon within the charged-pion loop. This identification is well defined for UV-finite contractions; the others rely on the convention above, which the model approximates by fixing the isospin-symmetric pion mass in (a) and adding the conversion term (b).

Table 2 gives infinite-volume model predictions for individual Wick contractions. We split the LD window at 1.5, 2.0 and 2.8 fm, typical cutoffs beyond which lattice collaborations reconstruct the correlator by other methods. Although all three contributions are formally $O(\alpha_{\mathrm{em}})$, the conversion term (b) is one order of magnitude larger than (a) and (c) and dominates the model sum, -7.1×10^{-10}. The UV-finite dumbbell contributes -4.7×10^{-10}, about two thirds of the model sum, compatible with lattice results $(-6.9\pm2.9)\times10^{-10}$ [23] and $(-6.4\pm1.3)\times10^{-10}$ [25]. The SD contribution is negligible and the ID contribution small. The window beyond 2.8 fm accounts for ∼30% of each contraction in Table 2. Beyond 1.0 fm, the dumbbell/total ratio is ∼ 0.7 in each window.

Additional $\pi\pi$ rescattering through the σ resonance, roughly estimated in the linear σ model with VMD pion form factors and $m_\sigma \simeq 500$ MeV, contributes about -0.3×10^{-10} to the full QED correction. Including the strange quark adds $\eta\gamma$, $\eta'\gamma$ and kaon-loop contributions of $O(10^{-11})$ or below, given the VMD parameters of Ref. [25]. Nevertheless, one should assign a very conservative 30% model uncertainty to the values in the "total" column of Tab. 2, covering omitted inelastic channels, matching assumptions and chiral behavior.

3 Conclusions

We provide phenomenological benchmarks for the light-quark Wick contractions and time windows of the electromagnetic HVP correction using a VMD model matched to certain quark topologies via LO PQChPT. The pion-mass conversion dominates the model sum, while about 30% of the UV-finite dumbbell lies beyond 2.8 fm. A comparison with BMW lattice data, including finite-volume effects, is in preparation.

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