The $B^{(*)}\bar{K}^{(*)}$ coupled-channel system in the hidden-gauge approach

J. Sánchez-Illana, R. Molina and Pan-Pan Shi

Jesus.Sanchez@ific.uv.es

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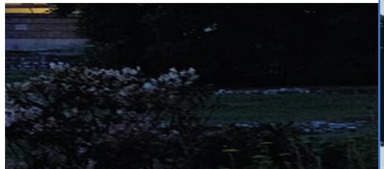

Content

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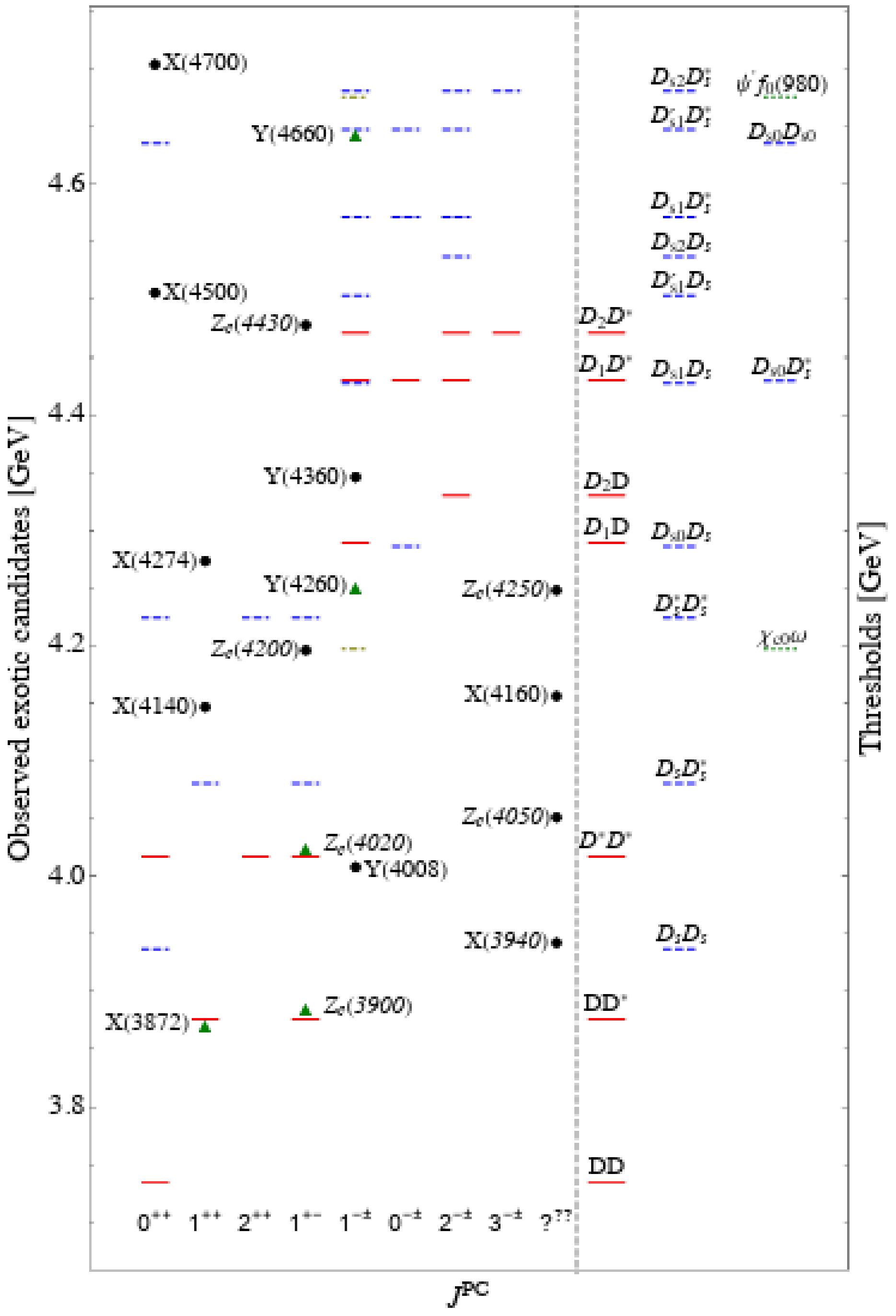
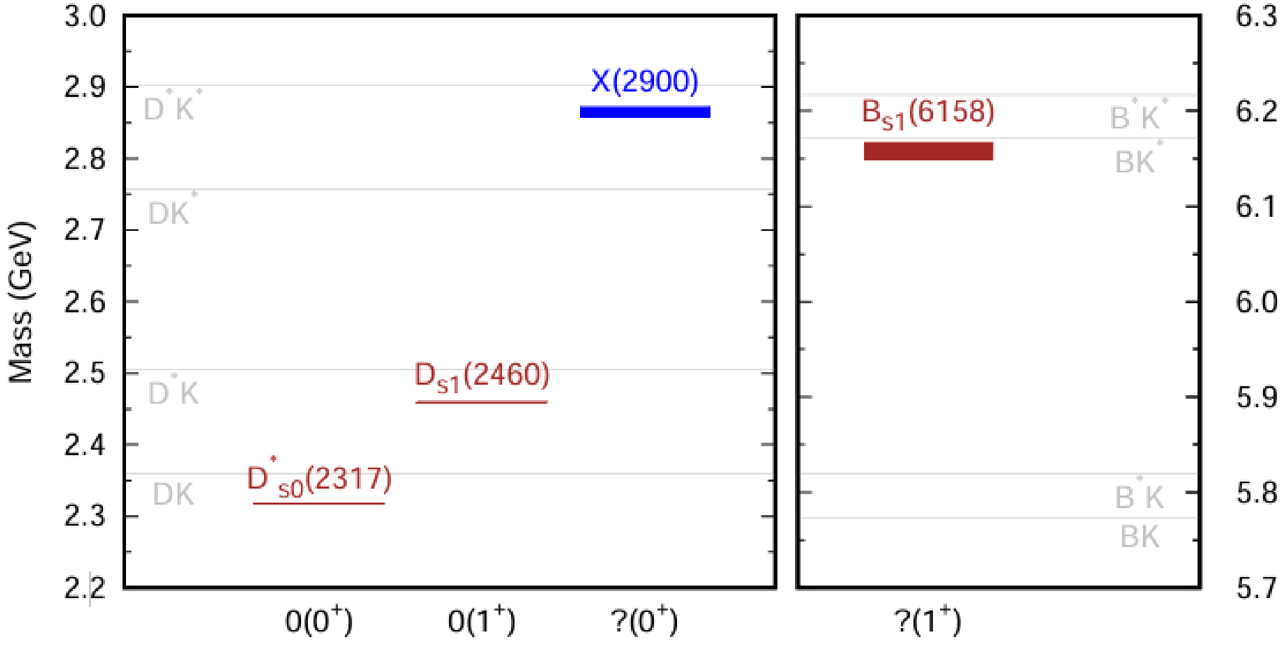
- Formalism
- Results
- Conclusions

