Inclusive production of $\Upsilon(1S, 2S, 3S)$ in proton-proton collisions

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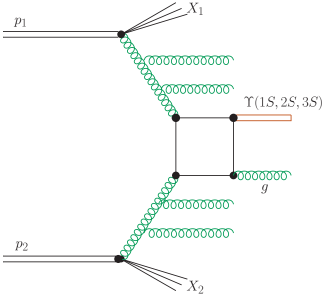
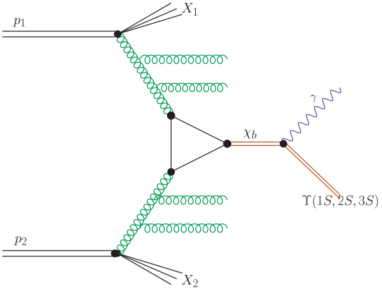
Outline

Introduction

- Some authors believe that the corresponding cross sections **are dominated by the so-called color-octet contribution**
- **Color-octet contribution** depend on unknown paramters that are often fitted to experimental data
- Some other authors expect that the **color-singlet contributions dominates**
- **Coresponding parameterss are known much better**
- **We calculate the color-singlet contribution** in the NRQCD k_t-factorization
- We compare our results for Υ(1S), Υ(2S) and Υ(3S) with experimental data from CMS and LHCb

The main color-singlet mechanism of production of $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ mesons





- There are different mechanisms:
  (a) direct produchtion, (b) feed down mechanisms
- We restrict to gluon-gluon fusion mechanism (high energy)
- We use unintegrated gluon distribution: KMR, JH and IN

Differential cross section

- The differential cross section in the k_t factorization can be writen as:

$$\frac{d\sigma(pp \to \Upsilon g X)}{dy_\Upsilon dy_g d^2p_{\Upsilon,t} d^2p_{g,t}} = \frac{1}{16\pi^2\hat{s}^2} \int \frac{d^2q_{1t}}{\pi} \frac{d^2q_{2t}}{\pi} \overline{|\mathcal{M}_{\mathbf{g^* g^* \to Vg}}|^2} \times$$
$$\times \delta^2 \left(\mathbf{q_{1t}} + \mathbf{q_{2t}} - \mathbf{p_{V,t}} - \mathbf{p_{g,t}}\right) \mathcal{F}_{\mathbf{g}}(\mathbf{x_1}, \mathbf{q^2_{1t}}, \mu^2_{\mathbf{F}}) \mathcal{F}_{\mathbf{g}}(\mathbf{x_2}, \mathbf{q^2_{2t}}, \mu^2_{\mathbf{F}})$$

- **We calculate the dominant color-singlet $gg \to Vg$ contribution taking into account transverse momenta of initial gluons**
- The corresponding matrix element squared for the $gg \to Vg$ is

$$|\mathcal{M}_{\mathbf{gg \to Vg}}|^2 \propto \alpha_s^3 |\mathbf{R(0)}|^2$$

S. P. Baranov, Phys. Rev. D **66** (2002) 114003

χ_b production

* In the k_t-factorization approach the leading-order **cross section for the χ_b meson production** can be written as:

$$\sigma_{\mathbf{pp}\to\chi_\mathbf{b}} = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\frac{d^2q_{1t}}{\pi}\frac{d^2q_{2t}}{\pi}\,\delta\left((q_1+q_2)^2 - M^2_{\chi_b}\right)\sigma_{gg\to\chi_b}(x_1,x_2,q_1,q_2)$$
$$\times\mathcal{F}_\mathbf{g}(\mathbf{x_1},\mathbf{q^2_{1t}},\mu^2_\mathbf{F})\mathcal{F}_\mathbf{g}(\mathbf{x_2},\mathbf{q^2_{2t}},\mu^2_\mathbf{F})$$

* The matrix element squared for the $gg\to\chi_b$ subprocess is

$$|\mathcal{M}_{\mathbf{gg}\to\chi_\mathbf{b}}|^2 \propto \alpha_\mathbf{s}^2|\mathbf{R}'(\mathbf{0})|^2$$

