# Doubly radiative $\eta^{(\prime)}$ decays: interplay of vector, scalar, and tensor contributions

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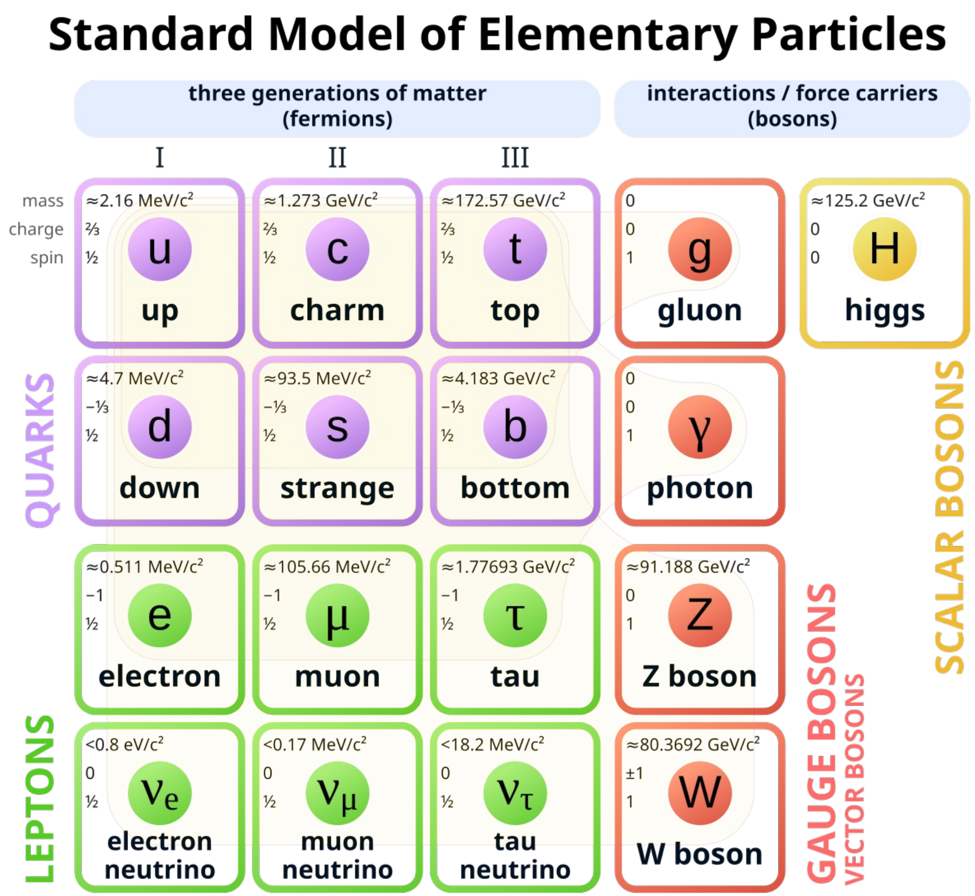
# Introduction

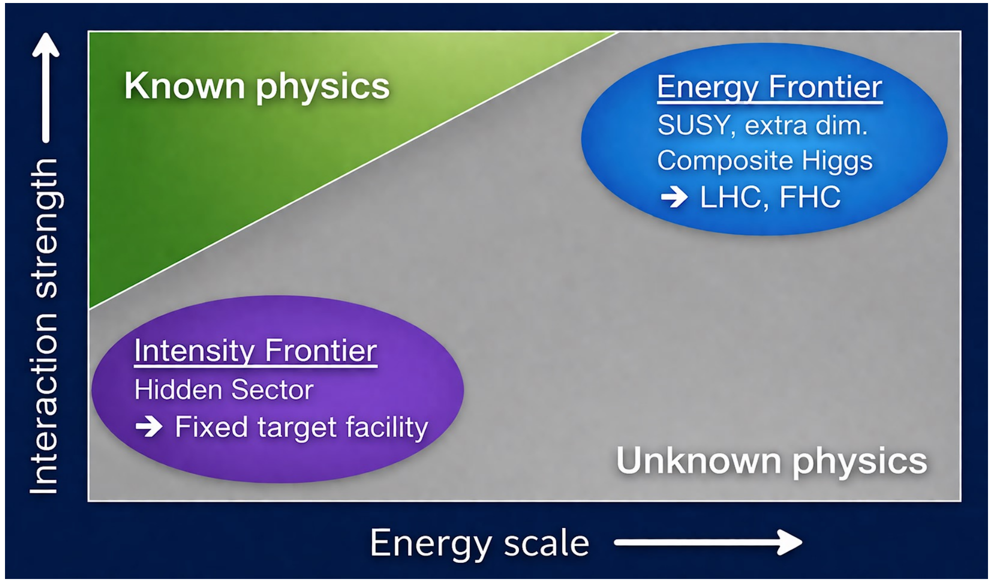
- The Standard Model (SM) of particle physics is one of the most successful scientific theories to date
  - Its current formulation was established in the 1970s
- The SM has received extensive experimental support
  - So far, no experimental result has conclusively contradicted the SM at the $5\sigma$ level
- Yet, we know the SM is incomplete, as it leaves several observed phenomena unexplained
  - It does not incorporate gravity
  - It does not provide a viable dark matter candidate
  - It does not fully explain the observed matter-antimatter asymmetry
  - …




https://en.wikipedia.org/wiki/Standard_Model

# Introduction

- Experimental strategies to search for BSM physics
  - Energy frontier: direct searches at high-energy colliders
    - Limited by cost and construction timescales of large facilities
    - Examples: LHC, FCC, ILC, CEPC
  - Intensity frontier: indirect searches via high-precision, high-intensity experiments
    - Provides sensitivity to small deviations from Standard Model predictions
    - Particularly powerful when SM predictions are precise or backgrounds are suppressed
    - Examples: Belle II, BESIII, KLOE-2, etc.




S. Alekhin et al. - arXiv:1504.04855

# Introduction

- The $\eta$ and $\eta'$ decays as an intensity frontier laboratory
  - The $\eta$ is a pseudo-Goldstone boson of the spontaneously broken chiral symmetry of QCD
  - The $\eta'$ is strongly affected by the $U(1)_A$ anomaly
  - The $\eta$ and $\eta'$ are eigenstates of $C$, $P$, $CP$, and $G$: $I^G J^{PC} = 0^+0^{-+}$
  - All their additive quantum numbers are zero $\Longrightarrow$ flavour conserving decays
  - Many strong and electromagnetic decays are suppressed at leading order by symmetries
  - Their decays are largely free from SM background

⇒ **Perfect laboratory for stress-testing the SM in search of BSM physics**

# Introduction

## Rich physics program at η,η′ factories

### Standard Model highlights

- Theory input for light-by-light scattering for $(g\text{-}2)_\mu$
- Extraction of light quark masses
- QCD scalar dynamics

### Fundamental symmetry tests

- P,CP violation
- C,CP violation

*[Kobzarev & Okun (1964), Prentki & Veltman (1965), Lee (1965), Lee & Wolfenstein (1965), Bernstein et al (1965)]*

### Dark sectors (MeV—GeV)

- Vector bosons (dark photon, B boson, X boson)
- Scalars
- Pseudoscalars (ALPs)

(Plus other channels that have not been searched for to date)

| Channel | Expt. branching ratio | Discussion |
|---|---|---|
| $\eta \to 2\gamma$ | 39.41(20)% | chiral anomaly, $\eta$–$\eta'$ mixing |
| $\eta \to 3\pi^0$ | 32.68(23)% | $m_d - m_u$ |
| $\eta \to \pi^0\gamma\gamma$ | $2.56(22) \times 10^{-4}$ | $\chi$PT at $O(p^6)$, leptophobic $B$ boson, light Higgs scalars |
| $\eta \to \pi^0\pi^0\gamma\gamma$ | $< 1.2 \times 10^{-3}$ | $\chi$PT, axion-like particles (ALPs) |
| $\eta \to 4\gamma$ | $< 2.8 \times 10^{-4}$ | $< 10^{-11}$[52] |
| $\eta \to \pi^+\pi^-\pi^0$ | 22.92(28)% | $m_d - m_u$, $C/CP$ violation, light Higgs scalars |
| $\eta \to \pi^+\pi^-\gamma$ | 4.22(8)% | chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, $P/CP$ violation |
| $\eta \to \pi^+\pi^-\gamma\gamma$ | $< 2.1 \times 10^{-3}$ | $\chi$PT, ALPs |
| $\eta \to e^+e^-\gamma$ | $6.9(4) \times 10^{-3}$ | theory input for $(g-2)_\mu$, dark photon, protophobic $X$ boson |
| $\eta \to \mu^+\mu^-\gamma$ | $3.1(4) \times 10^{-4}$ | theory input for $(g-2)_\mu$, dark photon |
| $\eta \to e^+e^-$ | $< 7 \times 10^{-7}$ | theory input for $(g-2)_\mu$, BSM weak decays |
| $\eta \to \mu^+\mu^-$ | $5.8(8) \times 10^{-6}$ | theory input for $(g-2)_\mu$, BSM weak decays, $P/CP$ violation |
| $\eta \to \pi^0\mu^+\ell^+\ell^-$ | $2.68(11) \times 10^{-4}$ | $C/CP$ violation, ALPs |
| $\eta \to \pi^+\pi^-e^+e^-$ | $< 3.6 \times 10^{-4}$ | theory input for doubly-virtual TFF and $(g-2)_\mu$, $P/CP$ violation, ALPs |
| $\eta \to \pi^+\pi^-\mu^+\mu^-$ | $< 3.6 \times 10^{-4}$ | theory input for doubly-virtual TFF and $(g-2)_\mu$, $P/CP$ violation, ALPs |
| $\eta \to e^+e^-e^+e^-$ | $2.40(22) \times 10^{-5}$ | theory input for $(g-2)_\mu$ |
| $\eta \to e^+e^-\mu^+\mu^-$ | $< 1.6 \times 10^{-4}$ | theory input for $(g-2)_\mu$ |
| $\eta \to \mu^+\mu^-\mu^+\mu^-$ | $< 3.6 \times 10^{-4}$ | theory input for $(g-2)_\mu$ |
| $\eta \to \pi^+\pi^-\pi^0\gamma$ | $< 5 \times 10^{-4}$ | direct emission only |
| $\eta \to \pi^\pm e^\mp \nu_e$ | $< 1.7 \times 10^{-4}$ | second-class current |
| $\eta \to \pi^+\pi^-$ | $< 4.4 \times 10^{-6}$ [53] | $P/CP$ violation |
| $\eta \to 2\pi^0$ | $< 3.5 \times 10^{-4}$ | $P/CP$ violation |
| $\eta \to 4\pi^0$ | $< 6.9 \times 10^{-7}$ | $P/CP$ violation |