Clues in the charm sector



F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, Rev. Mod. Phys. 90, 015004 (2018).

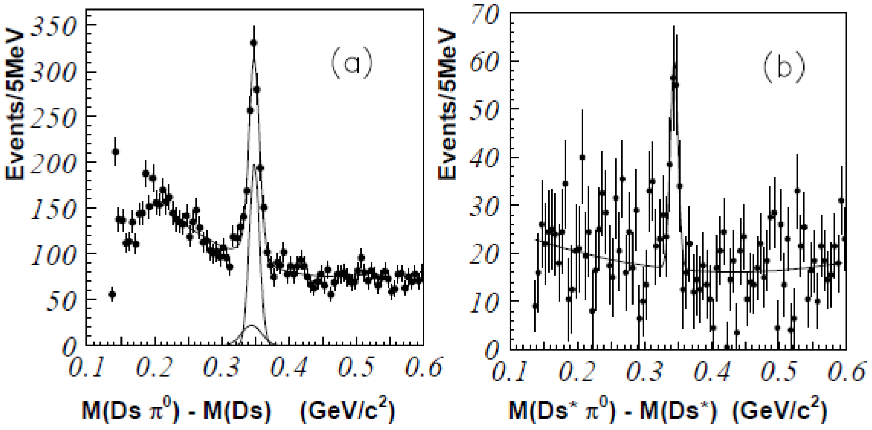
Lots of data



S. Y. Kong, J. T. Zhu, D. Song, and J. He, Phys. Rev. D 104, 094012 (2021).

Little data

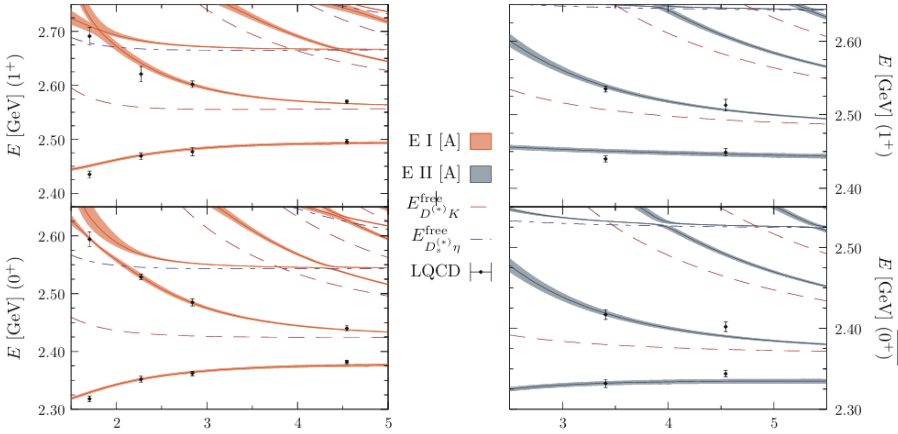
Clues in the charm sector



Y. Mikami, et al., Phys. Rev. Lett. 92 (2004) 012002.

$D_{s0}(2317), D_{s1}(2460)$

- $c\bar{s}$ state
- **Molecular DK, D*K state**
- Compact tetraquark
