

# Investigation of the $D\pi$ spectrum from a neural-network tuned heavy-quark action



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

18th International Workshop on Meson Physics, Kraków



---

Why study the  $D\pi$  spectrum?

Methodology

The  $D\pi$  spectrum

Current progress and outlook

## Why study the $D\pi$ spectrum?:

### The $D_0^*$ and its strange partner $D_{s0}^*(2317)$



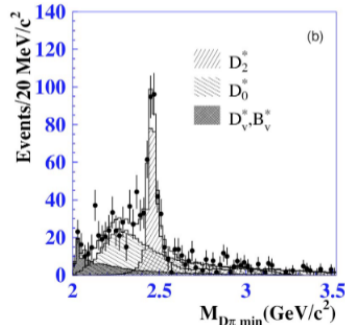
[Takahashi et al. (PDF), Int. J. Mod. Phys. A 41, 2630011 (2026)]

- Discovery of the broad  $D_0^*(2300)$  resonance in  $D\pi$  at BELLE and FOCUS and the narrow  $D_{s0}^*(2317)$  in  $DK$  at BABAR [BELLE

Collaboration PRD 69 112002 (2004)], [FOCUS Collaboration PLD (2004) 02 017], [BABAR Collaboration PR 90 242001 (2003)]

- Early-on: Phenomenology explained the low mass  $D_{s0}^*(2317)$  by the strong coupling to the  $DK$  threshold and predicted a broad  $D_0^*$  at a lighter mass [v. Beveren, Rupp, PRL 91 012003]

- $D$ -meson Goldstone boson scattering is studied using Femtoscopy at ALICE [ALICE Collaboration PRD 110032004 (2024)]



Taken from

[BELLE Collaboration PRD 69 112002 (2004)]

## Why study the $D\pi$ spectrum?:

### The $D_0^*$ and its strange partner $D_{s0}^*(2317)$



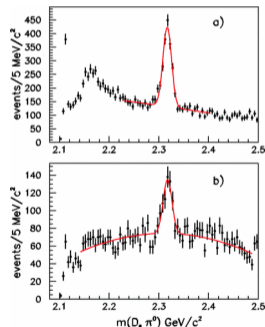
- Discovery of the broad  $D_0^*(2300)$  resonance in  $D\pi$  at BELLE and FOCUS and the narrow  $D_{s0}^*(2317)$  in  $DK$  at BABAR [BELLE

Collaboration PRD 69 112002 (2004)], [FOCUS Collaboration PLD (2004) 02 017], [BABAR Collaboration PR 90 242001 (2003)]

- Early-on: Phenomenology explained the low mass  $D_{s0}^*(2317)$  by the strong coupling to the  $DK$  threshold and predicted a broad  $D_0^*$  at a lighter mass [v. Beveren, Rupp, PRL 91 012003]

- $D$ -meson Goldstone boson scattering is studied using Femtoscopy at ALICE [ALICE Collaboration PRD 110032004 (2024)]

[Takahashi et al. (PDF), Int. J. Mod. Phys. A 41, 2630011 (2026)]



Taken from

[BABAR Collaboration PR 90 242001 (2003)]



### ■ ChPT

e.g. [Kolomeitsev, Lutz, *J. Phys. Lett. B* (2003) 10 118], [Guo et al. *PRD* 98 014510 (2018)], [Du et al., *PRL* 126 192001 (2021)] etc.

- Unitarized ChPT expects additional exotic states
- Talk right after by Zejian Zhuang

