Exclusive photoproduction of $\pi^+\pi^-$ pairs in diffractive photon-proton and in proton-proton collisions within the tensor-pomeron approach

Piotr Lebiedowicz (IFJ PAN)

THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

in collaboration with
Antoni Szczurek (IFJ PAN)
Otto Nachtmann (Univ. Heidelberg)

based on: JHEP 06 (2026) 015

MESON 2026, 18th International Workshop on Meson Physics

25-30.06.2026, KRAKÓW, POLAND

Introduction

- We study the production of $\pi^+\pi^-$ pairs in photon-proton collisions and central exclusive production (CEP) of such pairs in proton-proton collisions,

$$\gamma^{(*)} + p \to \pi^+ + \pi^- + p\,,$$
$$p + p \to p + \pi^+ + \pi^- + p\,.$$

  We are interested in high energies and small momentum transfers, that is, in the regime of Regge exchanges.

- An old problem there is to understand the shape of the $\rho^0(770)$ resonance and the $\pi^+\pi^-$ invariant mass region below ρ^0. Compared to the ρ^0 shape measured in e^+e^- annihilation there is a skewing of the ρ^0 shape observed in the reactions above.

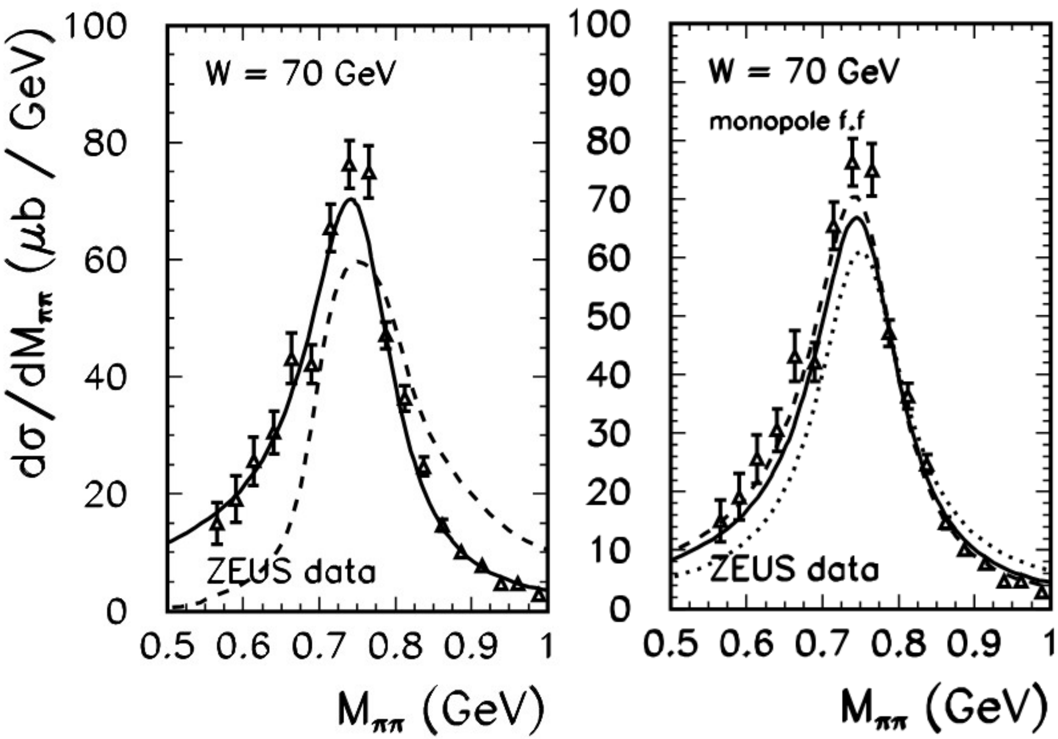
  Already a long time ago this skewing was attributed to the interference of the decay $\rho^0 \to \pi^+\pi^-$ with the non-resonant production of $\pi^+\pi^-$, the Drell-Söding (DS) term:
  S.D. Drell, *Production of particle beams at very high energies*, Phys. Rev. Lett. 5 (1960) 278,
  S.D. Drell, *Peripheral contributions to high-energy interaction processes,* Rev. Mod. Phys. 33 (1961) 458
  P. Söding, On the apparent shift of the *ρ meson mass in photoproduction,* Phys. Lett. 19 (1966) 702
  …

- In practice, the calculation of the continuum (DS) term is a tricky problem, not the least due to requirements of gauge invariance.
  J. Pumplin, *Diffraction dissociation and the reaction* $\gamma p \to \pi^+\pi^- p$, PRD 2 (1970) 1859
  A. Szczurek and A. Szczepaniak, PRD 71 (2005) 054005
  Ł. Bibrzycki *et al.* (JPAC), PRD 111 (2025) 014002
  …
  A. Bolz, C. Ewerz, M. Maniatis, O. Nachtmann, M. Sauter, A. Schöning, JHEP 01 (2015) 151
  P. Lebiedowicz, O. Nachtmann, A. Szczurek, JHEP 06 (2026) 015 **← this talk**

Introduction

Results from A. Szczurek and A. Szczepaniak

PHYSICAL REVIEW D **71**, 054005 (2005)



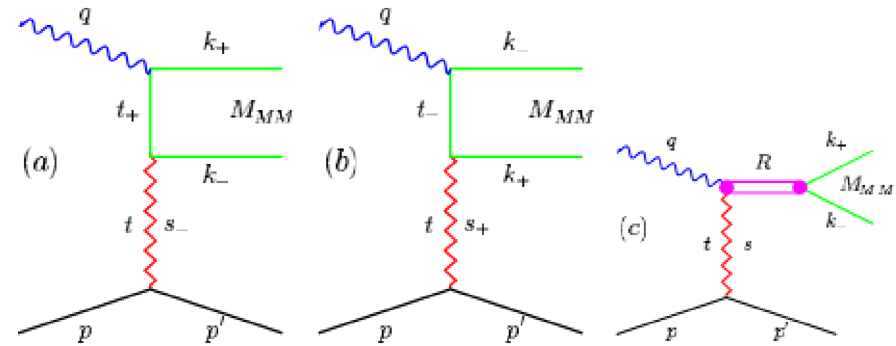
Left: ρ^0 + DS (solid line), ρ^0 (dashed line)

ZEUS data: M. Derrick et al., Z. Phys. C69 (1995) 39

Right: ρ^0 + DS contribution for three values of the off-shell form factor parameter, Λ = 0.5, 1.0, 2.0 GeV.

The exact form of the form factor of off-shell pion is not known. In principle, a good quality data would help to find the proper functional form.

- Gauge-invariant skewing mechanism



- The amplitude for the nonresonant (DS) component is

$$\mathcal{M}^{(DS)}_{\lambda_\gamma \lambda \to \lambda'}(s,t,s_+,t_+,s_-,t_-)$$
$$= V^{\gamma\pi^+}_{\lambda_\gamma} \frac{F(t_+)}{t_+ - m_\pi^2} \mathcal{M}^{\pi^- p}_{\lambda\lambda'}(s_-,t)$$
$$+ V^{\gamma\pi^-}_{\lambda_\gamma} \frac{F(t_-)}{t_- - m_\pi^2} \mathcal{M}^{\pi^+ p}_{\lambda\lambda'}(s_+,t) + \delta\mathcal{M}$$

$$V^{\gamma\pi^\pm}_{\lambda_\gamma} = \pm e(2k^\mu_\pm)\,\epsilon_\mu(\lambda_\gamma = \pm 1)$$

$$\mathcal{M}^{\pi^\pm p}_{\lambda\lambda'}(s_\pm,t) = i s_\pm\, \sigma^{\pi^\pm p}_{tot}(s_\pm)\, \exp\left(\frac{B}{2} t\right) \delta_{\lambda\lambda'}$$

DL parametrisation

$$F(t_\pm) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t_\pm}$$

- Except for the off-shell dependence determined by the the form factors $F(t_+)$ and $F(t_-)$ the DS model is essentially parameter free.

Tensor-pomeron model

- Tensor pomeron and vector odderon model [C. Ewerz, M. Maniatis, and O. Nachtmann, Annals Phys. 342 (2014) 31] has been constructed in order to describe soft high-energy hadronic reactions.

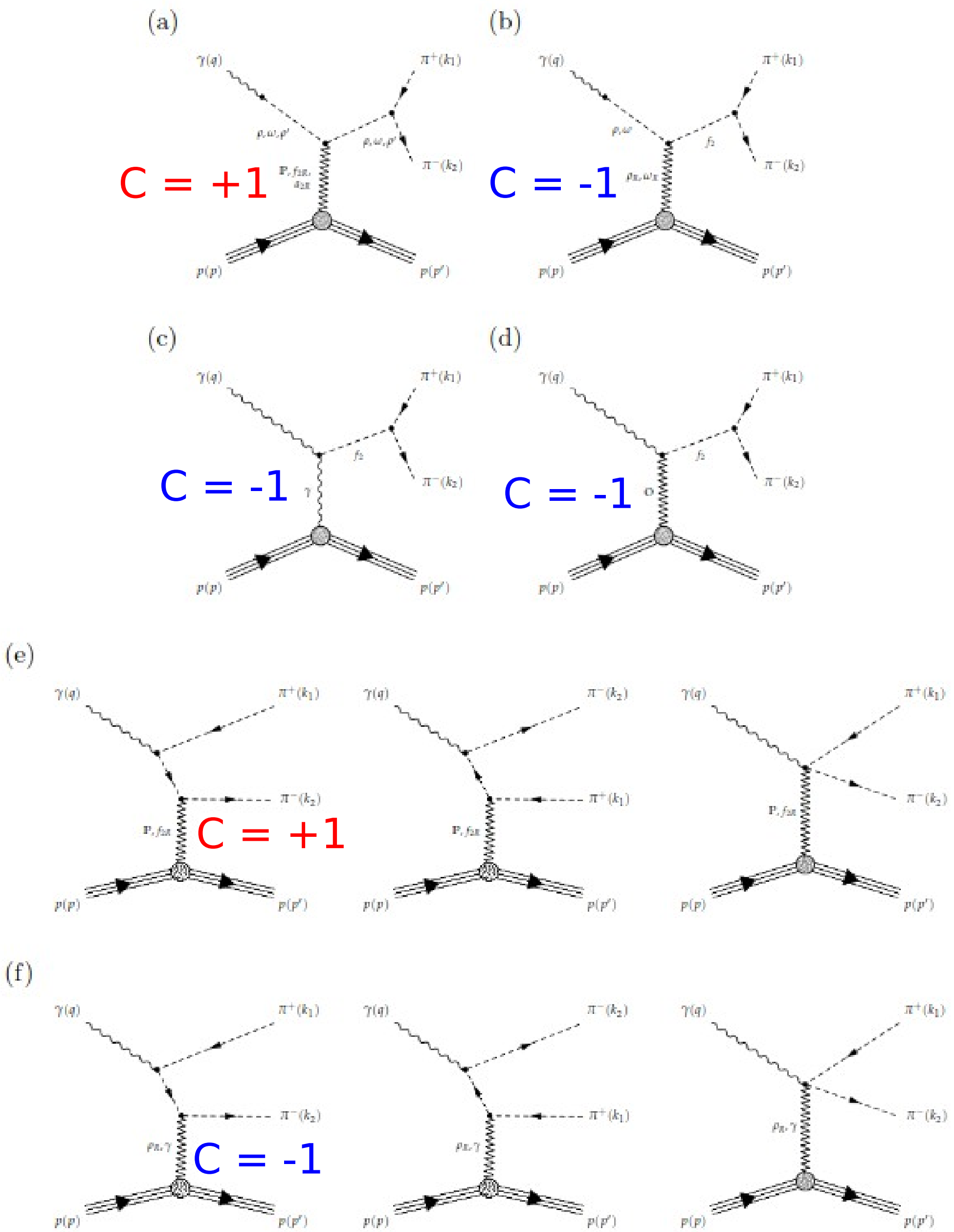
  The pomeron (IP) and C = +1 reggeons are described as effective rank-2 symmetric tensor exchanges, the odderon (O) and C = -1 reggeons as effective vector exchanges.

- It was applied to $\pi^+\pi^-$ photoproduction [A. Bolz, C. Ewerz, M. Maniatis, O. Nachtmann, M. Sauter, A. Schöning, JHEP 01 (2015) 151]

  → gauge-invariant mechanism with effective vertices derived from coupling Lagrangians

  → a common energy variable (s) was used in the respective Regge factors (propagators) of non-resonant amplitudes

  → in this way a skewing of the ρ^0 shape was achieved but compared to experiment it was not big enough !