*Gan, Kubis, Passemar, Tulin (2020)*

# Introduction

- $\eta$ / $\eta'$ factories

Wasa-at-COSY $\sim 3 \times 10^7$

KLOE-2 (Frascati) $\sim 10^9$

BESIII $\sim 4 \times 10^7$

HADES $\sim 4.5 \times 10^8$

CB@ MAMI $\sim 3 \times 10^8$

CMS $\eta \sim 10^{12}$

| Experiment | Technique | Total $\eta$ production |
|---|---|---|
| *GlueX@JLAB (running)* | $\gamma_{12\text{ GeV}}\ \ \mathrm{p} \to \eta\ \mathrm{X} \to$ neutrals | $5.5 \times 10^7/yr$ |
| *JEF@JLAB (approved)* | $\gamma_{12\text{ GeV}}\ \ \mathrm{p} \to \eta\ \mathrm{X} \to$ neutrals | $1.5 \times 10^8/yr$ |
| *HIAF (approved)* | | $\sim 10^{13}/yr$ |
| *REDTOP (proposed)* | $p_{1.8\text{ GeV}}\ \ \mathrm{Li} \to \eta\ \mathrm{X}$ | $3.4 \times 10^{13}/yr$ |

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# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

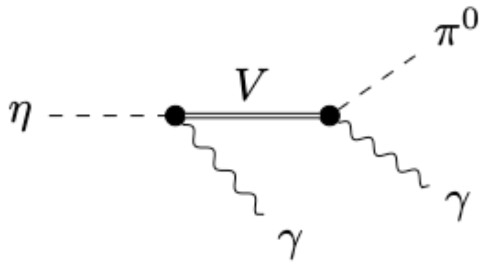
- SM motivation
  - To probe the non-perturbative regime of QCD and test $\chi PT$ and related chiral effective theories
- BSM motivation
  - Searching for BSM particles requires precise control of the SM background

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- Our approach: beyond $\chi PT$ via resonance dynamics
  - Inspired by the resonance saturation hypothesis, we model the resonance exchanges at amplitude level
    - Captures de physics encoded in LECs
    - Avoids relying on a fixed-order chiral truncation, effectively resumming resonance contributions
  - Vector resonances ($V$) [*Phys.Rev.D* 102 (2020) 3, 034026]
    - Vector Meson Dominance (VMD)
    - Dominant contribution, well-established phenomenology
  - Scalar resonances ($S$) [*Phys.Rev.D* 102 (2020) 3, 034026]
    - Linear Sigma Model
    - Full one-loop propagators for $\sigma$, $f_0$ and $a_0$ (Breit–Wigner inadequate for broad/near-threshold states)
  - Tensor resonances ($T$) [*Phys.Rev.D* 112 (2025) 11, 114009]
    - Chiral-inspired framework for $a_2$ exchange
    - Consistent with symmetry constraints

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- VMD to assess vector exchange contributions
  - Six diagrams corresponding to the exchange of $V = \rho^0, \omega, \phi$




$$\mathcal{A}^{\mathrm{VMD}}_{\eta\to\pi^0\gamma\gamma} = \sum_{V=\rho^0,\omega,\phi} g_{V\eta\gamma} g_{V\pi^0\gamma} \left[ \frac{(P\cdot q_2 - M_\eta^2)\{a\} - \{b\}}{D_V(t)} + \left\{ \begin{array}{c} q_2 \leftrightarrow q_1 \\ t \leftrightarrow u \end{array} \right\} \right]$$

Energy-dependent width in $\rho^0$ propagator

with Lorentz structures

$$\{a\} = (\epsilon_1\cdot\epsilon_2)(q_1\cdot q_2) - (\epsilon_1\cdot q_2)(\epsilon_2\cdot q_1)\,,$$
$$\{b\} = (\epsilon_1\cdot q_2)(\epsilon_2\cdot P)(P\cdot q_1) + (\epsilon_2\cdot q_1)(\epsilon_1\cdot P)(P\cdot q_2) - (\epsilon_1\cdot\epsilon_2)(P\cdot q_1)(P\cdot q_2) - (\epsilon_1\cdot P)(\epsilon_2\cdot P)(q_1\cdot q_2)$$

- The decays $\eta' \longrightarrow \pi^0(\eta)\gamma\gamma$ are formally identical with

$$M_\eta^2 \to M_{\eta'}^2, \text{ and } g_{V\eta\gamma} g_{V\pi^0\gamma} \to g_{V\eta'\gamma} g_{V\pi^0(\eta)\gamma}$$

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- VMD to assess vector exchange contributions
  - $g_{VP\gamma}$ extracted directly from experiment (**empirical couplings**)

$$\Gamma_{V\to P\gamma} = \frac{1}{3}\frac{g_{VP\gamma}^2}{32\pi}\left(\frac{M_V^2 - M_P^2}{M_V}\right)^3$$

$$\Gamma_{P\to V\gamma} = \frac{g_{VP\gamma}^2}{32\pi}\left(\frac{M_P^2 - M_V^2}{M_P}\right)^3$$

| Decay | BR | $\lvert g_{VP\gamma}\rvert$ GeV$^{-1}$ |
|---|---|---|
| $\rho^0 \to \pi^0\gamma$ | $(4.7 \pm 0.6) \times 10^{-4}$ | 0.22(1) |
| $\rho^0 \to \eta\gamma$ | $(3.00 \pm 0.21) \times 10^{-4}$ | 0.48(2) |
| $\eta' \to \rho^0\gamma$ | $(28.9 \pm 0.5)\%$ | 0.40(1) |
| $\omega \to \pi^0\gamma$ | $(8.40 \pm 0.22)\%$ | 0.70(1) |
| $\omega \to \eta\gamma$ | $(4.5 \pm 0.4) \times 10^{-4}$ | 0.135(6) |
| $\eta' \to \omega\gamma$ | $(2.62 \pm 0.13)\%$ | 0.127(4) |
| $\phi \to \pi^0\gamma$ | $(1.30 \pm 0.05) \times 10^{-3}$ | 0.041(1) |
| $\phi \to \eta\gamma$ | $(1.303 \pm 0.025)\%$ | 0.2093(20) |
| $\phi \to \eta'\gamma$ | $(6.22 \pm 0.21) \times 10^{-5}$ | 0.216(4) |

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- VMD to assess vector exchange contributions
  - $g_{VP\gamma}$ determined from a phenomenological model and a statistical fit to experimental data from $VP\gamma$ decays (**model-based couplings**)

*Phys.Lett.B* 807 (2020)

$$g_{\rho^0\pi^0\gamma} = \frac{1}{3}g\,, \quad g_{\omega\pi^0\gamma} = g\cos\phi_V\,, \quad g_{\phi\pi^0\gamma} = g\sin\phi_V\,,$$

$$g_{\rho^0\eta\gamma} = g z_{\rm NS}\cos\phi_P\,, \qquad g_{\rho^0\eta'\gamma} = g z_{\rm NS}\sin\phi_P\,,$$

$$g_{\omega\eta\gamma} = \frac{1}{3}g\left(z_{\rm NS}\cos\phi_P\cos\phi_V - 2\frac{\overline{m}}{m_s}z_{\rm S}\sin\phi_P\sin\phi_V\right),$$

$$g_{\omega\eta'\gamma} = \frac{1}{3}g\left(z_{\rm NS}\sin\phi_P\cos\phi_V + 2\frac{\overline{m}}{m_s}z_{\rm S}\cos\phi_P\sin\phi_V\right),$$

$$g_{\phi\eta\gamma} = \frac{1}{3}g\left(z_{\rm NS}\cos\phi_P\sin\phi_V + 2\frac{\overline{m}}{m_s}z_{\rm S}\sin\phi_P\cos\phi_V\right),$$

$$g_{\phi\eta'\gamma} = \frac{1}{3}g\left(z_{\rm NS}\sin\phi_P\sin\phi_V - 2\frac{\overline{m}}{m_s}z_{\rm S}\cos\phi_P\cos\phi_V\right),$$