- Mixed state

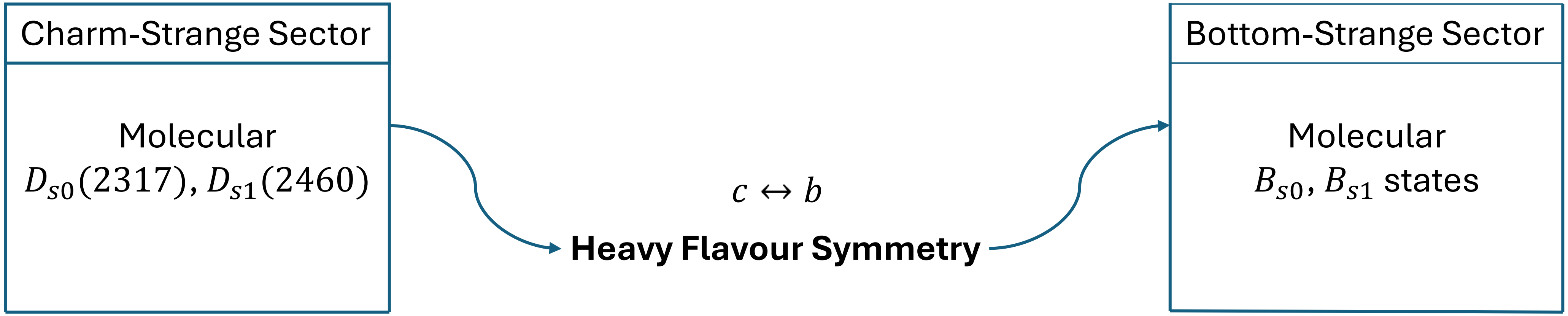


M. Albaladejo, P. Fernandez-Soler, J. Nieves, and P. G. Ortega, Eur. Phys. J. C 78, 722 (2018).

A. Martínez Torres, E. Oset, S. Prelovsek, and A. Ramos, Reanalysis of lattice QCD spectra leading to the D∗ s0(2317) and D∗ s1(2460), JHEP 05, 153, arXiv:1412.1706 [hep-lat].

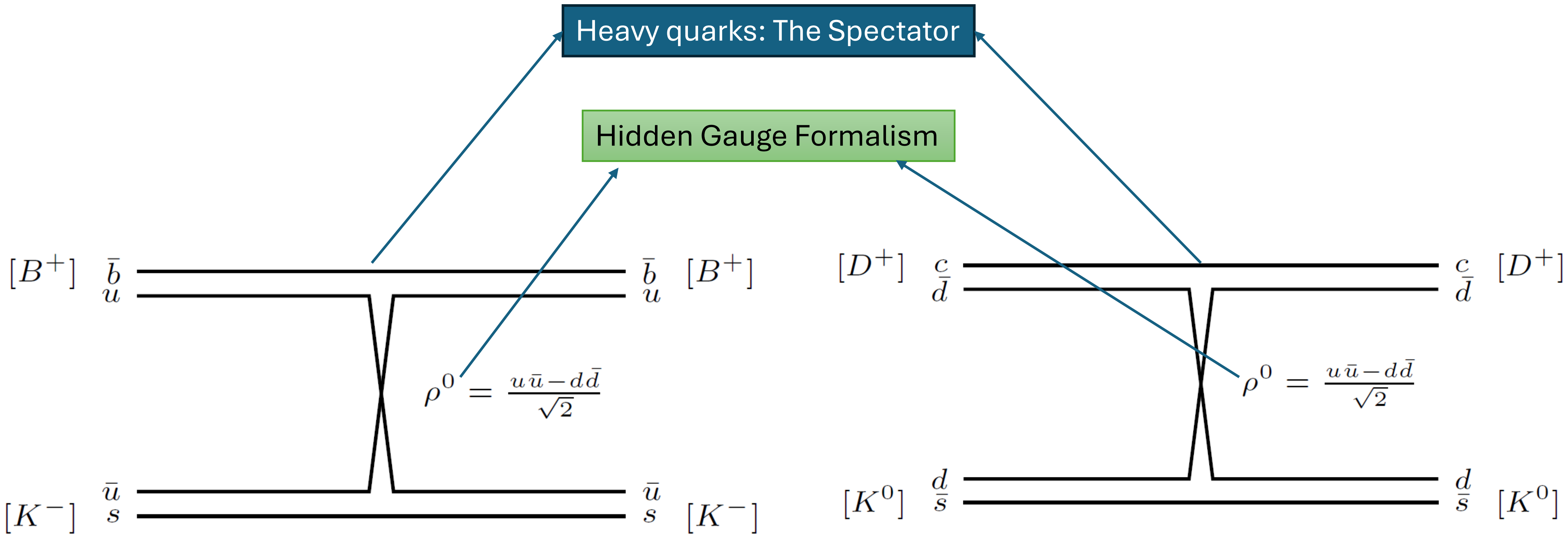
- $D_{s0}(2317)\sim72\% \pm 18\%$ molecular component.
- $D_{s1}(2460)\sim57\% \pm 27\%$ molecular component.