Tensor-pomeron model

Results from

A. Bolz, C. Ewerz, M. Maniatis, O. Nachtmann, M. Sauter, A. Schöning, JHEP 01 (2015) 151

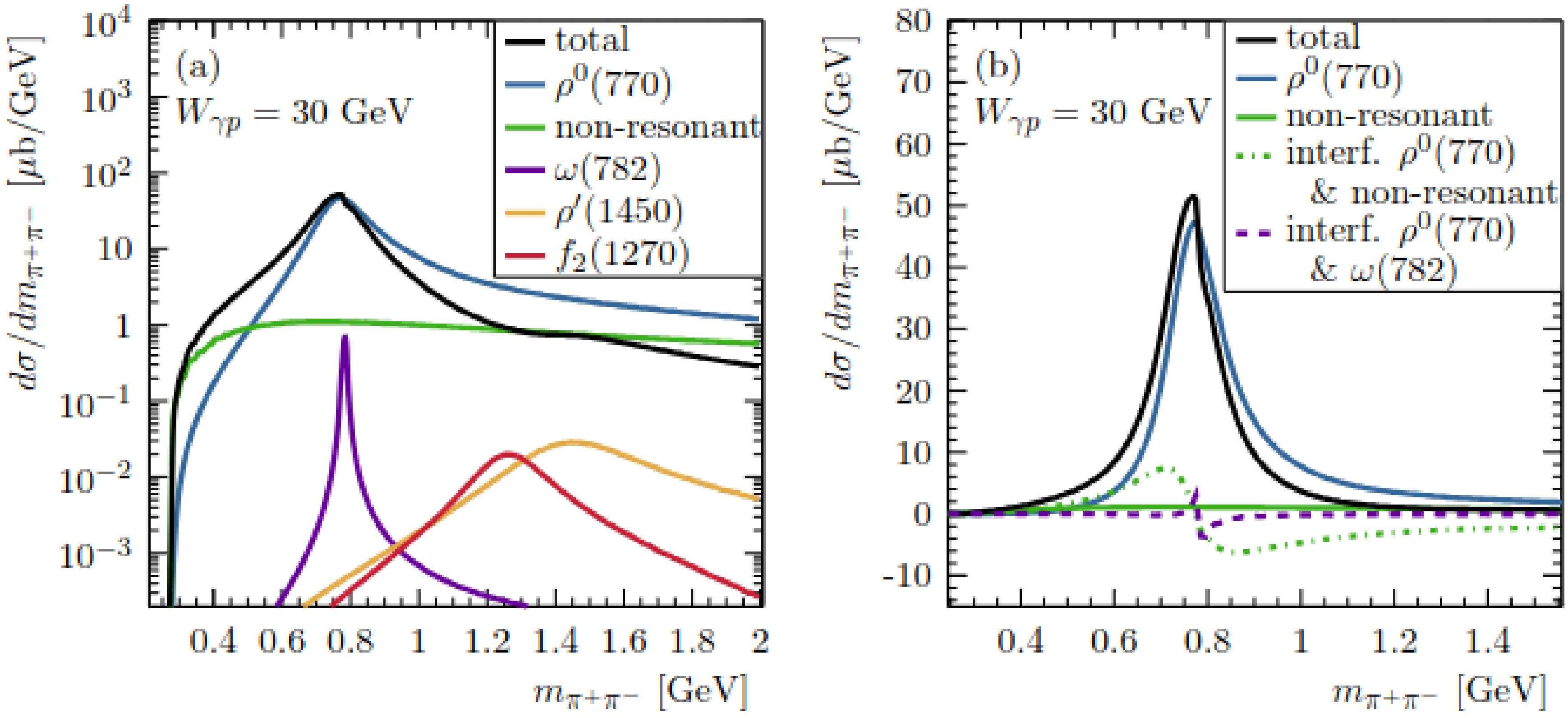
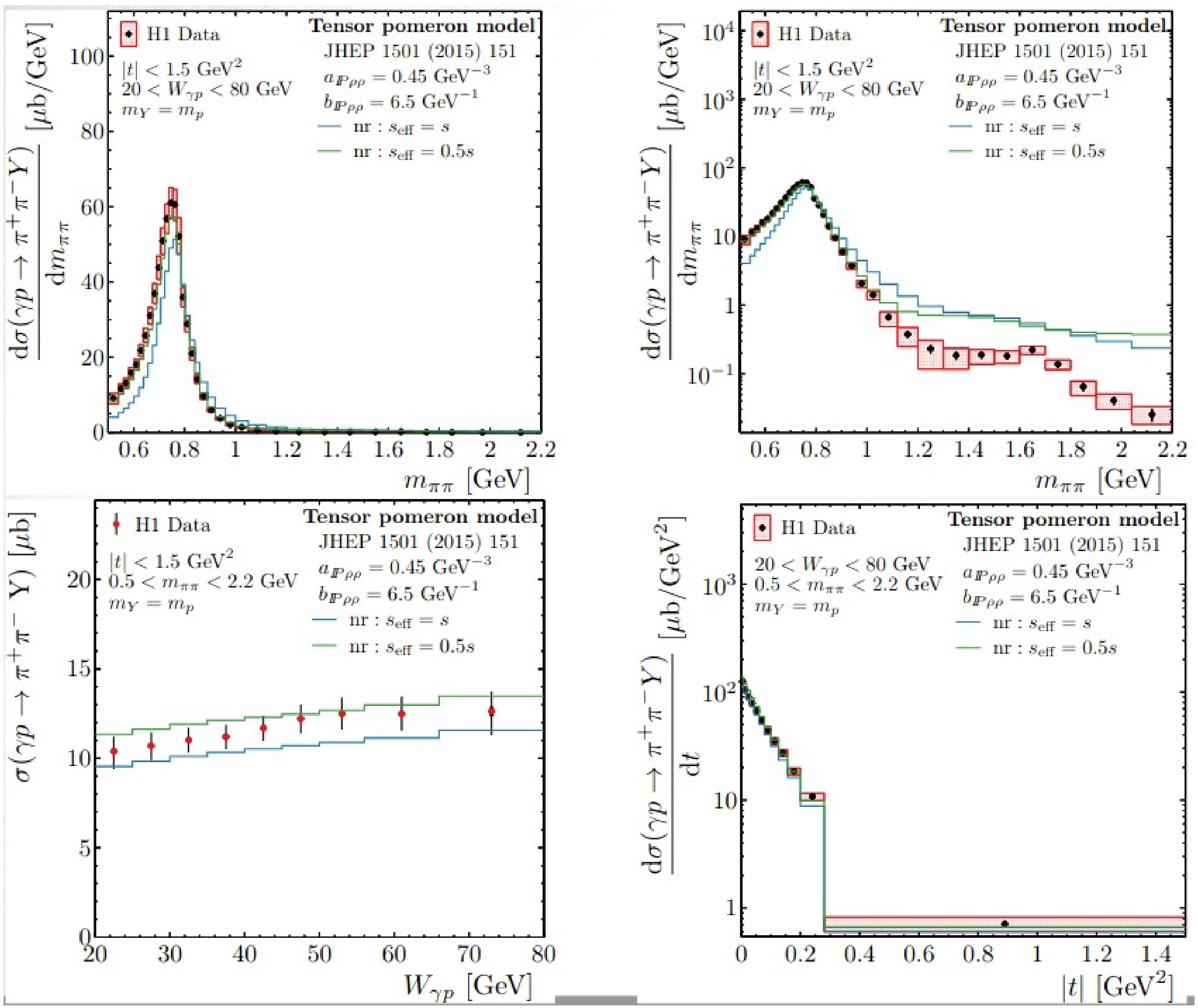
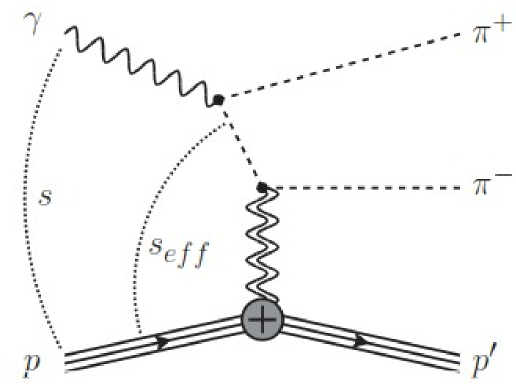


Figure 5. Differential cross sections $d\sigma/dm_{\pi^+\pi^-}$ $(\gamma p \to \pi^+\pi^- p)$ as function of $m_{\pi^+\pi^-}$ for fixed $W_{\gamma p} = 30\,\mathrm{GeV}$ and integrated over the range $-1\,\mathrm{GeV}^2 \leq t \leq 0$. (a) The full model, non-resonant contributions and the contributions from the resonances $\rho^0(770)$, $\omega(782)$, $f_2(1270)$ and $\rho'(1450)$ are shown. (b) Dominant contributions in the ρ mass region including the leading interferences of $\rho^0(770)$ with the non-resonant $\pi^+\pi^-$ production and the $\omega(782)$ meson are shown.

Towards better modelling, DS term...

- Comparison of modified Drell-Söding (DS) model to H1 data [H1 Collaboration, EPJC 80 (2020) 1169]. Results from A. Bolz talk "*Measurement of Exclusive $\pi^+\pi^-$ and ρ^0 Meson Photoproduction at HERA*" presented at **MESON 2021**.





- Better description of the H1 data can be achieved by modifying the DS term in such a way that the energy dependence in the Regge propagators is change from $s \to s_{\rm eff} = s/2$

- Good description of ρ^0 peak, low $M_{\pi\pi}$ range, $\sigma(W_{\gamma p})$ and $d\sigma/dt$ distribution.

- However, the DS model based on this somewhat arbitrary procedure should be considered as an effective model.

Towards better modelling

Production of $\pi^+\pi^-$ pairs in diffractive photon-proton and in proton-proton collisions revisited, in particular concerning the Drell-Söding contribution

Piotr Lebiedowicz,[1,*] Otto Nachtmann,[2,†] and Antoni Szczurek[1,3,‡]

[1]Institute of Nuclear Physics Polish Academy of Sciences, Radzikowskiego 152, PL-31342 Kraków, Poland
[2]Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany
[3]Institute of Physics, Faculty of Exact and Technical Sciences, University of Rzeszów, Pigonia 1, PL-35310 Rzeszów, Poland

JHEP 06 (2026) 015

We discuss the exclusive photoproduction of $\pi^+\pi^-$ pairs in photon-proton and in proton-proton collisions at high energies. The ρ^0, ω, $f_2(1270)$, and non-resonant (Drell-Söding) contributions are considered. The calculation is based on the tensor-pomeron model that includes not only the dominant pomeron exchange but also reggeon and odderon exchanges. In the Drell-Söding contribution we have different subenergies for the $\pi^+ p$ and $\pi^- p$ systems. In the method which we propose now we take this into account. Respecting the gauge-invariance constraints is then a nontrivial problem for which, however, we present a solution here. In the present paper we give in this way a substantial improvement of the calculations for real photoproduction of $\pi^+\pi^-$ from JHEP 01, 151 (2015), and we extend the calculations to low Q^2 electroproduction, $0 \leqslant Q^2 \leqslant 0.5$ GeV^2. The photo- and electroproduction amplitudes are then the basis for the calculation of central exclusive production (CEP) of $\pi^+\pi^-$ pairs in proton-proton collisions, where at least one proton participates in the CEP via a virtual-photon emission. The revised model leads to enhanced cross sections and gives an increased skewing of the ρ^0 spectral shape. For the $pp \to pp\pi^+\pi^-$ reaction, we calculate differential cross sections as function of the two-pion invariant mass, pion transverse momentum and pion pseudorapidity. Predictions of proton-pion and proton-pion-pion invariant mass distributions and the distribution in the proton-proton four-momentum transfer squared are also presented. This research is relevant in the context of ALICE, ATLAS, CMS, and LHCb measurements in pp collisions, even when the leading protons are not detected and instead only rapidity-gap conditions are checked experimentally. Our results can also serve as basis for the description of coherent $\pi^+\pi^-$ production in ultra-peripheral pA and AA collisions at the LHC. The formulas given in our paper can directly be used for the analysis of photoproduction and small-Q^2 electroproduction in ep collisions at high energies. Such data exist from the HERA experiments and will be obtained in the future at the electron-ion colliders.

→ improved calculation of the Drell-Söding (DS) contribution in the tensor-pomeron approach
→ we use for each diagram the appropriate energy variable in the Regge factors
→ we give a good calculation of the DS term which fulfils all requirements of QFT

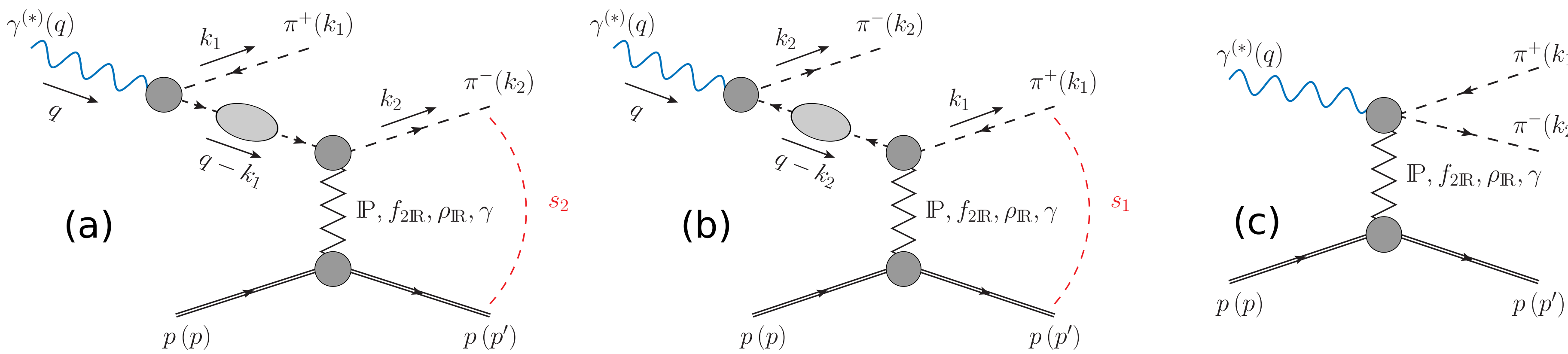
Theoretical formalism for $\gamma p \to \pi^+\pi^- p$

$$\gamma^{(*)}(q,\mu) + p(p,\mathfrak{s}) \to \pi^+(k_1) + \pi^-(k_2) + p(p',\mathfrak{s}')$$

$$\langle \pi^+(k_1), \pi^-(k_2), p(p',\mathfrak{s}')|\mathcal{T}|\gamma(q,\epsilon), p(p,\mathfrak{s})\rangle = \epsilon^\mu \mathcal{M}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)$$

for real photon $q^2 = 0$

Diagrams for non-resonant production of $\pi^+\pi^-$ pairs (Drell-Söding mechanism):



$$\mathcal{M}^{(\mathrm{DS})}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) = \mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) + \mathcal{M}^{(b)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) + \mathcal{M}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)$$

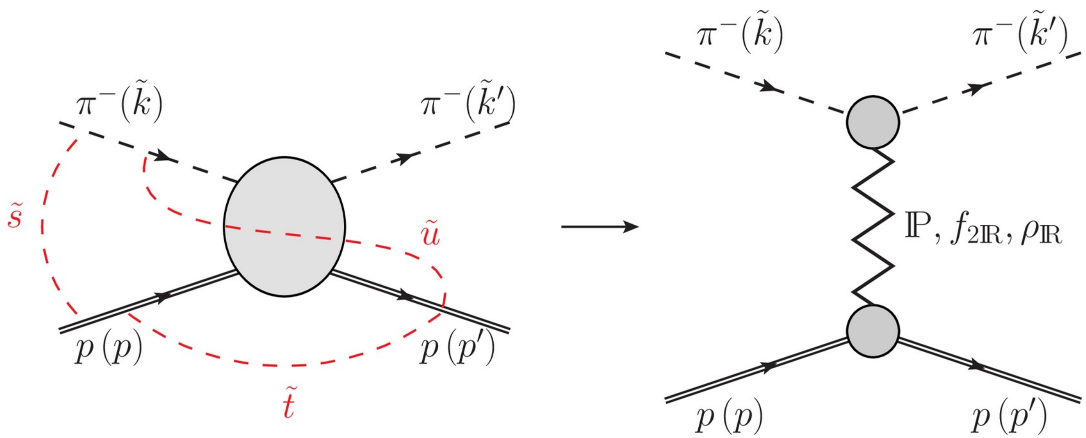
$$\mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) = e\widehat{\Gamma}^{(\gamma\pi\pi)}_\mu(k_1,k_1-q)\Delta_F[(k_1-q)^2]\mathcal{M}^{(0,\,a)}_{\mathfrak{s}',\mathfrak{s}}(k_2,p',q-k_1,p)$$

$e = \sqrt{4\pi\alpha_{\mathrm{em}}} > 0$
full pion-photon vertex function

full pion propagator

hadronic scattering amplitude, also denoted by $\mathcal{M}^{(\pi^-)}_{\mathfrak{s}',\mathfrak{s}}$

Pion-proton scattering (on shell)



$$\tilde{k} + p = \tilde{k}' + p'$$

$$\begin{aligned} \tilde{s} &= (\tilde{k}+p)^2 = (\tilde{k}'+p')^2\,, \\ \tilde{t} &= (\tilde{k}-\tilde{k}')^2 = (p-p')^2\,, \\ \tilde{u} &= (\tilde{k}-p')^2 = (p-\tilde{k}')^2\,, \\ \tilde{\nu} &= \frac{1}{4}(\tilde{s}-\tilde{u})\,. \end{aligned}$$

Amplitudes for pomeron exchange (considered as effective rank-2 symmetric tensor exchange)

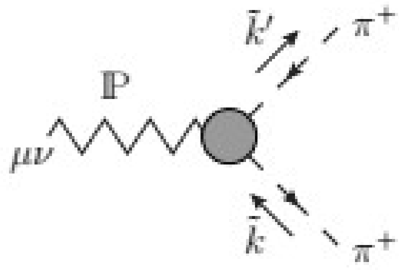
$$\begin{aligned} \mathcal{M}^{(\pi^\pm)}_{\mathfrak{s}',\mathfrak{s}}(\tilde{k}',p',\tilde{k},p)|_{\mathbb{P}} &= (-i)i\Gamma^{(\mathbb{P}\pi\pi)}_{\mu\nu}(\tilde{k}',\tilde{k})\, i\Delta^{(\mathbb{P})\mu\nu,\kappa\lambda}(2\tilde{\nu},\tilde{t})\; \bar{u}_{\mathfrak{s}'}(p')i\Gamma^{(\mathbb{P}pp)}_{\kappa\lambda}(p',p)u_{\mathfrak{s}}(p) \\ &= i\boxed{\mathcal{F}_{\mathbb{P}\pi p}(2\tilde{\nu},\tilde{t})}\; \bar{u}_{\mathfrak{s}'}(p')\left[2(\tilde{k}'+\tilde{k})^\nu\gamma_\nu(\tilde{k}'+\tilde{k},p'+p) - (\tilde{k}'+\tilde{k})^2 m_p\right]u_{\mathfrak{s}}(p) \end{aligned}$$

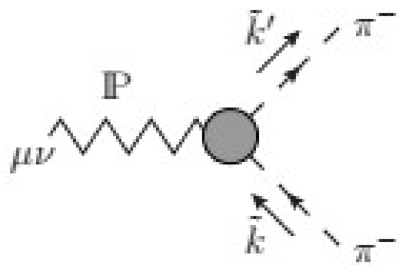
$$\boxed{\mathcal{F}_{\mathbb{P}\pi p}(2\tilde{\nu},\tilde{t})} = 2\beta_{\mathbb{P}\pi\pi}\, 3\beta_{\mathbb{P}NN}\, F_M(\tilde{t})\, F_1(\tilde{t})\, \underbrace{\frac{1}{8\tilde{\nu}}(-i\,2\tilde{\nu}\,\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(\tilde{t})-1}}_{\frac{1}{2}\left(-\frac{i}{2}\alpha'_{\mathbb{P}}\right)^{\alpha_{\mathbb{P}}(\tilde{t})-1}(16\tilde{\nu}^2)^{\frac{\alpha_{\mathbb{P}}(\tilde{t})-2}{2}}}$$

$$F_M(\tilde{t}) = \frac{\Lambda^2}{\Lambda^2 - \tilde{t}}\,, \qquad \Lambda^2 = 0.75\ \text{GeV}^2$$

proton Dirac form factor

Effective vertices and pomeron propagator

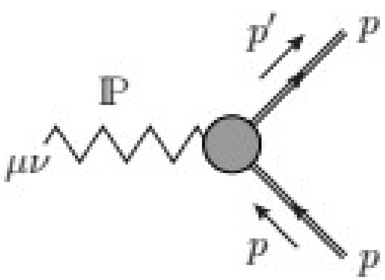




From C invariance we have the same vertex function for IPπ-π- and IPπ⁺π⁺.

