

$V \rightarrow 3\pi$ ($V = \omega, \phi, J/\psi$) and
 $V \rightarrow \pi^0 \gamma^*$ transition form factor
from Khuri-Treiman equations

JPAC:

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Phys.Rev.D 108, 014035 (2023);

2505.15309 [hep-ph]

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18th International Workshop on Meson Physics

Kraków, Poland, June 25-30, 2026



**18th International Workshop
on Meson Physics**

25th - 30th June 2026, Kraków, Poland

Organized by Jagiellonian University,
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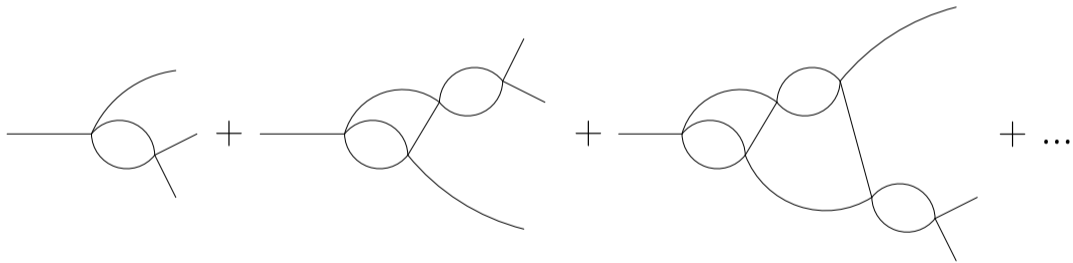


<https://meson.if.uj.edu.pl>



Three-particle decay

- In many decay processes one wants to take into account unitarity/FSI in the three possible channels
- Khuri-Treiman equations:
 - Include full rescattering effects



Khuri-Treiman formalism: citation summary

- Developed for $K \rightarrow 3\pi$

Pion-Pion Scattering and $K + /- \rightarrow 3\pi$ Decay

N.N. Khuri (Princeton, Inst. Advanced Study), S.B. Treiman (Princeton U.)

Aug 1, 1960

6 pages

Published in: *Phys.Rev.* 119 (1960) 1115-1121

DOI: [10.1103/PhysRev.119.1115](https://doi.org/10.1103/PhysRev.119.1115)

Citations per year



- Most prominent applications:
 - $\eta \rightarrow 3\pi$ [Kambor *et al.* 1995, Anisovich *et.al.* 1996, Descotes-Genon *et al.* 2014, JPAC 2015&17, Colangelo 2016, Albaladejo *et al.* 2017]
 - $\eta' \rightarrow \eta\pi\pi$ [Isken *et al.* 2017]
 - $\omega/\phi \rightarrow 3\pi, \pi\gamma^*$ [Kubis *et al.* 2012, JPAC'14'20'25, Dax *et al.* 2018] → this talk
 - $J/\psi \rightarrow 3\pi, \pi\gamma^*$ [JPAC 2023] → this talk
 - $e^+e^- \rightarrow 3\pi/\pi\gamma (a_\mu)$ [Hoferichter *et.al.* 2018&19, Hoid 2020]
 - $D \rightarrow K\pi\pi$ [Niecknig *et.al.* 2015]
 - $J^{PC} \rightarrow 3\pi$ [JPAC 2019, Stamen *et.al.* 2022]
 - $\pi_1(1600) \rightarrow 3\pi$ [JPAC *in progress*] → this talk

JPAC: Joint Physics Analysis Center

- Work in **theoretical/experimental/phenomenological** analysis
- Light/heavy meson **spectroscopy**
- Interaction with many **experimental collaborations** (BESIII, CLAS, GlueX, KLOE, LHCb, MAMI,...) and LQCD groups
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Three-body: definitions

- Decay amplitude for $V \rightarrow \pi^+ \pi^- \pi^0$ ($V = \omega, \phi, J/\psi$)

$$\mathcal{M}(s, t, u) = i\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu p_+^\nu p_-^\alpha p_0^\beta \mathcal{F}(s, t, u),$$

- Mandelstam variable, s -channel scattering angle and momenta

$$s = (p_+ + p_-)^2, \quad t = (p_0 + p_+)^2, \quad u = (p_0 + p_-)^2,$$

$$\cos \theta_s(s, t, u) = \frac{t - u}{4p(s)q(s)}, \quad \sin \theta_s(s, t, u) = \frac{\sqrt{\phi(s, t, u)}}{2\sqrt{s}p(s)q(s)},$$

$$p(s) = \frac{\lambda^{\frac{1}{2}}(s, m_{\pi^+}^2, m_{\pi^-}^2)}{2\sqrt{s}}, \quad q(s) = \frac{\lambda^{\frac{1}{2}}(s, m_{J/\psi}^2, m_{\pi^0}^2)}{2\sqrt{s}},$$

$$|\mathcal{M}(s, t, u)|^2 = \frac{1}{4} (2\sqrt{s} \sin \theta_s p(s) q(s))^2 |\mathcal{F}(s, t, u)|^2,$$

Khuri-Treiman representation of the amplitude

- s -channel partial-wave (pw) expansion of the amplitude

$$F(s, t, u) = \sum_{J=0}^{\infty} (p(s) q(s))^{J-1} P'_J(z_s) f_J(s),$$

- Khuri-Treiman/isobar decomposition of the amplitude

$$F(s, t, u) = \sum_{J=0}^{J_{\max}} (p(s) q(s))^{J-1} P'_J(z_s) F_J(s) + (s \leftrightarrow t) + (s \leftrightarrow u),$$

- Consider only $J = 1$ isobars

$$F(s, t, u) = F_1(s) + F_1(t) + F_1(u),$$

- pw projection of the KT decomposition

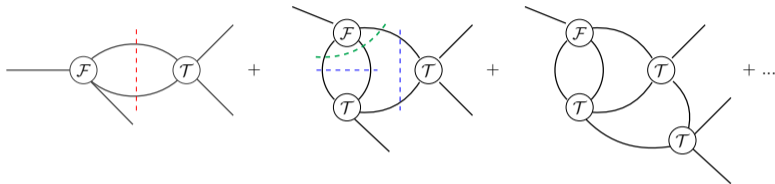
$$f_1(s) = F_1(s) + \hat{F}_1(s), \quad \hat{F}_1(s) = 3 \int_{-1}^1 \frac{dz_s}{2} (1 - z_s^2) F_1(t(s, z_s)),$$

