MESON 2026

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# Scattering information from production experiments


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# Hadron-hadron scattering

- Resonances live in (coupled-channel) scattering, but scattering experiments (beam, target) are exceptional;
  - need production experiments: phase (shifts), ERE parameters
- Probability conservation ⇒ Unitarity of the S matrix ⇒ partial-wave 2-body T-matrix element

$$\mathrm{Im}\, T_L^{-1}(s) = -\rho(s)\Theta(s - s_{\mathrm{thr.}}), \quad \rho(s) \equiv \frac{k}{8\pi\sqrt{s}}$$

  - Effective range expansion (ERE) H. Bethe (1949)

$$T_L(s) = \frac{e^{i\delta_L(s)}\sin\delta_L(s)}{\rho(s)} = \frac{8\pi\sqrt{s}}{k\cot\delta_L(s) - ik}, \qquad k\cot\delta_L(s) = k^{-2L}\left(\frac{1}{a_L} + \frac{1}{2}r_L k^2 + \ldots\right)$$

    - Convergence radius: Taylor expansion ⇒ set by the singularity closest to the threshold, e.g.,
      - ✧ branch point of the left-hand cut from particle-exchange or amplitude zero; M.-L. Du et al., PRL 131 (2023) 131903; talk by S. Dawid

        for an extension overcoming this problem, M.-L. Du, FKG, B. Wu, PRL 135 (2025) 011903

        modification due to Adler zero M.-L. Du, FKG, C. Hanhart, F. Herren, B. Kubis, R. van Tonder, EPJC 85 (2025) 1289
      - ✧ momentum scale from the next neglected threshold
    - For S-wave, the most important low-energy scattering observables:

$a_0 = \frac{T_L(s=s_{\mathrm{thr.}})}{8\pi(m_1+m_2)}$: scattering length, ∝ interaction strength at the threshold