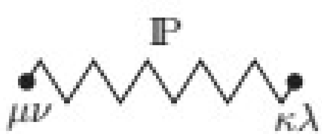
$$i\Gamma_{\mu\nu}^{(\mathbb{P}\pi\pi)}(\tilde{k}',\tilde{k}) = -i2\beta_{\mathbb{P}\pi\pi}F_M[(\tilde{k}'-\tilde{k})^2]\left[(\tilde{k}'+\tilde{k})_\mu(\tilde{k}'+\tilde{k})_\nu - \frac{1}{4}g_{\mu\nu}(\tilde{k}'+\tilde{k})^2\right],$$

$$\beta_{\mathbb{P}\pi\pi} = 1.76\ \mathrm{GeV}^{-1},$$



$$i\Gamma_{\mu\nu}^{(\mathbb{P}pp)}(p',p) = -i3\beta_{\mathbb{P}NN}F_1[(p'-p)^2]\left[\frac{1}{2}\gamma_\mu(p'+p)_\nu + \frac{1}{2}\gamma_\nu(p'+p)_\mu - \frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right],$$

$$\beta_{\mathbb{P}NN} = 1.87\ \mathrm{GeV}^{-1},$$

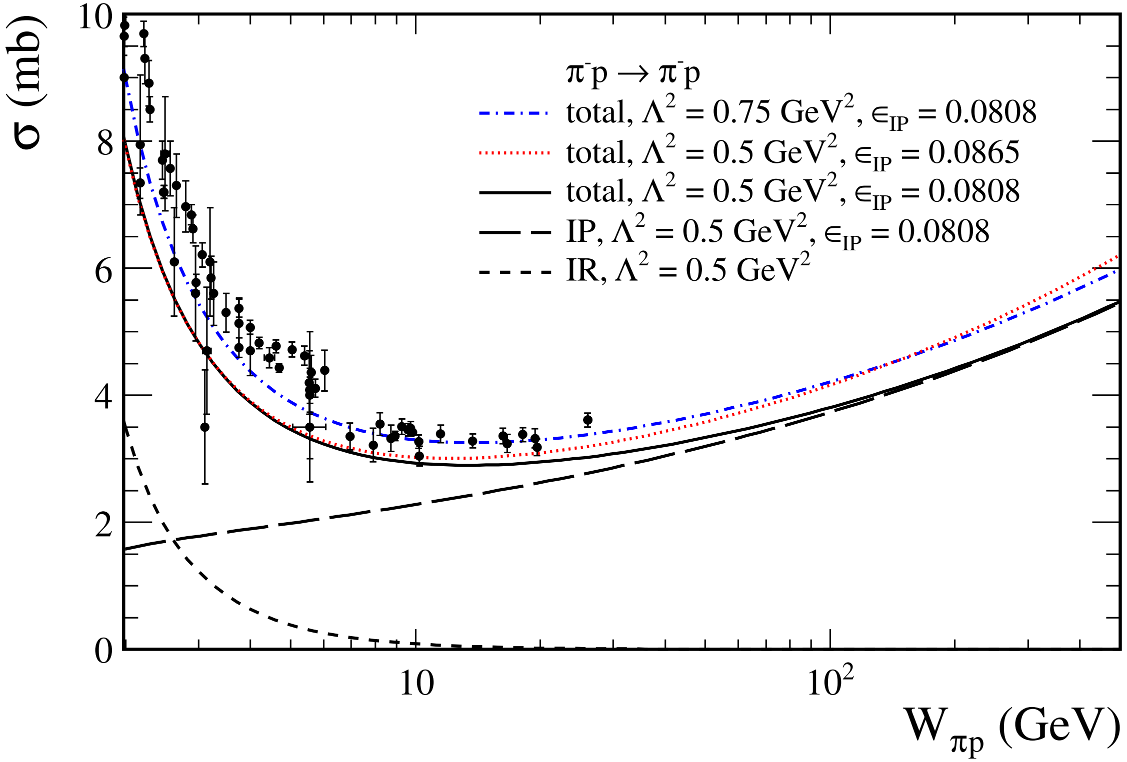


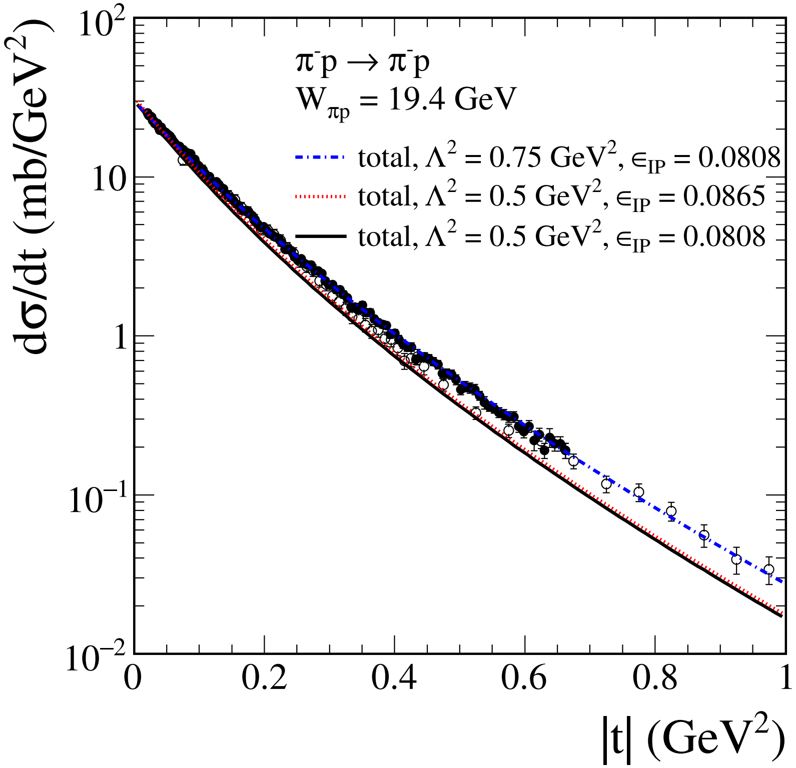
$$i\Delta_{\mu\nu,\kappa\lambda}^{(\mathbb{P})}(2\tilde{\nu},\tilde{t}) = \frac{1}{8\tilde{\nu}}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)(-i\,2\tilde{\nu}\,\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(\tilde{t})-1},$$

$$\alpha_{\mathbb{P}}(\tilde{t}) = 1 + \epsilon_{\mathbb{P}} + \alpha'_{\mathbb{P}}\tilde{t},$$

$$\epsilon_{\mathbb{P}} = 0.0808, \qquad \alpha'_{\mathbb{P}} = 0.25\ \mathrm{GeV}^{-2};$$

Pion-proton scattering



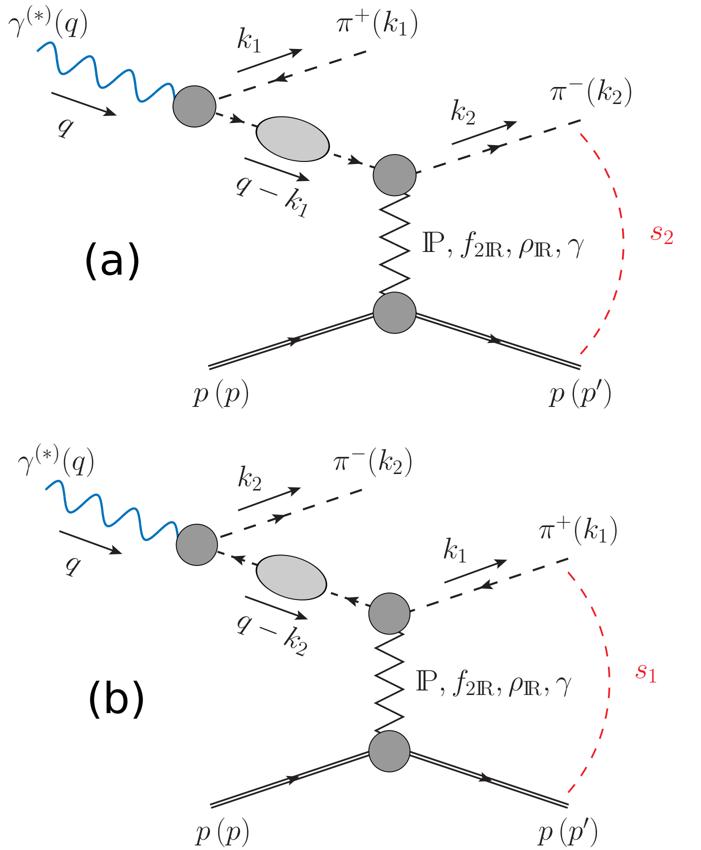


Complete results (total) and individual components, IP and IR, obtained for different values of cut-off parameter Λ

$$F_M(\tilde{t}) = \frac{\Lambda^2}{\Lambda^2 - \tilde{t}}\,, \quad \Lambda^2 = (0.5 - 0.75)\ \mathrm{GeV}^2$$

and intercept of Pomeron: ϵ_{IP} = 0.0808 and 0.0865.

The larger value of ϵ_{IP} is motivated by our analysis of the elastic *pp* scattering and by comparing the model to the TOTEM data.



$$q + p = k_1 + k_2 + p'$$

$$
\begin{aligned}
s &= (q+p)^2 = (k_1+k_2+p')^2\,, \\
t &= (p-p')^2 = (q-k_1-k_2)^2\,, \\
s_1 &= (p'+k_1)^2 = (p+q-k_2)^2\,, \\
u_1 &= (p-k_1)^2 = (p'-q+k_2)^2\,, \\
s_2 &= (p'+k_2)^2 = (p+q-k_1)^2\,, \\
u_2 &= (p-k_2)^2 = (p'-q+k_1)^2\,, \\
M_{\pi\pi}^2 &= (k_1+k_2)^2 = (p-p'+q)^2\,; \\
\nu_1 &= \frac{1}{4}(s_1-u_1) \\
&= \frac{1}{4}\left[(p+p',k_1-k_2)+(p+p',q)\right]\,, \\
\nu_2 &= \frac{1}{4}(s_2-u_2) \\
&= \frac{1}{4}\left[(p+p',k_2-k_1)+(p+p',q)\right]\,.
\end{aligned}
$$

We have

$$
\begin{aligned}
\nu_1^2 &= \frac{1}{16}\left[(p+p',q)^2+(p+p',k_1-k_2)^2+2(p+p',k_1-k_2)(p+p',q)\right], \\
\nu_2^2 &= \frac{1}{16}\left[(p+p',q)^2+(p+p',k_1-k_2)^2-2(p+p',k_1-k_2)(p+p',q)\right],
\end{aligned}
$$

and we define

$$
\begin{aligned}
\bar{\nu}^2 &= \frac{1}{2}(\nu_1^2+\nu_2^2)\,, \\
\varkappa &= \frac{2(q,p+p')(p+p',k_1-k_2)}{16\bar{\nu}^2}\,.
\end{aligned}
$$

$$
\begin{aligned}
|\varkappa| &\leqslant 1\,, \\
16\nu_1^2 &= 16\bar{\nu}^2(1+\varkappa)\,, \\
16\nu_2^2 &= 16\bar{\nu}^2(1-\varkappa)\,.
\end{aligned}
$$

$$\mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) = e\widehat{\Gamma}^{(\gamma\pi\pi)}_{\mu}(k_1,k_1-q)\Delta_F[(k_1-q)^2]\mathcal{M}^{(0,\,a)}_{\mathfrak{s}',\mathfrak{s}}(k_2,p',q-k_1,p)$$

For photons of small virtuality, $Q^2 = -q^2 < 0.5\ \mathrm{GeV}^2$ we set

description of low-x DIS and DVCS HERA data
→ predominant soft-pomeron exchange;
Britzger *et al.*, PRD 100 (2019) 114007
Lebiedowicz *et al.*, PLB 835 (2022) 137947

$$\widehat{\Gamma}^{(\gamma\pi\pi)}_{\mu}(k_1,k_1-q)\,\Delta_F\left[(k_1-q)^2\right]|_{k_1^2=m_\pi^2} = \frac{(2k_1-q)_\mu}{-2k_1\cdot q+q^2+i\varepsilon}F_M(q^2) - q_\mu\frac{1-F_M(q^2)}{q^2}$$

where $F_M(q^2) = \dfrac{m_0^2}{m_0^2-q^2}\,, \quad m_0^2 = 0.5\ \mathrm{GeV}^2$

is a simple representation of the pion electromagnetic form factor.

For real photons ($q^2 = 0$) this is an exact result up to an irrelevant gauge term.
For small photon virtualities this represents our model assumption.

We get for diagram (a) for IP exchange:

$$\mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)|_{\mathbb{P}} = ie\left[\frac{(2k_1-q)_\mu}{-2k_1\cdot q+q^2+i\varepsilon}F_M(q^2) - q_\mu\frac{1-F_M(q^2)}{q^2}\right]\mathcal{F}_{\mathbb{P}\pi p}(2\nu_2,t)$$
$$\times\left[2(k_2-k_1+q)^\nu(k_2-k_1+q,p'+p)\bar{u}_{\mathfrak{s}'}(p')\gamma_\nu u_{\mathfrak{s}}(p) - (k_2-k_1+q)^2 m_p\bar{u}_{\mathfrak{s}'}(p')u_{\mathfrak{s}}(p)\right]$$

$$\mathcal{F}_{\mathbb{P}\pi p}(2\nu_2,t) = \mathcal{F}_{\mathbb{P}\pi p}(2\bar{\nu},t)\left[1+(2-\alpha_{\mathbb{P}}(t))\frac{\varkappa}{2}g\left(\frac{2-\alpha_{\mathbb{P}}(t)}{2},\varkappa\right)\right]$$

$$g(\lambda,\varkappa) = \frac{(1-\varkappa)^{-\lambda}-1}{\lambda\varkappa}$$

In a completely analogous way we get amplitude for diagram (b).

At high energies we use the approximation: $\bar{u}_{\mathfrak{s}'}(p')\gamma_\nu u_{\mathfrak{s}}(p) \simeq (p'+p)_\nu\,\delta_{\mathfrak{s}',\mathfrak{s}}$

Gauge invariance requires

$$q^\mu \mathcal{M}^{(\mathrm{DS})}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) = 0 \quad \Longrightarrow \quad q^\mu \mathcal{M}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}} = -q^\mu \mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}} - q^\mu \mathcal{M}^{(b)}_{\mu,\mathfrak{s}',\mathfrak{s}}$$

Using the generalised Ward identity

$$(k'-k)^\mu\, \widehat{\Gamma}^{(\gamma\pi\pi)}_\mu(k',k) = \Delta_F^{-1}(k'^2) - \Delta_F^{-1}(k^2)$$ and normalisation conditions for pion propagator

$$\Delta_F^{-1}(m_\pi^2) = 0\,,\ \frac{\partial}{\partial k^2}\Delta_F^{-1}(k^2)|_{k^2=m_\pi^2} = 1$$

we find

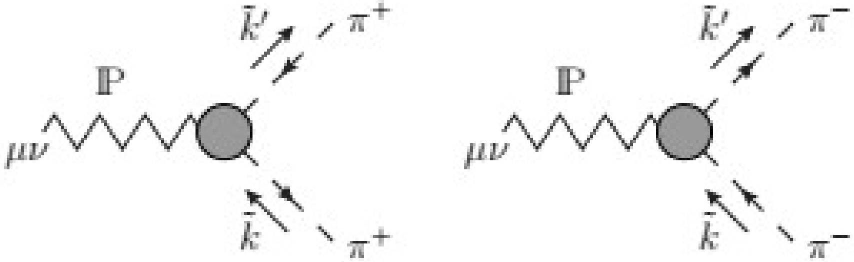
$$q^\mu \mathcal{M}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) = e\Big[\mathcal{M}^{(0,\,a)}_{\mathfrak{s}',\mathfrak{s}}(k_2,p',q-k_1,p) - \mathcal{M}^{(0,\,b)}_{\mathfrak{s}',\mathfrak{s}}(k_1,p',q-k_2,p)\Big]$$

The r.h.s. of this equation can be written in a way that is explicitly ∝ q^μ.
Then we drop q^μ on both sides and get our result for (c) term:

$$\begin{aligned}
\mathcal{M}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)|_{\mathbb{P}} &= 2ie\mathcal{F}_{\mathbb{P}\pi p}(2\bar{\nu},t)\Big\{2\delta_\mu{}^\nu(k_2-k_1,p'+p) + 2(p'+p)_\mu(k_2-k_1)^\nu \\
&+(p'+p)_\mu(2-\alpha_{\mathbb{P}}(t))\frac{(p'+p,k_1-k_2)}{16\bar{\nu}^2}\left[g\left(\frac{2-\alpha_{\mathbb{P}}(t)}{2},\varkappa\right)(k_2-k_1+q)^\nu(k_2-k_1+q,p'+p)\right. \\
&\left.+g\left(\frac{2-\alpha_{\mathbb{P}}(t)}{2},-\varkappa\right)(k_2-k_1-q)^\nu(k_2-k_1-q,p'+p)\right]\Big\}\bar{u}_{\mathfrak{s}'}(p')\gamma_\nu u_{\mathfrak{s}}(p) \\
&+2ie\mathcal{F}_{\mathbb{P}\pi p}(2\bar{\nu},t)\Big\{-2(k_2-k_1)_\mu - (p'+p)_\mu\frac{2-\alpha_{\mathbb{P}}(t)}{2}\frac{(p'+p,k_1-k_2)}{16\bar{\nu}^2} \\
&\times\left[g\left(\frac{2-\alpha_{\mathbb{P}}(t)}{2},\varkappa\right)(k_2-k_1+q)^2 + g\left(\frac{2-\alpha_{\mathbb{P}}(t)}{2},-\varkappa\right)(k_2-k_1-q)^2\right]\Big\}m_p\bar{u}_{\mathfrak{s}'}(p')u_{\mathfrak{s}}(p)
\end{aligned}$$

Introduction of form factors $F_\pi(t_\pi)$.

