Searching for the pseudoscalar glueball with fractional determinants

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Abstract. The pseudoscalar glueball is closely linked to the axial anomaly, making its decays a natural place to study anomalous meson interactions. After a brief account of the search and of the BESIII evidence for $X(2370)$, I discuss the determinant interaction and its recent extension (arXiv:2610.00484) to fractional powers of the normalized determinant phase. The emphasis is on the connection between the gluonic vacuum and hadronic decays, and on the opportunities opened by the new experimental results and this new class of anomalous interactions.

1 Brief recall of the pseudoscalar glueball

Gluons interact with each other. This simple statement, which distinguishes QCD from an Abelian gauge theory, suggests an equally simple question: can gluons form hadrons without valence quarks? Glueballs are the answer expected from the nonperturbative dynamics of QCD. Finding them, however, has proved much harder than anticipating their existence; for recent overviews, see Refs. [1, 2].

In Yang–Mills (YM) theory, glueballs form the physical spectrum of the pure gauge theory and, as such, have been established in various lattice calculations [3, 4] (see also the comparison in Ref. [5]). In QCD with dynamical quarks, they can mix with quark–antiquark and multihadron configurations. Experimentally, the aim is therefore to identify a predominantly gluonic resonance, rather than a perfectly pure glueball. Neither a mass match nor a single favorable decay mode can settle this issue.

The pseudoscalar sector has its own long history. The resonances once grouped under $\eta(1440)$, and in particular $\eta(1405)$ and $\eta(1475)$ [6], attracted considerable attention as glueball candidates; see the review in Ref. [7]. Chiral mixing schemes involving a pseudoscalar glueball and radially excited η and η' mesons explored this possibility [8]. Modern pure-gauge lattice calculations instead place the lightest 0^{-+} glueball near 2.5–2.6 GeV, well above the scalar ground state [3, 4]. Hence, a predominantly gluonic state around 1.4 GeV is difficult to reconcile with that spectrum. This does not forbid a gluonic admixture in a lighter meson, but it changes where one should look for the dominant glueball state. In addition, the pseudoscalar glueball is also peculiar because it is linked to the axial anomaly [9, 10], which represents a fundamental nonperturbative feature of QCD.

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The recent BESIII measurements (see below) make this discussion timely and allow a more ambitious question: can the decays of a glueball candidate tell us something about the axial anomaly itself? This is the perspective developed in Ref. [11], which provides the main thread of this contribution.

2 What do we know about $X(2370)$?

BESIII first observed $X(2370)$ in $J/\psi \to \gamma\pi^+\pi^-\eta'$ in 2011 [12]. The partial-wave analysis of $J/\psi \to \gamma K_S^0 K_S^0 \eta'$ subsequently established $J^{PC} = 0^{-+}$ [13]. Further observations in $K_S^0 K_S^0 \pi^0$ and $\pi^0\pi^0\eta$ have enlarged the available decay information. The combined resonance parameters reported in the latter study are $M_X = 2359^{+13}_{-14}$ MeV and $\Gamma_X = 170^{+44}_{-29}$ MeV [14].

The recent BESIII study brings together several arguments for a predominantly gluonic interpretation: sizable radiative J/ψ production, several three-pseudoscalar decay modes, and suppression of $K^*(892)\bar{K}$ and of the radiative channels $\gamma\omega$ and $\gamma\phi$ [15]. These are important pieces of evidence. Their interpretation still involves assumptions about flavor symmetry, phase space and competing quark-model assignments; a flavor-singlet pattern alone is not a measurement of the glueball fraction.

Figure 1 places the candidate alongside representative lattice results. The comparison gives useful orientation, but it is not an identification: the lattice points belong to pure Yang–Mills theory, whereas $X(2370)$ is a resonance in full QCD. The mass motivates the search; the detailed production and decay properties must do the rest.

For testing decay models, the measured products $\mathcal{B}(J/\psi \to \gamma X)\mathcal{B}(X \to f)$ are particularly useful. In ratios of two such products, the common radiative production factor cancels. One can therefore compare relative partial widths without knowing the absolute production rate of $X(2370)$. The channels $K\bar{K}\pi$, $\eta\pi\pi$, $\eta'\pi\pi$ and $\eta' K\bar{K}$ now provide such a test [15].