* For running coupling constants we choose:

$$\alpha_s^2 \to \alpha_s(\mu_1^2)\alpha_s(\mu_2^2)$$

where $\mu_1^2 = \mathbf{max}(\mathbf{q^2_{1t}},\mathbf{m^2_t})$ and $\mu_2^2 = \mathbf{max}(\mathbf{q^2_{2t}},\mathbf{m^2_t})$

B. A. Kniehl, D. V. Vasin, V. A. Saleev; Phys. Rev. D **73** (2006) 074022

Cross section for χ_b

- After some manipulation:

$$\sigma_{\mathbf{pp}\to\chi_\mathbf{b}} = \int dy d^2p_t d^2q_t \frac{1}{s\mathbf{x_1 x_2}} \frac{1}{m^2_{t,\chi_b}} \overline{|\mathcal{M}_{\mathbf{g^* g^* \to \chi_b}}|^2} \mathcal{F}_\mathbf{g}(\mathbf{x_1}, \mathbf{q^2_{1t}}, \mu^2_\mathbf{F}) \mathcal{F}_\mathbf{g}(\mathbf{x_2}, \mathbf{q^2_{2t}}, \mu^2_\mathbf{F})/4$$

- Which can be also used to calculate rapidity and transverse momentum distribution of the χ_b mesons
- In the last equation:

$$\mathbf{p_t} = \mathbf{q_{1t}} + \mathbf{q_{2t}} \qquad \mathbf{q_t} = \mathbf{q_{1t}} - \mathbf{q_{2t}}$$

$$\mathbf{x_1} = \frac{\mathbf{m_{t,\chi_b}}}{\sqrt{\mathbf{s}}} \exp(\mathbf{y}) \qquad \mathbf{x_2} = \frac{\mathbf{m_{t,\chi_b}}}{\sqrt{\mathbf{s}}} \exp(-\mathbf{y})$$

- The factor $\frac{1}{4}$ is the jacobian of transformation from $(\mathbf{q_{1t}}, \mathbf{q_{2t}})$ to $(\mathbf{p_t}, \mathbf{q_t})$ variables

KMR UGDF

The Kimber, Martin and Ryskin method (KMR UGDF) to construct unintegrated parton distributions from the conventional DGLAP parton distributions proceeds as:

$$\mathcal{F}(x, \boldsymbol{\kappa}^2) = \frac{\partial}{\partial Q^2}\left[xg(x, Q^2)\right]|_{Q^2=\kappa^2}$$

$$\frac{\partial a(x, \kappa^2, \mu^2)}{\log \kappa^2} = T_a(\kappa^2, \mu) \times \left[\frac{\alpha_s(\kappa^2)}{2\pi}\int_x^{1-\Delta}\sum_{a'} P_{aa'}(z)\ \ a'\left(\frac{x}{z}, \kappa^2\right) dz\right]$$

where $P_{aa'}(z)$ are splitting functions. The functions $a'\left(x, \kappa^2\right)$ are the collinear parton densities, $a' = xg$ or $xq, x\bar{q}$.
The virtual corrections are resumed via **Sudakov-type form factors**:

$$\log\ T_q(\kappa^2, \mu^2) = -\int_{\kappa^2}^{\mu^2}\frac{dp_t^2}{p_t^2}\frac{\alpha_s(p_t^2)}{2\pi}\int_0^{z_{max}} dz\ P_{qq}(z)$$

$$\log\ T_g(\kappa^2, \mu^2) = -\int_{\kappa^2}^{\mu^2}\frac{dp_t^2}{p_t^2}\frac{\alpha_s(p_t^2)}{2\pi}\left[n_f\int_0^1 dz\ P_{qg}(z) + \int_{z_{min}}^{z_{max}} dz\ z\ P_{gg}(z)\right]$$