We assume that the IPππ vertex depends on the off-shellness of the pions.



$$i\Gamma^{(\mathbb{P}\pi\pi)}_{\mu\nu}(\tilde{k}',\tilde{k}) = -i2\beta_{\mathbb{P}\pi\pi}F_M[(\tilde{k}'-\tilde{k})^2]\left[(\tilde{k}'+\tilde{k})_\mu(\tilde{k}'+\tilde{k})_\nu - \frac{1}{4}g_{\mu\nu}(\tilde{k}'+\tilde{k})^2\right],$$

$$F[(\tilde{k}'-\tilde{k})^2, \tilde{k}'^2 - m_\pi^2, \tilde{k}^2 - m_\pi^2]$$

As normalisation condition we take F(0,0,0) = 1.

$$F_M(t) = \frac{\Lambda^2}{\Lambda^2 - t}$$

$$F_\pi(t_{\pi,i}) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t_{\pi,i}}$$

$$t_{\pi,i} = (q-k_i)^2, \quad i = 1,2$$

For $k^2 = m_\pi^2$ we assume $F[t, 0, (q-k)^2 - m_\pi^2] \to F_M(t)F_\pi(t_\pi)$

For *γp → π⁺π⁻ p* we get

$$\begin{aligned}
\mathcal{N}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) &= F_\pi(t_{\pi,1})\mathcal{M}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) \\
\mathcal{N}^{(b)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) &= F_\pi(t_{\pi,2})\mathcal{M}^{(b)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) \\
\mathcal{N}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) &= \frac{1}{2}\left[F_\pi(t_{\pi,1}) + F_\pi(t_{\pi,2})\right]\mathcal{M}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p) \\
&+ (k_2-k_1)_\mu \frac{F_\pi(t_{\pi,1}) - F_\pi(t_{\pi,2})}{(t_{\pi,1}-m_\pi^2)-(t_{\pi,2}-m_\pi^2)} e\left[\mathcal{M}^{(\pi^-)}_{\mathfrak{s}',\mathfrak{s}} + \mathcal{M}^{(\pi^+)}_{\mathfrak{s}',\mathfrak{s}}\right]
\end{aligned}$$

Our result for the DS term considering a general $\mathbb{P}\pi\pi$ vertex function is

$$\mathcal{N}^{(\mathrm{DS})}_{\mu,\mathfrak{s}',\mathfrak{s}} = \mathcal{N}^{(a)}_{\mu,\mathfrak{s}',\mathfrak{s}} + \mathcal{N}^{(b)}_{\mu,\mathfrak{s}',\mathfrak{s}} + \mathcal{N}^{(c)}_{\mu,\mathfrak{s}',\mathfrak{s}}$$

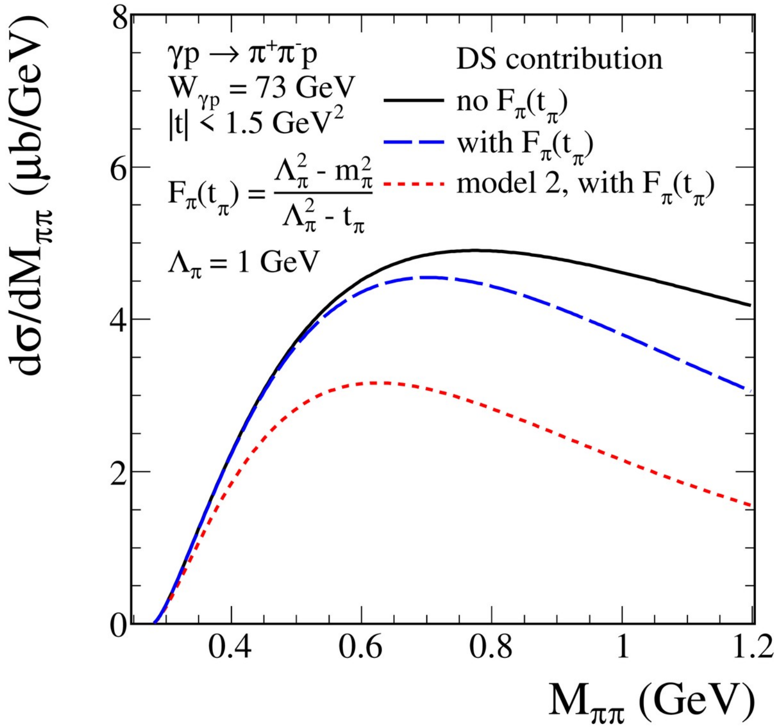
Comparison with Pumplin's ansatz

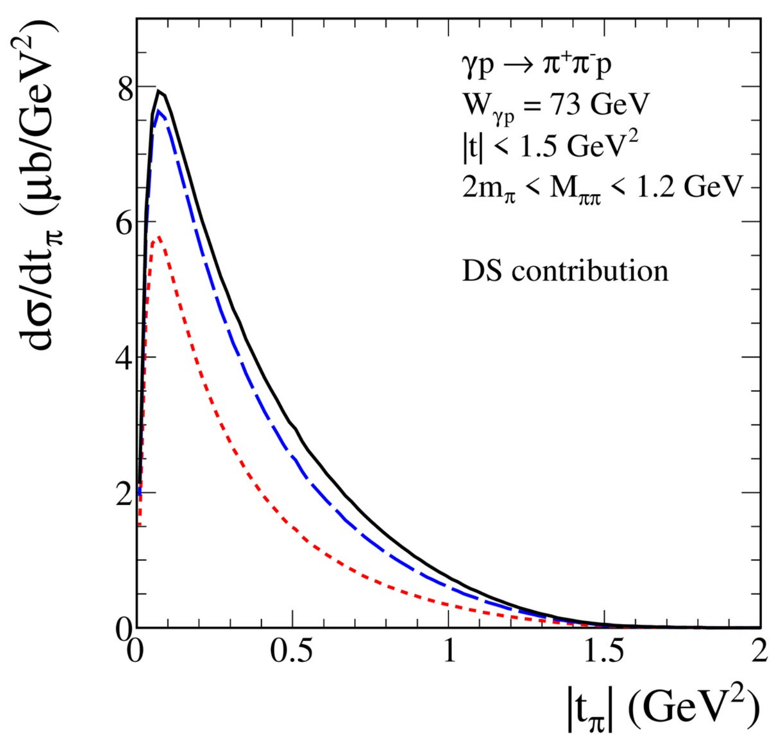
J. Pumplin, PRD 2 (1970) 1859
used in, e.g.,
A. Szczurek and A. Szczepaniak, PRD 71 (2005) 054005
Ł. Bibrzycki *et al.* (JPAC), PRD 111 (2025) 014002

For $\gamma p \to \pi^+\pi^- p$ we have

$$\mathcal{M}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)|_{\rm Pumplin} = e\Bigg[\left(\frac{k_{2\mu}}{(q,k_2)} - \frac{(p+p')_\mu}{(q,p+p')}\right)F_\pi(t_{\pi,2})\,\mathcal{M}^{(\pi^+)}_{\mathfrak{s}',\mathfrak{s}}(k_1,p',q-k_2,p) - \left(\frac{k_{1\mu}}{(q,k_1)} - \frac{(p+p')_\mu}{(q,p+p')}\right)F_\pi(t_{\pi,1})\,\mathcal{M}^{(\pi^-)}_{\mathfrak{s}',\mathfrak{s}}(k_2,p',q-k_1,p)\Bigg]$$

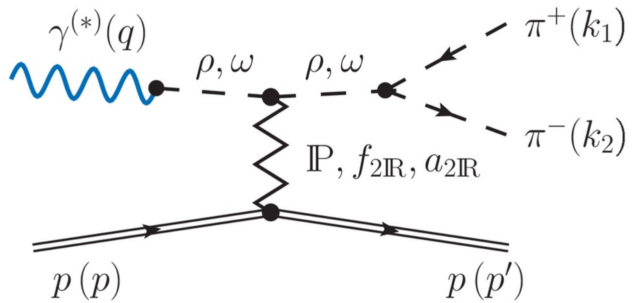
- Pumplin's prescription → 'ad hoc' ansatz not backed by any QFT calculation
- Significant difference between our approach and Pumplin's (model 2):



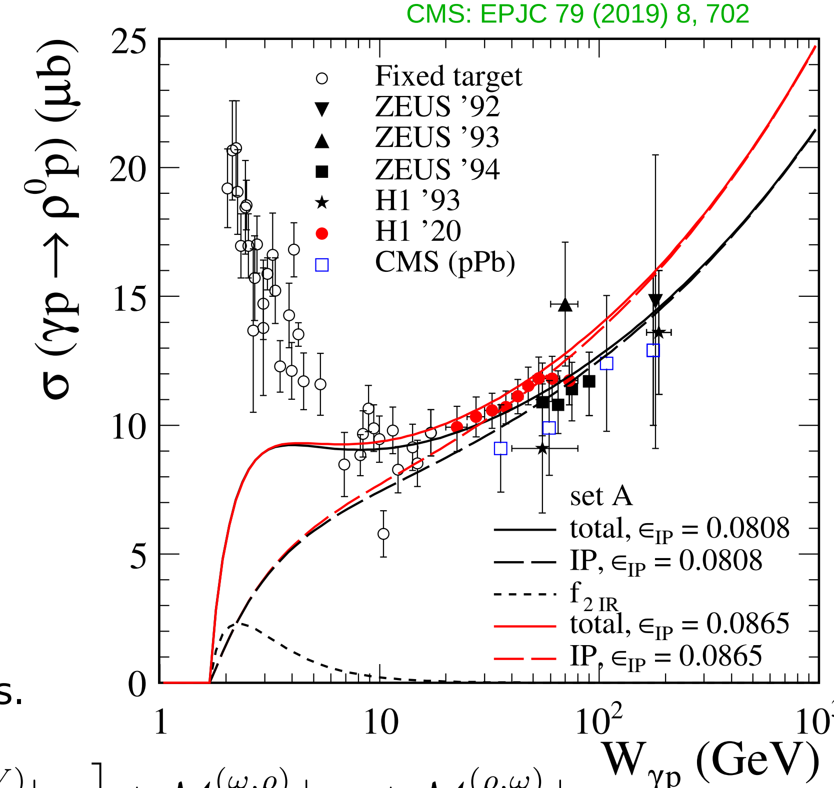


Resonant $\pi^+\pi^-$ production via ρ and ω scattering on the proton

ZEUS '94: EPJC 2 (1998) 247
H1 '20: EPJC 80 (2020) 12, 1189
CMS: EPJC 79 (2019) 8, 702



- Amplitudes includes IP and secondary C = +1 IR exchanges
- ρ-ω interference effect included only in the final state via propagator mixing and the explicit $\omega \to \pi^+\pi^-$ decay
- Numerical values of coupling constants occurring in vertices and vertex form factors were obtained from comparison of the model to experimental data; JHEP 01 (2015) 151, PRD 91 (2015) 074023, PRD 101 (2020) 094012.
- In JHEP 01 (2015) 151, a model for real photons was given. We extended the model for the case of slightly virtual photons.



$$\mathcal{M}^{(\mathrm{res})}_{\mu,\mathfrak{s}',\mathfrak{s}}(k_1,k_2,p',q,p)|_{\mathbb{P}+f_{2\mathbb{R}}+a_{2\mathbb{R}}} = \sum_{\substack{V=\rho,\omega,\\ V'=\rho,\omega}} \left[\mathcal{M}^{(V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}}|_{\mathbb{P}} + \mathcal{M}^{(V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}}|_{f_{2\mathbb{R}}}\right] + \mathcal{M}^{(\omega,\rho)}_{\mu,\mathfrak{s}',\mathfrak{s}}|_{a_{2\mathbb{R}}} + \mathcal{M}^{(\rho,\omega)}_{\mu,\mathfrak{s}',\mathfrak{s}}|_{a_{2\mathbb{R}}}$$

$$\mathcal{M}^{(V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}}|_{\mathbb{P}} = \frac{i}{4}\, e\, s\, F_1(t)\, F_M(t)\, \tilde{F}^{(V)}(k^2)\, g_{V'\pi\pi} \left[\mathcal{K}^{(0,V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}} V^{(0,V)}_{\mathbb{P}} - \mathcal{K}^{(2,V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}} V^{(2,V)}_{\mathbb{P}}\right] \tilde{F}^{(V)}(q^2) \underbrace{(-m_V^2)\Delta^{(V,V)}_T(q^2)}_{\cong \frac{m_V^2}{m_V^2 - q^2}}$$

$$\mathcal{K}^{(i,V',V)}_{\mu,\mathfrak{s}',\mathfrak{s}} = \frac{1}{s^2}(k_1-k_2)^\nu \Delta^{(V',V)}_T(k^2)\, \Gamma^{(i)}_{\nu\mu\kappa\lambda}(k,-q)\, \bar{u}_{\mathfrak{s}'}(p')\gamma^\kappa\, (p'+p)^\lambda\, u_{\mathfrak{s}}(p)$$

Melikhov, Nachtmann, Nikonov, Paulus, EPJC 34 (2004) 345

with two (i = 0 ,2) rank-four tensor functions

$$V^{(0,V)}_{\mathbb{P}} = \gamma_V^{-1}\, 6\, \beta_{\mathbb{P}NN}\, a_{\mathbb{P}VV}(-i\, s\, \alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