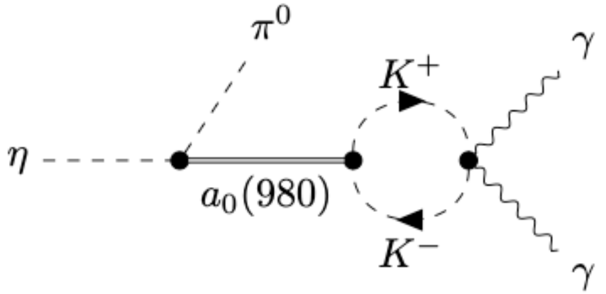
$$g = 0.70 \pm 0.01\ \mathbf{GeV}^{-1}\,, \quad z_{\rm S}\overline{m}/m_s = 0.65 \pm 0.01\,,$$

$$\phi_P = (41.4 \pm 0.5)^\circ\,, \qquad \phi_V = (3.3 \pm 0.1)^\circ\,,$$

$$z_{\rm NS} = 0.83 \pm 0.02\,.$$

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- LσM to assess scalar exchange contributions
  - $\chi PT$ loops complemented with scalar meson poles. For the $\eta \longrightarrow \pi^0\gamma\gamma$ decay, one obtains




$$\mathcal{A}^{\mathrm{L}\sigma\mathrm{M}}_{\eta\to\pi^0\gamma\gamma} = \frac{2\alpha}{\pi}\frac{1}{M^2_{K^+}}L(s_K)\{a\} \times \mathcal{A}^{\mathrm{L}\sigma\mathrm{M}}_{K^+K^-\to\pi^0\eta}$$

where

$$\mathcal{A}^{\mathrm{L}\sigma\mathrm{M}}_{K^+K^-\to\pi^0\eta} = \frac{1}{2f_\pi f_K}\left\{(s-M^2_\eta)\frac{M^2_K - M^2_{a_0}}{D_{a_0}(s)}\cos\phi_P + \frac{1}{6}\left[(5M^2_\eta + M^2_\pi - 3s)\cos\phi_P - \sqrt{2}(M^2_\eta + 4M^2_K + M^2_\pi - 3s)\sin\phi_P\right]\right\}$$

$$L(z) = -\frac{1}{2z} - \frac{2}{z^2}f\left(\frac{1}{z}\right), \qquad f(z) = \begin{cases} \frac{1}{4}\left(\log\frac{1+\sqrt{1-4z}}{1-\sqrt{1-4z}} - i\pi\right)^2 & \text{for } z < \frac{1}{4} \\ -\left[\arcsin\left(\frac{1}{2\sqrt{z}}\right)\right]^2 & \text{for } z > \frac{1}{4} \end{cases}$$

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- L$\sigma$M to assess scalar exchange contributions
  - The complete one-loop propagator is used to parameterise the propagation of the $\sigma$, $f_0$ and $a_0$ scalar resonances

$$D(s) = s - M_R^2 + \mathrm{Re}\Pi(s) - \mathrm{Re}\Pi(M_R^2) + i\mathrm{Im}\Pi(s)$$

where $M_R$ is the renormalised mass of the scalar meson and $\Pi(s)$ is the 1PI two-point function

- The amplitude for the $\eta' \longrightarrow \pi^0\gamma\gamma$ decay gets contribution from the exchange of an $a_0$
- The amplitude for the $\eta' \longrightarrow \eta\gamma\gamma$ decay receives contributions from the exchange of a $\sigma$ and an $f_0$

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- Theoretical predictions vs. measurements
  - The square amplitude with vector and scalar exchange contributions is

$$|\mathcal{A}|^2 = |\mathcal{A}^{\mathrm{VMD}}|^2 + |\mathcal{A}^{\mathrm{L}\sigma\mathrm{M}}|^2 + 2\mathrm{Re}\mathcal{A}^{*\mathrm{VMD}}\mathcal{A}^{\mathrm{L}\sigma\mathrm{M}}$$

  - Summary of predictions and contributions

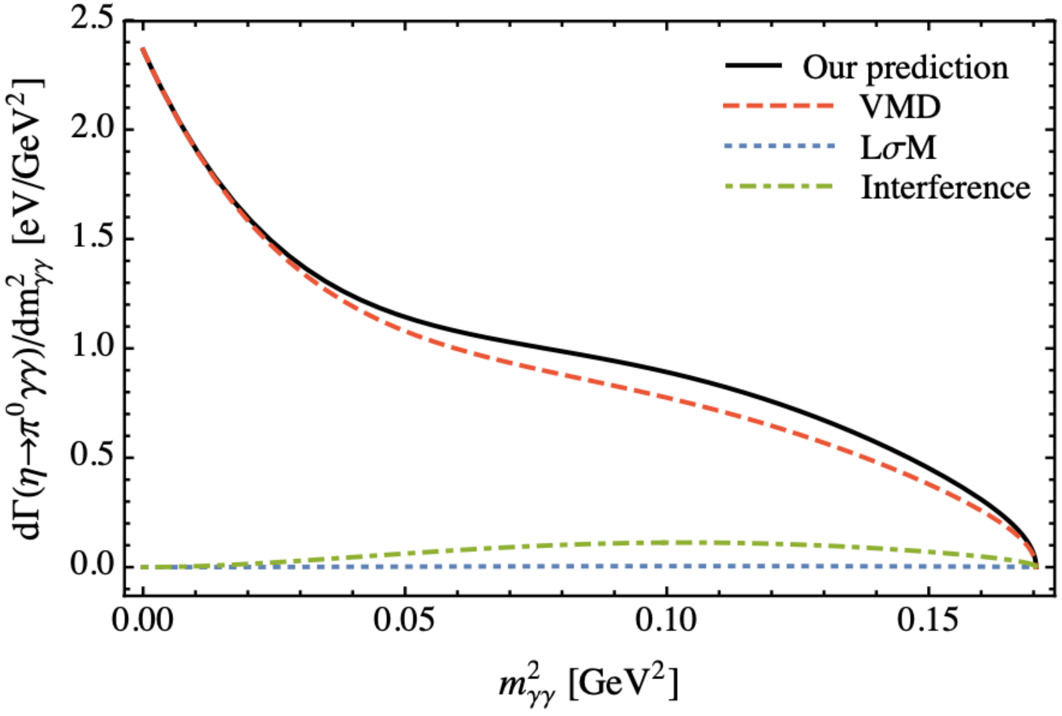
Results from *Phys.Rev.D 102 (2020)*

| Decay | Couplings | Chiral-loop | L$\sigma$M | VMD | $\Gamma$ | $\mathrm{BR}_{\mathrm{th}}$ | $\mathrm{BR}_{\mathrm{exp}}$ |
|---|---|---|---|---|---|---|---|
| $\eta \to \pi^0\gamma\gamma$ (eV) | Empirical | $1.87\times10^{-3}$ | $5.0\times10^{-4}$ | 0.16(1) | 0.18(1) | $1.35(8)\times10^{-4}$ | $2.56(22)\times10^{-4}$ [*Particle Data Group*] |
| | M-B | $1.87\times10^{-3}$ | $5.0\times10^{-4}$ | 0.16(1) | 0.17(1) | $1.30(1)\times10^{-4}$ | |
| $\eta' \to \pi^0\gamma\gamma$ (keV) | Empirical | $1.1\times10^{-4}$ | $1.3\times10^{-4}$ | 0.57(3) | 0.57(3) | $2.91(21)\times10^{-3}$ | $3.20(7)(23)\times10^{-3}$ [*Phys.Rev.D* 96 (2017) 1, 012005] |
| | M-B | $1.1\times10^{-4}$ | $1.3\times10^{-4}$ | 0.70(4) | 0.70(4) | $3.57(25)\times10^{-3}$ | |
| $\eta' \to \eta\gamma\gamma$ (eV) | Empirical | $1.4\times10^{-2}$ | 3.15 | 21.2(1.2) | 21.7(1.2) | $1.17(8)\times10^{-4}$ | $8.25(3.41)(0.72)\times10^{-5}$ [*Phys.Rev.D* 100 (2019) 5, 052015] |
| | M-B | $1.4\times10^{-2}$ | 3.15 | 19.1(1.0) | 19.8(1.0) | $1.07(7)\times10^{-4}$ | |

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- Theoretical predictions vs. measurements for $\eta \longrightarrow \pi^0\gamma\gamma$
  - Updated prediction $\mathrm{BR} = 1.36(18) \times 10^{-4}$
  - Vector exchanges dominate: 93%
    - $\rho$: 27% of the signal
    - $\omega$: 21% of the signal
    - $\phi$: 0% of the signal
    - $\rho$-$\omega$-$\phi$ interference: 52% of the signal
  - Scalar exchanges negligible: ~0%
  - Vector-scalar interference: 7%

Updated with latest empirical couplings



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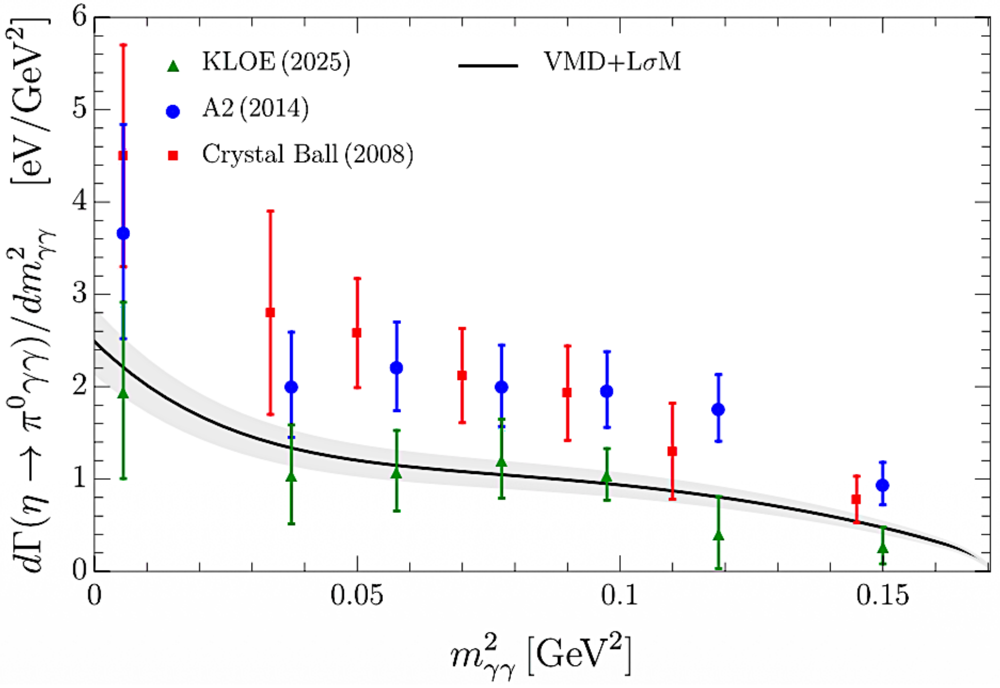
# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- Theoretical predictions vs. measurements for $\eta \longrightarrow \pi^0\gamma\gamma$
  - Shape of the spectrum is well captured
    - Normalisation offset with respect to
      - A2 $[\mathrm{BR} = 2.54(27) \times 10^{-4}]$
      - Crystall Ball $[\mathrm{BR} = 2.21(24)(47) \times 10^{-4}]$
    - Good agreement with KLOE-2, $\mathrm{BR} = 0.98(11)(13)\times 10^{-4}$
      [*JHEP* 01 (2026) 091]
  - **Discrepancy between measurements** from different experimental collaborations
    - Experimental situation needs to be settled (e.g., JEF, REDTOP)