### ■ Lattice QCD:

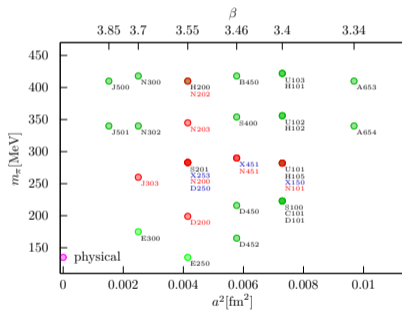
- First exploration using  $\bar{\psi}\Gamma\psi$ -type and  $D\pi$ -type operators [Mohler et al. *PRD* 87 034501 (2013)]
- Hadron Spectrum Collaboration: Several different  $K$  matrix parametrizations and coupled channel analysis  
→ States show normal mass ordering  $m_{D_0^*} < m_{D_{s0}^*}$  [Gayer et al. *JHEP* 07 (2021) 123]
- Study of a pole in the  $S$ -wave sextet amplitude asserted by unitrized ChPT [Gregory et al. *POS Lattice2021* 1.396.0211 (2022)]
- Pion mass dependence investigated [Yan et al. *PRD* 111 014503 (2024)]
- HadSpeC again: More extensive study of the sextet pole [Lang et al. *JHEP* 07 (2025) 060]

## Methodology:

### Coordinated Lattice Simulations (CLS) gauge field ensembles



- 2+1 flavor ensembles with improved Wilson action
- Two sets of ensembles
  - ▣  $Tr(M) = \text{const}$
  - ▣  $m_S = \text{const}$
- Study discretization effects around  $m_\pi \approx 280$  MeV
- Study of  $m_\pi$  dependence for  $\beta = 3.55$
- This talk: Data on X451



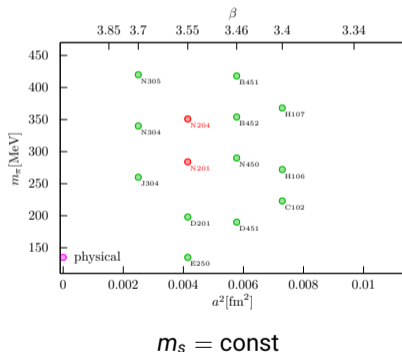
$Tr(M) = \text{const}$

## Methodology:

### Coordinated Lattice Simulations (CLS) gauge field ensembles



- 2+1 flavor ensembles with improved Wilson action
- Two sets of ensembles
  - ▣  $Tr(M) = \text{const}$
  - ▣  $m_s = \text{const}$
- Study discretization effects around  $m_\pi \approx 280$  MeV
- Study of  $m_\pi$  dependence for  $\beta = 3.55$
- This talk: Data on X451





[Aoki et al. *Prog.Theor.Phys.* 109 383 (2003), [El-Khadra et al. *PRD* 55 3933 (1997)]

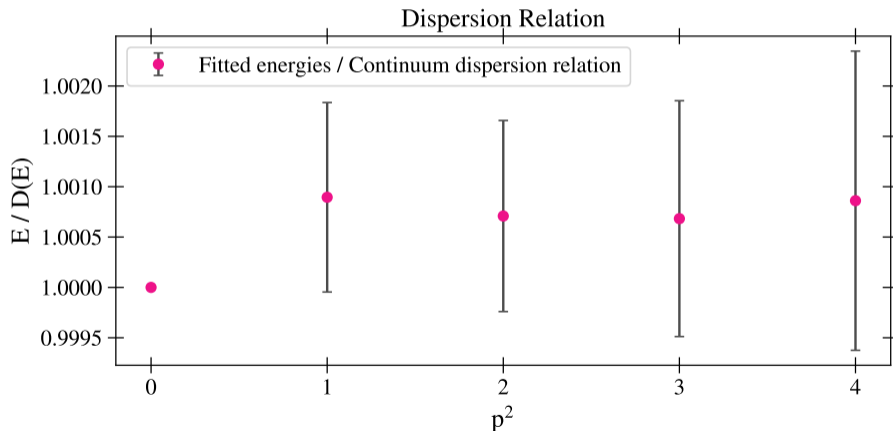
- 5 parameter heavy-quark action
- Good choice of these parameters minimizes discretization effects.
- Several tuning procedures for these parameters
- The employed fixing procedure: Neural network tuning using experimental charmonium ground states and spin-averaged dispersion relation