— $F_1(s)$: right-hand cut (RHC)

— $\hat{F}_1(s)$: left-hand cut (given by the RHC of the crossed channels $F_1(t), F_1(u)$)

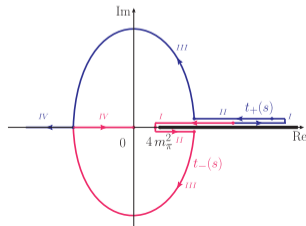
Unitarity and analyticity

- Unitarity relation for the isobar amplitude $F_1(s)$



$$\text{disc}F_1(s) = 2i \left(F_1(s) + \hat{F}_1(s) \right) \sin \delta(s) e^{-i\delta(s)} \theta(s - 4m_\pi^2),$$

- Complications: integration contour for $\hat{F}_1(s)$
- Classic' strategy (Khuri-Treiman, 1960)
 - deform path of angular integral to avoid crossing branch cuts
- Alternative approach (Gasser and Rusetsky, 2018)
 - deform path of dispersion relation

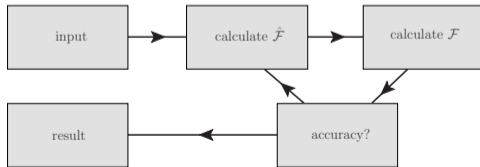


KT equations: DR, solutions, subtractions

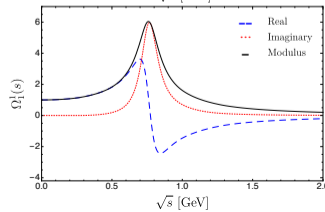
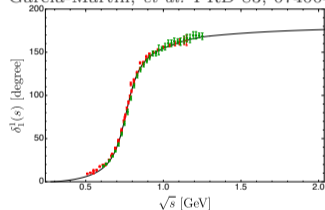
- Unsubtracted dispersion relation:

$$F_1(s) = \frac{1}{2\pi i} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{disc } F_1(s')}{s' - s}, \quad F_1(s) = \Omega_1(s) \left(a + \frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds' \sin \delta_1(s') \hat{F}_1(s')}{s' |\Omega_1(s')| (s' - s)} \right),$$

- $\delta_1(s)$: $\pi\pi$ phase taken as input:
- Solution by numerical iteration
 - Initial input: $F_1 = \Omega_1(s)$



Garcia-Martin, *et al.* PRD 83, 074004 (2011)



KT equations: solutions for $\omega \rightarrow 3\pi$

- Once-subtracted dispersion relations

$$F_1(s) = \Omega_1(s) \left(a + b' s + \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{(s')^2} \frac{\sin \delta_1(s') \hat{F}_1(s')}{|\Omega_1(s')| (s' - s)} \right),$$

- Sum rule:

$$b_{\text{sr}} \equiv b'/a = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{(s')^2} \frac{\sin \delta_1(s') \hat{F}_1(s')/a}{|\Omega_1(s')|}.$$

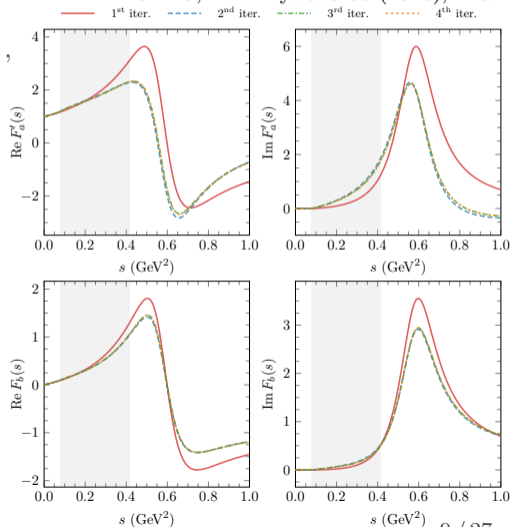
- Solution

$$F_1(s) = a [F_a(s) + b F_b(s)],$$

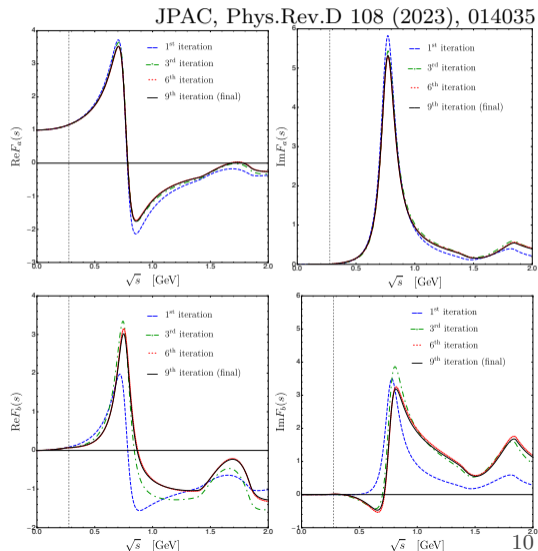
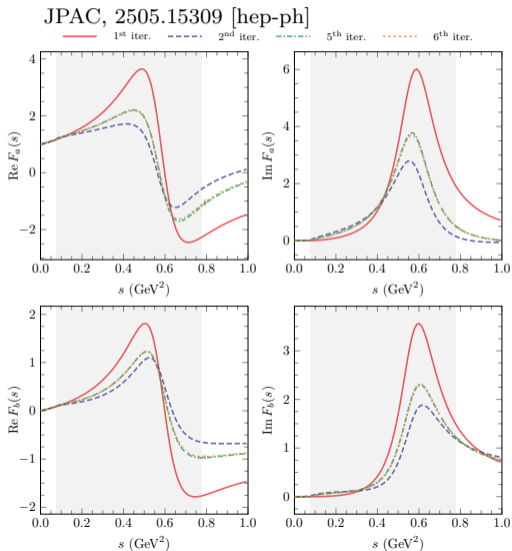
$$F_a(s) = \Omega_1(s) \left[1 + \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\sin \delta_1(s') \hat{F}_a(s')}{|\Omega_1(s')| (s' - s)} \right],$$

$$F_b(s) = \Omega_1(s) \left[s + \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\sin \delta_1(s') \hat{F}_b(s')}{|\Omega_1(s')| (s' - s)} \right].$$