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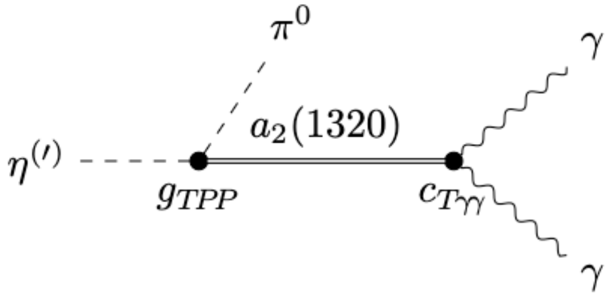
# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- Motivation for including the $a_2(1320)$ tensor meson
  - The $a_2(1320)$ couples strongly to $\gamma\gamma$ and $\pi\eta$, making the tensor exchange a natural contribution to $\eta \longrightarrow \pi^0\gamma\gamma$
  - It is observed in the crossed channel $\gamma\gamma \rightarrow \pi^0\eta$ (Belle), so a consistent description of both processes requires its inclusion
  - Scalar contributions have been shown by multiple independent analyses to be subdominant and insufficient to explain discrepancies. The tensor meson is the next natural candidate to explore
  - The KLOE-2 vs A2/Crystal Ball tension and the subleading role of scalars motivate including tensor exchange

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- $a_2(1320)$ tensor exchange contribution
  - The Feynman diagram contributing to the process is




  - The chiral Lagrangians for the tensor mesons required in the calculations are

$$\mathcal{L}_{TPP} = g_{TPP}\langle T_{\mu\nu}\{u^\mu, u^\nu\}\rangle, \qquad \mathcal{L}_{T\gamma\gamma} = c_{T\gamma\gamma}\langle T_{\mu\nu}\Theta_\gamma^{\mu\nu}\rangle$$

where

$$u_\mu = i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger), \qquad u = e^{i\frac{\Phi}{\sqrt{2}F_\pi}}$$

$$\Theta_\gamma^{\mu\nu} = f^\mu_{+\alpha} f_+^{\alpha\nu} + \frac{1}{4} g^{\mu\nu} f_+^{\rho\sigma} f_{+\rho\sigma},$$

$$f_+^{\mu\nu} = e(uQu^\dagger + u^\dagger Q u)F^{\mu\nu}, \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Careful with order of indices!

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- The $g_{TPP}$ and $c_{T\gamma\gamma}$ couplings are determined experimentally through

$$\Gamma(a_2 \to \pi\eta) = \frac{8p^5(m_{a_2}, m_\pi, m_\eta)}{15\pi m_{a_2}^2}\frac{|g_{TPP}|^2}{F_\pi^4}\cos^2\phi_P \longrightarrow |g_{TPP}| = 21.5(1.0)\ \text{MeV}$$

$$\Gamma(a_2 \to \pi\eta') = \frac{8p^5(m_{a_2}, m_\pi, m_{\eta'})}{15\pi m_{a_2}^2}\frac{|g_{TPP}|^2}{F_\pi^4}\sin^2\phi_P \longrightarrow |g_{TPP}| = 22.4(1.9)\ \text{MeV}$$

and

$$\Gamma(a_2 \to \gamma\gamma) = \frac{8\pi}{45}\alpha^2 m_{a_2}^3 |c_{T\gamma\gamma}|^2 \longrightarrow |c_{T\gamma\gamma}| = 1.2 \times 10^{-4}\ \text{MeV}^{-1}$$

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- The amplitude for $\eta \longrightarrow \pi^0\ (a_2 \longrightarrow \gamma\gamma)$ is

$$\mathcal{A}^{a_2}_{\eta\to\pi^0\gamma\gamma} = \frac{32e^2}{3F_\pi^2} g_{TPP} c_{T\gamma\gamma} \cos\phi_P P^\mu p^\nu \frac{P_{\mu\nu,\rho\sigma}(s)}{m_{a_2}^2 - s - i m_{a_2}\Gamma_{a_2}(s)} \times \left\{ g^{\rho\alpha} q_1^\beta q_2^\sigma + g^{\sigma\beta} q_1^\rho q_2^\alpha - g^{\rho\alpha} g^{\sigma\beta}(q_1\cdot q_2) - g^{\alpha\beta} q_1^\rho q_2^\sigma + \frac{1}{2} g^{\rho\sigma}[g^{\alpha\beta}(q_1\cdot q_2) - q_1^\beta q_2^\alpha] \right\} \epsilon^*_\alpha(q_1)\epsilon^*_\beta(q_2)$$

where

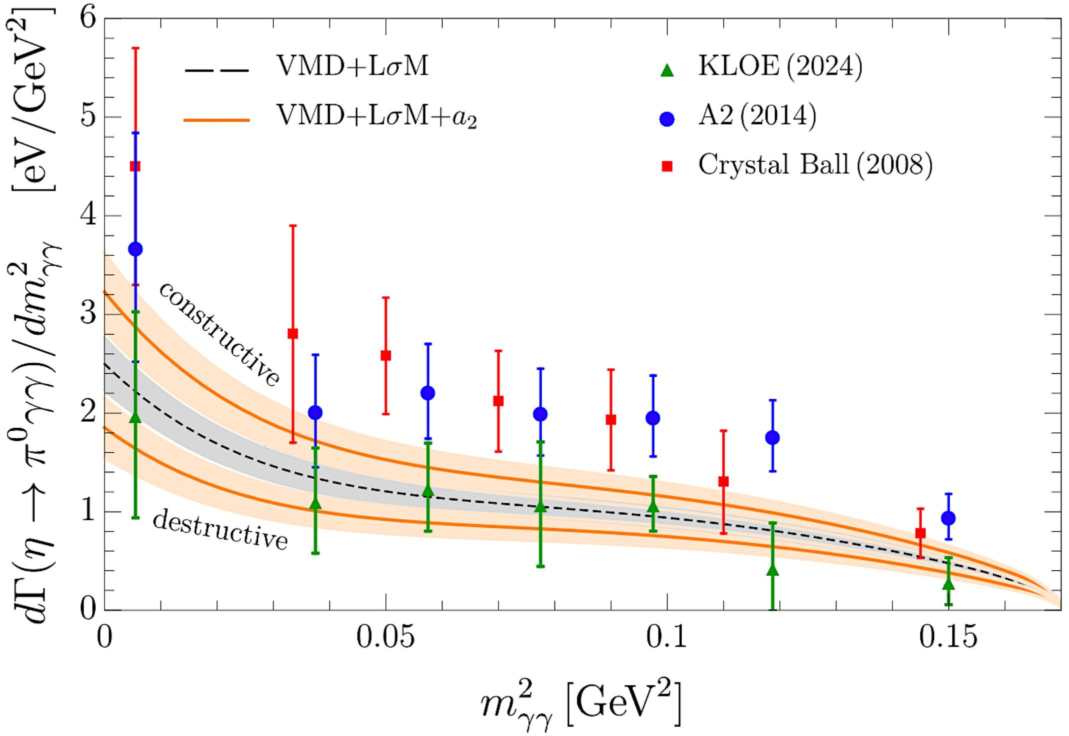
$$P_{\mu\nu,\rho\sigma}(q) = \frac{1}{2}(P_{\mu\rho}P_{\nu\sigma} + P_{\nu\rho}P_{\mu\sigma}) - \frac{1}{3}P_{\mu\nu}P_{\rho\sigma}, \qquad P_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{m_{a_2}^2}$$

and $P(p)$ is the four-momentum of the $\eta(\pi^0)$

- Likewise, the amplitude for $\eta' \longrightarrow \pi^0\ (a_2 \longrightarrow \gamma\gamma)$ is obtained by making the replacements $m_\eta \rightarrow m_{\eta'}$ and $\cos\phi_P \rightarrow \sin\phi_P$