Clues in the charm sector



WHAT EFT’s HAVE TO SAY ABOUT THIS?

Formalism


Heavy quarks: The Spectator
Hidden Gauge Formalism
$[B^+]$ $\bar{b}$ u
$\bar{b}$ u $[B^+]$
$\bar{u}$ s $[K^-]$
$[K^-]$ $\bar{u}$ s
$\rho^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$
$[D^+]$ c $\bar{d}$
c $\bar{d}$ $[D^+]$
$[K^0]$ d $\bar{s}$
d $\bar{s}$ $[K^0]$
$\rho^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$

Formalism

- We have

$$\mathcal{L}_{VVV} = ig\langle (V^\nu \partial_\mu V_\nu - \partial_\mu V^\nu V_\nu) V^\mu \rangle,$$

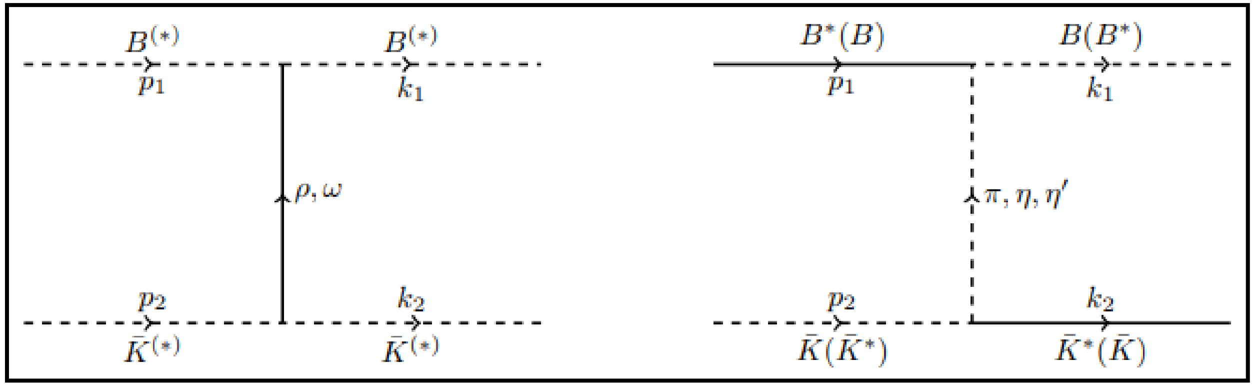
$$\mathcal{L}_{VPP} = -ig\langle [P, \partial_\mu P] V^\mu \rangle.$$

where

$$g = \frac{M_V}{2f}.$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ & B^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 & B^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' & B_s^0 \\ B^- & \bar{B}^0 & \bar{B}_s^0 & \eta_b \end{pmatrix}$$

$$V_\mu = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} & B^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} & B^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi & B_s^{*0} \\ B^{*-} & \bar{B}^{*0} & \bar{B}_s^{*0} & \Upsilon \end{pmatrix}_\mu$$



Formalism

TABLE I. Potential of the $B^{(*)}\bar{K}^{(*)}$ channels and $B^*\bar{K} - B\bar{K}^*$ coupled channel.