JPAC, Eur.Phys.J.C 80 (2020), 1107

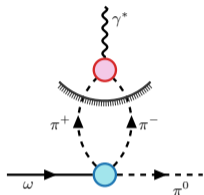


KT equations: solutions for $\phi \rightarrow 3\pi$ and $J/\psi \rightarrow 3\pi$



$V \rightarrow \pi^0 \gamma^*$ transition form factor

- Two-pion contribution to the discontinuity to $\omega \rightarrow \pi^0 \gamma^*$:



$$\text{disc } f_{\omega\pi^0}(s) = i \frac{p^3(s)}{6\pi\sqrt{s}} F_{\pi}^{V*}(s) f_1^{\omega \rightarrow 3\pi}(s') \theta(s - 4m_{\pi}^2),$$

$$F_{\pi}^V(s) = \Omega(s)$$

- One-subtracted dispersion relation for $\omega \rightarrow \pi^0 \gamma^*$:

$$f_{\omega\pi^0}(s) = |f_{\omega\pi^0}(0)| e^{i\phi_{\omega\pi^0}(0)} + \frac{s}{12\pi^2} \int_{4m_{\pi}^2}^{\infty} \frac{ds'}{(s')^{3/2}} \frac{p^3(s') F_{\pi}^{V*}(s') f_1^{\omega \rightarrow 3\pi}(s')}{(s' - s)},$$

$$\Gamma(\omega \rightarrow \pi^0 \gamma) = \frac{e^2(m_{\omega}^2 - m_{\pi^0}^2)^3}{96\pi m_{\omega}^3} |f_{\omega\pi^0}(0)|^2$$

Combined analysis of $\omega \rightarrow 3\pi$ and $\omega \rightarrow \pi^0\gamma^*$

- We have a total of five parameters: $|a|$, $b = |b|e^{i\phi_b}$, $f_{\omega\pi^0}(0) = |f_{\omega\pi^0}(0)|e^{i\phi_{\omega\pi^0}(0)}$
- We minimize the following χ^2 function:

$$\chi^2 = \left(\frac{\alpha^{\text{exp}} - \alpha^{\text{th}}}{\sigma_{\alpha^{\text{exp}}}}\right)^2 + \left(\frac{\beta^{\text{exp}} - \beta^{\text{th}}}{\sigma_{\beta^{\text{exp}}}}\right)^2 + \left(\frac{\gamma^{\text{exp}} - \gamma^{\text{th}}}{\sigma_{\gamma^{\text{exp}}}}\right)^2 + \left(\frac{\Gamma_{\omega \rightarrow 3\pi}^{\text{exp}} - \Gamma_{\omega \rightarrow 3\pi}^{\text{th}}}{\sigma_{\Gamma_{\omega \rightarrow 3\pi}^{\text{exp}}}}\right)^2$$

$$+ \sum_i \left(\frac{|f_{\omega\pi}^{\text{exp},i}|^2 - |f_{\omega\pi}^{\text{th}}(s_i)|^2}{\sigma_{f_{\omega\pi}^{\text{exp},i}}}\right)^2 + \left(\frac{\Gamma_{\omega \rightarrow \pi^0\gamma}^{\text{exp}} - \Gamma_{\omega \rightarrow \pi^0\gamma}^{\text{th}}}{\sigma_{\Gamma_{\omega \rightarrow \pi^0\gamma}^{\text{exp}}}}\right)^2$$

- $\omega \rightarrow \pi^+\pi^-\pi^0$ Dalitz parameters: $\left(X = \frac{t-u}{\sqrt{3}R_\omega} = \sqrt{Z} \cos \phi, Y = \frac{s_c-s}{R_\omega} = \sqrt{Z} \sin \phi\right)$

$$|\mathcal{F}(Z, \phi)|^2 = |N|^2 \left[1 + 2\alpha Z + 2\beta Z^{3/2} \sin 3\phi + 2\gamma Z^2 + \mathcal{O}(Z^{5/2})\right]$$

Reference	$\alpha \times 10^3$	$\beta \times 10^3$	$\gamma \times 10^3$
WASA@COSY [PLB 770, 418 (2017)]	133(41)	37(54)	—
BESIII [PRD98, 112007 (2018)]	120.2(8.1)	29.5(9.6)	—
BESIII [PRD98, 112007 (2018)]	111(18)	25(10)	22(29)

Combined analysis of $\omega \rightarrow 3\pi, \pi^0\gamma^*$

- Values of the fitted Dalitz parameters:

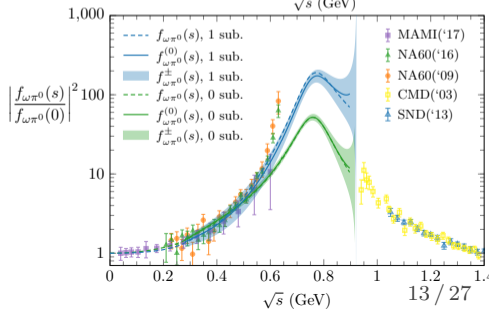
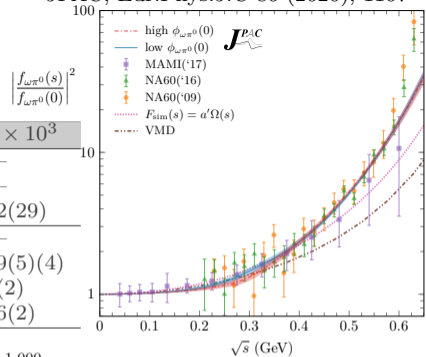
Reference	$\alpha \times 10^3$	$\beta \times 10^3$	$\gamma \times 10^3$
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BESIII [PRD98, 112007 (2018)]	111(18)	25(10)	22(29)
KT 1 sub (this work)	120.1(7.7)(0.7)	30.2(4.3)(2.5)	—
KT 1 sub (this work)	109(14)(2)	26(6)(2)	19(5)(4)
KT unsub [EPJC 72, 2014 (2012)]	77(4)	26(2)	5(2)
Omnès [EPJC 72, 2014 (2012)]	116(4)	28(2)	16(2)

- Subtraction constant:

$$b_{\text{Fit}} \simeq 2.65(35)e^{1.70(27)i} \text{ GeV}^{-2}$$

vs

$$b_{\text{SR}} \simeq 0.55e^{0.15i} \text{ GeV}^{-2}$$

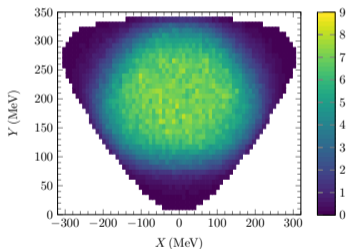


Combined analysis of $\phi \rightarrow 3\pi$ and $\phi \rightarrow \pi^0\gamma^*$