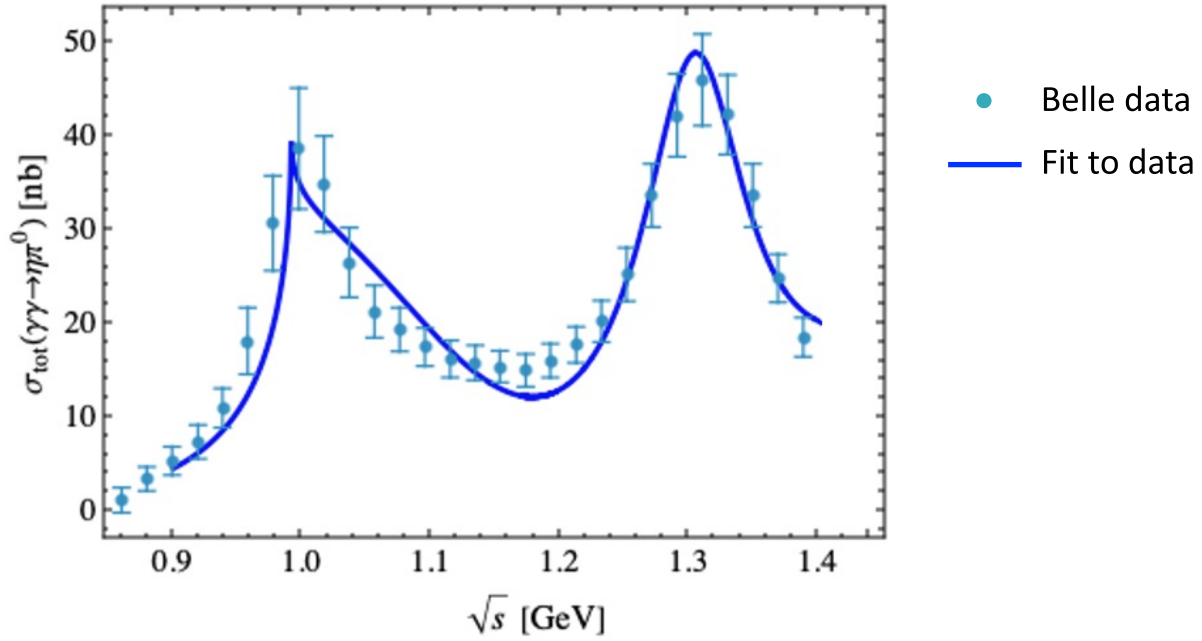
# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- Diphoton invariant mass distribution for $\eta \longrightarrow \pi^0\gamma\gamma$



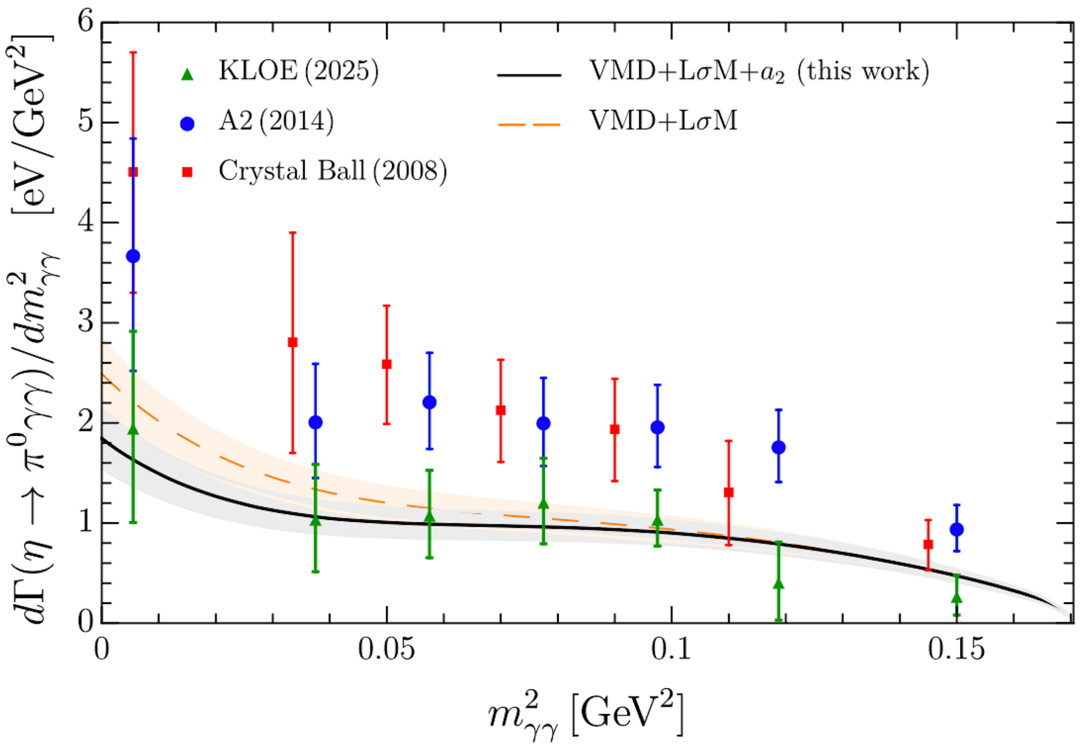
# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

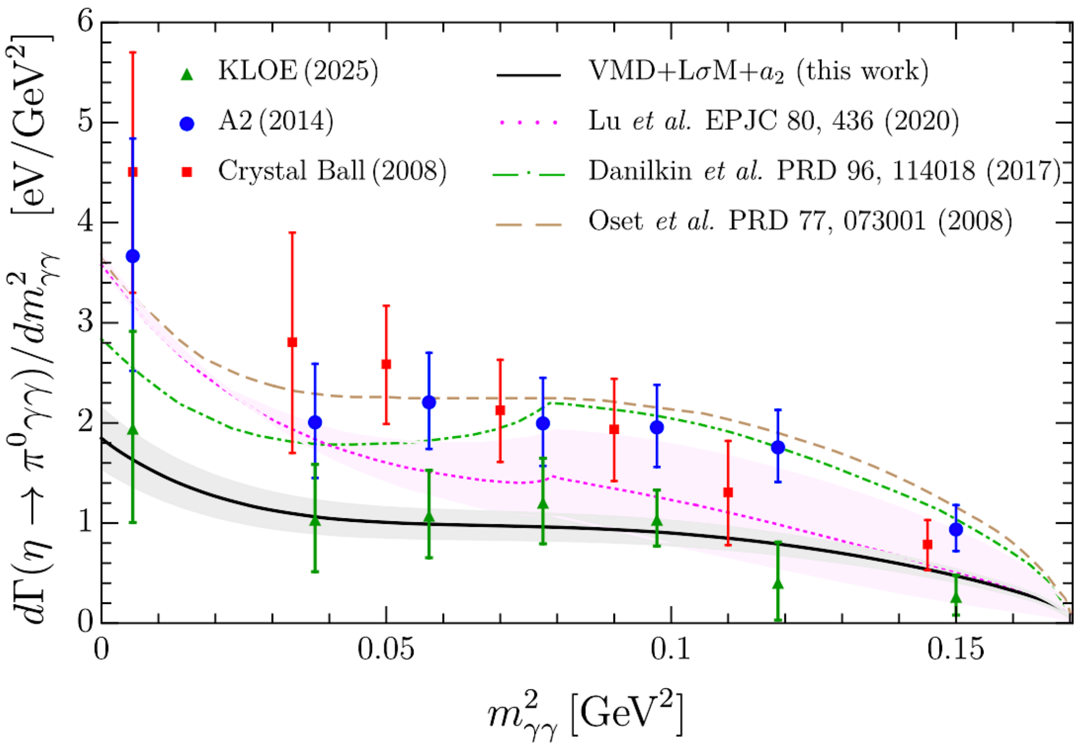
- Unlike $\eta \longrightarrow \pi^0\gamma\gamma$, $\gamma\gamma \longrightarrow \eta\pi^0$ scattering data fix the sign of the vector-tensor interference, which is found to be destructive



# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- Our prediction for $\eta \longrightarrow \pi^0\gamma\gamma$: BR $= 1.17(17) \times 10^{-4}$



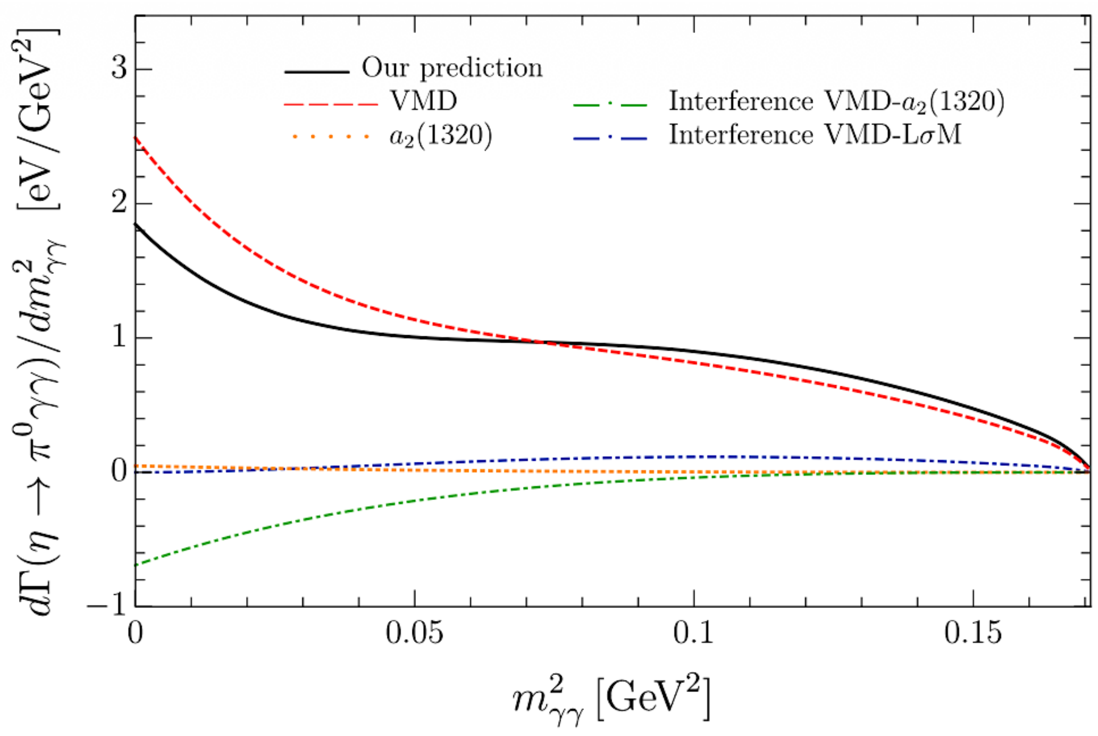






*Phys.Rev.D 112 (2025) 11, 114009*

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- Our prediction for $\eta \longrightarrow \pi^0\gamma\gamma$: $\mathrm{BR} = 1.17(17) \times 10^{-4}$




| Term | Value (in eV) |
|---|---|
| Vectors (V) | 0.1661 |
| Scalars (S) | 0.0005 |
| $a_2$ | 0.0020 |
| V-S interference | 0.0121 |
| V-$a_2$ interference | −0.0270 |
| $\Gamma_{\mathrm{th}}$ | 0.1538 |
| $\mathrm{BR}_{\mathrm{th}}$ | $1.17(17) \times 10^{-4}$ |
| $\mathrm{BR}_{\mathrm{exp}}$ (PDG) | $2.55(22) \times 10^{-4}$ |
| $\mathrm{BR}_{\mathrm{exp}}$ (KLOE-2) | $0.98(11_{\mathrm{stat}})(14_{\mathrm{syst}}) \times 10^{-4}$ |

