Quark mass dependence of the $T_{cc}(3875)^+$ pole

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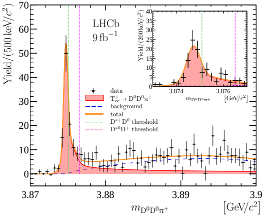
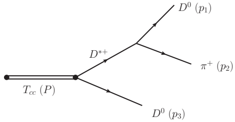
$T_{cc}^{+}(3875)$ signal in $D^0 D^0 \pi^+$ **LHCb, Nature (2022)**



$\delta m_{\text{exp}} = -360 \pm 40^{+4}_{-0}$ KeV;

$\Gamma = 48 \pm 2^{+0}_{-14} keV$

Close to the $D^{*+} D^0 / D^{*0} D^+$



$\Gamma \simeq 43$ KeV

Feijoo, Liang, Oset, PRD104 (2021) HGF

Possible role of three-body effects. It can be taken into account by considering

- π-exchange
- Decay of $D^* \to D\pi$

(LHCb) Nat. com. 13 (2022)

ML Du, Baru, Dong, Filing, FK Guo, Hanhart, Nefediev, Nieves, Wang, PRD105 (2022)

See also S. Dawid talk

Lattice QCD simulations on the T_{cc}

- Padmanath, Prelovsek, PRL129 (2022), $m_\pi \simeq 280$ MeV
- S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun and R. Zhang, PLB833 (2022), $m_\pi = 349$ MeV. Attractive for $I = 0$ with ρ-exchange being dominant and repulsive for $I = 1$. (Similar conclusion in Meng, Ortiz-Pacheco, Baru, Epelbaum, Padmanath and Prelovsek, PRD111 (2025)). Consistent with HGF, Molina, Branz, Oset, PRD82 (2010).
- Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, PRL131 (2023), see also PLB729 (2014), $m_\pi = 146, 411$ MeV
- Collins, Nefediev, Padmanath, Prelovsek, PRD109 (2024), $m_D \simeq 1.7 - 2.4$ GeV
- Whyte, Wilson, Thomas, PRD111, (2025), $m_\pi = 391$ MeV

π-exchange ($V_\pi \sim \frac{q\cdot\epsilon q\cdot\epsilon'}{u-m_\pi^2}$) can lead to

- $S - D$ mixing (it is found negligible), Whyte et al.
- Left-hand-cut starting at $p^2_{lhc} = \frac{1}{4}(p_\pi^{0\,2} - m_\pi^2)$, with $p_\pi^0 \simeq m_D^* - m_D$. For some unphysical pion masses the pole can be close to the lhc.

Meng, Baru, Epelbaum, Filin and Gasparyan, PRD 109 (2024)

EFT (contact+π-exchange) See also discussion in Collins et al., PRD109'24

Global analysis of LatticeQCD data

F. Gil-Dominguez, Giachino and Molina, PRD111, (2025)

- ρ exchange in the finite volume, $\rho + \pi$ in the infinite volume
- Can the LQCD energy levels be described with ρ-meson exchange? (HGF)
- Is the effect of the π-exchange seen in the LQCD data? What is the role of the π exchange after the continuous extrapolation is done and in the physical pion mass?

Sources of quark mass dependence:

- $D^{(*)}$ masses
- DD^* scattering amplitude
- ρ-meson mass

Quark mass dependence of the $D(D^*)$ mesons

Heavy Hadron Chiral Perturbation Theory (HHχPT)

E. Jenkins, NPB412 (1994); Gil-Domínguez, Molina PLB (2023)

$$\frac{1}{4}(D+3D^*) = m_H + \alpha_a - \sum_{X=\pi,K,\eta} \beta_a^{(X)} \frac{M_X^3}{16\pi f^2} + \sum_{X=\pi,K,\eta} \left(\gamma_a^{(X)} - \lambda_a^{(X)}\alpha_a\right) \frac{M_X^2}{16\pi^2 f^2} \log\left(M_X^2/\mu^2\right) + c_a$$

$$(D^* - D) = \Delta + \sum_{X=\pi,K,\eta} \left(\gamma_a^{(X)} - \lambda_a^{(X)}\Delta\right) \frac{M_X^2}{16\pi^2 f^2} \log\left(M_X^2/\mu^2\right) + \delta c_a$$