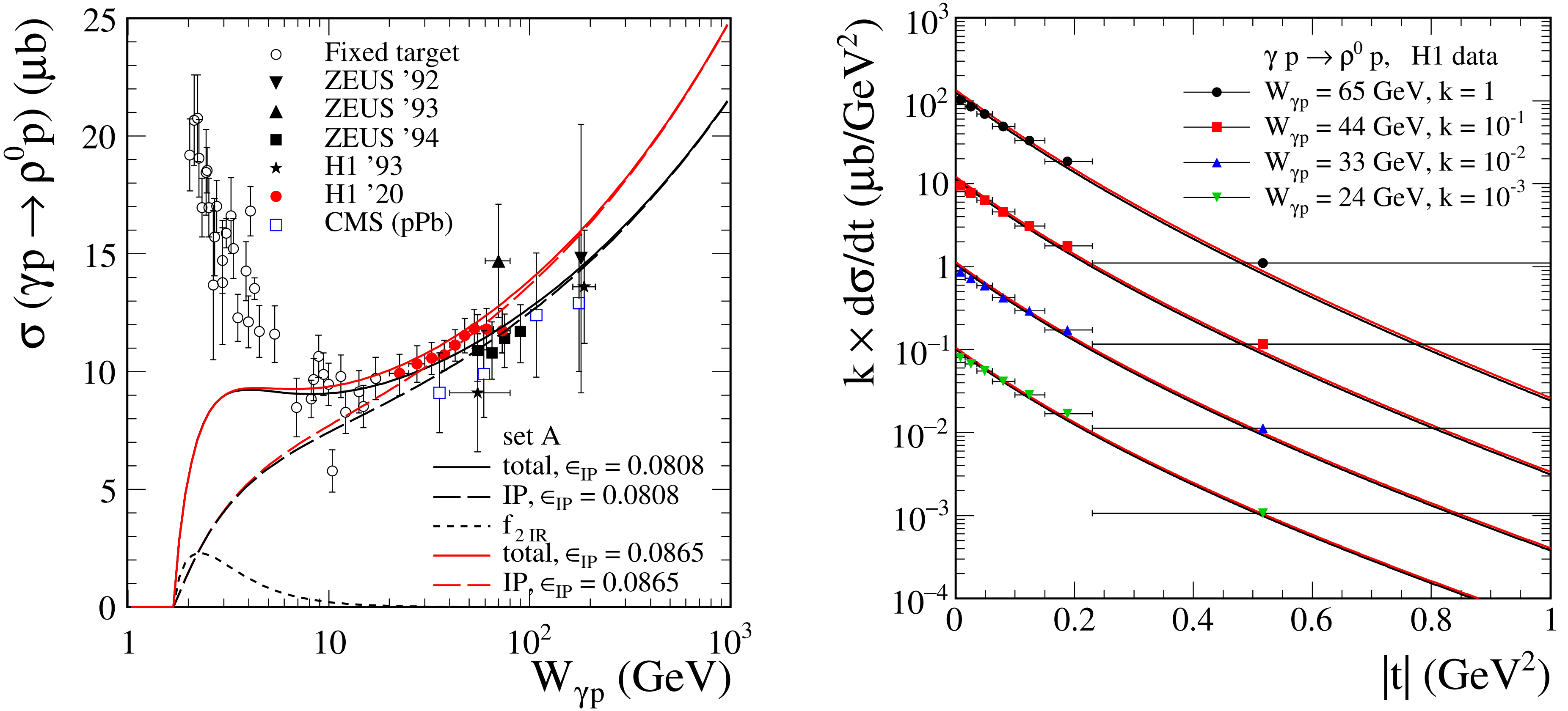
$$V^{(2,V)}_{\mathbb{P}} = \gamma_V^{-1}\, 3\, \beta_{\mathbb{P}NN}\, b_{\mathbb{P}VV}(-i\, s\, \alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$\tilde{F}^{(V)}(k^2) = \left[1 + \frac{k^2(k^2 - m_V^2)}{\Lambda_V^4}\right]^{-n_V}$$

$$2m_\rho^2\, a_{\mathbb{P}\rho\rho} + b_{\mathbb{P}\rho\rho} = 4\beta_{\mathbb{P}\pi\pi} = 7.04\ \mathrm{GeV}^{-1}$$

set A: $a_{\mathbb{P}\rho\rho} = 0.45\ \mathrm{GeV}^{-3}$ $\quad b_{\mathbb{P}\rho\rho} = 6.5\ \mathrm{GeV}^{-1}$

Comparison with H1 data

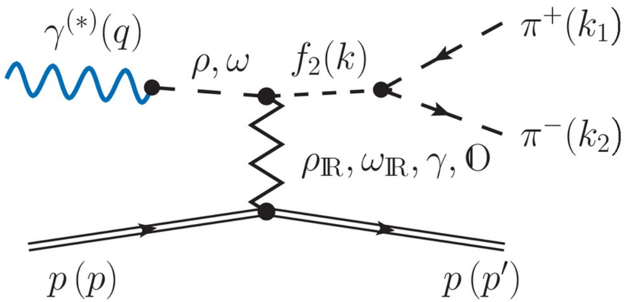


- At high energies the dominant contribution comes from the Pomeron (IP) exchange. For $W_{\gamma p} > 100$ GeV the cross section is completely dominated by the IP exchange.

- Calculations for two values of the ϵ_{IP} parameter in the Pomeron trajectory are shown.

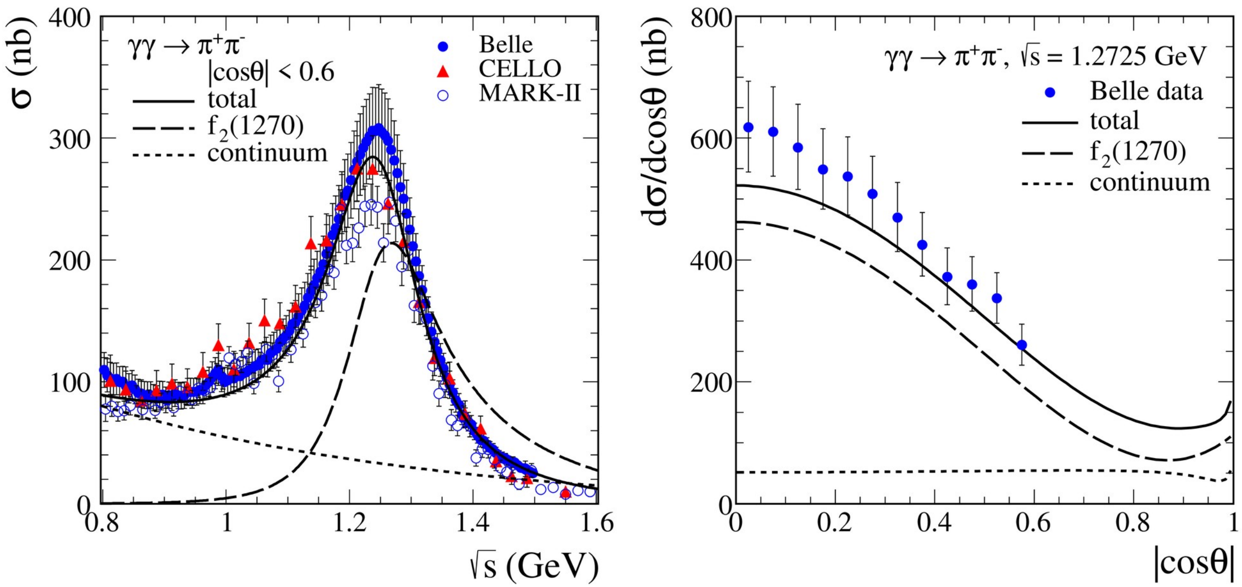
- In the right panel: the differential cross sections $d\sigma/dt$ together with the H1 data. Here the results are scaled by a factor k for displaying purposes.

Resonant $\pi^+\pi^-$ production via $f_2(1270)$

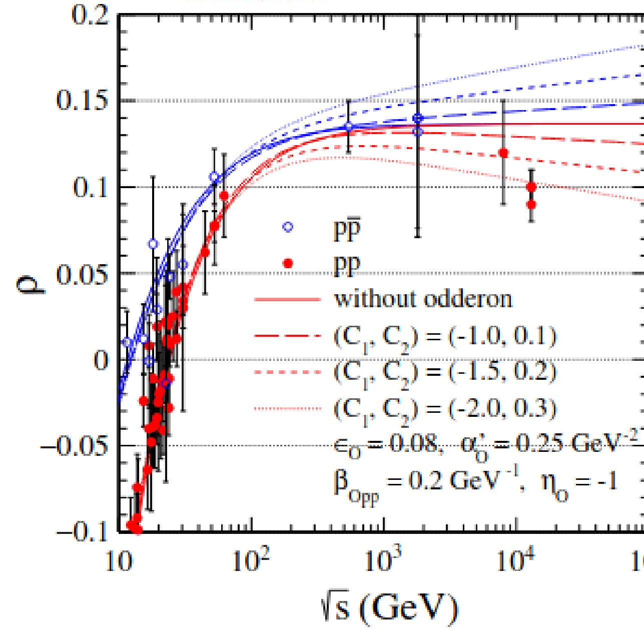


- Production of f_2 resonance can occur by the C = -1 reggeons and Odderon considered as effective vector exchanges, and by the photon (Primakoff effect)
- $f_2\pi\pi$ and $f_2\gamma\gamma$ coupling constants and form factors with cut-off parameters were estimated by comparing model results for the $\gamma\gamma \to \pi^+\pi^-$ reaction with the Belle data
- For the coupling parameters of $V_{I\!R}Vf_2$ we have assumed here that are the same size as VVf_2
- From study of *pp* and $p\bar{p}$ scattering [PRD 106 (2022) 034023] we found that a double-pole ansatz for the Odderon seems to be preferred → a better description of TOTEM data for ρ = 0.1

Belle data: T. Mori *et al.*, J. Phys. Soc. Jap. 76 (2007) 074102

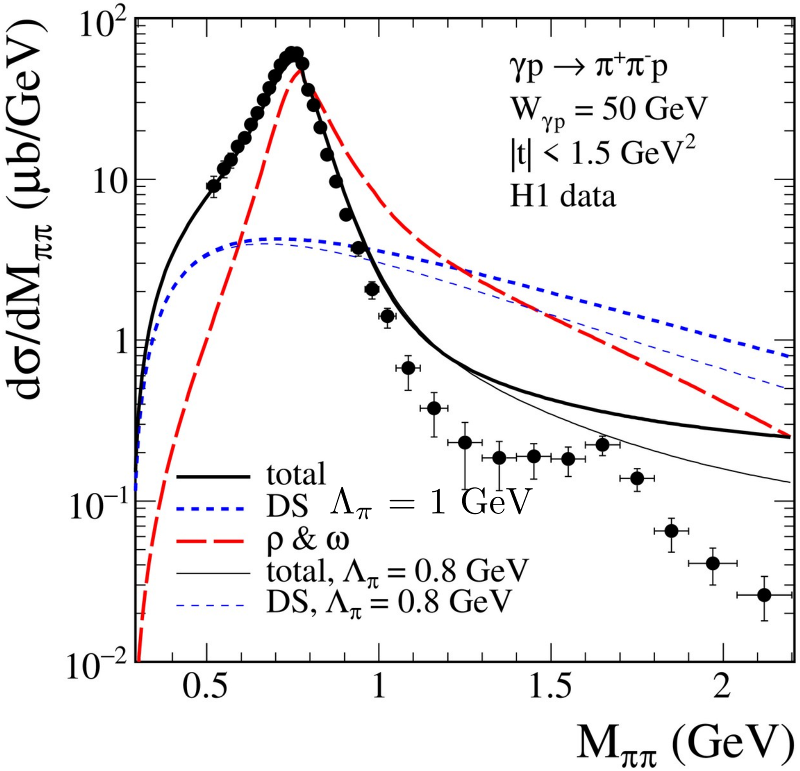


$$\rho = \frac{\mathrm{Re}\mathcal{T}(s,0)}{\mathrm{Im}\mathcal{T}(s,0)}$$



Lebiedowicz, Nachtmann, Szczurek, PRD 106 (2022) 034023

Comparison with H1 data

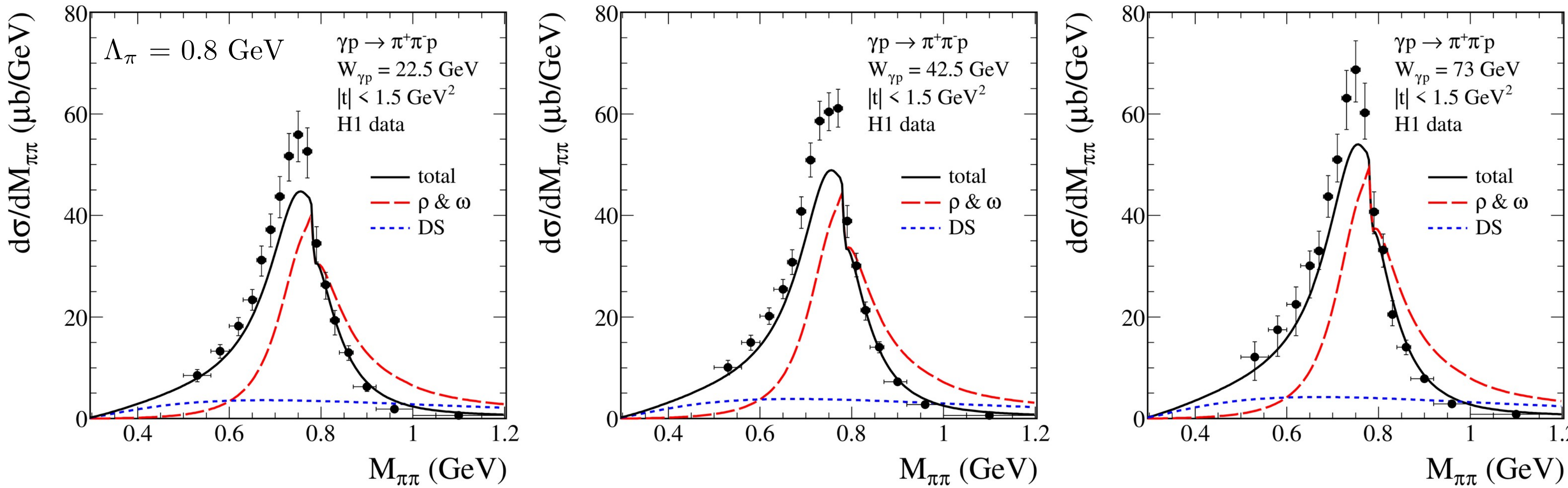


Data from Andreev et al. (H1 Collaboration), Eur. Phys. J. C80 (2020) 1189

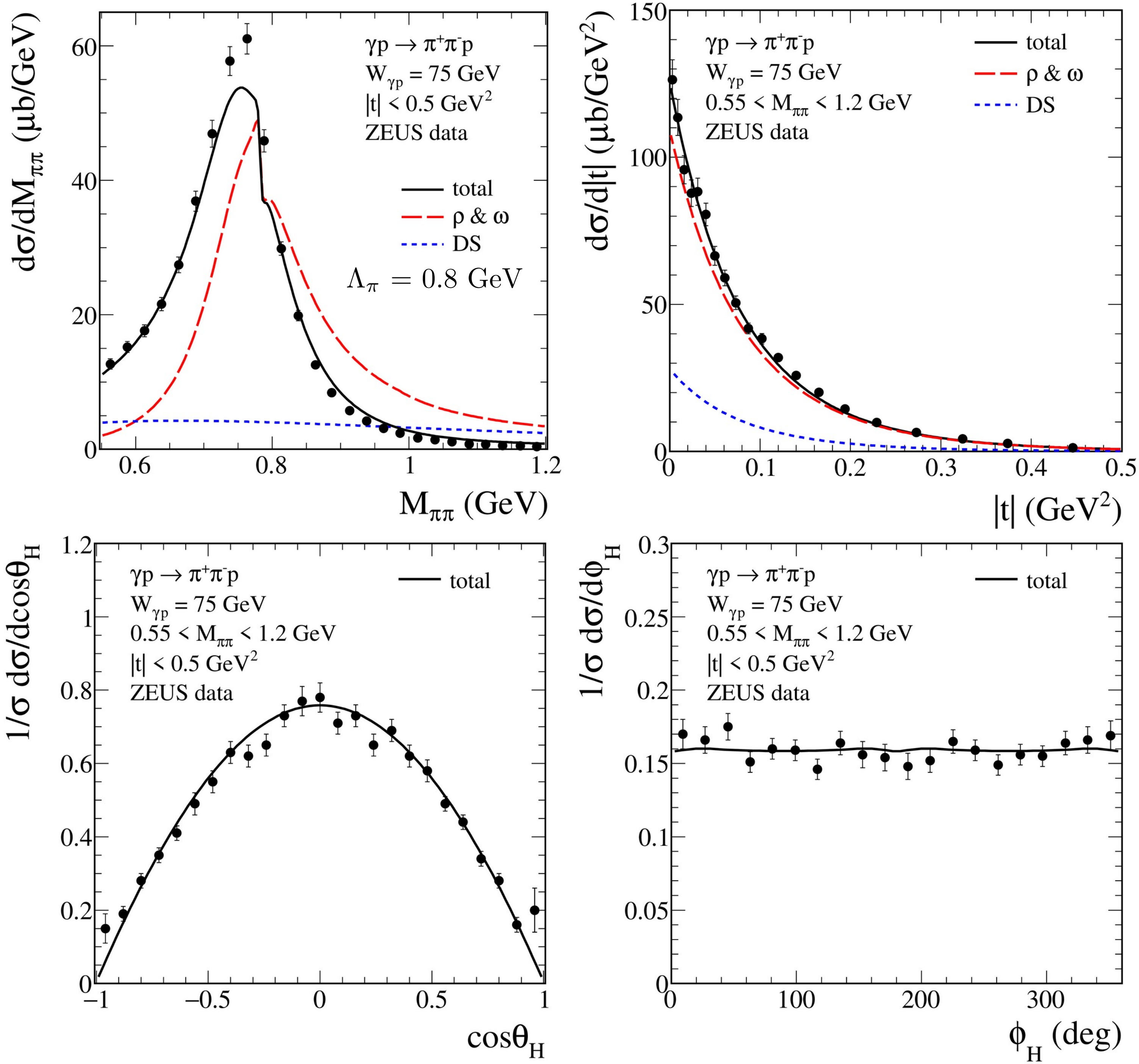
Data were measured in the kinematic region
20 < $W_{\gamma p}$ < 80 GeV and |t| < 1.5 GeV2,
in the bottom panels for 20 < $W_{\gamma p}$ < 25 GeV,
40 < $W_{\gamma p}$ < 45 GeV, and 66 < $W_{\gamma p}$ < 80 GeV.
Our calculations are for <$W_{\gamma p}$>.