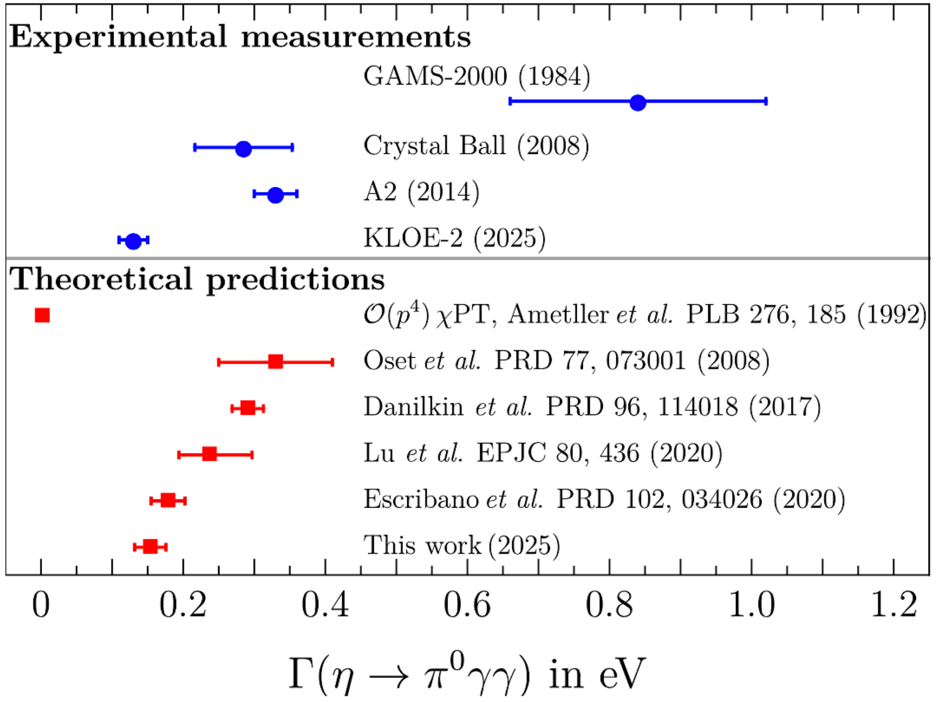
Using updated $g_{VP\gamma}$ values



*Phys.Rev.D 112 (2025) 11, 114009*

# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

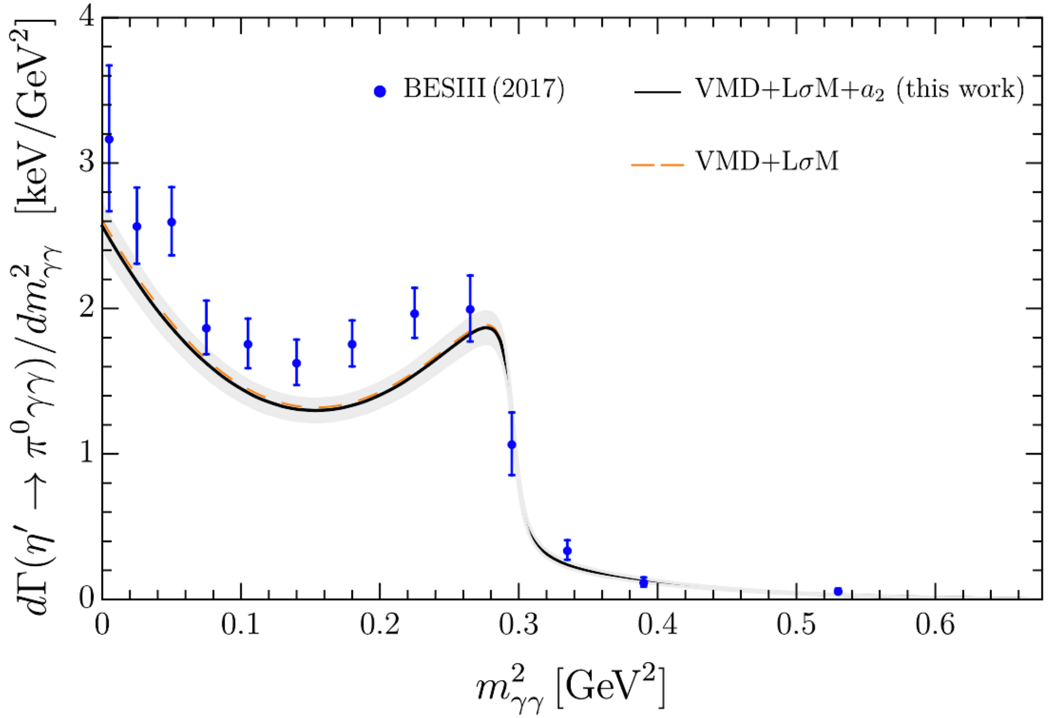
- $\eta \longrightarrow \pi^0\gamma\gamma$: Experimental and Theoretical Status
  - Our final prediction, including vector, scalar, and tensor resonance exchanges, is $\text{BR} = 1.17(17) \times 10^{-4}$
    [*Phys.Rev.D* 112 (2025) 11, 114009]
  - In very good agreement with KLOE-2 experiment $\text{BR} = 0.98(11)(14) \times 10^{-4}$
    [*JHEP* 01 (2026) 091]
  - In tension ($\sim 4.3\sigma$) with A2 Collaboration measurement $\text{BR} = 2.54(27) \times 10^{-4}$
    [*Phys.Rev.C* 90 (2014) 2, 025206]
  - **Both theoretical predictions and experimental measurements show significant spread**



# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- Theoretical predictions vs. measurements for $\eta' \longrightarrow \pi^0\gamma\gamma$
  - Updated prediction $\mathrm{BR} = 2.78(21) \times 10^{-3}$
  - Vector exchanges dominate: 100%
    - $\rho$: 5% of the signal
    - $\omega$: 78% of the signal
    - $\phi$: ~0% of the signal
    - $\rho$-$\omega$-$\phi$ interference: 17% of the signal
  - Scalar exchanges negligible: ~0%
  - Vector-scalar interference: ~0%
  - Tensor contribution negligible: ~0%
  - Prediction compatible with BESIII measurement, $\mathrm{BR} = 3.10(7)(23) \times 10^{-3}$
    [*Phys.Rev.D* 96 (2017) 1, 012005]

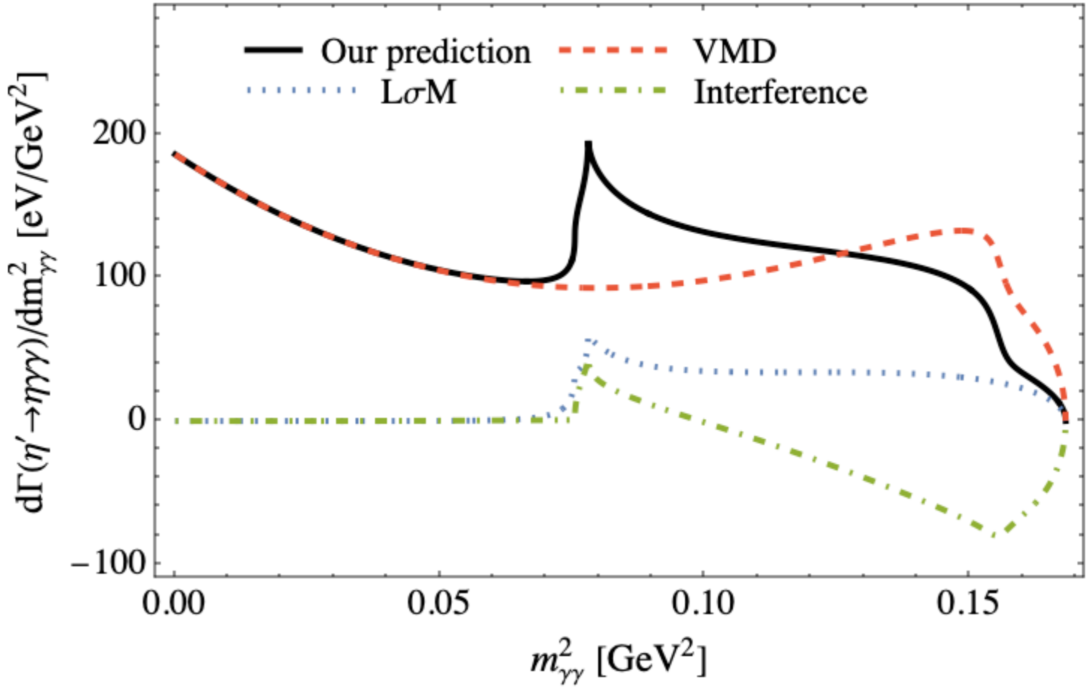
Updated result
*Phys.Rev.D 112 (2025)*



# Rare $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$ decays in the SM

- Theoretical predictions vs. measurements for $\eta' \longrightarrow \eta\gamma\gamma$
  - Updated prediction $\mathrm{BR} = 1.17(8) \times 10^{-4}$
  - Vector exchanges dominate: 96%
    - $\rho$: 57% of the signal
    - $\omega$: 15% of the signal
    - $\phi$: 1% of the signal
    - $\rho$-$\omega$-$\phi$ interference: 23% of the signal
  - Scalar exchanges NOT negligible: 16%
  - Vector-scalar interference: -12%
  - Prediction compatible with BESIII
    - $\mathrm{BR} = 8.25(3.41)(0.72) \times 10^{-5}$
      [*Phys.Rev.D* 100 (2019) 5, 052015]
    - $\mathrm{BR} < 1.33 \times 10^{-4}$ at 90% C.L.
      [*Phys.Rev.D* 100 (2019) 5, 052015]
  - We await the experimental release of the $m^2_{\gamma\gamma}$ spectrum