$\mu = 770$ MeV

$$\left.\begin{aligned} \frac{1}{4}(D+3D^*) &= m_H + f(\sigma, a, b, c, d) \\ (D^* - D) &= \Delta + g(\Delta^{(\sigma)}, \Delta^{(a)}) \end{aligned}\right\}$$

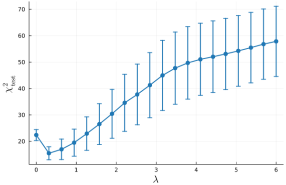
9 parameters, but different collaborations/scale settings, $7 + 2 \times 7 = 21$ parameters, ~ 80 data points

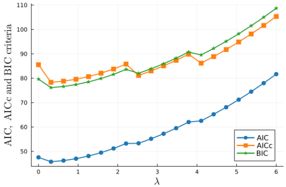
ETMC, PACS, HSC, CLS, RQCD, S.Prelovsek, MILC

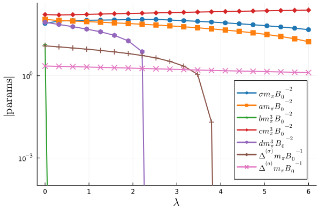
$D(D^*)$ quark mass dependence

LASSO + information criteria;

$$\chi_P^2 = \chi^2 + \lambda \sum_i^n |p_i|; \quad \text{Data} = \text{Training}\,(70\%) + \text{Test}\,(30\%) \qquad (1)$$







Some of the parameters are not relevant

Plots for ETMC data analysis

$D(D^*)$ quark mass dependence

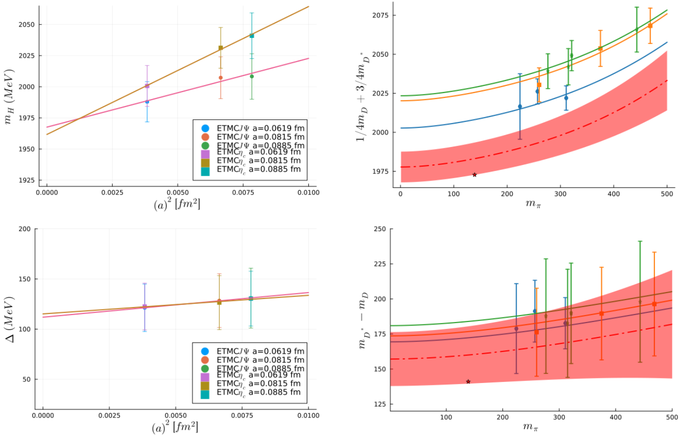


Figure: Extrapolation to the physical point of the ETMC data. $m_H = m'_H + r_H a^2$, $\Delta = \Delta' + r_\Delta a^2$.

Global analysis

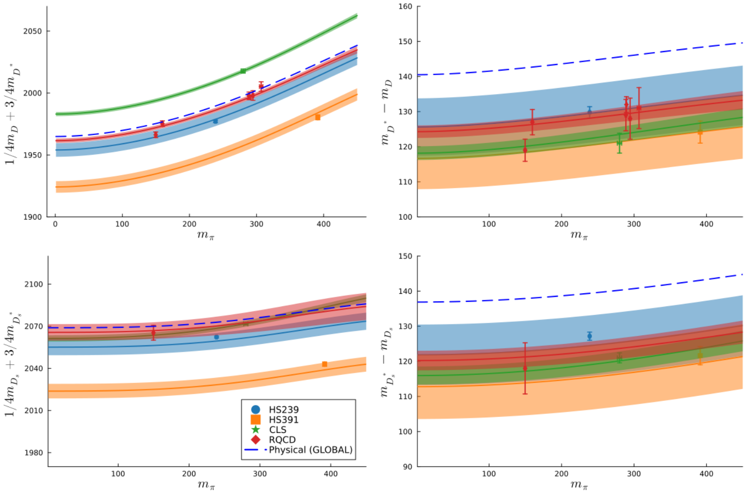


Figure: Results of the D, D^*, D_s and D_s^* meson masses for the global analysis of some collaborations.