- Tuning of model parameters (form factors) needed, especially at higher $M_{\pi\pi}$
- $f_2(1270)$ contribution is negligible
- The structure visible at $M_{\pi\pi}$ ~1.5 GeV provides evidence for the existence of the ρ(1450) and ρ(1700) mesons (BaBar, LHCb)

- possible better description using: (1) $\epsilon_{\mathbb{P}} = 0.0808 \to 0.0865$ or (2) $b_{\mathbb{P}\rho\rho} = 6.5\,\mathrm{GeV}^{-1} \to 7.04\,\mathrm{GeV}^{-1}$



Comparison with ZEUS data



Data from Breitweg et al. (ZEUS Collaboration), Eur. Phys. J. C2 (1998) 247

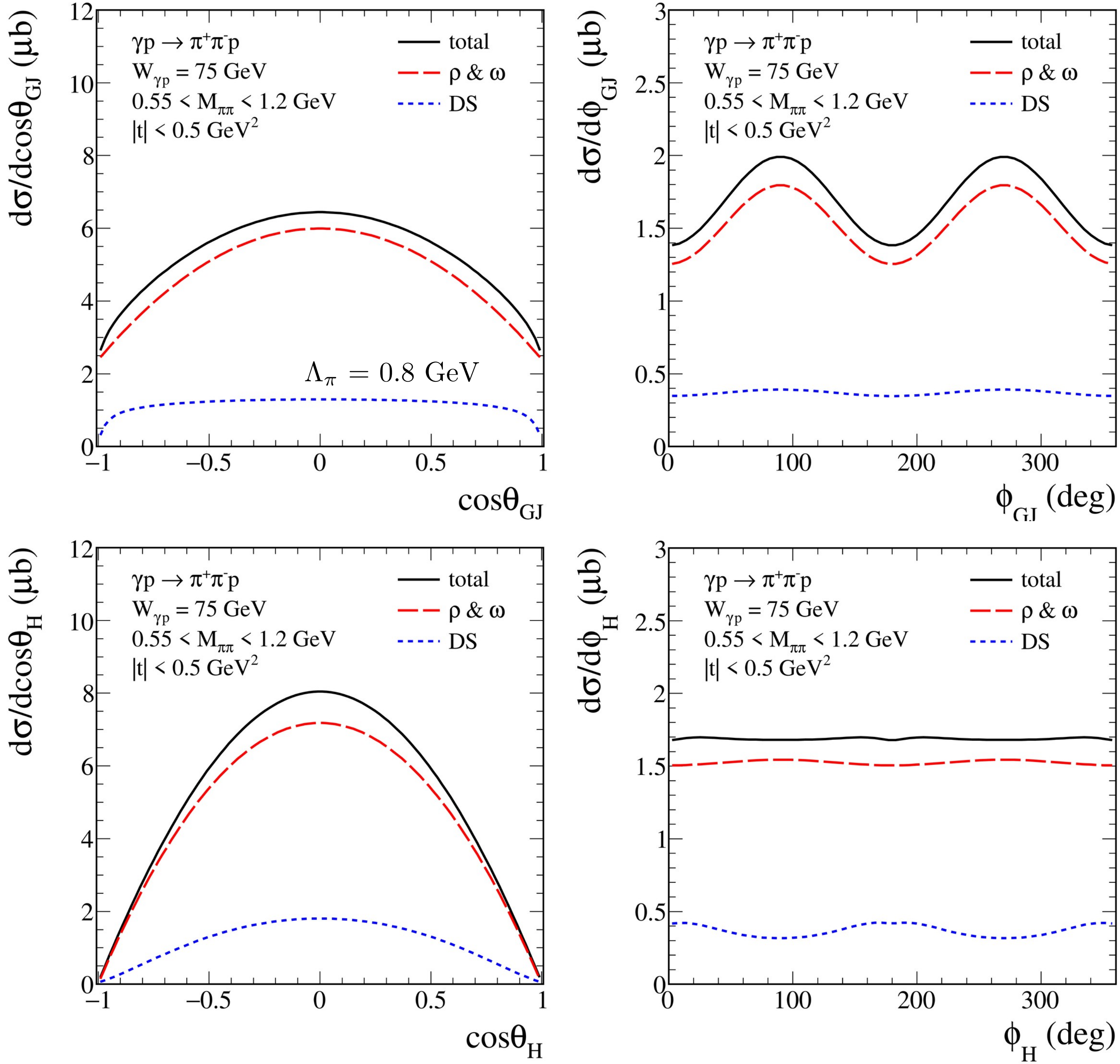
Data were measured in the kinematic region $50 < W_{\gamma p} < 100$ GeV, $|t| < 0.5$ GeV^2, $0.55 < M_{\pi\pi} < 1.2$ GeV.

Our results are for $W_{\gamma p} = 75$ GeV.

In the helicity frame, the direction of the recoiling proton (p') defines the negative z-axis.

In the photon-Jackson frame (GJ), the direction of the photon is parallel to the z-axis.

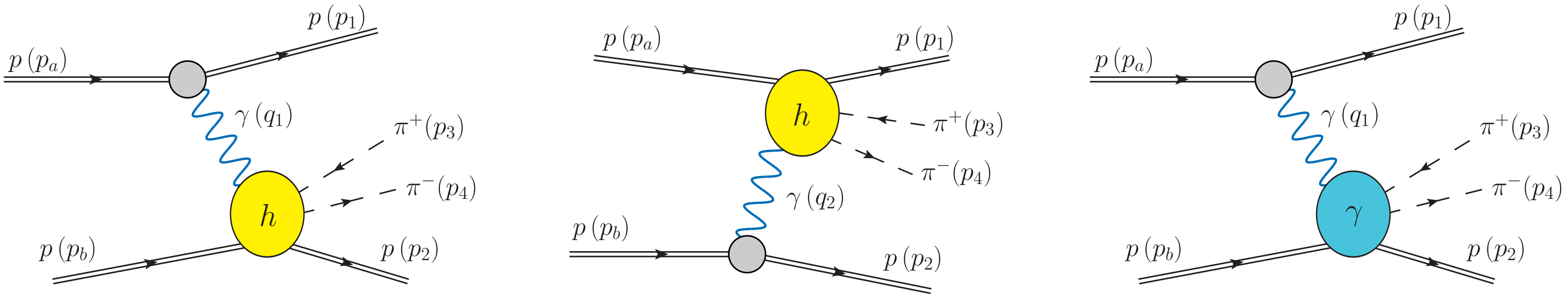
Results



γp → π⁺π⁻p
W_γp = 75 GeV
0.55 < M_ππ < 1.2 GeV
|t| < 0.5 GeV²
total
ρ & ω
DS
Λ_π = 0.8 GeV
dσ/dcosθ_GJ (μb)
cosθ_GJ
dσ/dφ_GJ (μb)
φ_GJ (deg)
dσ/dcosθ_H (μb)
cosθ_H
dσ/dφ_H (μb)
φ_H (deg)

$pp \to pp\ \pi^+\pi^-$

$$p(p_a, \lambda_a) + p(p_b, \lambda_b) \to p(p_1, \lambda_1) + \pi^+(p_3) + \pi^-(p_4) + p(p_2, \lambda_2)$$



Complete amplitude for central exclusive photoproduction of $\pi^+\pi^-$ pairs:

$$\mathcal{M}_{pp\to pp\pi^+\pi^-} = \mathcal{M}^{(\gamma h)}_{pp\to pp\pi^+\pi^-} + \mathcal{M}^{(h\gamma)}_{pp\to pp\pi^+\pi^-} + \mathcal{M}^{(\gamma\gamma)}_{pp\to pp\pi^+\pi^-}$$

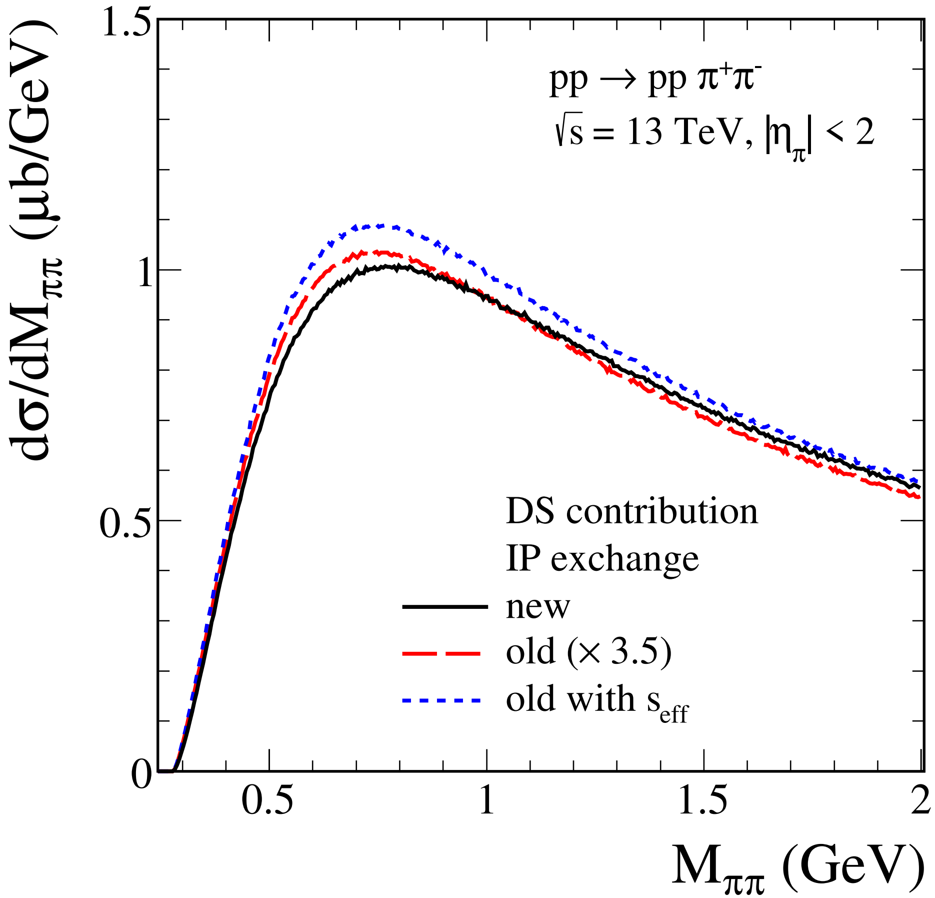
$$\mathcal{M}^{(\gamma h)}_{pp\to pp\pi^+\pi^-} = \bar{u}(p_1)\Gamma^{(\gamma pp)\mu}u(p_a)\frac{1}{t_1}\Big(\mathcal{M}^{(\mathrm{res})}_{\mu}|_{\mathbb{P}+f_{2\mathbb{R}}+a_{2\mathbb{R}}} + \mathcal{M}^{(f_2)}_{\mu}|_{\rho_{\mathbb{R}}+\omega_{\mathbb{R}}+\mathbb{O}} + \mathcal{M}^{(\mathrm{DS})}_{\mu}|_{\mathbb{P}+f_{2\mathbb{R}}+\rho_{\mathbb{R}}}\Big)$$

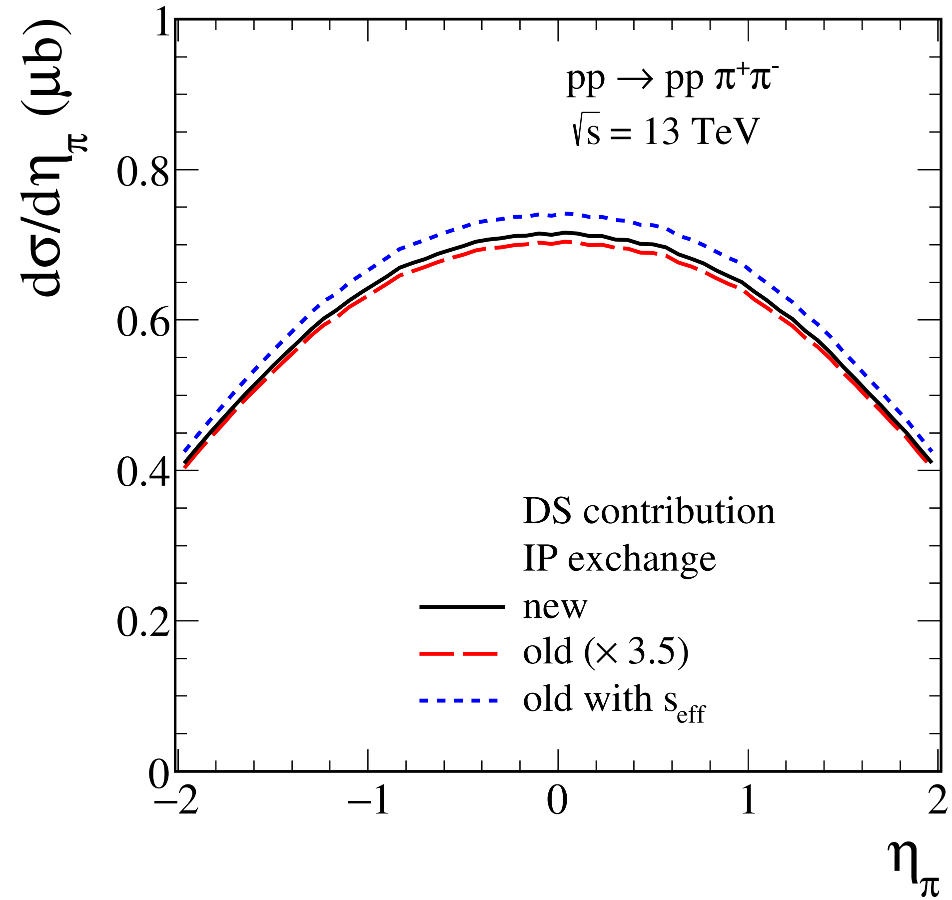
$$\mathcal{M}^{(\gamma\gamma)}_{pp\to pp\pi^+\pi^-} = \bar{u}(p_1)\Gamma^{(\gamma pp)\mu}u(p_a)\frac{1}{t_1}\Big(\mathcal{M}^{(f_2)}_{\mu}|_{\gamma} + \mathcal{M}^{(\mathrm{DS})}_{\mu}|_{\gamma}\Big)$$

- Eventually we should also include absorption corrections due to the proton-proton interactions to the Born amplitudes above. This absorption reduces the cross section for photoproduction processes by about 10% at LHC energies. These effects depend on the kinematic conditions in a particular experiment.

- We consider the kinematic regime where at least one photon exchange is involved, that is, at least one proton gets only a very small deviation. We neglect purely diffractive IP-IP, IP-IR, and IR-IR contributions that were discussed, e.g., in PRD93 (2016) 054015.