Channel (J^P)	Potential
$B\bar{K}$ (0^+)	$\frac{g^2}{2}\left(3D_{su}(m_\rho) + D_{su}(m_\omega)\right)$
$B^*\bar{K} - B\bar{K}^*$ (1^+)	$V_{11} = -\frac{g^2}{2}\left(3D_{su}(m_\rho) + D_{su}(m_\omega)\right) \varepsilon(p_1)\cdot\varepsilon^*(k_1)$ $V_{12} = \frac{2g^2}{3}\left(-4D_t(m_\eta) + D_t(m_{\eta'}) - 9D_\pi\right) k_1\cdot\varepsilon(p_1)\ p_2\cdot\varepsilon^*(k_2)$ $V_{21} = \frac{2g^2}{3}\left(-4D_t(m_\eta) + D_t(m_{\eta'}) - 9D_\pi\right) k_2\cdot\varepsilon(p_2)\ p_1\cdot\varepsilon^*(k_1)$ $V_{22} = -\frac{g^2}{2}\left(3D_{su}(m_\rho) + D_{su}(m_\omega)\right) \varepsilon(p_2)\cdot\varepsilon^*(k_2)$
$B^*\bar{K}^*$ $(0^+/1^+/2^+)$	$\frac{g^2}{2}\left(3D_{su}(m_\rho) + D_{su}(m_\omega)\right) \varepsilon(p_1)\cdot\varepsilon^*(k_1)\ \varepsilon(p_2)\cdot\varepsilon^*(k_2)$

Formalism

Lippmann-Schwinger equation:

$$T_{ij}(E,p,k) = V_{ij}(E,p,k) + \sum_k \int_\Lambda \frac{d^3\vec{q}}{(2\pi)^3} V_{ik}(E,p,q) I_k(E,q) T_{kj}(E,q,k),$$

where

$$I_k(E,q) = \frac{\omega_1(q)+\omega_2(q)}{2\omega_1\omega_2} \cdot \frac{1}{E^2-\left(\omega_1(q)+\omega_2(q)\right)^2+i\epsilon}, \quad I(E,q) = \frac{1}{4\omega_{\bar{K}^*}\omega_{B^{(*)}}} \cdot \frac{1}{P^0-\omega_{\bar{K}^*}-\omega_{B^{(*)}}+i\frac{\Gamma}{2}}.$$

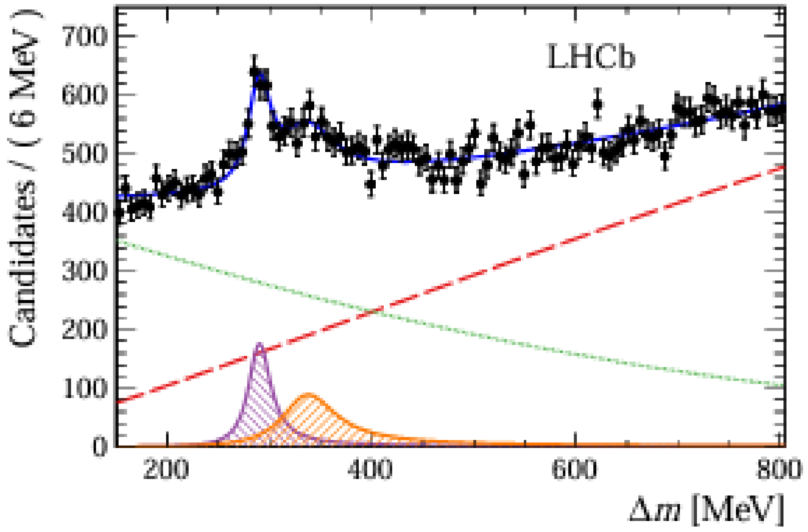
Three-body cut

$$\boldsymbol{T} = \frac{\boldsymbol{V}}{I - \boldsymbol{VG}}$$

$$\det(I - \boldsymbol{VG}) = 0$$

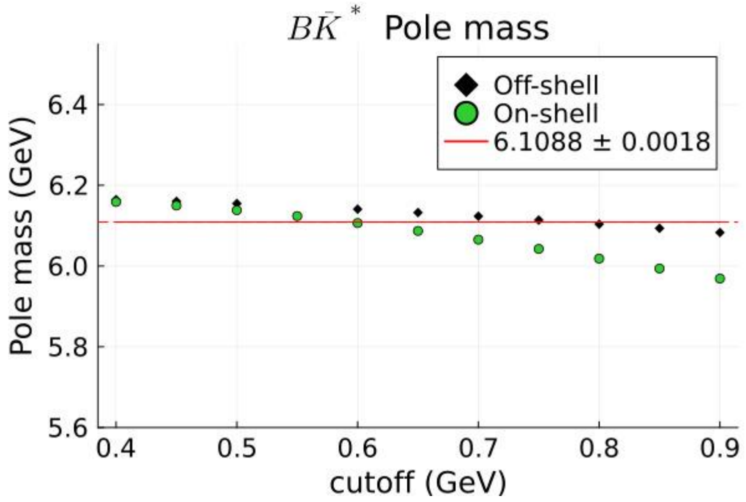
- Bound state.
- Virtual state.
- Resonance.