Results for the T_{cc}

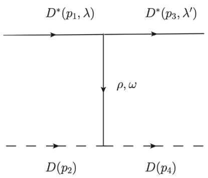
Lagrangian HGF   Bando, Kugo, Yamawaki, PRL54

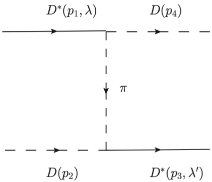
$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}_{III} \quad (2)$$

$$\mathcal{L}^{(2)} = \frac{1}{4} f^2 \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle, \; \mathcal{L}_{III} = -\frac{1}{4}\langle V_{\mu\nu} V^{\mu\nu} \rangle + \frac{1}{2} M_V^2 \langle [V_\mu - \frac{i}{g}\Gamma_\mu]^2 \rangle$$

Some of the vertices: $\mathcal{L}_{V\gamma} = -M_V^2 \frac{e}{g} A_\mu \langle V^\mu Q \rangle$, $g = \frac{M_V}{2f}$

$\mathcal{L}_{VPP} = -ig\langle V^\mu [P, \partial_\mu P] \rangle$, $\mathcal{L}_{III}^{(3V)} = ig\langle (\partial_\mu V_\nu - \partial_\nu V_\mu) V^\mu V^\nu \rangle, \ldots$





$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu$$

Results for the T_{cc}

(including only vector meson exchange)

$$V^{\rho(I=0)}_{\lambda,\lambda'}(p,p') = -g^2 \frac{(p_1+p_3)_\mu (p_2+p_4)^\mu}{t-m_\rho^2}\, \epsilon_{\lambda,\nu}(p_1)\epsilon^{*\,\nu}_{\lambda'}(p_3) \qquad (3)$$

and $g = g_1 + g_2\, m_\pi^2$. Quark mass dependence of m_ρ, Molina, Elvira, JHEP20

Col.	a	L	m_π
Padmanath22,Collins24	0.086	2 – 3	280
HSC24	0.120	1.9 – 2.9	391
CLQCD Chen22	0.152	2.4	349

Col.	a	m_π	a_0^{-1}
HALQCD23	0.0846	146	0.05
HALQCD14	0.0907	411	2.34

Infinite Volume

Scattering amplitude (Bethe-Salpeter):

$$T^{-1} = V_0^{-1} - G\,. \tag{4}$$

where $V_0 \equiv V_{(L=0)}$ is the s-wave projection.

$$G_j = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_j^2 + i\epsilon} \frac{1}{(P-q)^2 - M_j^2 + i\epsilon}\,. \tag{5}$$

The loop function can be evaluated in the Dimensional Regularization (DR) or cutoff scheme (q_{max}). Oller, Meissner, PLB01:

$$G_i(s) = \frac{1}{16\pi^2}\left\{a_i(\mu) + \ln\frac{M_i^2}{\mu^2} + \frac{m_i^2 - M_i^2 + s}{2W^2}\ln\frac{m_i^2}{M_i^2}\right.$$
$$+\frac{\bar{q}_i}{\sqrt{s}}\left[\ln(\ \ s - (M_i^2 - m_i^2) + 2\bar{q}_i\sqrt{s}) + \ln(\ \ s + (M_i^2 - m_i^2) + 2\bar{q}_i\sqrt{s})\right.$$
$$\left.\left. - \ln(-s + (M_i^2 - m_i^2) + 2\bar{q}_i\sqrt{s}) - \ln(-s - (M_i^2 - m_i^2) + 2\bar{q}_i\sqrt{s})\right]\right\},$$