Corrected complete propagators



# Table of contents

# Conclusions

- SM results for $\eta^{(\prime)} \longrightarrow \pi^0(\eta)\gamma\gamma$: vector/scalar exchanges (VMD, L$\sigma$M) and tensor $a_2(1320)$ included via a chiral model
- $\eta \longrightarrow \pi^0\gamma\gamma$
    - In excellent agreement with latest KLOE-2 measurement
    - Vector-tensor interference relevant at low $m^2_{\gamma\gamma}$, identified as destructive from scattering data (Belle)
    - Experimental tension among CB, A2, KLOE-2 calls for more precise measurements
- $\eta' \longrightarrow \pi^0\gamma\gamma$
    - In good agreement with BESIII measurement
    - Tensor contribution negligible
- $\eta' \longrightarrow \eta\gamma\gamma$
    - In good agreement with BESIII measurement

# Backup slides

# Backup slides

- Many of the strong and EM $\eta$ and $\eta'$ decays are suppressed at leading order due to symmetries
  - Strong decays at lowest order
    - $\eta^{(\prime)} \to 2\pi$ and $\eta \to 4\pi$ violate $P$ and $CP$ invariance
    - $\eta^{(\prime)} \to 3\pi$ are suppressed by $G$-parity conservation
  - First order EM decays
    - $\eta^{(\prime)} \to 2\gamma$ are suppressed because they proceed through the anomaly
    - $\eta^{(\prime)} \to 3\gamma$ violate $C$ invariance
    - $\eta \to \pi^0\gamma$ violates $C$ invariance and angular momentum conservation
    - $\eta \to 2\pi^0\gamma$ and $\eta \to 3\pi^0\gamma$ violate $C$ invariance
    - $\eta \to \pi^+\pi^-\gamma$ is suppressed because it proceeds through the anomaly
    - $\eta^{(\prime)} \to \pi^0 l^+ l^-$ violate $C$ as single-$\gamma$ process
    - $\eta' \to \eta l^+ l^-$ violate $C$ as single-$\gamma$ process

# Backup slides

- Updated $g_{VP\gamma}$ values from $V \to P\gamma$ and $P \to V\gamma$ decays

[*Phys.Rev.D 102 (2020) 3, 034026*]

| Decay | BR | $\lvert g_{VP\gamma}\rvert$ GeV$^{-1}$ |
|---|---|---|
| $\rho^0 \to \pi^0\gamma$ | $(4.7 \pm 0.6) \times 10^{-4}$ | 0.22(1) |
| $\rho^0 \to \eta\gamma$ | $(3.00 \pm 0.21) \times 10^{-4}$ | 0.48(2) |
| $\eta' \to \rho^0\gamma$ | $(28.9 \pm 0.5)\%$ | 0.40(1) |
| $\omega \to \pi^0\gamma$ | $(8.40 \pm 0.22)\%$ | 0.70(1) |
| $\omega \to \eta\gamma$ | $(4.5 \pm 0.4) \times 10^{-4}$ | 0.135(6) |
| $\eta' \to \omega\gamma$ | $(2.62 \pm 0.13)\%$ | 0.127(4) |
| $\phi \to \pi^0\gamma$ | $(1.30 \pm 0.05) \times 10^{-3}$ | 0.041(1) |
| $\phi \to \eta\gamma$ | $(1.303 \pm 0.025)\%$ | 0.2093(20) |
| $\phi \to \eta'\gamma$ | $(6.22 \pm 0.21) \times 10^{-5}$ | 0.216(4) |

[*Phys.Rev.D 112 (2025) 11, 114009*]

| Decay | BR | $\lvert g_{VP\gamma}\rvert$ GeV$^{-1}$ |
|---|---|---|
| $\rho^0 \to \pi^0\gamma$ | $(4.7 \pm 0.8) \times 10^{-4}$ | 0.22(2) |
| $\rho^0 \to \eta\gamma$ | $(3.00 \pm 0.21) \times 10^{-4}$ | 0.48(2) |
| $\eta' \to \rho^0\gamma$ | $(29.48 \pm 0.35)\%$ | 0.393(7) |
| $\omega \to \pi^0\gamma$ | $(8.33 \pm 0.25)\%$ | 0.71(1) |
| $\omega \to \eta\gamma$ | $(4.5 \pm 0.4) \times 10^{-4}$ | 0.136(6) |
| $\eta' \to \omega\gamma$ | $(2.52 \pm 0.07)\%$ | 0.122(2) |
| $\phi \to \pi^0\gamma$ | $(1.33 \pm 0.05) \times 10^{-3}$ | 0.041(1) |
| $\phi \to \eta\gamma$ | $(1.306 \pm 0.024)\%$ | 0.2096(20) |
| $\phi \to \eta'\gamma$ | $(6.23 \pm 0.21) \times 10^{-5}$ | 0.216(4) |

# Backup slides

- Complete one-loop propagator

$$D(s)=\frac{i}{s-M_0^2+\Pi(s)}=\frac{i}{s-M_R^2+\mathrm{Re}\Pi(s)-\mathrm{Re}\Pi(M_R^2)+i\mathrm{Im}\Pi(s)}\,,$$ with $$M_R^2\equiv M_0^2-\mathrm{Re}\Pi(M_R^2)$$

- The pole mass and pole width of the resonance, $M_p$ and $\Gamma_p$, are found from the pole equation on the second Riemann sheet

$$\Delta(s_p)=s_p-M_R^2+\mathrm{Re}\Pi_+(s_p)-\mathrm{Re}\Pi_+(M_R^2)+i\mathrm{Im}\Pi_+(s_p)=0\,,$$

with

$$\Pi_+(s)=\mathrm{Re}R(s)-\mathrm{Im}I(s)+i[\mathrm{Im}R(s)+\mathrm{Re}I(s)]$$ and $$s_p\equiv\left(M_p-\frac{i}{2}\Gamma_p\right)^2$$

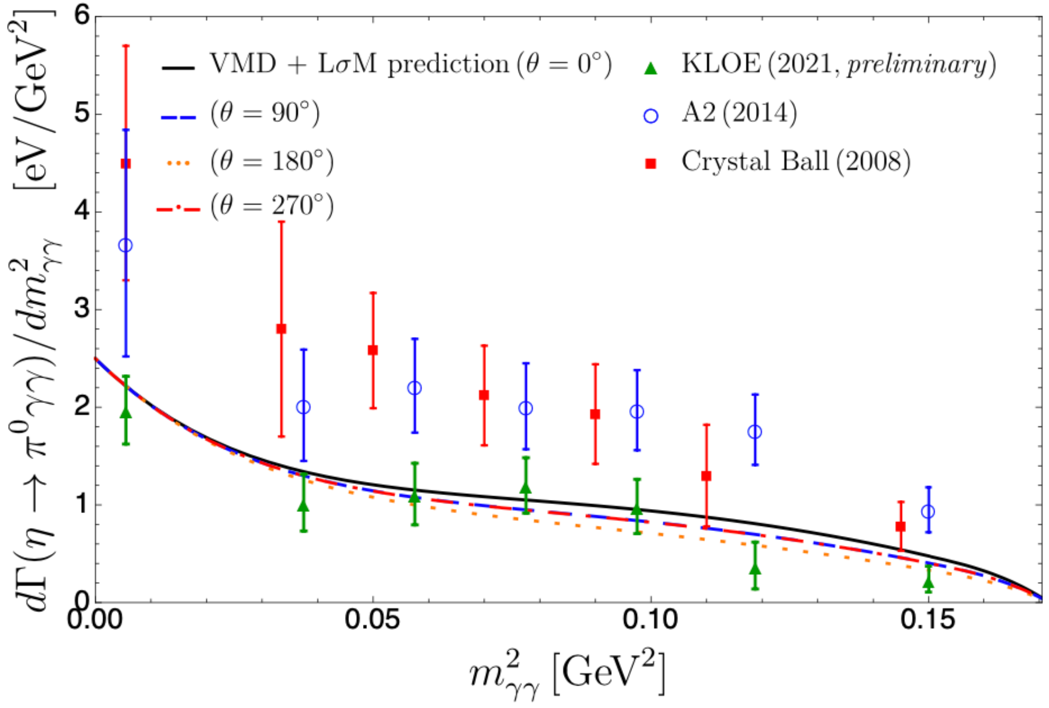
where $\Pi_+(s)$ denotes the self-energy analytically continued to the second Riemann sheet