$$\begin{aligned}
 D_{xy} = & \delta_{xy} - \kappa_C \left[ \sum_i (r_s - \nu \gamma_i) U_i(\mathbf{x}) \delta_{\mathbf{x}+i, \mathbf{y}} + (r_s + \nu \gamma_i) U_i^\dagger(\mathbf{x}) \delta_{\mathbf{x}, \mathbf{y}+i} \right] \\
 & - \kappa_C \left[ (r_t - \gamma_t) U_t(\mathbf{x}) \delta_{\mathbf{x}+t, \mathbf{y}} + (r_t + \gamma_t) U_t^\dagger(\mathbf{x}) \delta_{\mathbf{x}, \mathbf{y}+t} \right] \\
 & - \kappa_C \left[ c_B \sum_{i,j} \sigma_{ij} F_{ij}(\mathbf{x}) + c_E \sum_i \sigma_{it} F_{it}(\mathbf{x}) \right] \delta_{xy} .
 \end{aligned}$$

[Hudspith, Mohler, *PRD* 106 034508 (2022)]

Parameter	$\kappa_C$	$r_s$	$\nu$	$c_E$	$c_B$
Value	0.118 10(4)	1.894(4)	2.052(5)	1.1043(12)	1.0926(12)

# Dispersion relation of the $D$ meson





e.g. [Gattringer, Lang, Book "Quantum chromodynamics on the lattice" (2009)]

$$\langle \bar{O}_2(t_f) O_1(t_i) \rangle = \frac{1}{Z} \int \mathcal{D}[\Psi, \bar{\Psi}] \mathcal{D}[U] e^{-S_F[\Psi, \bar{\Psi}, U] - S_G[U]} \bar{O}_2[\Psi, \bar{\Psi}, U] O_1[\Psi, \bar{\Psi}, U]$$

- Excited-state contamination at early times and fast growing statistical noise at large time separation  
→ limited temporal fit range
- An optimal operator basis will include single-hadron as well as multi-hadron operators  
→ Well-chosen operator basis has maximal overlap with states of interest  
→ Enhances signal quality and suppresses higher state contributions



[Peardon et al. PRD 80 054506 (2009)], [Morningstar et al. PRD 83 114505]

- Distillation provides a separable smearing procedure for efficient calculations of correlation functions
  - Stochastic distillation makes propagator calculations in large volumes feasible
  - Better volume/#eigenvalue-scaling for tensor contractions
- **Step 1:** Computation of the perambulators, which encode the quark propagation
- **Step 2:** Computation of the hadron source and sink functions
- **Step 3:** The correlator is obtained by contracting the hadron functions according to the Wick theorem
- E.g. the resulting single meson correlator takes the following form

$$C_M^{(2)}(t_0, t) = \text{Tr} \left[ \Phi^B(t_0) \tau(t_0, t) \Phi(t) \tau(t, t_0) \right] + \text{Tr} \left[ \Phi^A(t) \tau(t, t) \right] \text{Tr} \left[ \Phi^B(t) \tau(t_0, t_0) \right]$$



e.g. [Michael, Teasdale, *Nuc. Phys. B* 0550-3213(83)90674-0], [ALPHA collaboration et al. *JHEP*04(2009)094]

- How do we extract the spectrum from the correlator matrix? → Variational method
- It is employed through the Generalized Eigenvalue Problem (GEVP)

$$C(t)\vec{v}_n(t, t_0) = \lambda_n(t, t_0)C(t_0)\vec{v}_n(t, t_0)$$

- The eigenvectors  $\vec{v}_n(t, t_0)$  define combinations of the SH and MH operators
- At leading order  $\lambda_n$  is expressed in terms of the energy of the  $n$ -th state

$$\lambda_n(t, t_0) = A_n e^{-E_n(t-t_0)} [1 + \mathcal{O}(e^{-\Delta E_n t})]$$

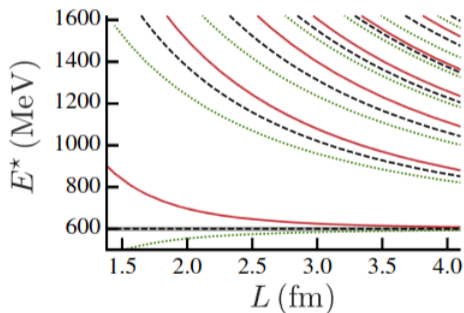
- On lattice: Rotational symmetry is broken → operators belong to a set of irreducible representations (irreps) of octahedral group  $O_h$