Infinite Volume

Oller, PPNP20:

$$a(\mu) = -\frac{2}{m_i + M_i}\left[m_i \log(1 + \sqrt{1 + \frac{m_i^2}{q_{\rm max}^2}}) + M_i \log(1 + \sqrt{1 + \frac{M_i^2}{q_{\rm max}^2}})\right] + 2\log(\frac{\mu}{q_{\rm max}}). \quad (6)$$

with $\mu = 630$ MeV (Oset, Ramos, NPA98). Analytical continuation of G:

$$G_i^{II}(s) = G_i(s) + i\,\frac{q_{\rm cm}}{4\pi\sqrt{s}}, \qquad {\rm Im}q_{\rm cm} > 0. \quad (7)$$

Near to the resonance region:

$$T_{ij} \simeq \frac{g_i g_j}{s - s_0}, \quad (8)$$

g_i coupling to the channel i. The phase shift is:

$$p\cot\delta_j = -8\pi E(T_{jj})^{-1} + i\,p_j\,, \quad (9)$$

Finite volume

Finite volume scattering amplitude:

$$\tilde{T}^{-1} = V_0^{-1} - \tilde{G}\,, \tag{10}$$

Loop function $\tilde{G}$ (DR),

$$\widetilde{G}(P^0, \vec{P}) = G^{DR}(P^0, \vec{P}) + \lim_{q_{\rm max}\to\infty} \Delta G(P_0, \vec{P}, q_{\rm max}), \tag{11}$$

G^{DR}: loop function in the infinite volume, $\Delta G = \tilde{G}^{co} - G^{co}$.
Loop function (cutoff):

$$\tilde{G}^{co} = \frac{1}{L^3}\sum_{n}^{q_{\rm max}} \frac{E}{P_0} I(q^*)\,, \tag{12}$$

q^*: center-of-mass (CM) momentum, $\vec{q}_1{}^* + \vec{q}_2{}^* = 0$, $P^\mu = q_1^\mu + q_2^\mu$,
$s = P_0^2 - \vec{P}^{\,2}$.

Finite volume

The function $I(q)$ reads,

$$I(\vec{q}\,) = \frac{\omega_1(q) + \omega_2(q)}{2\omega_1(q)\omega_2(q)\left[P_0^2 - (\omega_1(q) + \omega_2(q))^2 + i\epsilon\right]}\ , \qquad (13)$$

with $\omega_i = \sqrt{q^2 + m_i^2}$, and $q = |\vec{q}\,|$.

The energy levels are given by,

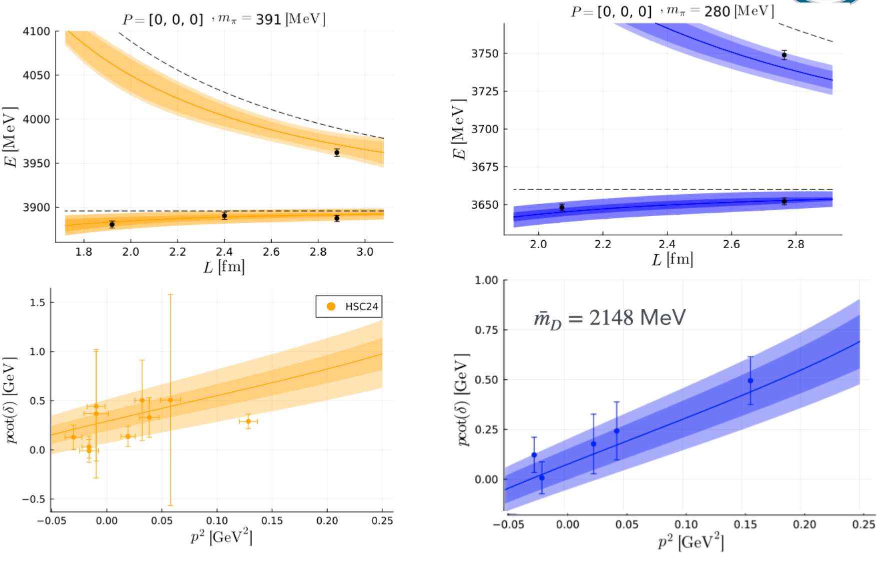
$$\det(\delta_{ll'}\delta_{mm'} - V_l\tilde{G}_{lm,l'm'}) = 0\ . \qquad (14)$$

We take into account the covariance matrix of the energy levels from the LQCD simulation.