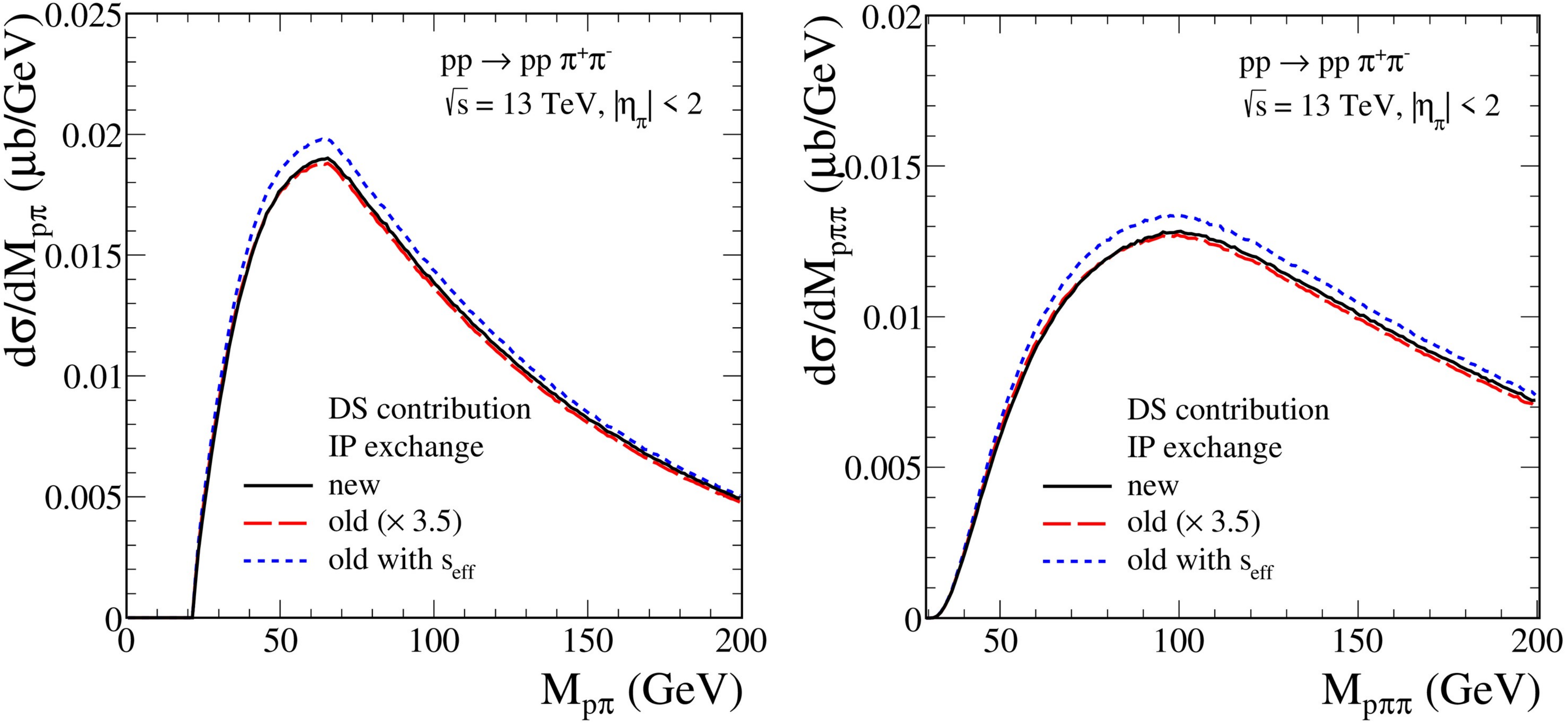
Results





- Cross section for our new Drell-Söding (DS) contribution [JHEP 06 (2026) 015] is larger by a factor of 3.5 compared to old result [PRD91 (2015) 074023] with a common energy variable ($s_{eff} = s = M^2_{p\pi\pi}$) in the Pomeron propagator.
- In order to improve the old model, one can use $s_{eff} = s/2$ instead of s. This procedure leads to description of data for the $\gamma p \to \pi^+\pi^- p$ reaction measured by the H1 Collaboration.
- In these calculations for $pp \to pp\pi^+\pi^-$ reaction, we have neglected the off-shellness of the pions, $F_\pi(t_\pi) = 1$.

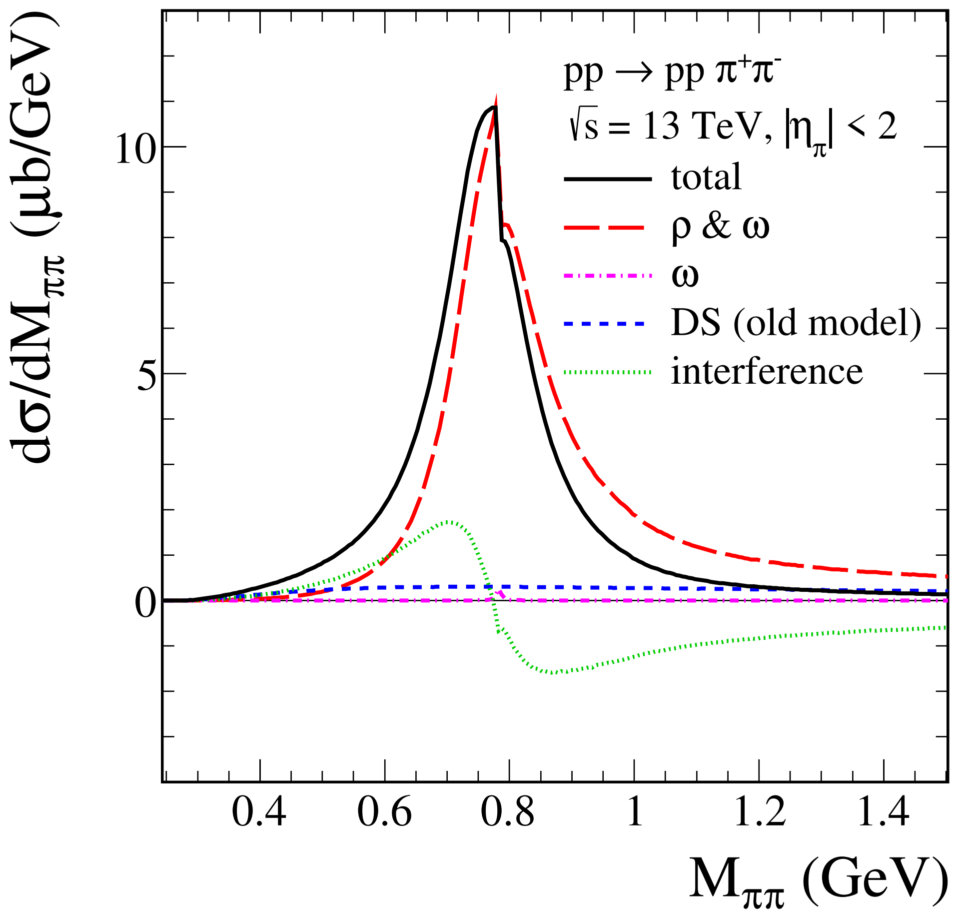
Results



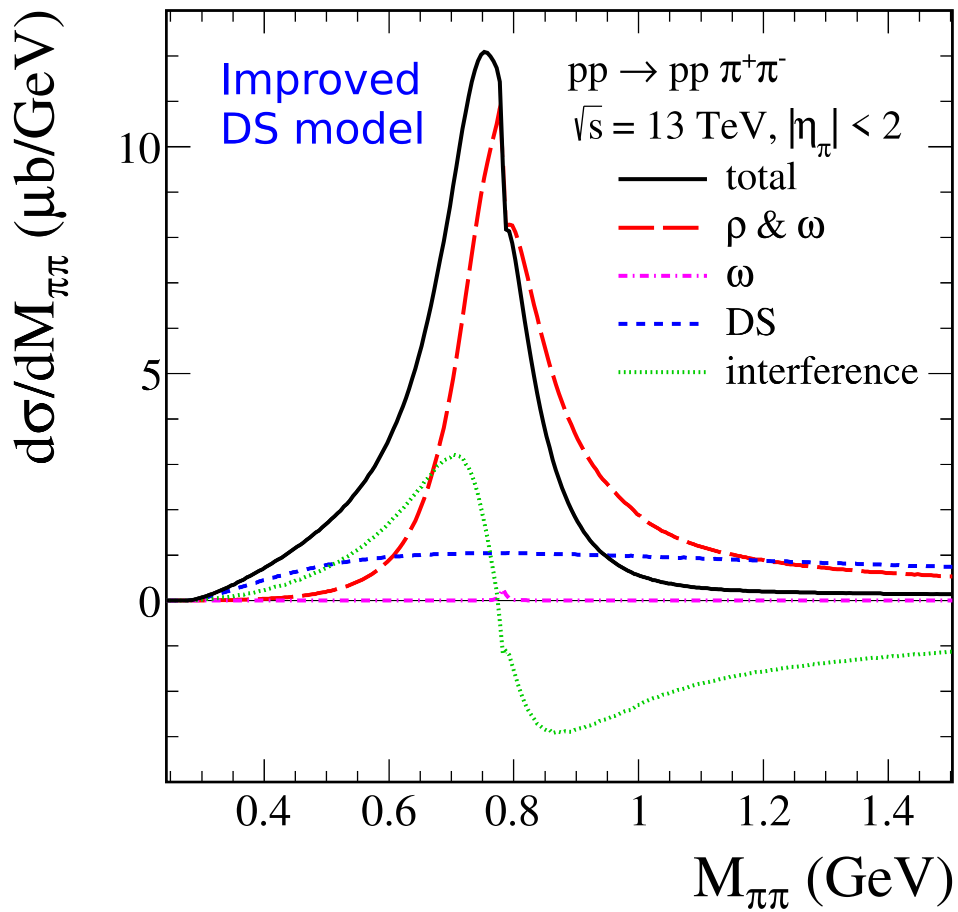
- Predictions of proton-pion and proton-pion-pion invariant mass distributions.

Results

PRD91 (2015) 074023

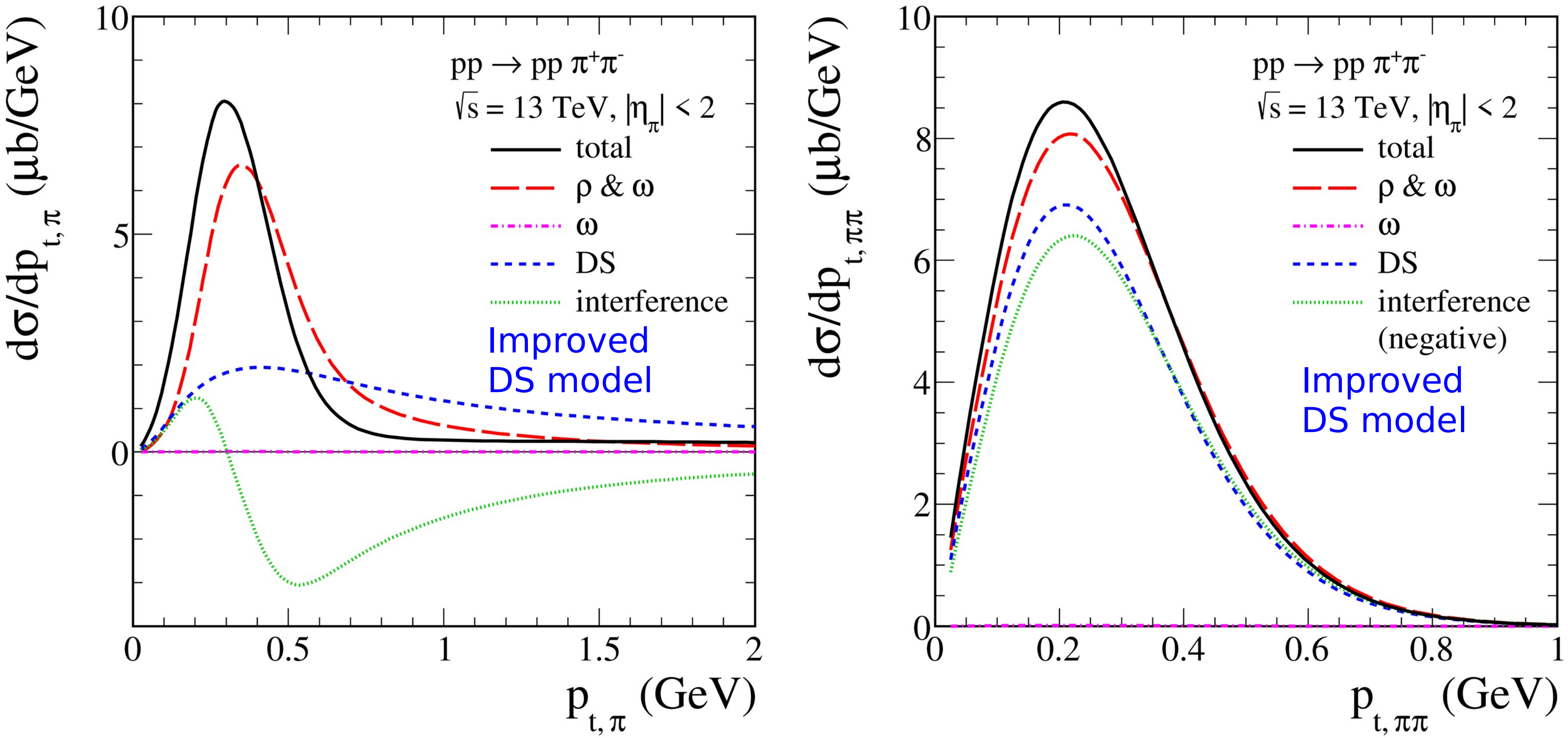


JHEP 06 (2026) 015



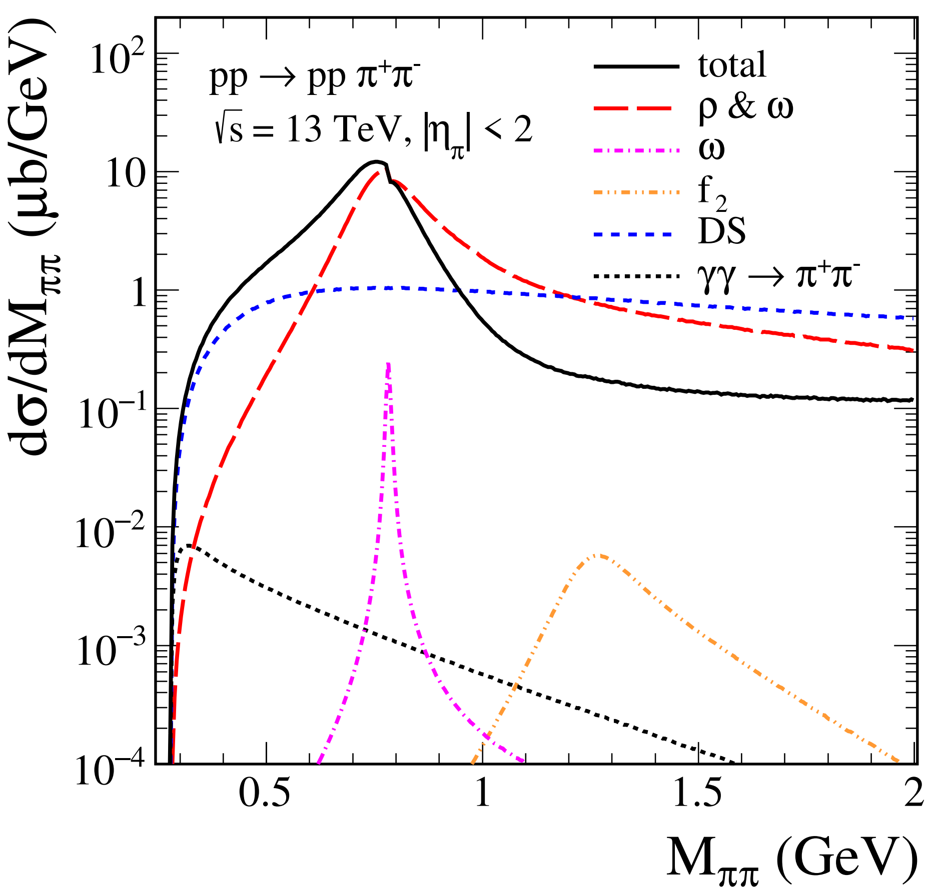
- Improved DS model leads to enhanced cross section and gives an increased skewing of the ρ^0 spectral shape (caused by the interference of the ρ^0 and $\pi^+\pi^-$ continuum)
- *ρ-ω* interference effect is also clearly exposed

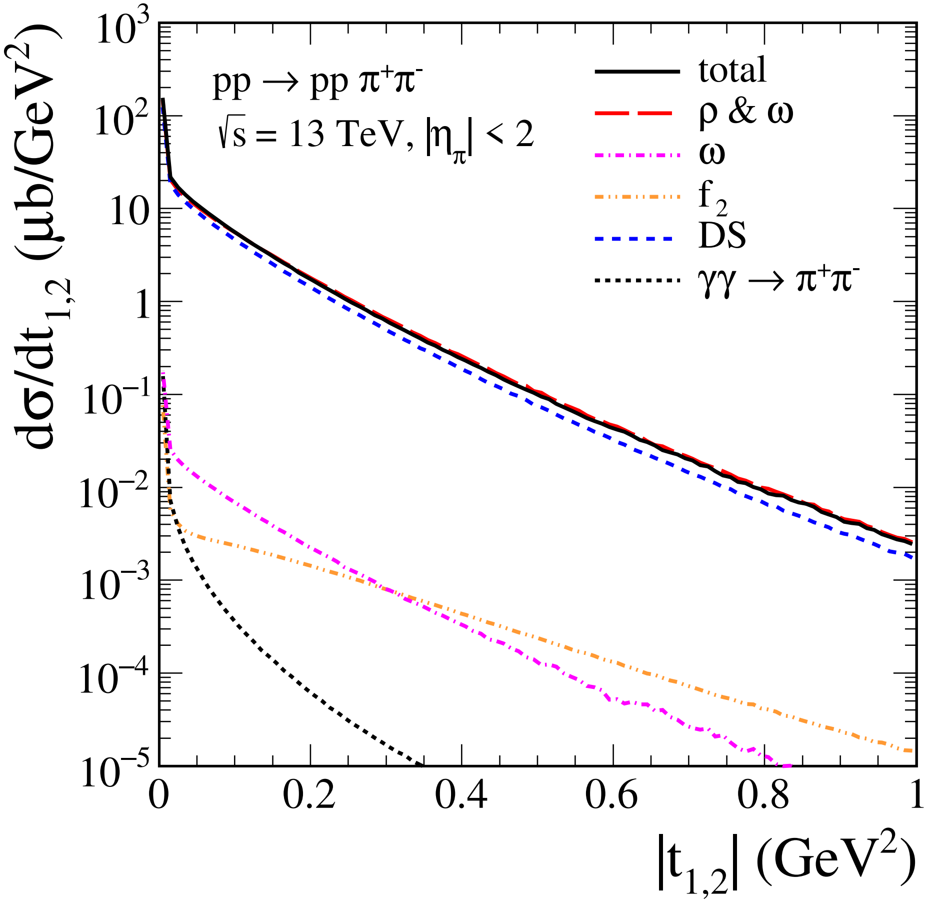
Results



- Distributions in transverse momentum of the pion and in transverse momentum of the $\pi^+\pi^-$ pair.