M. A. Kimber, A. D. Martin and M. G. Ryskin; Rys. Rev. **D63** (2001) 114027

IN UGDF

The Ivanov-Nikolaev unintegrated gluon distribution (IN UGD) is a phenomenological model of the proton structure, describing the gluon density in terms of both the proton's longitudinal momentum fraction and its transverse momentum.
They suggested the extrapolation of **hard UGDF** at large $\boldsymbol{\kappa}^2$ and of **soft UGDF** at moderate and small $\boldsymbol{\kappa}^2$ can be written as:

$$\mathcal{F}(x,\boldsymbol{\kappa}^2) = \mathcal{F}^{(B)}_{soft}(x,\boldsymbol{\kappa}^2)\frac{\boldsymbol{\kappa}_s^2}{\boldsymbol{\kappa}^2+\boldsymbol{\kappa}_s^2} + \mathcal{F}_{hard}(x,\boldsymbol{\kappa}^2)\frac{\boldsymbol{\kappa}^2}{\boldsymbol{\kappa}^2+\boldsymbol{\kappa}_h^2}\,.$$

Interaction of large dipoles has been modeled by the non-perturbative, soft mechanism with energy-independent dipole cross section. Specific form of soft mechanism has been driven by exchange of two nonperturbative gluons.

I.P.Ivanov and N.N.Nikolaev Phys. Rev. **D65** (2002) 054004.

JH UGDF

Jung - Hautmann gluon distribution (JH UGDF) numerically solves the integral CCFM (Ciafaloni-Catani-Fiorani-Marchesini) evolution equation. It describes how the gluon distribution function $\mathcal{A}(x, k_t, p)$ varies with the hardness scale p:

$$\mathcal{A}(x, k_t, p) =$$

$$= \mathcal{A}_0(x, k_t, p) \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(p - zq) \times \Delta(p, zq) \mathcal{P}(z, q, k_t) \mathcal{A}(\frac{x}{z}, k_t + (1 - z)q, q)$$

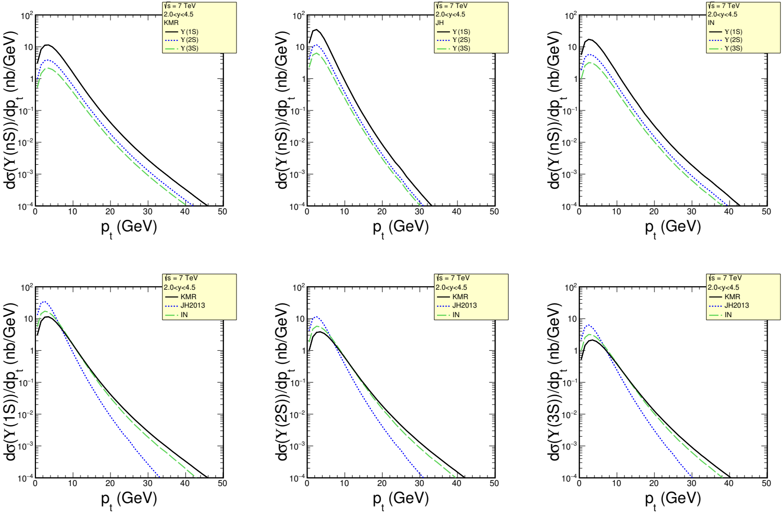
The first term in the right hand side of above equation is the contribution of the non-resolvable branchings between the starting scale q_0 and the evolution scale p, and is given by:

$$\mathcal{A}_0(x, k_t, p) = \mathcal{A}_0(x, k_t, p) \Delta(p, q_0)$$

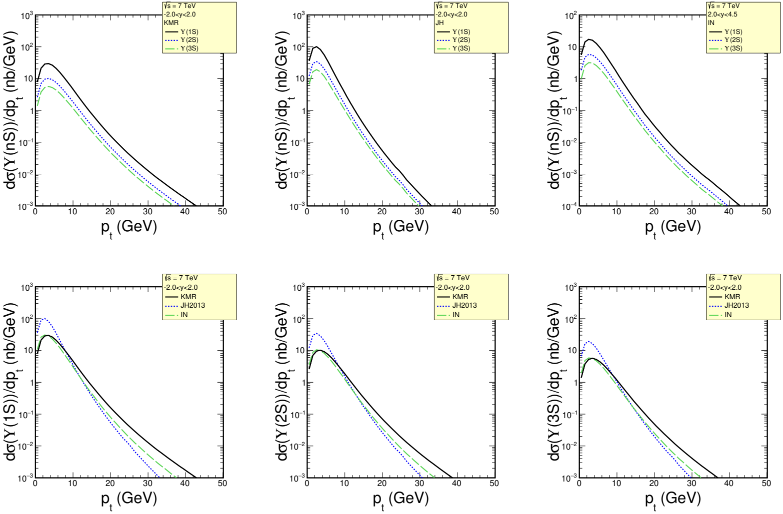
Where Δ is the Sudakov form factor, and $\mathcal{A}_0(x, k_t, p)$ is the starting distribution at scale q_0 . The integral term in the right hand side gives the k_t - dependent branchings in terms of the Sudakov form factor and unintegrated splitting function $\mathcal{P}$. The gluons' average momentum does not change with p.