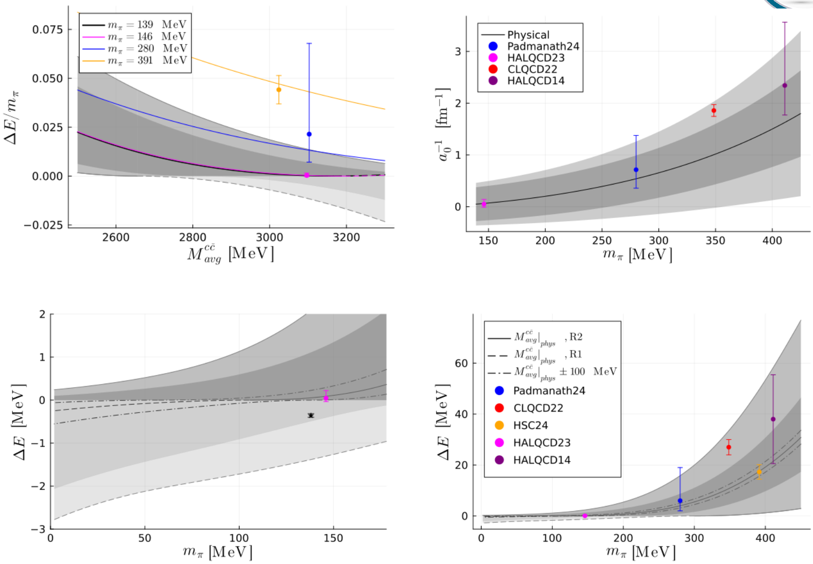
$$\chi^2 = \Delta E^T C_E^{-1} \Delta E \qquad (15)$$

$q_{\rm max} = 612 \pm 29$ MeV, $g_0 = 3.13 \pm 0.10$, $g_2 m^2_{\rm phys} = -0.057 \pm 0.058$

Results for the T_{cc}



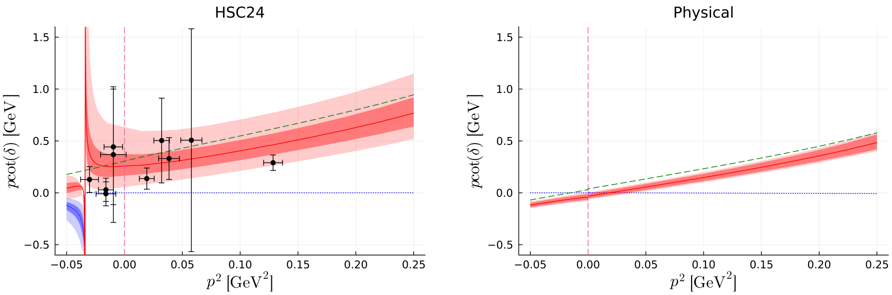
Results for the T_{cc}



Results for the T_{cc}

Momentum dependent BS equation including $\pi + \rho$, finite width D^*

$$V^{\pi\,(I=0)}_{\lambda,\lambda'}(p,p') \quad = \quad \frac{3}{4}\, g^2_{D^* D\pi}\, \frac{e^{u/\Lambda^2}}{u - m_\pi^2}\, (2p_4 - p_1)_\mu \epsilon_\lambda^{\ \mu}(p_1)\ (2p_2 - p_3)_\nu \epsilon^{*\ \nu}_{\lambda'}(p_3)$$