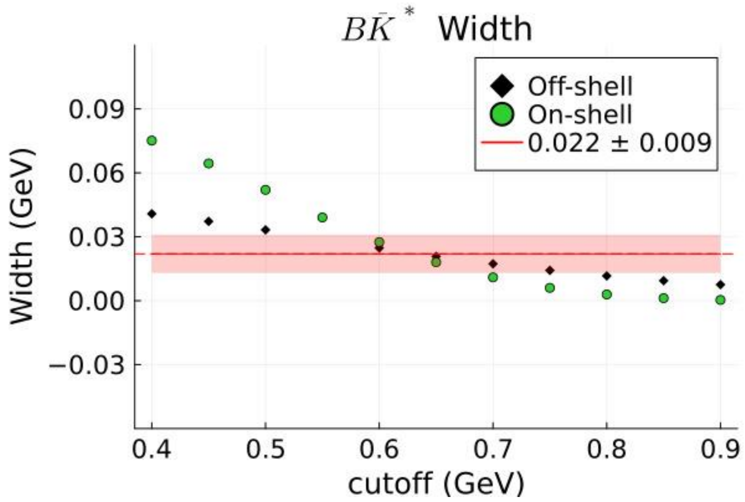
Results



R. Aaij et al. (LHCb),Eur. Phys. J. C 81, 601 (2021), arXiv:2010.15931 [hep-ex]

If decays in $B^{*+}K^-$,
$m_1 = (6108.8 \pm 1.8)$ MeV,
$\Gamma_1 = (22 \pm 9)$ MeV,
$m_2 = (6158 \pm 9)$ MeV,
$\Gamma_2 = (72 \pm 43)$ MeV.





OFF-SHELL $\xrightarrow{\chi^2/_{d.o.f} = 1.15}$ $\Lambda = 775$ MeV

ON-SHELL $\xrightarrow{\chi^2/_{d.o.f} = 0.72}$ $\Lambda = 593$ MeV

Results

$m_1 = (6108.8 \pm 1.8)$ MeV,
$\Gamma_1 = (22 \pm 9)$ MeV,
$m_2 = (6158 \pm 9)$ MeV,
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TABLE II. Predicted pole positions of the $B^{(*)}\bar{K}^{(*)}$ systems with the on-shell potentials. In the fourth column, "+" and "−" indicate that the poles lie on the physical and unphysical RS relative to the corresponding two-body threshold, respectively. The quantity g_i in the fifth column denotes the S-wave effective coupling. The sixth column, labeled "Thr.", represents the relevant threshold. The binding energy E_b is defined as the difference between the real part of the pole position and the relevant threshold.

J^P	Channel	Pole [MeV]	RS	g_i [GeV]	Thr. [MeV]	E_b [MeV]
0^+	$B\bar{K}$	5758.9	$(+)$	23.0	5775.0	−16.1
1^+	$B^*\bar{K}$	5804.0	$(+)$	23.0	5820.0	−16.0
1^+	$B\bar{K}^*$	$6109.0 - 7.2i$	$(+)$	$39.5 - 2.5i$	6173.0	−64.0
$0^+/1^+/2^+$	$B^*\bar{K}^*$	$6154.1 - 7.2i$	$(+)$	$38.8 - 2.8i$	6218.0	−63.9

TABLE III. Predicted pole positions of the $B^{(*)}\bar{K}^{(*)}$ systems with the off-shell potentials.

J^P	Channel	Pole [MeV]	RS	g_i [GeV]	Thr. [MeV]	E_b [MeV]
0^+	$B\bar{K}$	5757.2	$(+)$	24.8	5775.0	−17.8
1^+	$B^*\bar{K}$–$B\bar{K}^*$	5802.4	$(+,+)$	25.0 −1.7	5820.0	−17.6
		$6108.8 - 6.4i$	$(-,+)$	$0.2 + 0.5i$ $-28.7 + 1.6i$	6173.0	−64.2
$0^+/1^+/2^+$	$B^*\bar{K}^*$	$6155.8 - 6.4i$	$(+)$	$45.1 - 2.2i$	6218.0	−62.2