- Parameters for the scalar resonances

| | | |
|---|---|---|
| $M_{\sigma R}=498$ MeV | $M_{f_0 R}=990$ MeV | $M_{a_0 R}=980$ MeV |
| $M_{\sigma p}=550$ MeV | $M_{f_0 p}=990$ MeV | $M_{a_0 p}=980$ MeV |
| $\Gamma_{\sigma p}=400$ MeV | $\Gamma_{f_0 p}=15$ MeV | $\Gamma_{a_0 p}=100$ MeV |

# Backup slides

- Relative phase between the VMD and LσM amplitudes in $\eta \longrightarrow \pi^0\gamma\gamma$

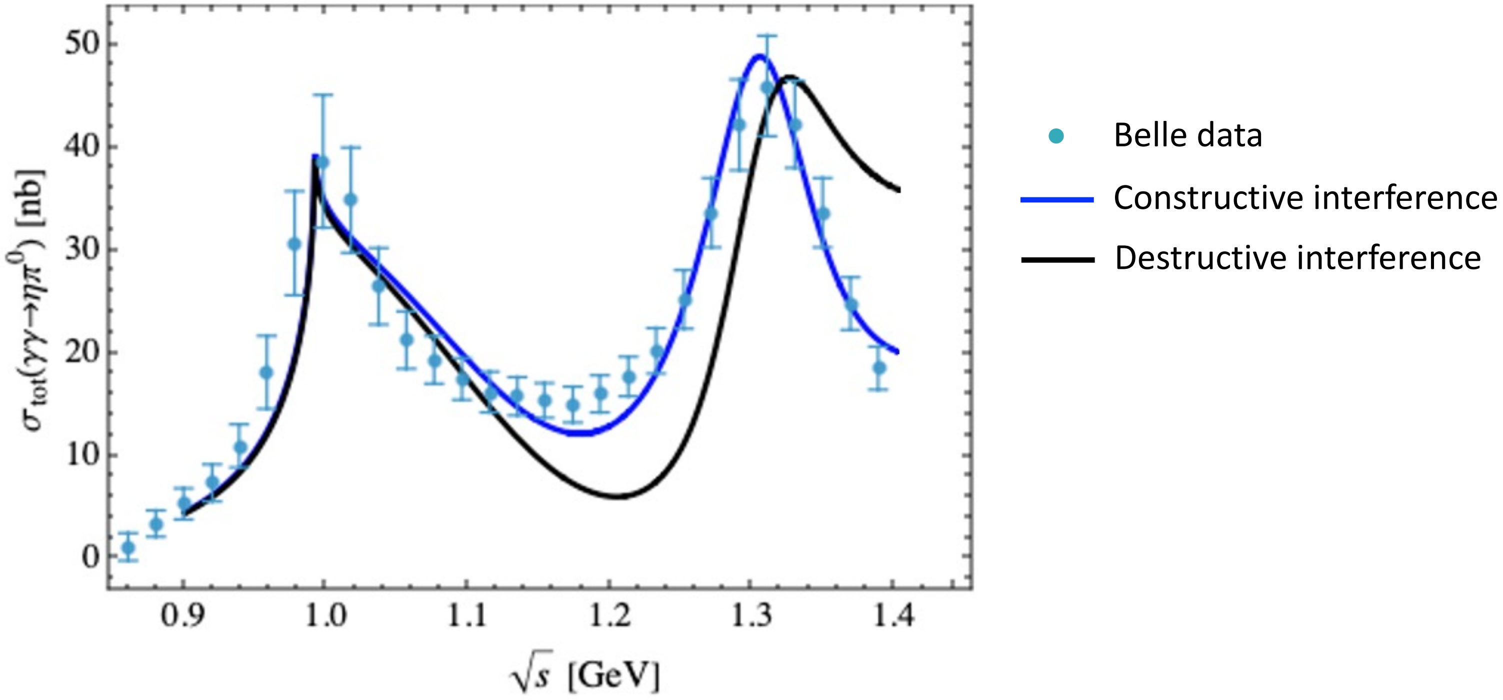




*Phys.Rev.D 102 (2020) 3, 034026*

# Backup slides

- Probing the relative phase of the tensor-exchange contribution via $\gamma\gamma \longrightarrow \eta\pi^0$



# Backup slides

- Comparison of theoretical approaches for $\eta \longrightarrow \pi^0\gamma\gamma$ in the SM

| | **This work (2025)** | **Oset et al. (2008)** | **Danilkin et al. (2017)** | **Lu & Moussallam (2020)** |
|---|---|---|---|---|
| **Approach** | VMD + L$\sigma$M + chiral tensor Lagrangian; amplitude computed directly in the decay region | Chiral unitary (Bethe–Salpeter) + VMD loops + anomalous terms | Dispersion theory on $\gamma\gamma \to \eta\pi^0$, continued by crossing symmetry to the decay region | Muskhelishvili–Omnès on $\gamma\gamma \to \eta\pi^0/K_SK_S$ + analytic continuation |
| **Vectors** | VMD $\rho, \omega, \phi$; empirical PDG 2024 couplings; energy-dependent width for the $\rho$ resonance | VMD $\rho, \omega$ (no $\phi$); universal $SU(3)$ couplings normalized to 2007 BRs | VMD $\rho, \omega$ (no $\phi$); universal or individual coupling; no full propagator | VMD $\rho, \omega, \phi$ in $d$-wave; couplings fitted to Belle $\gamma\gamma \to \eta\pi^0$ and $K_SK_S$ data |
| **Scalar ($a_0$)** | L$\sigma$M (subdominant, minimal effect) | Dynamically generated (Bethe–Salpeter, $\pi\eta/K\bar{K}$); sign of $a_0$ fixed for first time | Coupled-channel Omnès for scattering; scalars not retained in decay region; decay prediction: $\chi$PT NLO loops + VMD only | Muskhelishvili–Omnes two-channel; soft-photon/soft-pion theorems imposed; simultaneous $\pi\eta$ and $K_SK_S$ fit |
| **Tensor ($a_2$)** | Included with chiral Lagrangians; sign fixed by $\gamma\gamma \to \pi^0\eta \Rightarrow$ **destructive interference** | **Not included** | BW in scattering amplitude; Blatt–Weisskopf factors omitted; $a_2$ not continued to the decay region | BW in d-wave with fitted couplings; **interference sign** not discussed in decay region |
| **Additional contributions** | None (resonance exchanges provide all-order estimates) | $\chi$PT loops + VMD loop diagrams + anomalous terms – potential **overcounting** of vector contributions | $\chi$PT kaon loops included, yielding $\sim 0.010$ eV - larger than found by others; added in positive interference with VMD to reproduce decay data | Isospin-violating $\pi^+\pi^-$ cusp included |
| **Main strength** | Simplicity, all-order resonances; updated couplings; tensor phase determined by $\gamma\gamma \to \pi^0\eta$ | First dynamical generation of $a_0$ – fixes scalar sign unambiguously | Rigorous $s$-wave (analyticity, unitarity); no free hadronic parameters | State-of-the-art dispersive $s$-wave; constrained by $\pi\eta$ and $K_SK_S$ data |
| **Main weakness** | Scalar sector treated with L$\sigma$M (less sophisticated than dispersive methods) – but scalar contribution is subdominant | Mix of VMD, loops, and anomalous terms without a clear organising principle; uses older radiative BRs | Crude $d$-wave treatment relative to $s$-wave; vector $\phi$ missing; continuation to the decay region not fully validated; uses older radiative BRs | Hadronic cutoff $\Lambda_S$ governing left-hand cut; $d$-wave treatment crude relative to $s$-wave; two degenerate fit solutions – one argued to be unphysical |
| **Predicted $\Gamma$ (eV)** | $\mathbf{0.154 \pm 0.022}$ | $0.33 \pm 0.08$ | $0.291 \pm 0.022$ | $0.237^{+0.060}_{-0.043}$ |
| **Predicted BR** | $\mathbf{(1.17 \pm 0.17) \times 10^{-4}}$ | $(2.52 \pm 0.61) \times 10^{-4}$ | $(2.22 \pm 0.19) \times 10^{-4}$ | $(1.81^{+0.46}_{-0.33}) \times 10^{-4}$ |
| **Agreement with KLOE-2 (2026)** | **Excellent** ($\sim 0.8\sigma$) | Disfavoured ($\sim 8\sigma$ above) | Disfavoured ($\sim 7\sigma$ above) | High (factor $\sim 2.4$, $\sim 4\sigma$, above KLOE-2 central value) |

Table 1: Comparison of theoretical approaches for $\eta \to \pi^0\gamma\gamma$. Experimental BRs: KLOE-2 (2025) BR $= 0.98(11_{\text{stat}})(14_{\text{syst}}) \times 10^{-4}$; PDG (2024) BR $= 2.55(22) \times 10^{-4}$.

# Backup slides

- The REDTOP experiment
  - Concept: low-energy fixed-target meson factory operating in the high-intensity precision frontier
  - Setup: 1.8 GeV proton beam on a target of 10 Li/Be foils, producing $\sim 10^{14}$ $\eta$ mesons in three years
  - Production: $\eta/\eta'$ production via intranuclear baryonic resonances [Δ, N(1440), N(1535), ...] in nucleon-nucleon collisions
  - Detection principle: Prompt Cherenkov light from charged decay products, while most background particles remain below Cherenkov threshold
  - Physics opportunities :
    - New particle searches (BSM vectors, scalars, heavy neutral leptons, ALPs, …)
    - Test of conservation laws ($C/CP$ symmetries, lepton flavour universality)
    - Non-perturbative QCD (light-quark masses, form factors, ...)
    - Muon polarimetry (polarisation in decays, $CP$ violation, …)