F. Hautmann and H. Jung, Nucl. Phys. **B883**, (2014) 1-19.

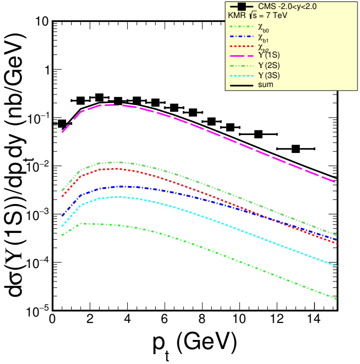
Transverse momentum distributon (2.0<y<4.5)

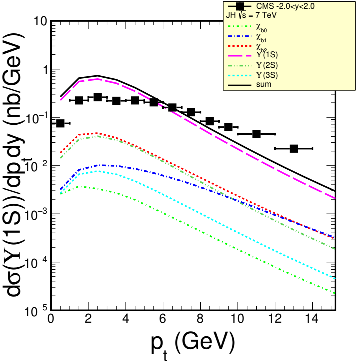


Transverse momentum distributon (-2.0<y<2.0)



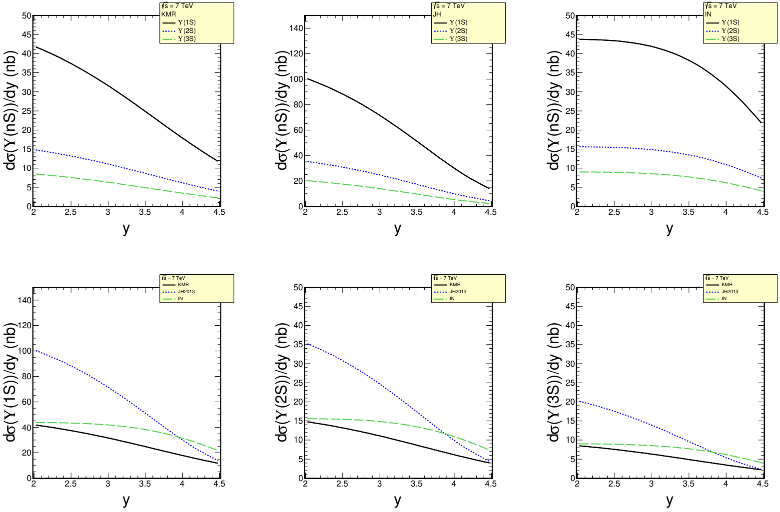
Transverse momentum distributon (-2.0<y<2.0)



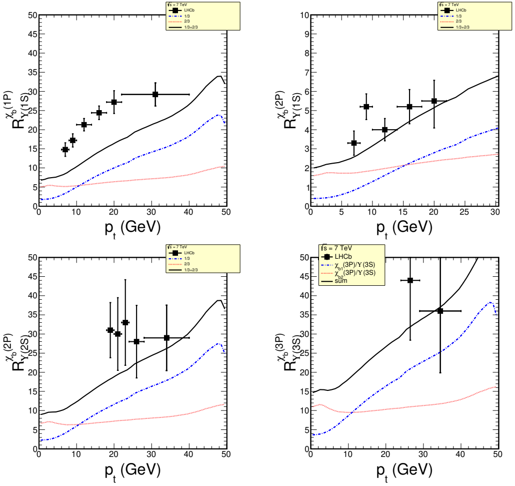


- Phys. Rev. **D83** (2011) 112004.