Results

Three-body effect

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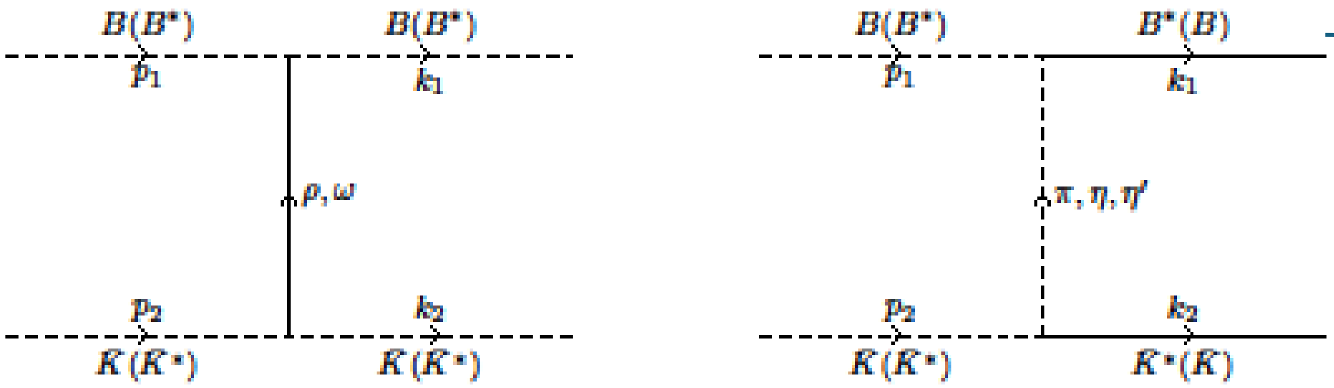
Results



$$V_{11} = \frac{g^2}{2}\left(3D_{su}(m_\rho) + D_{su}(m_\omega)\right),$$

$$V_{12} = 4g^2\left(-\frac{2}{3}D_t(m_\eta) + \frac{1}{6}D_t(m_{\eta'}) - \frac{3}{2}D_\pi\right)p_1\varepsilon(k_1)p_2\varepsilon^*(k_2),$$

$$V_{21} = 4g^2\left(-\frac{2}{3}D_t(m_\eta) + \frac{1}{6}D_t(m_{\eta'}) - \frac{3}{2}D_\pi\right)k_1\varepsilon(p_1)k_2\varepsilon^*(p_2),$$

$$V_{22} = \frac{g^2}{2}\left(3D_{su}(m_\rho) + D_{su}(m_\omega)\right)\varepsilon(p_1)\cdot\varepsilon^*(k_1)\varepsilon(p_2)\cdot\varepsilon^*(k_2).$$

F. Gil-Domínguez, A. Giachino y R. Molina, "Quark mass dependence of the $T_{cc}(3875)^+$", Physical Review D 111, 016029, 2025.

$g_{B^*BP} \approx 6.3$

For $J = 0$:

$$V = \begin{pmatrix} V_{11}^{SS} & 0 & V_{12}^{SS} & V_{12}^{SD} \\ 0 & 0 & 0 & 0 \\ V_{21}^{SS} & 0 & V_{22}^{SS} & V_{22}^{SD} \\ V_{21}^{DS} & 0 & V_{22}^{DS} & V_{22}^{DD} \end{pmatrix}$$

For $J = 1$:

$$V = \begin{pmatrix} V_{22}^{SS} & V_{22}^{SD} \\ V_{22}^{DS} & V_{22}^{DD} \end{pmatrix}$$

For $J = 2$:

$$V = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & V_{11}^{DD} & V_{12}^{DS} & V_{12}^{DD} \\ 0 & V_{21}^{SD} & V_{22}^{SS} & V_{22}^{SD} \\ 0 & V_{21}^{DD} & V_{22}^{DS} & V_{22}^{DD} \end{pmatrix}$$

Conclusions

Using HGF, taking into account the three-body effect when $\bar{K}^*$ is envolved, we have solved the LS equation in the on-shell and off-shell approach.

The cutoff is determined by reproducing the pole position of $B_{sJ}(6063)^0$, interpreting it as a $B\bar{K}^*$ molecule.

Our results support the existence of six $B^{(*)}\bar{K}^{(*)}$ molecules:

1. Two poles near $B\bar{K}$ and $B^*\bar{K}$ thresholds.
2. A pole close to the $B\bar{K}^*$. Possibly corresponds to $B_{sJ}(6063)^0$.
3. Three poles in the $B^*\bar{K}^*$ system with $J^P = 0^+, 1^+, 2^+$. The $J^P = 1^+$ molecule may be related to the $B_{sJ}(6114)^0$ state.