3 The determinant: a natural starting point

The connection to the anomaly is direct. The gluonic topological charge density has $J^{PC} = 0^{-+}$ and enters the divergence of the flavor-singlet axial current. It can therefore interpolate a pseudoscalar glueball. At the hadronic level, scalar and pseudoscalar mesons are collected in

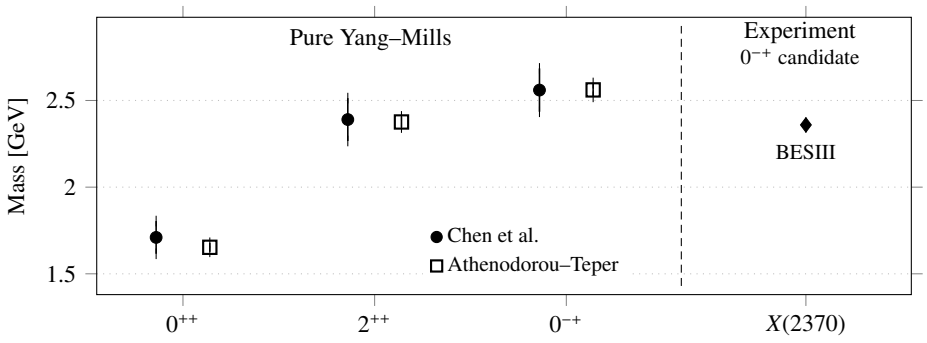


Figure 1. The lowest scalar, tensor and pseudoscalar glueball masses from Chen et al. [3] and Athenodorou–Teper [4], using their respective scale settings as collected in Ref. [5]. Chen et al.'s two quoted errors are combined in quadrature; the Athenodorou–Teper errors shown are statistical. The separate experimental column shows the $X(2370)$ mass [14], not its width.

$\Phi = \mathcal{S} + i\mathcal{P}$, with $\Phi \to U_L \Phi U_R^\dagger$ [21]. The determinant is invariant under $SU(3)_L \times SU(3)_R$, but not under the axial $U(1)_A$ transformation. The familiar 't Hooft interaction [9, 16]

$$\mathcal{L}_A = -\xi_1 \left(\det\Phi + \det\Phi^\dagger\right) \tag{1}$$

realizes this anomalous symmetry breaking in a mesonic theory. Its instanton origin and extensions to other mesons have been investigated in Refs. [17, 18].

A corresponding pseudoscalar-glueball field P couples to mesons as:

$$\mathcal{L}_{P\Phi}^{\det} = ic_{P\Phi} P \left(\det\Phi - \det\Phi^\dagger\right). \tag{2}$$

This Hermitian interaction preserves parity and the chiral symmetry, but not the axial $U(1)_A$ symmetry. Ref. [19] studied this interaction in 2013 within the extended Linear Sigma Model (eLSM), whose vacuum phenomenology and broader applications are described in Refs. [20, 21].

The calculations in Ref. [19] found a prominent $K\bar{K}\pi$ channel, nonzero $\eta^{(\prime)}\pi\pi$ channels, and no three-pion decay in the isospin-symmetric treatment. It also included scalar–pseudoscalar final states, whose subsequent decays can feed the same three-body channels. Later, Ref. [18] related the anomalous coupling to a dilute-instanton-gas estimate, permitting estimates of absolute partial widths.

The determinant is thus well motivated, but its simplest form need not be the whole story. Higher anomalous couplings can have important consequences, including for the chiral phase transition [22]. Another extension, the polydeterminant, combines different mesonic matrices and is useful for interactions involving excited multiplets and nonzero spin [23].

Very recently, fractional determinant powers address a different question: the structure of the anomalous interaction inherited from the θ dependence of the gluonic vacuum [11, 24].

4 From the θ vacuum to fractional determinants

The central idea of Ref. [11] is to start with the gluonic vacuum; see Refs. [25, 26] for recent reviews. The scalar glueball describes the trace anomaly through a dilaton potential, while the pseudoscalar glueball couples to the CP-odd part of the θ dependence. The interactions are expanded in harmonics of θ/N_c, the relevant large-N_c variable on a given vacuum branch. This also connects the θ dependence of the vacuum energy, condensate and glueball masses [11]. Remarkably, good agreement with lattice YM results of Refs. [28, 29] is achieved.

The mesonic fields enter through the anomalous combination (e.g. Refs. [30, 31])

$$\Theta = \theta - i \ln U_\Phi, \qquad U_\Phi \equiv \frac{\det\Phi}{|\det\Phi|}. \tag{3}$$

Thus it is the phase of the determinant that is combined with θ. Replacing θ by Θ in the gluonic interactions, and setting the scalar glueball to its vacuum value, generates couplings of the form

$$\mathcal{L}_{P\Phi}^{\text{phase}} = iP \sum_{n\geq 1} b_n \left[e^{in\theta/N_c} U_\Phi^{n/N_c} - e^{-in\theta/N_c} (U_\Phi^*)^{n/N_c}\right]. \tag{4}$$

The real coefficients b_n contain the gluonic couplings and vacuum scales. For three colors, the first determinant powers are $1/3$, $2/3$ and 1. Fractional powers are therefore not inserted merely as a phenomenological alternative: they emerge from the way the anomalous mesonic phase enters the gluonic vacuum [11].

Two comments are important. First, a single branch depending on θ/N_c is not generally 2π periodic. The physical vacuum energy is obtained by selecting the lowest of the branches

Table 1. Ratios of the measured and predicted three-pseudoscalar decay channels, normalized to $K\bar{K}\pi$. The experimental ratios are obtained from the BESIII product branching fractions [15]. The fractional-phase predictions are from Ref. [11]; the ordinary determinant results are from Ref. [19].