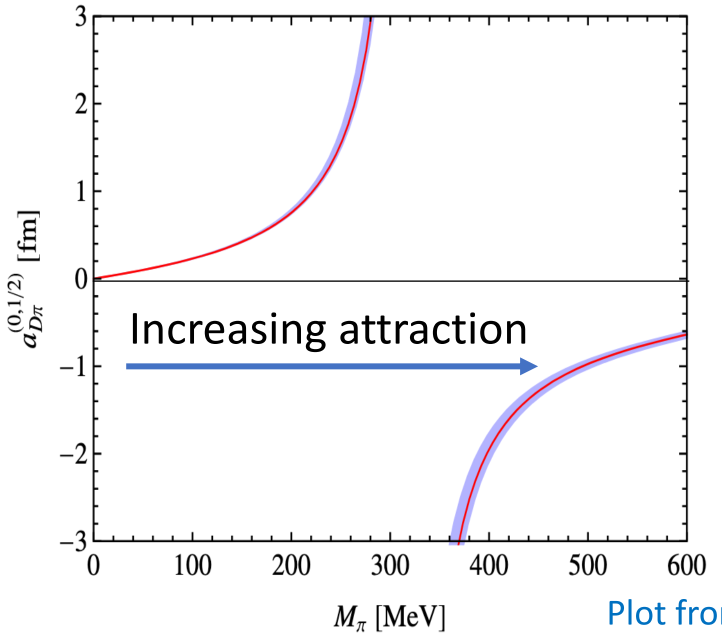
$r_0$: effective range

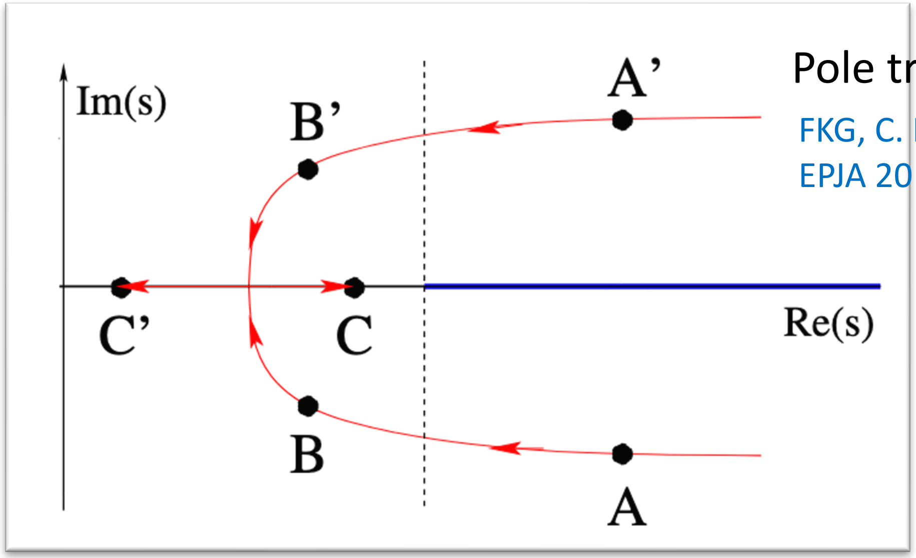
# Scattering length

- Scattering length approximation: $T_0^{-1} \propto \frac{1}{a_0} - ik$, pole at $k = -i/a_0$
  - $a_0 < 0$: bound state pole, or repulsive
  - $a_0 > 0$: attractive insufficient to bind, can be near-threshold virtual state pole
  - Unitarity limit: $|a| = \infty$, zero-energy bound state at the threshold
  - NN scattering lengths: $a_{3S1} = -5.4$ fm (deuteron), $a_{1S0} = -23.7$ fm (virtual state)
  - Important information about the nature of pole: Weinberg's compositeness relation

Talk by C. Hanhart

- Increasing attraction strength (e.g., by adjusting the quark masses), $a_0$ can change sign




Pole trajectory

FKG, C. Hanhart, U.-G. Meißner, EPJA 20 (2009) 179

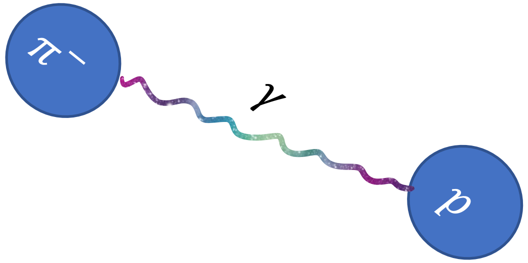
Plot from L. Liu, K. Orginos, FKG, C. Hanhart, U.-G. Meißner, PRD 87 (2013) 014508

# Scattering length measurements

- Ideal case, cross section at the threshold: $\sigma = 4\pi a^2$, e.g., $np$, $nA$
- Measuring phase shift, constraining low-energy amplitudes: $a_0 = \lim_{k\to 0} \frac{\tan\delta_0(k)}{k}$, e.g.,
  - $pp$ (subtracting Coulomb effect)
  - Isoscalar $\pi\pi$, from $K \to \pi^+\pi^-\ell\nu_\ell$
    - Watson's final-state interaction (FSI) theorem: $\delta_L = \text{phase of form factor}$ — K. M. Watson (1952)
    - Measure phase shift difference $\delta_0^0 - \delta_1^1$, + Roy eq. ⇒ the most precise extraction of $\pi\pi$ isoscalar $a_0$ — G. Colangelo et al., NPB 603 (2001) 125; NA48/2, EPJC 70 (2010) 635; …
- Hadronic atoms — For a review, see J. Gasser, A. Rusetsky, Phys.Rept. 456 (2008) 167
  - Strong interaction ($a_0$) induced energy shift from Coulomb binding energy
  - Decay width
  - Deser-Goldberger-Baumann-Thirring (DGBT) formula — S. Deser, M. Goldberger, K. Baumann, W. Thirring (1954)

$$\Delta E_1^{\text{str}} - \frac{i}{2}\Gamma_1 = -\frac{2\pi}{\mu}\underbrace{\left|\tilde{\Psi}_{10}(0)\right|^2}_{=\alpha^3\mu^3/\pi} a_0 + O\left(\alpha^4\right)$$