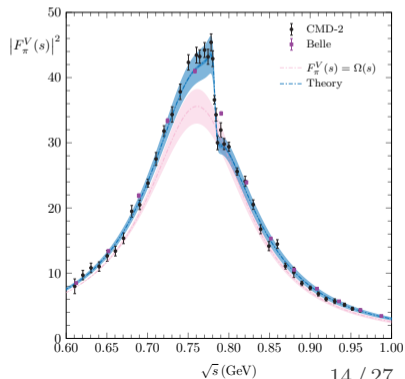
- We minimize the following χ^2 function:

$$\chi^2 = \left(\frac{N_{\text{events},i}^{\text{exp}} - N_{\text{events},i}^{\text{th}}}{\sigma_{N_{\text{events},i}^{\text{exp}}}} \right)^2 + \left(\frac{\Gamma_{\phi \rightarrow 3\pi}^{\text{exp}} - \Gamma_{\phi \rightarrow 3\pi}^{\text{th}}}{\sigma_{\Gamma_{\phi \rightarrow 3\pi}^{\text{exp}}}} \right)^2 + \sum_i \left(\frac{|f_{\phi\pi}^{\text{exp},i}|^2 - |f_{\phi\pi}^{\text{th}}(s_i)|^2}{\sigma_{f_{\phi\pi}^{\text{exp},i}}} \right)^2 + \left(\frac{\Gamma_{\phi \rightarrow \pi^0\gamma}^{\text{exp}} - \Gamma_{\phi \rightarrow \pi^0\gamma}^{\text{th}}}{\sigma_{\Gamma_{\phi \rightarrow \pi^0\gamma}^{\text{exp}}}} \right)^2$$

- $\phi \rightarrow \pi^+\pi^-\pi^0$ Dalitz-plot [KLOE PLB 757, 362 (2016)]:

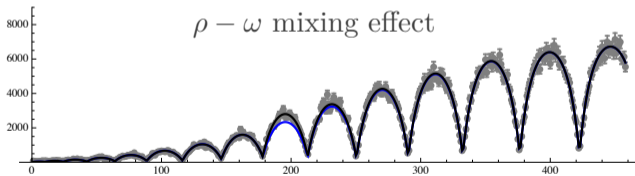
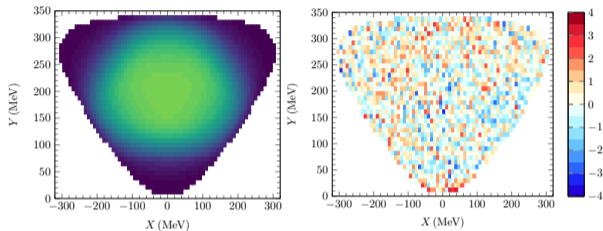


- $\rho - \omega$ mixing: $F_1(s) \rightarrow F_1(s) + a \frac{m_\omega^2}{m_\omega^2 - s - i\sqrt{s}\Gamma_\omega}$
- $F_\pi^V(s) = P(s)G_\omega(s)\Omega(s)$



Combined analysis of $\phi \rightarrow 3\pi$ and $\phi \rightarrow \pi^0\gamma^*$

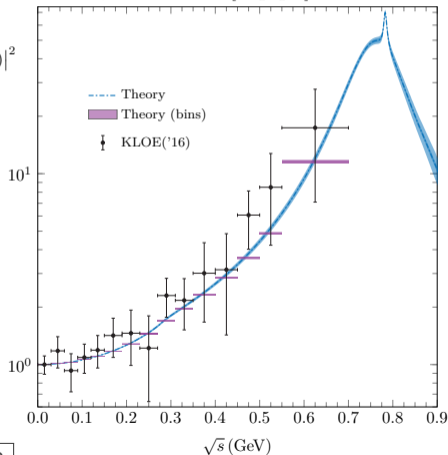
- Fits to Dalitz plot and TFF:



- Subtraction constant:

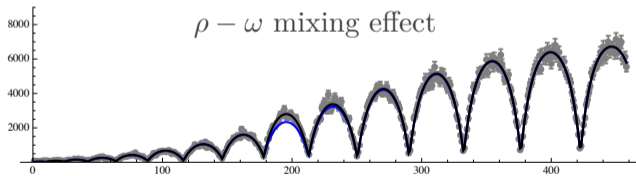
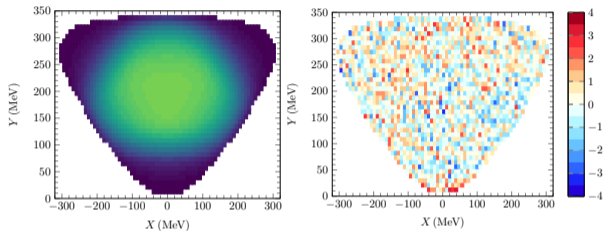
$$b_{\text{Fit}} = 0.76e^{0.43i} \text{ GeV}^{-2} \text{ vs } b_{\text{sr}} = 0.79e^{0.69i} \text{ GeV}^{-2}$$

JPAC, 2505.15309 [hep-ph]



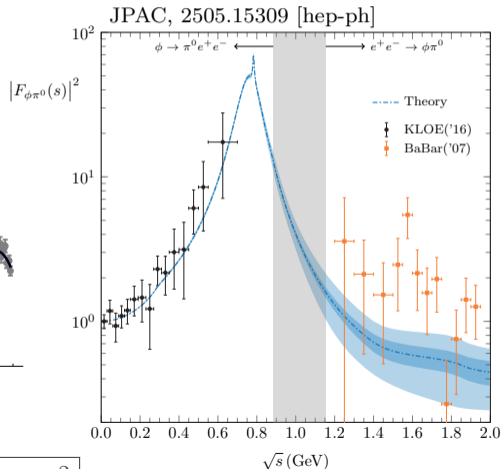
Combined analysis of $\phi \rightarrow 3\pi$ and $\phi \rightarrow \pi^0\gamma^*$

- Fits to Dalitz plot and TFF:



- Subtraction constant:

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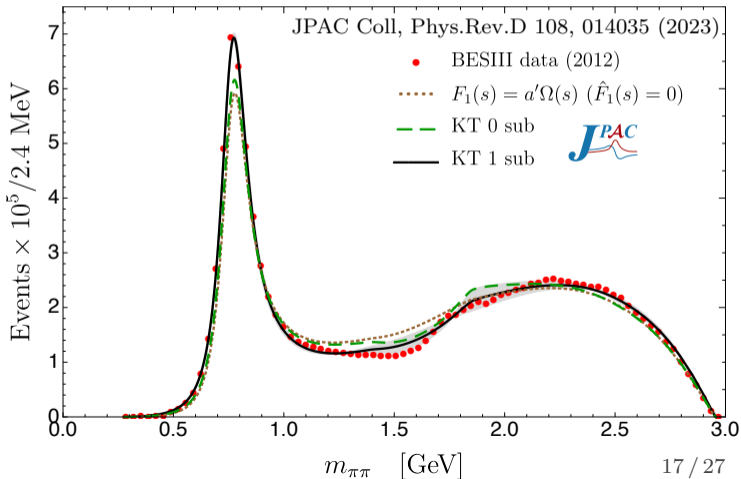
$J/\psi \rightarrow 3\pi$ BESIII data analysis

- Dominated by the ρ
- BESIII data [PLB 710 (2012)]
- JPAC analysis:
 - Two-body
 - KT unsub basic features
 - KT 1-sub improves the description

$$b_{\text{fit}} = 0.198(35)e^{i2.675(300)} \text{ GeV}^{-2}$$

vs

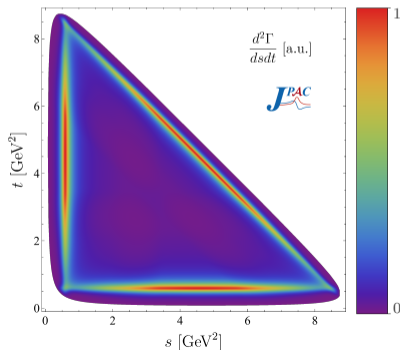
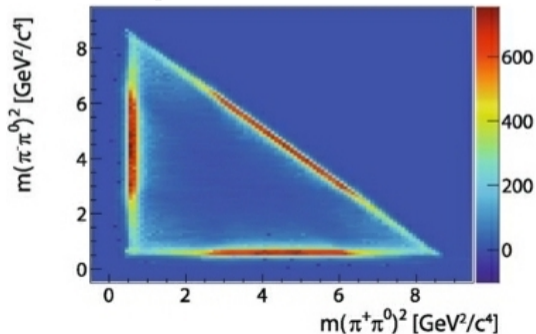
$$b_{\text{sr}} = 0.141e^{i2.32} \text{ GeV}^{-2}$$



JPAC $J/\psi \rightarrow 3\pi$ BESIII data analysis

- Dominated by the ρ
- BESIII data [PLB 710 (2012)]
- JPAC analysis:

— Dalitz plot distribution similar to experimental one



Contribution of the F -wave

- Isobar decomposition of the amplitude including F -wave:

$$F(s, t, u) = F_1(s) + F_1(t) + F_1(u) \\ + (p(s)q(s))^2 P_3'(z_s) F_3(s) + (p(t)q(t))^2 P_3'(z_t) F_3(t) + (p(u)q(u))^2 P_3'(z_u) F_3(u),$$

- $F_{1,3}(s)$ is the P, F -wave isobar amplitude
- Discontinuity of the F -wave:

$$\text{disc } F_3(s) = 2i \left(F_3(s) + \hat{F}_3(s) \right) \sin \delta_3(s) e^{-i\delta_3(s)} \theta(s - 4m_\pi^2),$$

- We neglect $\hat{F}_3(s)$ (for simplicity)

$$F_3(s) = p_3(s)\Omega_3(s), \quad \Omega_3(s) = \exp \left[\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\delta_3(s')}{s' - s} \right],$$

Model of the F -wave: $\rho_3(1690)$ exchange

- Exchange of a $\rho_3(1690)$ in the s -channel with an energy-dependent width:

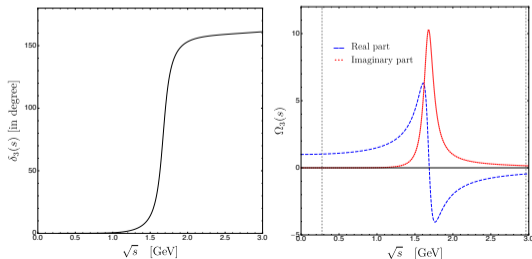
$$F_3(s)|_{\text{BW}} = \frac{m_{\rho_3}^2}{m_{\rho_3}^2 - s - im_{\rho_3}\Gamma_{\rho_3}^{\ell=3}(s)}, \quad \Gamma_R^\ell(s) = \frac{\Gamma_R m_R}{\sqrt{s}} \left(\frac{p(s)}{p(m_R^2)} \right)^{2\ell+1} \left(F_R^\ell(s) \right)^2,$$

$$p(s) = \frac{\sqrt{s}}{2} \sigma_\pi(s), \quad F_R^{\ell=3}(s) = \sqrt{\frac{z_0(z_0 - 15)^2 + 9(2z_0 - 5)^2}{z(z - 15)^2 + 9(2z - 5)^2}}, \quad z = r_R^2 p^2(s), \quad z_0 = r_R^2 p^2(m_{\rho_3}^2),$$

- Extraction of the F -wave phase and Omnès F -wave function:

$$\tan \delta_3(s) = \frac{\text{Im}F_3(s)|_{\text{BW}}}{\text{Re}F_3(s)|_{\text{BW}}},$$

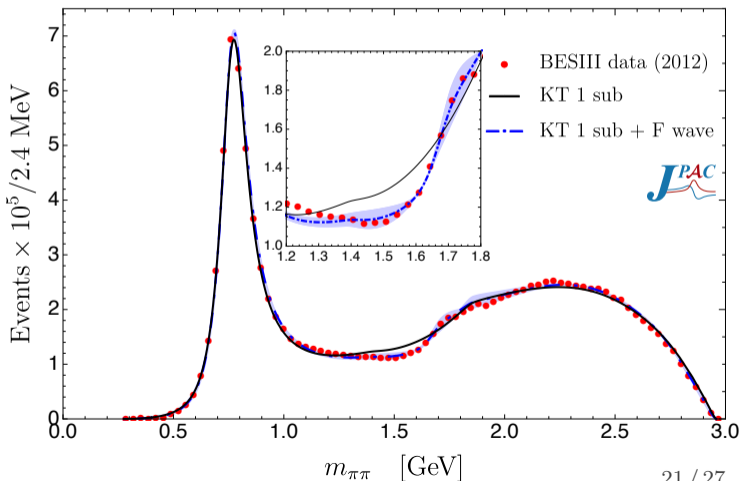
$$\Omega_3(s) = \exp \left[\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\delta_3(s')}{s' - s} \right],$$