Results for the T_{cc}

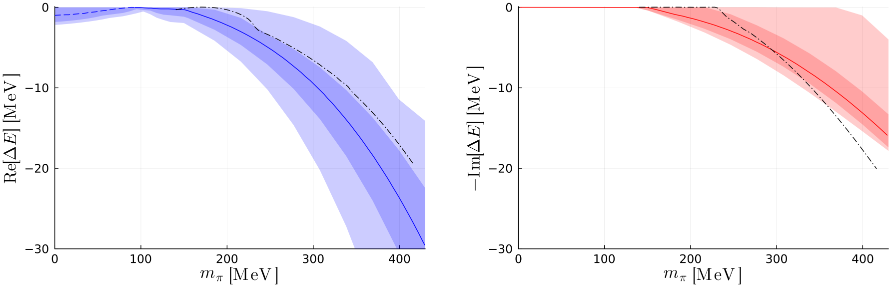
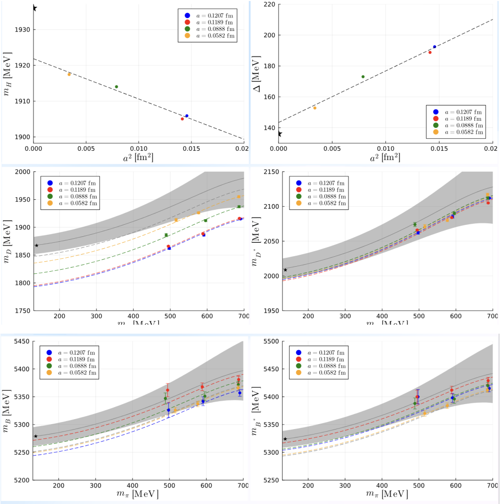


Figure: Dependence of the real and imaginary parts of the pole with the pion mass for the physical charm quark mass trajectory. The dash-dotted black line corresponds to the result of Abolnikov2024.

See also M.L. Du, Baru, Dong, F.K. Guo, et al, PRD22, M.L. Du, Filin, Baru, F.K. Guo et al., PRL23; Meng, Baru et al., PRD24

Prediction for the T_{bc} state

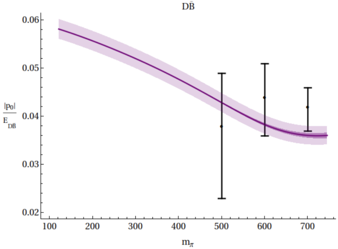
Lattice data analysis on the work by Radhakrishnan, Padmanath and Mathur, PRD110, 3 (2024) for $D\bar{B}$ scattering for pion masses m_π = 500 – 700 MeV.



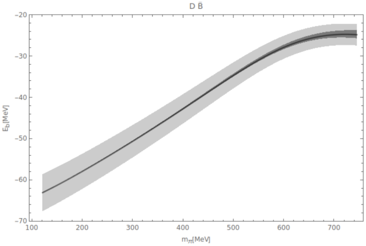
Prediction for the T_{bc} state

Continuum extrapolation

Pion mass [MeV]	500	600	7
Lattice D mass [MeV]	1880±30	1900±20	193



Physical extrapolation



Our physical results:

m_π [MeV]	500	600	700	140 (physical)
E_b [MeV]	-34.642	-27.829	-24.774	-61.857

See also the recent analysis of Abolnikov, Meng, Baru et al., PRD, 113, 114041 (2026) where the analysis from C. Alexandrou data, PRL (2024) gives shallow bound states. Needs further study.

Conclusions

- The combination of finite volume-EFT with LQCD is an useful tool to study the quark mass dependence of resonances
- Our study also supports the molecular interpretation of the T_{cc} as generated by the DD^* interaction
- The results of the LQCD simulation for the T_{cc} are compatible with a virtual state that is transitioning into a bound state for a pion mass close to physical and with the vector meson exchange being dominant, Gil-Dominguez, Giachino and Molina, PRD25
- We have also explored the $D\bar{B}$ scattering and obtained deeply bound states