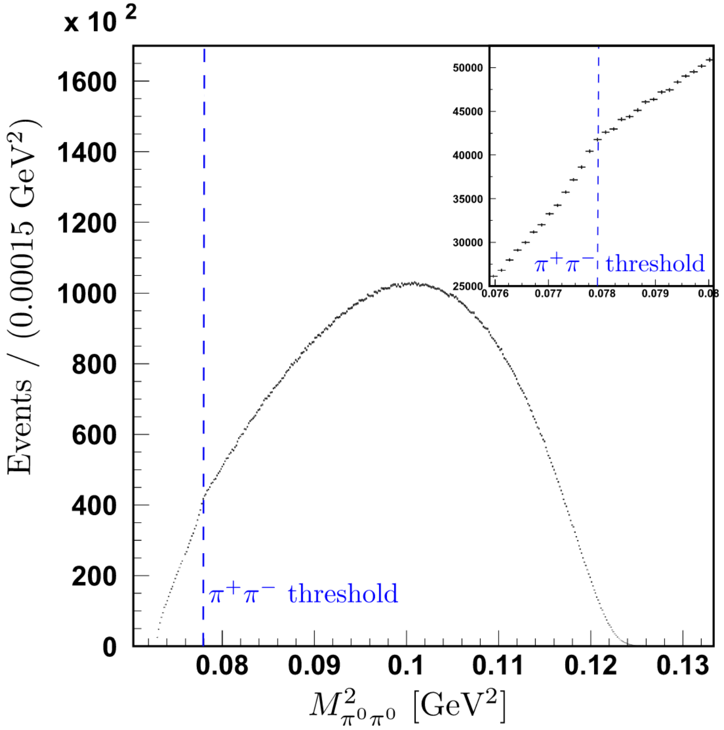
Coulomb wave function at the origin

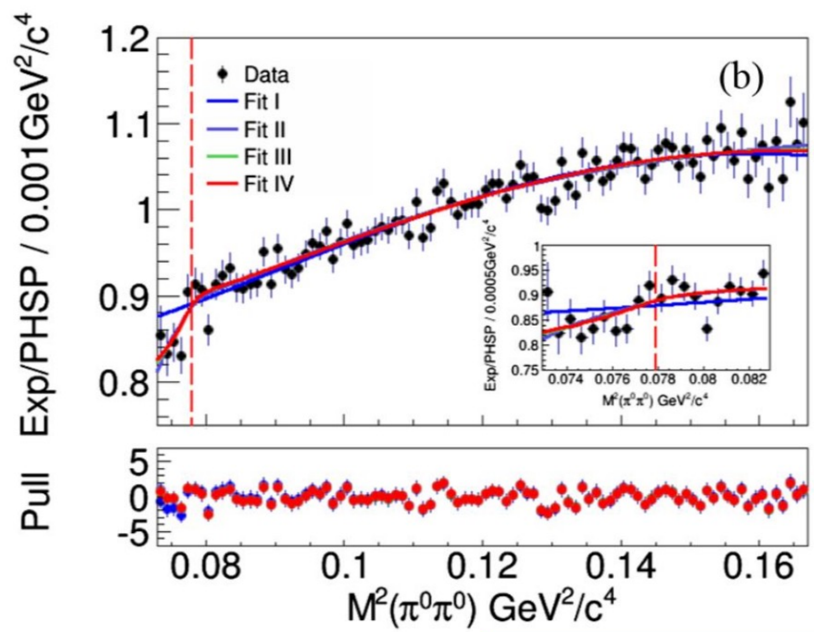


# Scattering length measurements

- Threshold cusp
  - Cusp location exactly fixed at the threshold, masses complete known, strength determined by $a_0$
  - Cusp at the $\pi^+\pi^-$ threshold in $K^\pm \to \pi^\pm\pi^0\pi^0$ P. Budini, L. Fonda (1961); N. Cabibbo (2004)
  - $K^\pm \to \pi^\pm\pi^0\pi^0 + K \to \pi^+\pi^-\ell\nu_\ell \Rightarrow$ most precise measurements of $\pi\pi$ scattering lengths NA48/2 (2009, 2010)