Rapidity distributon

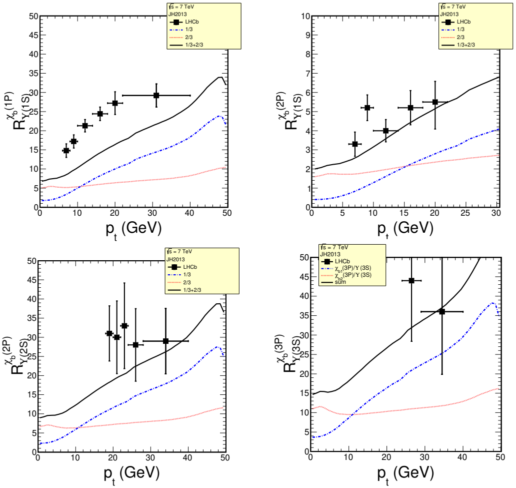


Ratio $\chi_b(mS)/\Upsilon(nS)$ - KMR



- Eur. Phys. J **C74** (2014) 3092.

Ratio $\chi_b(mS)/\Upsilon(nS)$ - JH



- Eur. Phys. J **C74** (2014) 3092.

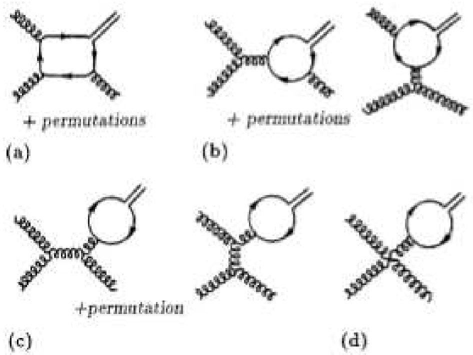
Conclusions

- **We have calculated the color-singlet contribution** in the NRQCD k_t-factorization
- We have calculation for three different model of UGDF: KMR, JH and IN
- **We have compared our results with CMS and LHCb data for Υ and χ_b productions**
- **These are preliminary results** and I have hope there will be more soon and that they will be published is one of the journals.
- New experimental data: CMS 2026 - CERN-EP-2026-004

Thank you for your attention !!!

Backup

Matrix elements for Υ



$$\mathcal{M}_{\mathbf{a}}(\mathbf{gg} \to \mathbf{Vg}) = tr\{\epsilon_{\mathbf{1}}(\mathbf{p_b} - \mathbf{k_1} + m_b)\epsilon_{\mathbf{2}} \times (-\mathbf{p_b} - \mathbf{k_3} + m_b)\epsilon_{\mathbf{3}} J(S,L)\}C \times tr\{T^a T^b T^c T^d\}[k_1^2 - 2(p_b k_1)]^{-1} \times [k_3^2 - 2(p_{\bar{b}} k_3)]^{-1} + 5\ permutations$$

S. P. Baranov, Phys. Rev. D **66** (2002) 114003

Matrix elements for Υ

$$\mathcal{M}_{\mathbf{b}}(\mathbf{gg} \to \mathbf{Vg}) = tr\{\gamma_\mu(p_{\bar{b}} - k_3 + m_b)\epsilon_{\mathbf{3}} J(S,L)\} \times G^3(k_1, \epsilon_{\mathbf{1}}, k_2, \epsilon_{\mathbf{2}}, -k, \mu) C f^{abe} \times tr\{T^e T^c T^d\}[k^2]^{-1} \times [k_3^2 - 2(p_{\bar{b}} k_3)]^{-1} + 5\ permutations$$

$$\mathcal{M}_{\mathbf{c}}(\mathbf{gg} \to \mathbf{Vg}) = tr\{\gamma_\mu J(S,L)\} G^3(k_1, \epsilon_{\mathbf{1}}, k_2, \epsilon_{\mathbf{2}}, -k, \mu) \times G^3(-k_3, -\epsilon_{\mathbf{3}}, -p, -\epsilon, -k, \nu) C f^{abe} f^{cfe} \times tr\{T^f T^d\}[k^2]^{-1} \times [m^2]^{-1} + 2\ permutations$$

$$\mathcal{M}_{\mathbf{d}}(\mathbf{gg} \to \mathbf{Vg}) = tr\{\gamma_\nu J(S,L)\} G^{(4)A,B,C}(\epsilon_{\mathbf{1}}, \epsilon_{\mathbf{2}}, \epsilon_{\mathbf{3}}, \nu) C \times tr\{T^f T^d\}[k^2]^{-1}[m^2]^{-1}$$