- **An observation:** Multi-particle energies, in a finite volume, are shifted with regard to the free energy levels  $\leftrightarrow$  Interaction of the particles
- Scattering amplitude(s) encoded in these energy shifts
- The Lüscher method rigorously relates the finite-energy levels to the infinite-volume scattering amplitudes in 2 particle scattering

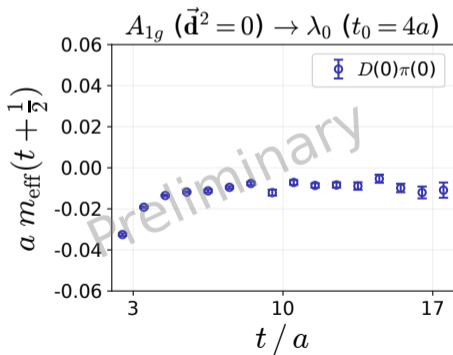
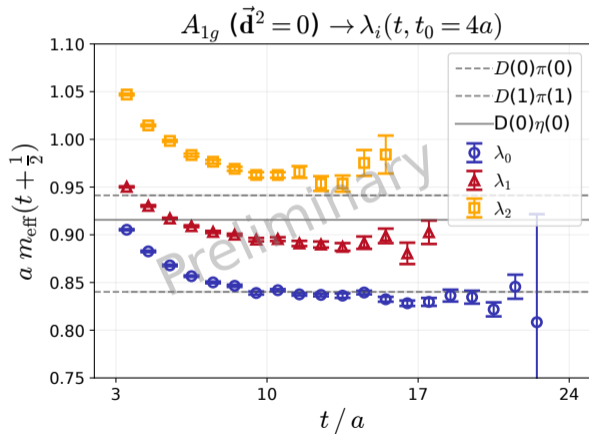
$$\det \left[ F^{-1}(E_{cm}, \vec{p}; L) + \mathcal{M}(E_{cm}) \right] = 0$$

[Lüscher *Commun. Math. Phys.* 105 (1986) 153], [Nucl. Phys. B 354 (1991) 531],  
Nucl. Phys. B 364 (1991) 237

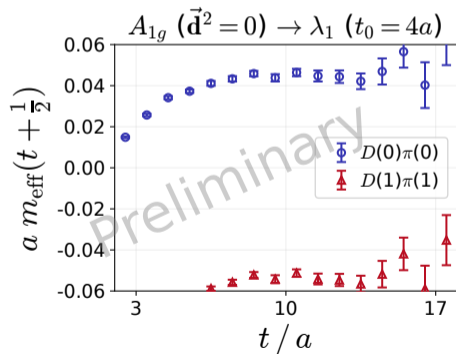
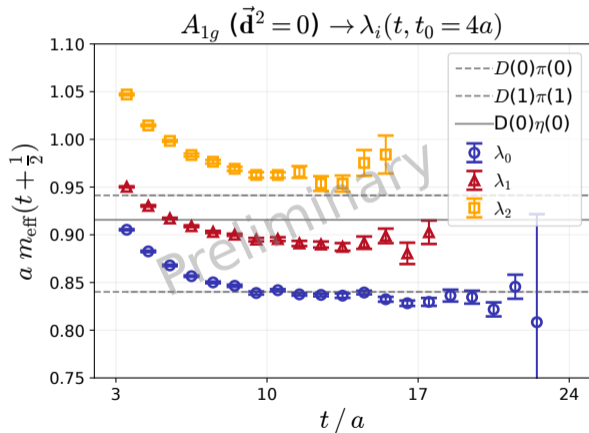


Taken from review  
[Briceño et al. *RMP* 90 025001 (2018)]