THANK YOU FOR YOUR ATTENTION

SS wave expansión for $B^*\bar{K}^*$

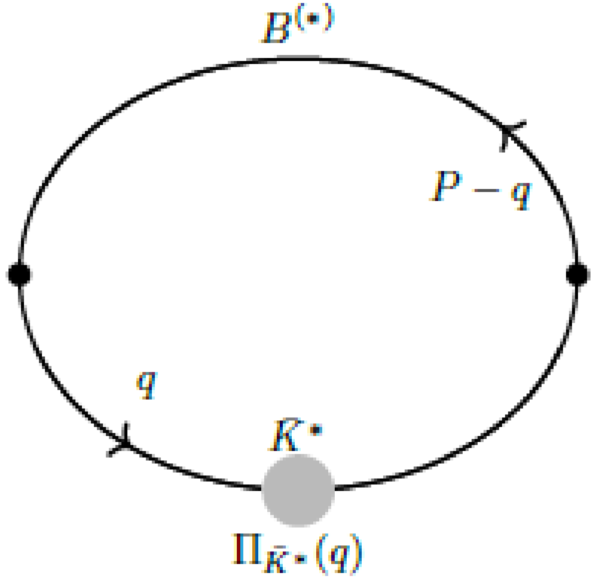
$$V_0(p,k) = \int_{-1}^{1} dz\ D(s,t,u)\left[\frac{1}{2} - \frac{pkz}{6m_{p_1}m_{k_1}} - \frac{pkz}{6m_{p_2}m_{k_2}} + \frac{p^2k^2}{6m_{p_1}m_{k_1}m_{p_2}m_{k_2}}\right],$$

$$V_1(p,k) = \int_{-1}^{1} dz\ D(s,t,u)\left[\frac{1}{2} - \frac{pkz}{6m_{p_1}m_{k_1}} - \frac{pkz}{6m_{p_2}m_{k_2}}\right],$$

$$V_2(p,k) = \int_{-1}^{1} dz\ D(s,t,u)\left[\frac{1}{2} - \frac{pkz}{6m_{p_1}m_{k_1}} - \frac{pkz}{6m_{p_2}m_{k_2}} + \frac{p^2k^2}{30m_{p_1}m_{k_1}m_{p_2}m_{k_2}}(3z^2-1)\right],$$

Loop

$$G(P^0) \approx \int \frac{d^4p}{(2\pi)^4} \frac{1}{4\omega_{\bar{K}^*}\omega_{B^{(*)}}} \cdot \frac{1}{q^0 - \omega_{\bar{K}^*} + i\frac{\Gamma}{2}} \cdot \frac{1}{P^0 - q^0 - \omega_{B^{(*)}} + i\varepsilon}.$$



We integrate the q^0 component,

$$G(P^0) = \int \frac{dq}{8\pi^2\omega_{\bar{K}^*}\omega_{B^{(*)}}} \cdot \frac{\vec{q}^2}{P^0 - \omega_{\bar{K}^*} - \omega_{B^{(*)}} + i\frac{\Gamma}{2}}.$$

Riemann Sheets

The on-Shell momentum p_k^{on} is determined by the zeros of the denominator of the Green's function.

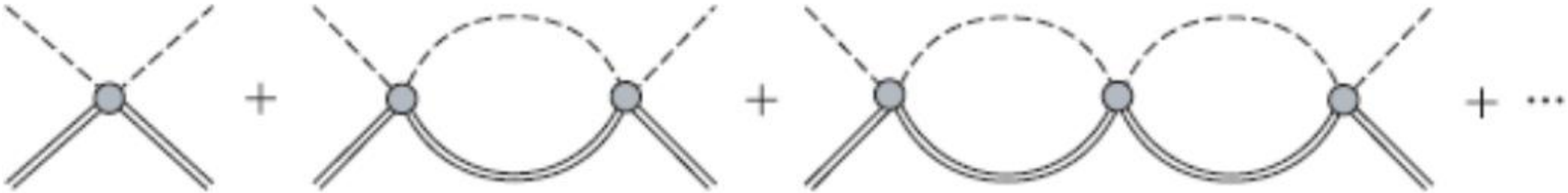
$$G_k^{on,-}(E) = G_k^{on,+}(E) + \frac{ip_k^{on}}{4\pi E},$$
$$G_k^{on,-}(E) = \int \frac{d^3q}{(2\pi)^3} I_k(E,q).$$

Considering the second RS of the $B^{(*)}K\pi$ three-body cut, the momentum-dependent partial width can be replaced by

$$\Gamma_i(M) \to -\Gamma_i(M) \text{ for } \Im(M) < 0.$$

Lippmann-Schwinger unitarization

$$T = V + V\,G\,V + V\,G^2\,V + \ldots = V + V\,G\,T$$



Nagahiro H. et al., "Hidden gauge formalism for the radiative decays of axial-vector mesons", Phys. Rev. D 79, 014015(2009); arXiv:0809.0943[hep-ph].