Results



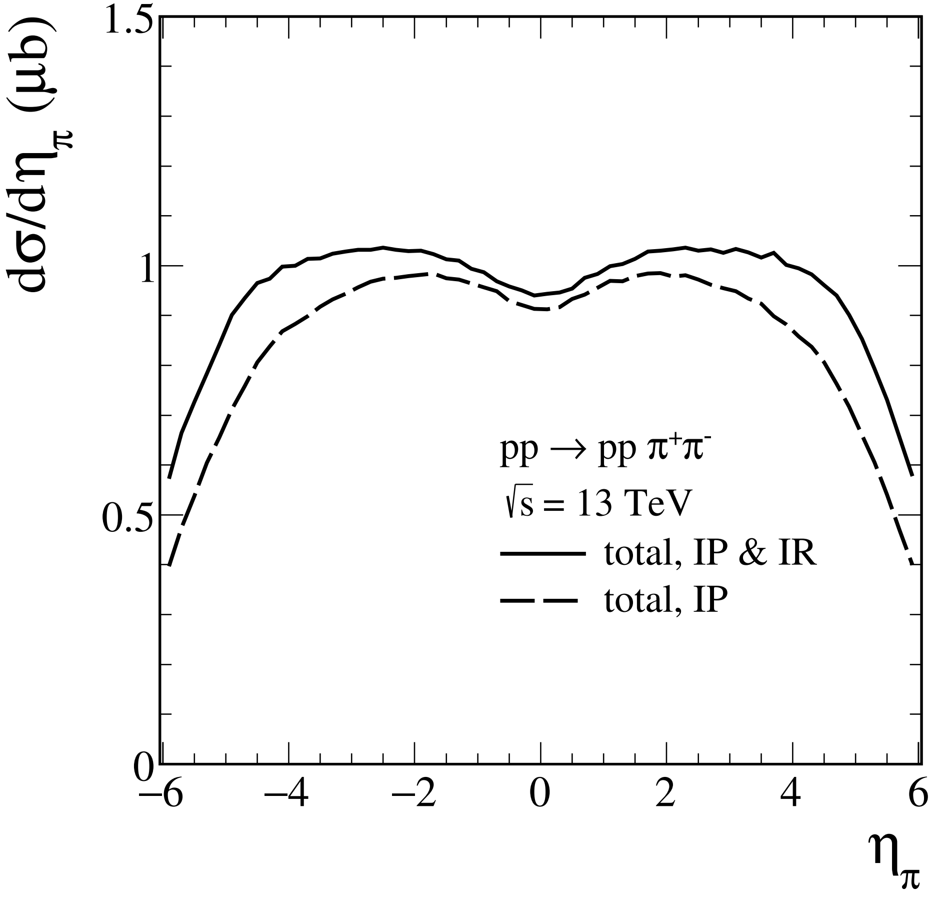


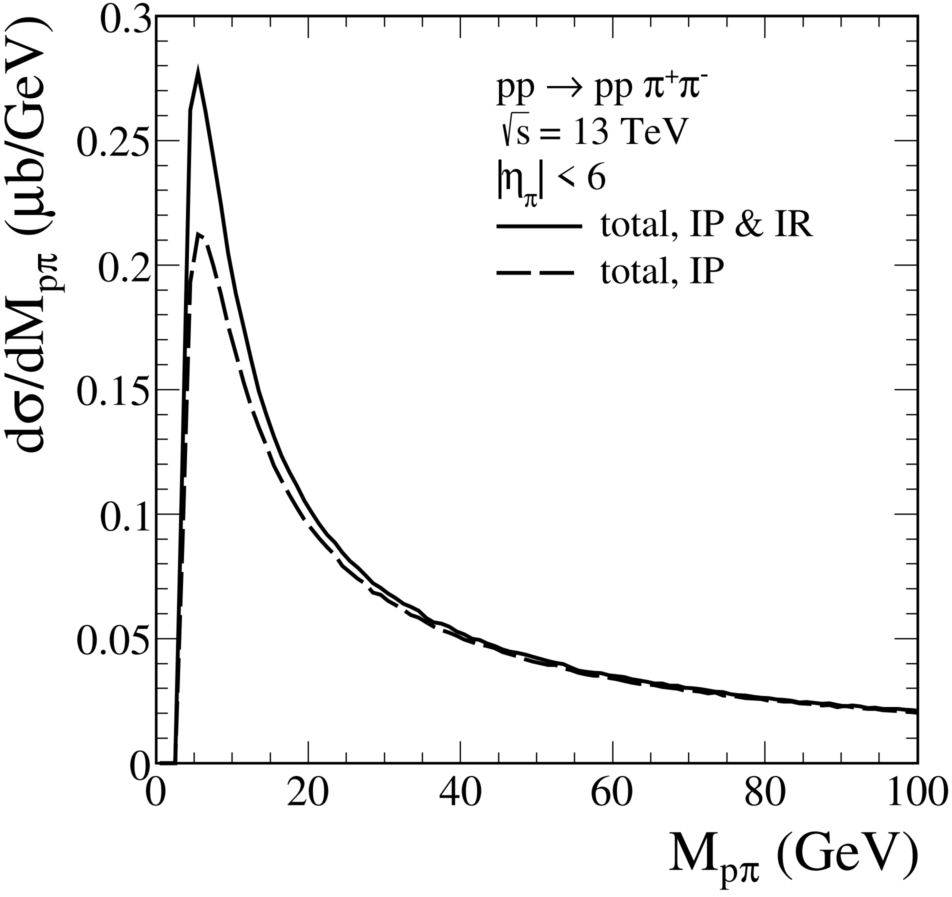
- Complete result (total) and the resonant and non-resonant individual contributions.
- $f_2(1270)$ production is more than 3 orders of magnitude smaller than $\rho(770)$.
  The production of higher-mass vector mesons decaying into $\pi^+\pi^-$ could easily be added here.

- In the DS term, the off-shell pion form-factor should also be introduced.

- Non-resonant DS contribution ($\gamma\gamma \to \pi^+\pi^-$) is very small.
  The distributions in $t_{1,2}$ are strongly peaked at very small $|t_{1,2}|$. This is caused by the factors $1/t_{1,2}$ from the photon propagators.

Results





- Secondary reggeon exchanges contribute mainly at backward and forward pion pseudorapidity regions that correspond to low proton-pion invariant mass regions.

  The cut at $|\eta_{\pi}| < 2$ eliminates small proton-pion subenergies and we have $M_{p\pi} > 20$ GeV.

Summary

- We have proposed (Lebiedowicz, Nachtmann, Szczurek, JHEP 06 (2026) 015) a new model for the non-resonant (Drell-Söding, DS) contribution to the reactions:

$$\gamma^{(*)} + p \to \pi^+ + \pi^- + p\,,$$

$$p + p \to p + \pi^+ + \pi^- + p\,.$$

- The calculations have been done within the tensor-pomeron approach including the secondary reggeon exchanges. Complete model incorporates both non-resonant and resonant ($\rho(770)$, ω, f_2) contributions.

- For the calculations of DS contribution, the correct Regge variable 2ν was used instead of the squared energy variable s. We have obtained from the gauge invariance relation a solution for the diagram (c) which is satisfactory from the QFT point of view. This improves results presented in JHEP 01 (2015) 15 for real photoproduction of $\pi^+\pi^-$ pairs and in PRD 91 (2015) 074023 for the reaction $pp \to pp\pi^+\pi^-$ (revised DS model gives a larger cross section by a factor of 3.5).

  The $\rho(770)$ + DS interference effect is more pronounced and leads to larger skewing of the observed spectral shape of $\rho(770)$.

- We have presented a preliminary comparison of improved model with some selected results from H1 and ZEUS experimental data. We invite the experimentalists to make a detailed comparison of the data from HERA to our present model.

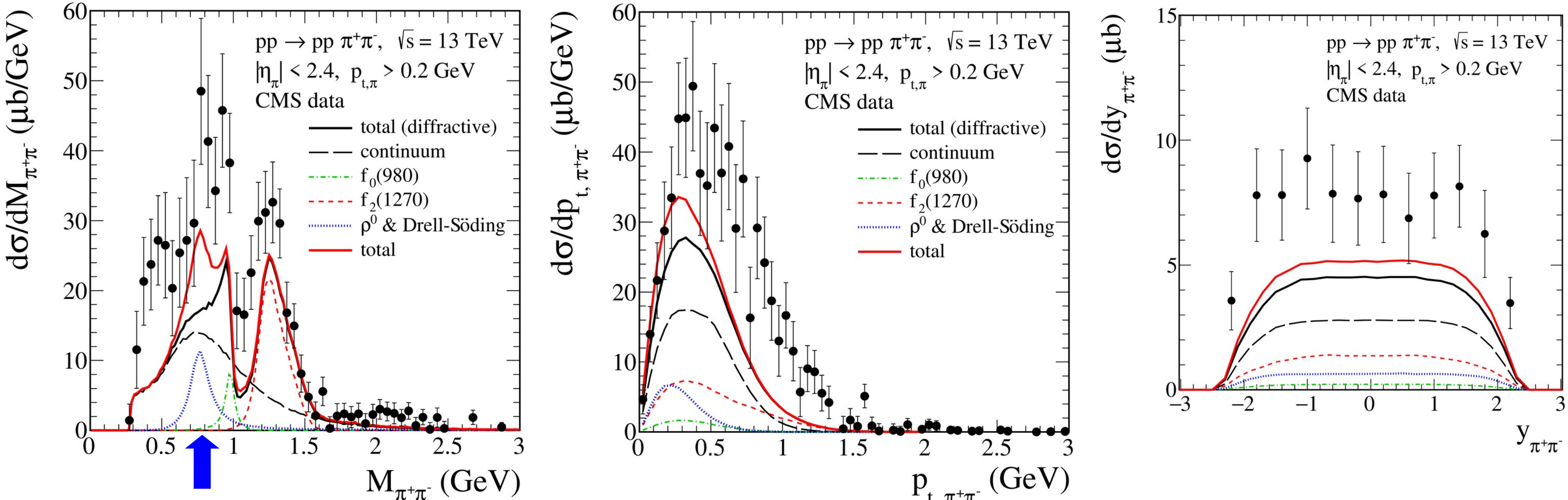
  Tuning of model parameters (form factors) needed, especially at higher $M_{\pi\pi}$.
  The contributions of ρ(1450) and ρ(1700) should be taken into account.

- Our findings should be relevant for the measurements of $pp \to pp\pi^+\pi^-$ reaction by the ALICE, ATLAS, CMS, and LHCb Collaborations at the LHC, when the leading protons are not detected and only rapidity-gap conditions are checked, and for coherent $\pi^+\pi^-$ photoproduction in ultra-peripheral pA and AA collisions. Future measurements at the electron-ion colliders (EIC and LHeC) would be very helpful to improve our understanding of non-perturbative processes.

Thank you for your attention !

$pp \to pp\, \pi^+\pi^-$

- **Comparison to CMS data, Eur. Phys. J. C 80 (2020) 718**
  → rapidity gap method (gaps between the $\pi^+\pi^-$ system and the outgoing protons) – no proton tagging
  This measurement is not fully exclusive → the data include, to some extent, a contribution related to the dissociation of one or both protons.

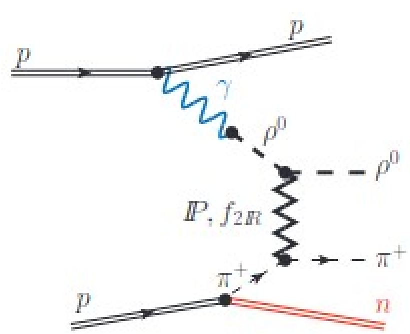


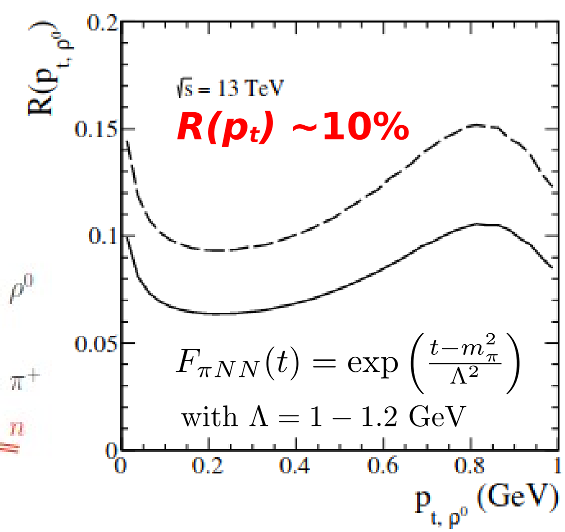
photoproduction contribution

- PL, Nachtmann, Szczurek, PRD95 (2017) 034036
  Drell-Hiida-Deck type mechanism with centrally produced ρ^0 associated with a very forward/backward πN system

$$\text{Plotted is } R(p_{t,\rho^0}) = \frac{d\sigma_{pp\to pN\rho^0\pi}/dp_{t,\rho^0}}{d\sigma_{pp\to pp\rho^0}/dp_{t,\rho^0}}$$

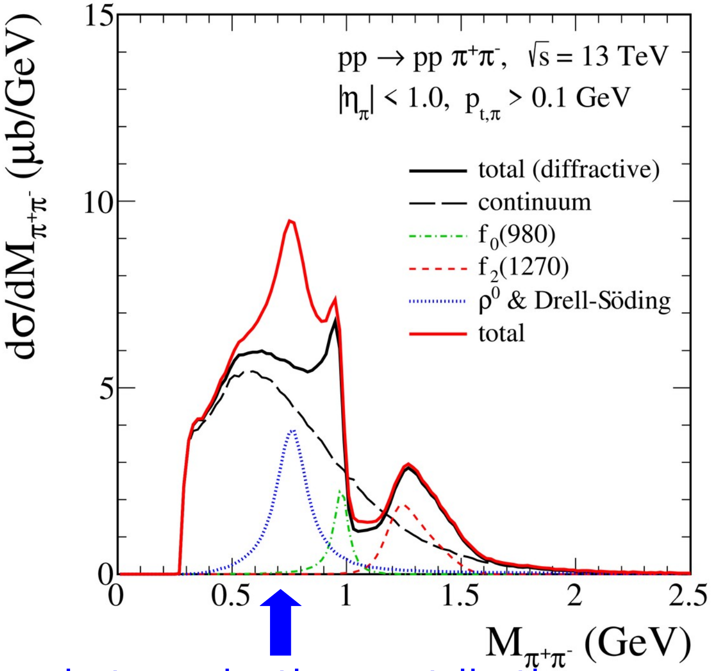
where $pN\rho^0\pi$ stands for $pn\rho^0\pi^+$ and $pp\rho^0\pi^0$





$pp \to pp\, \pi^+\pi^-$

PRELIMINARY RESULTS



photoproduction contribution