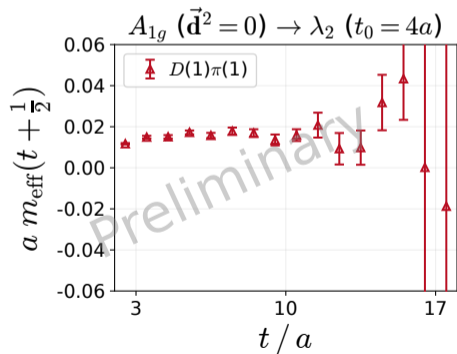
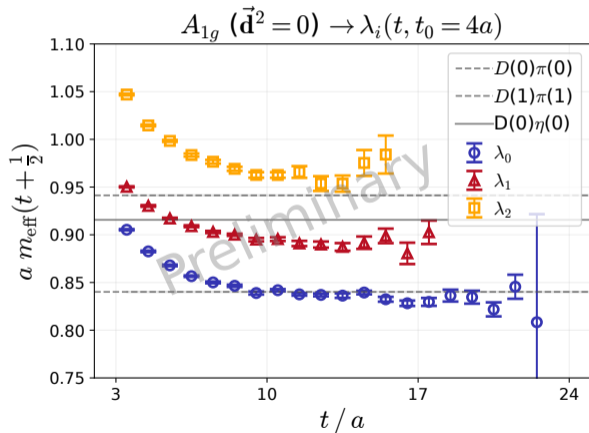
# The $D\pi$ spectrum:



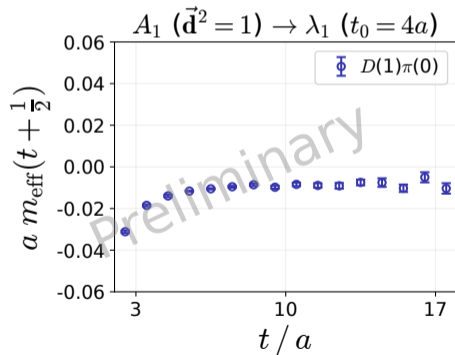
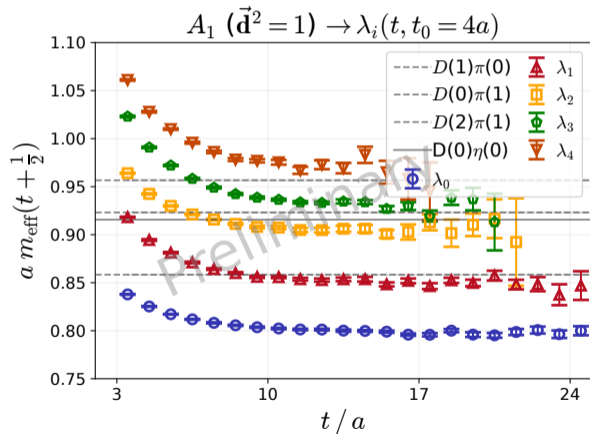
# The $D\pi$ spectrum:



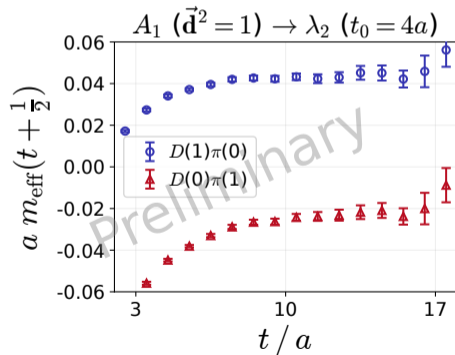
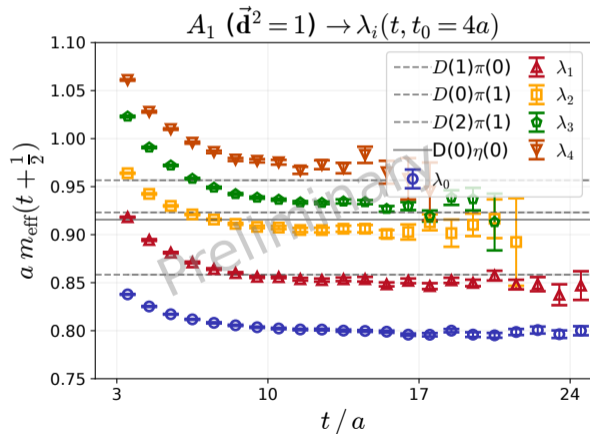
# The $D\pi$ spectrum:



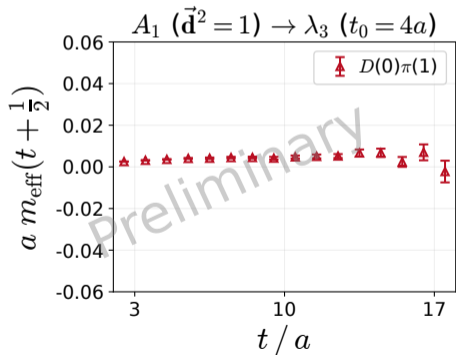
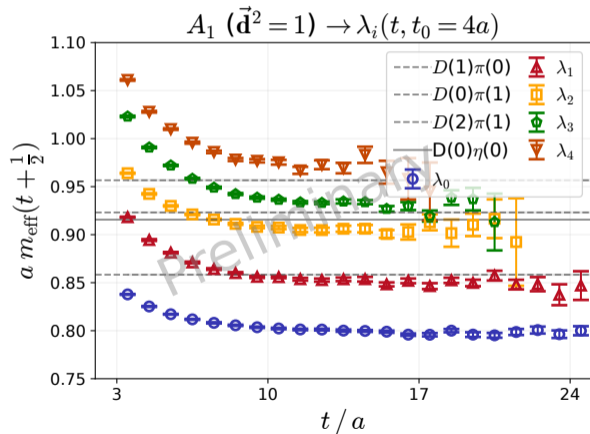
# The $D\pi$ spectrum:



# The $D\pi$ spectrum:



# The $D\pi$ spectrum:



## What did we already achieve?



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

- Bench mark test for our action: Dispersion relation behaves as expected
- Computed all perambulator matrices encoding quark propagation for all previously shown ensembles
- All necessary code modifications have for the full analysis have been implemented:
  - ▣ All quantum numbers, flavor labels etc.
  - ▣ All necessary Wick contractions
  - ▣ An extensive operator basis
- We started the Lüscher analysis for two ensembles X451 and N451



- Full analysis on all aforementioned CLS ensembles to control the full set of systematic uncertainties
- Inclusion of the  $D\eta$  and  $D_s\bar{K}$  channels
- Extension to the  $DK$  system, in order to study the  $D_{s0}^*$
- Perform the analysis with various amplitude parametrizations

---

# Thank you for your attention

**I'd also like to thank my collaborators:**

Daniel G. Mohler, Renwick J. Hudspith,  
Colin J. Morningstar, Bárbara Cid-Mora,  
Fernando Alvarado

## The tuning states



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

State	$\eta_c$	$J/\psi$	$\chi_{c0}$	$\chi_{c1}$	$h_c$
$\Gamma$	$\gamma_5$	$\gamma_i$	<b>1</b>	$\gamma_i \gamma_t$	$\gamma_i \gamma_j$
$J^{PC}$	$0^{-+}$	$1^{--}$	$0^{++}$	$1^{++}$	$1^{+-}$
Experiment [GeV]	2.9839	3.096 916	3.414 71	3.510 72	3.525 49



$A1g (d^2 = 0)$	$A1 (d^2 = 1)$	$A1 (d^2 = 2)$	$A1 (d^2 = 3)$
$D(0)\pi(0)$	$D(1)\pi(0)$	$D(2)\pi(0)$	$D(3)\pi(0)$
$D(1)\pi(1)$	$D(0)\pi(1)$	$D(1)\pi(1)$	$D(2)\pi(1)$
	$D(2)\pi(1)$	$D(3)\pi(1)$	$D(1)\pi(2)$
$2\bar{\psi}\psi$	$4\bar{\psi}\psi$	$4\bar{\psi}\psi$	$4\bar{\psi}\psi$