$$a_0^0 M_{\pi^+} = 0.2210 \pm 0.0047_{\rm stat} \pm 0.0040_{\rm syst}\,, \quad a_0^2 M_{\pi^+} = -0.0429 \pm 0.0044_{\rm stat} \pm 0.0028_{\rm syst}$$




evidence of cusp in in $\eta' \to \pi^0\pi^0\eta$

BESIII, PRL130 (2023) 081901

Talk by X.-L. Kang

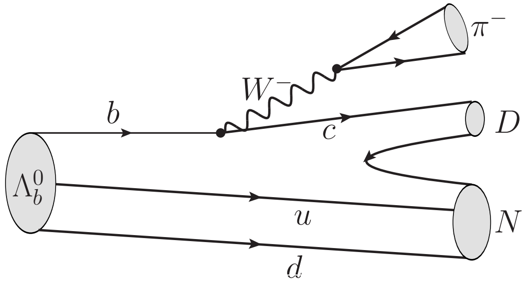
For a review, see FKG, X.-H. Liu, S. Sakai, PPNP 112 (2020) 103757

# Threshold cusp: $ND$ scattering lengths

S. Sakai, FKG, B. Kubis, PLB 808 (2020) 135623

- Threshold cusps can be described model-independently using nonrelativistic effective field theory (NREFT)
- From LHCb measurement of $\Lambda_b \to pD^0\pi^-$

T.D. Cohen, B.A. Gelman, U. von Kolck (2004)




  - NREFT for coupled channels: $pD^0$, $nD^+$

$$t_{ij} = \left[(1 - vG)^{-1}v\right]_{ij}$$

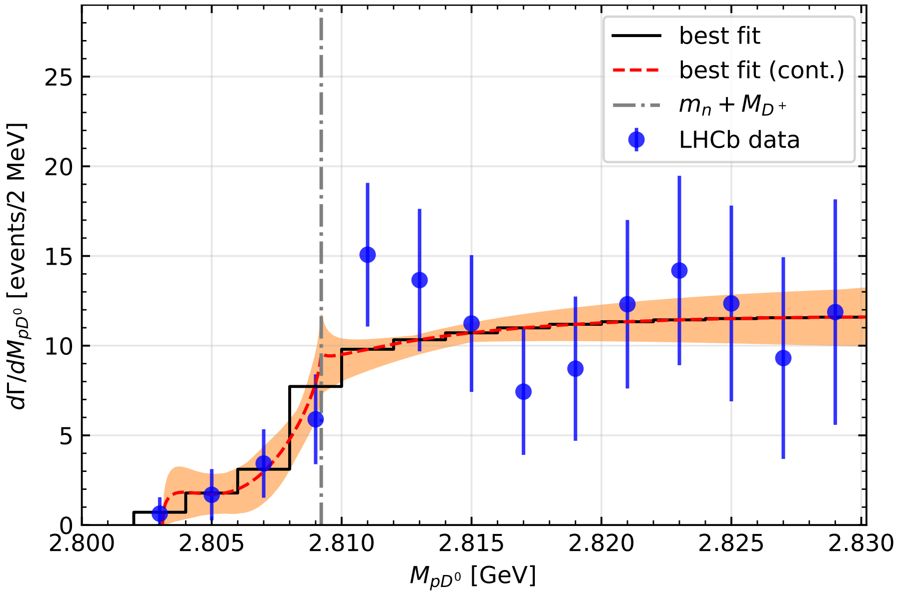
$$= \frac{2\pi}{\det}\begin{pmatrix} \frac{1}{\mu_1}\left(-\frac{1}{a_{22}} + ik_2\right) & -\frac{1}{a_{12}\sqrt{\mu_1\mu_2}} \\ -\frac{1}{a_{12}\sqrt{\mu_1\mu_2}} & \frac{1}{\mu_2}\left(-\frac{1}{a_{11}} + ik_1\right) \end{pmatrix}_{ij},$$

$$\det = \left(\frac{1}{a_{12}}\right)^2 - \left(\frac{1}{a_{11}} - ik_1\right)\left(\frac{1}{a_{22}} - ik_2\right),$$