Ratio	Experiment	Fractional phase	Ordinary determinant
$\Gamma_{\eta\pi\pi}/\Gamma_{K\bar{K}\pi}$	$0.98^{+0.37}_{-0.39}$	1.34	0.36
$\Gamma_{\eta'\pi\pi}/\Gamma_{K\bar{K}\pi}$	$0.60^{+0.18}_{-0.31}$	0.69	0.19
$\Gamma_{\eta' K\bar{K}}/\Gamma_{K\bar{K}\pi}$	$0.120^{+0.045}_{-0.044}$	0.096	0.023

related by $\theta \to \theta + 2\pi k$; a 2π shift permutes these branches. Second, the modulus $|\det\Phi|$ in Eq. (3) depends on the meson fields. It is part of the interaction, not a constant that can simply be absorbed into the coupling. Keeping this dependence changes the decay vertices compared with a frozen-modulus approximation [11].

In the flavor-symmetric limit, the vacuum matrix is proportional to the identity, $\Phi_0 = v\,\mathbf{1}$. Setting aside scalar fluctuations, one writes $\Phi = \Phi_0 + i\mathcal{P}$ and expands about this nonzero vacuum to obtain the glueball-meson vertices. Flavor breaking and the detailed vertices are treated in Ref. [11]; implications for the chiral phase transition are discussed in Ref. [24].

5 What do decays tell us?

The recent experimental progress makes the anomalous interaction testable. Within the direct three-pseudoscalar decay approximation, the normalized determinant phase leads to a significantly improved description of the available $X(2370)$ decay data as compared to the ordinary determinant vertex. The ratios in Table 1 are evaluated at $\theta = 0$, using $M_X = 2.359$ GeV, the physical isospin-averaged pseudoscalar masses, the mixing angle $\varphi = -44.6°$, and $f_\pi = 92.2$ MeV, $f_K = 110$ MeV. (Thus, flavor-symmetry breaking is retained.) The full field-dependent factor $|\det\Phi|$ is kept throughout. Since only ratios are considered, the common production factor and overall coupling normalization cancel.

The direct tree-level amplitudes are momentum independent and are integrated over the physical three-body phase space, including the appropriate charge multiplicities and identical-particle factors [11]. The resulting ratios are collected in Table 1. For the corresponding fit to the four charge-summed product branching fractions, statistical and systematic uncertainties are combined in quadrature and the experimental inputs are treated as uncorrelated, since no covariance matrix is available.

Fitting one common normalization gives

$$\text{ordinary determinant (Eq. (2))}: \qquad \chi^2/\text{d.o.f.} = 13.2/3,$$

whereas for the normalized determinant phase one finds

$$\text{normalized determinant phase (Eq. (4))}: \qquad \chi^2/\text{d.o.f.} = 1.82/3.$$

Thus, the fractional interaction provides a substantially better description of the available decay pattern.

The harmonic universality has a simple origin. At cubic order, the normalized determinant phase contains a single-trace and a triple-trace contribution. The latter involves only the isoscalar fields η_N and η_S and hence does not contribute to the four channels entering Table 1. Consequently, all four direct amplitudes are proportional to the same harmonic-dependent combination

$$\sum_{n\geq 1} n b_n,$$

which cancels from their ratios. Every individual harmonic, as well as any combination for which this coefficient is nonzero, therefore predicts the same ratios [11]. Note, the coefficients b_n are not individually determined by the lattice input considered here, which mainly constrains the gluonic vacuum sector.

Thus, the presently measured channels test the normalized-phase structure without selecting a particular harmonic. The all-isoscalar modes $\eta\eta\eta$ and $\eta\eta\eta'$ are different, since the triple-trace term also contributes and makes them sensitive to the harmonic content of the interaction [11]. BESIII has already searched for $X(2370) \to \eta\eta\eta'$ and reported an upper limit [27], making improved measurements of these channels especially interesting.

A more complete treatment should eventually include glueball–quarkonium mixing, scalar-mediated decays, and final-state interactions. The observed $f_0(980)\eta'$ and $a_0(980)\pi$ contributions indicate that such effects are phenomenologically relevant [13, 14].

6 Outlook

The resonance $X(2370)$ has the required quantum numbers, lies in the relevant mass region and is now observed in several decay channels. The question is no longer only whether a peak lies near a lattice mass: its decays can also test the interaction used to describe it.

The ordinary determinant provides a natural starting point. Fractional powers of its normalized phase connect this phenomenology to the θ vacuum [11]. Establishing a predominantly gluonic interpretation still requires a combined understanding of production, decays and mixing. But this resonance may teach us not only where the glueball is, but also how the axial anomaly is realized in its hadronic interactions.

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