JPAC $J/\psi \rightarrow 3\pi$ BESIII data analysis

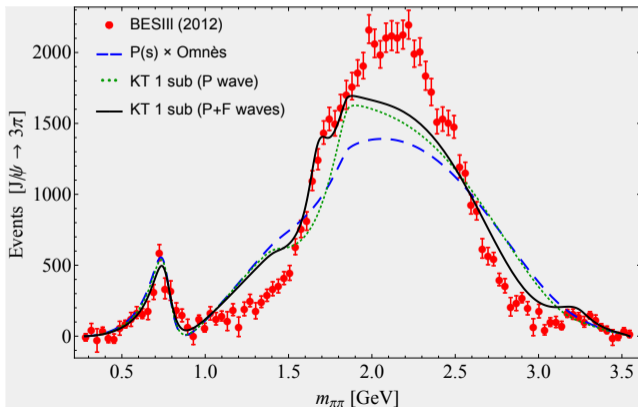
- Dominated by the ρ
- BESIII data [PLB 710 (2012)]
- JPAC analysis:
 - Two-body
 - KT unsub basic features
 - KT 1-sub improves the description
 - KT 1-sub+F-wave describe better $m_{\pi\pi} \sim 1.5$ GeV.

JPAC Coll, Phys.Rev. D108, 014035 (2023)



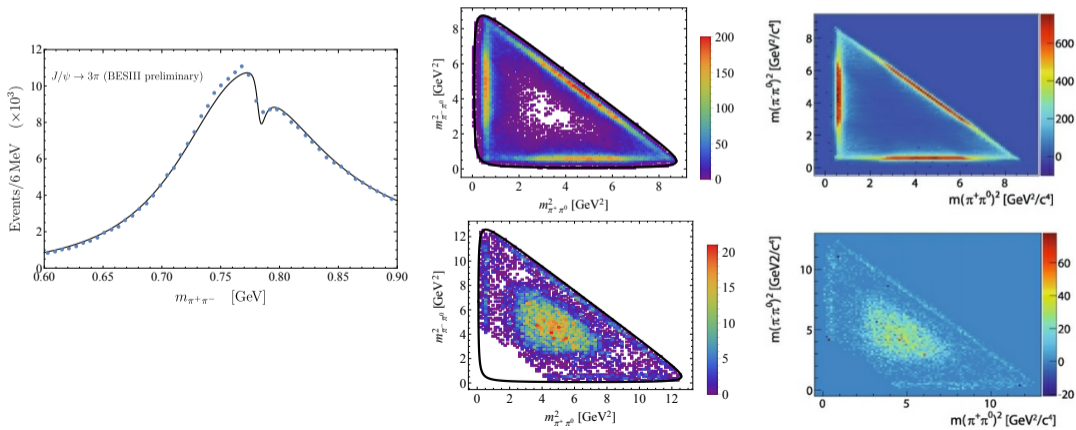
$\psi(2S) \rightarrow \pi^+\pi^-\pi^0$ decays

- Completely analogous formalism
- Larger phase space,
 $m_{\psi(2S)} = 3.68610 \text{ GeV}$
- BESIII data [PLB 710 (2012)]
- Why does the $\psi(2S) \rightarrow \pi^+\pi^-\pi^0$ Dalitz plot differ so dramatically from $J/\psi \rightarrow \pi^+\pi^-\pi^0$?
 - $\rho(2150)$?
 - Coupled-channels?



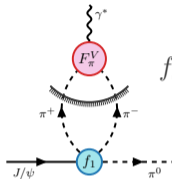
$J/\psi \rightarrow 3\pi$: future analyses

- Until 2022, BESIII collected 10 billion J/ψ events
 - Expect to see roughly 200 million $J/\psi \rightarrow 3\pi$ events (vs ~ 1.9 million in 2012)
 - High-precision ρ - ω mixing extraction (JPAC, ongoing)



$J/\psi \rightarrow \pi^0 \gamma^*$ transition form factor

- Dispersive representation (once-subtracted)

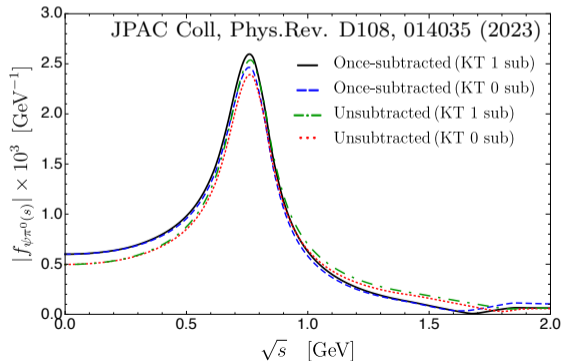


$$f_{\omega\pi^0}(s) = |f_{J/\psi\pi^0}(0)| e^{i\phi_{J/\psi\pi^0}(0)} + \frac{s}{12\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds'}{(s')^{3/2}} \frac{p^3(s') F_\pi^{V*}(s') f_1^{J/\psi \rightarrow 3\pi}(s')}{(s' - s)},$$

- $|f_{J/\psi\pi^0}(0)|$ from data:

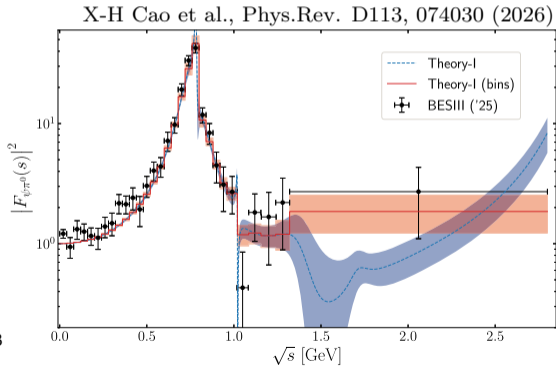
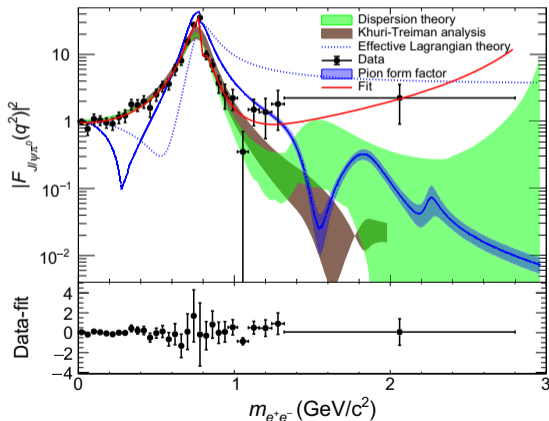
$$\Gamma(J/\psi \rightarrow \pi^0 \gamma) = \frac{e^2 (m_{J/\psi}^2 - m_{\pi^0}^2)^3}{96\pi m_{J/\psi}^3} |f_{J/\psi\pi^0}(0)|^2,$$

- $\phi_{\omega\pi^0}(0)$ only free parameter
- No data, $\phi_{\omega\pi^0}(0) = 0$



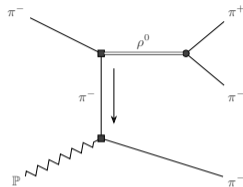
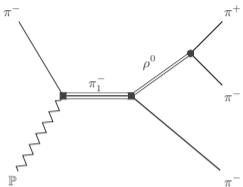
$J/\psi \rightarrow \pi^0 \gamma^*$ transition form factor

- Recent measurement by BESIII [Phys.Rev.D 112 (2025) 1, L011101]

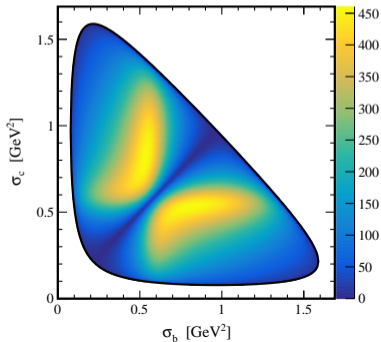
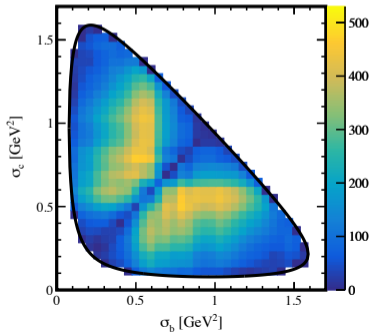


Exotic $\pi_1(1600) \rightarrow 3\pi$ KT analysis (preliminary)

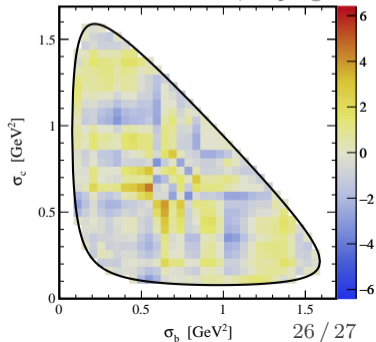
- Exotic with $J^{PC} = 1^{-+}$



COMPASS, 2108.01744



JPAC, in progress



Outlook

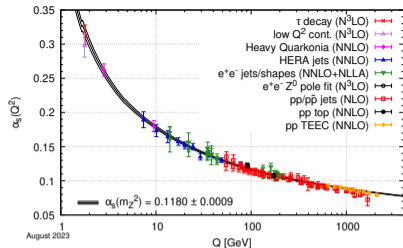
- **Khuri-Treiman equations:**
 - Dispersive representation for **3-particle Final-State Interactions**
 - Based on fundamental principles: **analyticity, unitarity and crossing symmetry**
 - Input: $\pi\pi$ scattering **phase shifts**
 - Resonance shape affected by **left-hand cuts / 3-body effects**
 - **Predictive power** (subtraction constants)
 - Experimental **data** well described

QCD: open questions

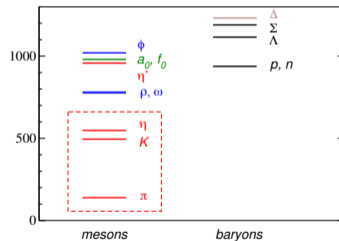
- **Asymptotic freedom** at high energies (“weak QCD”)
 - **Confinement** at low energies (“strong QCD”), no quarks+gluons, only hadrons
 - **Hadron spectrum and interactions:** why these states? scattering, decays of hadrons?
- (perturbative) QCD does not answer any of these questions!

Remedies:

- Effective Field Theories: symmetries, separation of scales
- Dispersion Theory: unitarity, analyticity, crossing symmetry
- Lattice QCD: approach to solving a discretized version of QCD on a computer

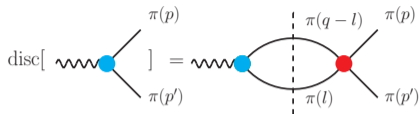


Mass [MeV]



Warm-up: the pion vector form factor

- **Unitarity:**



$$\text{disc}F_\pi(s) = 2i\text{Im}F_\pi(s) = 2i\sigma_\pi(s)F_\pi(s)t_1^{1*}(s) = 2iF_\pi(s)\sin\delta_1^1(s)e^{-i\delta_1^1(s)},$$

- Analytic solution, **Omnès** equation [Omnès, Nuovo Cimento 8, 316 (1958)]

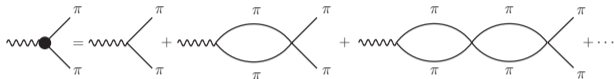
$$F_\pi(s) = \frac{1}{2\pi i} \int_{4m_\pi^2}^{\infty} \frac{\text{disc}F_\pi(s')}{s' - s} ds',$$

$$F_\pi(s) = P(s)\Omega_1^1(s), \quad \Omega_1^1(s) = \frac{f(s)}{f(0)} = \exp \left\{ \frac{s}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{s'(s' - s)} \right\},$$