S. P. Baranov, Phys. Rev. D **66** (2002) 114003

Matrix elements for χ_b

$$\overline{|\mathcal{A}(g^\star + g^\star \to \mathcal{H}[^3P_0^{(1)}]|^2} = \frac{8}{3}\pi^2\alpha_s^2 \frac{\langle \mathcal{O}^{\mathcal{H}}[^3\mathbf{P}_0^{(1)}]\rangle}{M^5} \mathbf{F}^{[^3\mathbf{P}_0]}(\mathbf{t_1}, \mathbf{t_2}, \varphi)$$

$$\overline{|\mathcal{A}(g^\star + g^\star \to \mathcal{H}[^3P_1^{(1)}]|^2} = \frac{16}{3}\pi^2\alpha_s^2 \frac{\langle \mathcal{O}^{\mathcal{H}}[^3\mathbf{P}_1^{(1)}]\rangle}{M^5} \mathbf{F}^{[^3\mathbf{P}_1]}(\mathbf{t_1}, \mathbf{t_2}, \varphi)$$

$$\overline{|\mathcal{A}(g^\star + g^\star \to \mathcal{H}[^3P_2^{(1)}]|^2} = \frac{32}{45}\pi^2\alpha_s^2 \frac{\langle \mathcal{O}^{\mathcal{H}}[^3\mathbf{P}_2^{(1)}]\rangle}{M^5} \mathbf{F}^{[^3\mathbf{P}_2]}(\mathbf{t_1}, \mathbf{t_2}, \varphi)$$

where

$$\langle \mathcal{O}^{\chi_{cJ}}[^3\mathbf{P}_J^{(1)}]\rangle = \mathbf{2N_c(2J+1)|R'(0)|^2}$$

B. A. Kniehl, D. V. Vasin, V. A. Saleev; Phys. Rev. D **73** (2006) 074022

Matrix elements for χ_b

$$\mathbf{F}^{[^3\mathbf{P}_0]}(\mathbf{t_1}, \mathbf{t_2}, \varphi) = \frac{2}{9}\frac{M^2\left(M^2+|\mathbf{p}_t|^2\right)^2\left[(3M^2+t_1+t_2)\cos\varphi+2\sqrt{t_1t_2}\right]^2}{(M^2+t_1+t_2)^4}$$

$$\mathbf{F}^{[^3\mathbf{P}_1]}(\mathbf{t_1}, \mathbf{t_2}, \varphi) = \frac{2}{9}\frac{M^2\left(M^2+|\mathbf{p}_t|^2\right)^2\left[(t_1+t_2)^2\sin^2\varphi+M^2\left(t_1+t_2-2\sqrt{t_1t_2}\cos\varphi\right)\right]}{(M^2+t_1+t_2)^4}$$

$$\mathbf{F}^{[^3\mathbf{P}_2]}(\mathbf{t_1}, \mathbf{t_2}, \varphi) = \frac{1}{3}\frac{M^2}{(M^2+t_1+t_2)^4}(M^2+|\mathbf{p}_t|^2)^2\{3M^4+3M^2(t_1+t_2)+4t_1t_2 + (t_1+t_2)^2\cos^2\varphi+2\sqrt{t_1t_2}[3M^2+2(t_1+t_2)]\cos\varphi\}$$

where $\mathbf{p}_t = \mathbf{q}_{1t} + \mathbf{q}_{2t}$
and $\varphi = \varphi_1 - \varphi_2$ is the angle between $\mathbf{q}_{1t}$ and $\mathbf{q}_{2t}$ so

$$|\mathbf{p}_t|^2 = t_1 + t_2 + 2\sqrt{t_1t_2}\cos\varphi$$

B. A. Kniehl, D. V. Vasin, V. A. Saleev; Phys. Rev. D **73** (2006) 074022