- Polynomial $P(s)$, not fixed by unitarity: matched to EFT or by data

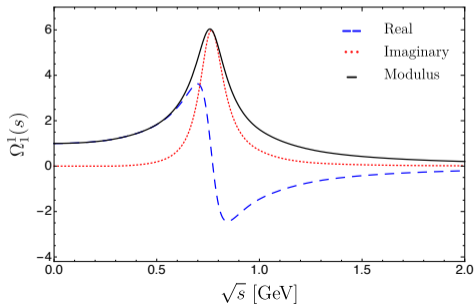
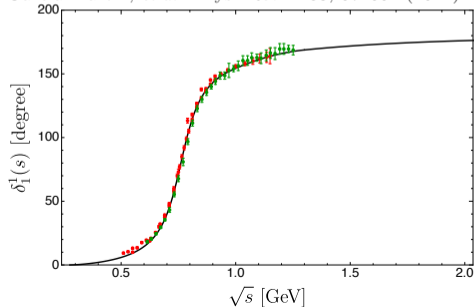
Omnès equation

- Diagrammatic interpretation



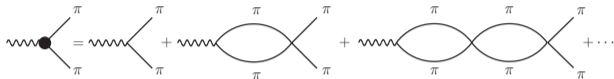
- Solution: depends solely on the P -wave phase shift of $\pi\pi$

Garcia-Martin, *et.al.* Phys. Rev. D83, 074004 (2011)



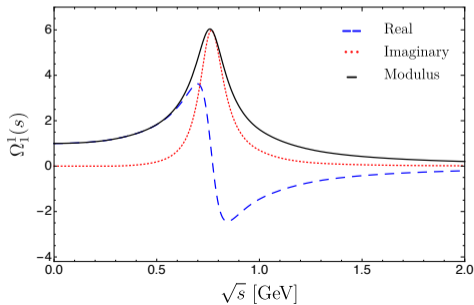
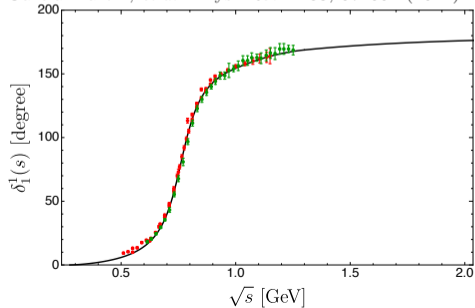
Omnès equation

- Diagrammatic interpretation



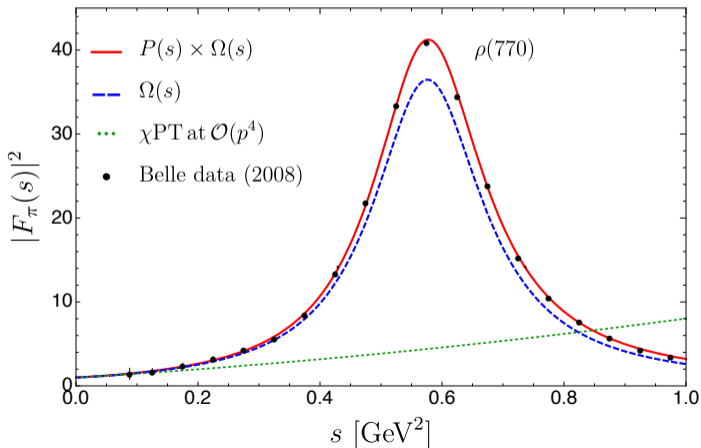
- Solution: depends solely on the P -wave phase shift of $\pi\pi$

Garcia-Martin, *et.al.* Phys. Rev. D83, 074004 (2011)



The pion vector form factor

- $P(s) = 1 + \alpha s$, with $\alpha = 0.11 \text{ GeV}^{-2}$ due to inelasticities

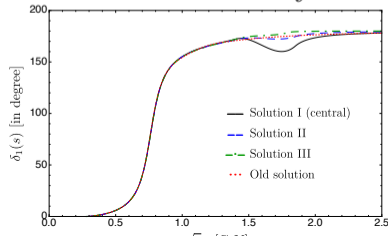
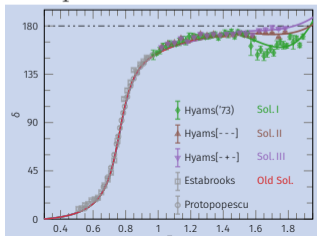


KT equations: DR, solutions, subtractions

- Unsubtracted dispersion relation:

$$F_1(s) = \frac{1}{2\pi i} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{disc } F_1(s')}{s' - s}, \quad F_1(s) = \Omega_1(s) \left(a + \frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\sin \delta_1(s') \hat{F}_1(s')}{|\Omega_1(s')| (s' - s)} \right),$$

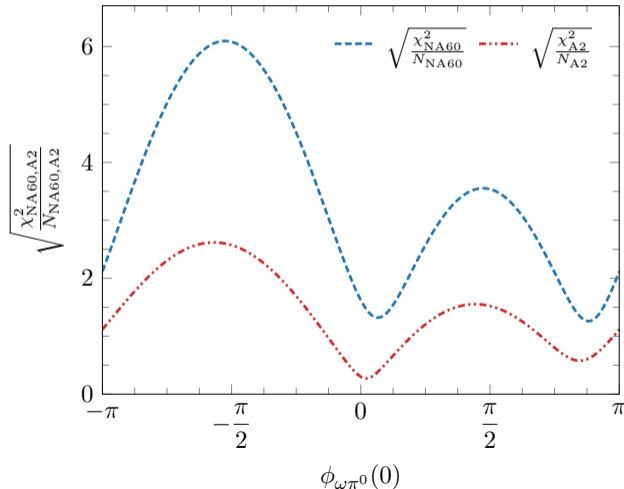
- $\delta_1(s)$: $\pi\pi$ phase taken as input:
 - Old solution [Garcia-Martin, *et.al.* Phys. Rev. D83, 074004 (2011)]
 - New solutions [Pelaez, Rodas, Ruiz De Elvira, Eur. Phys. J. C79, 1008 (2019)]
 - **Central** analysis performed with solution I for $\delta_1(s)$
 - The spread in the other solutions: **theoretical uncertainty**



On the $\phi_{\omega\pi^0}$

- Two solution for $\phi_{\omega\pi^0}$

JPAC Collaboration, Eur.Phys.J. C80 (2020) no.12, 1107



KT 1 sub vs Omnès

- Accidental cancellation of $|F(s, t, u)|^2$

JPAC Collaboration, Eur.Phys.J. C80 (2020) no.12, 1107