  - Cusp at the $nD^+$ threshold

$$\frac{d\Gamma_{\Lambda_b \to \pi^- pD^0}}{dM_{pD^0}} = \mathcal{N} p_{\pi^-} p_{D^0} \left| t_{pD^0,pD^0} + t_{nD^+,pD^0} \right|^2$$

isoscalar source for $ND$




LHCb Run-1 data: JHEP 05 (2017) 030

  - Both isoscalar and isovector $ND$ scattering lengths were extracted with only $\sim \mathcal{O}(100)$ events

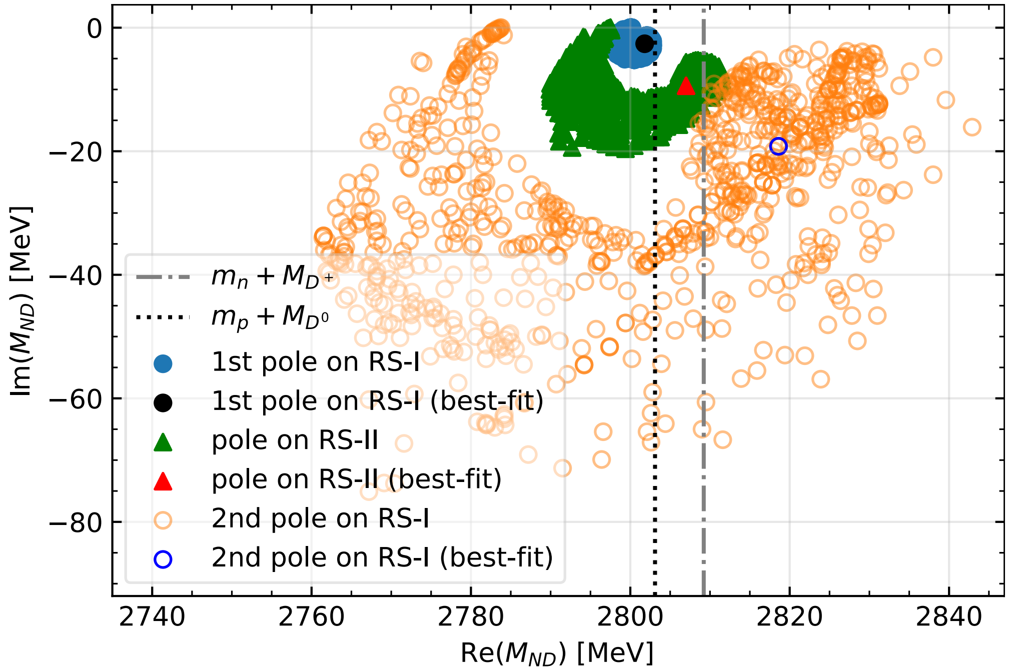
# Threshold cusp: $ND$ scattering lengths

S. Sakai, FKG, B. Kubis, PLB 808 (2020) 135623

- Both isoscalar and isovector $ND$ scattering lengths were extracted with only $\sim \mathcal{O}(100)$ events

| $a_{ND}$[fm] | Our result | SU(4) | SU(4) | SU(8) | Meson-exchange model |
|---|---|---|---|---|---|
| $I=0$ | $-0.79^{+0.66}_{-0.61}$ | -0.43 | $-0.57+i0.001$ | $0.004+i0.002$ | $-0.41+i0.04$ |
| $I=1$ | $-3.8^{+1.4}_{-2.0}+i2.7^{+1.6}_{-2.7}$ | -0.41 | $-1.47+i0.65$ | $0.33+i0.05$ | $-2.07+i0.57$ |
| | | M.Lutz et al. (2006) | T.Mizutani et al. (2006) | C.García-Recio et al. (2006) | J.Haidenbauer et al. (2011) |

- Poles of amplitude extracted:
  - Isovector: $\Sigma_c(2800)$ ?
  - Isoscalar: inconclusive, error too large

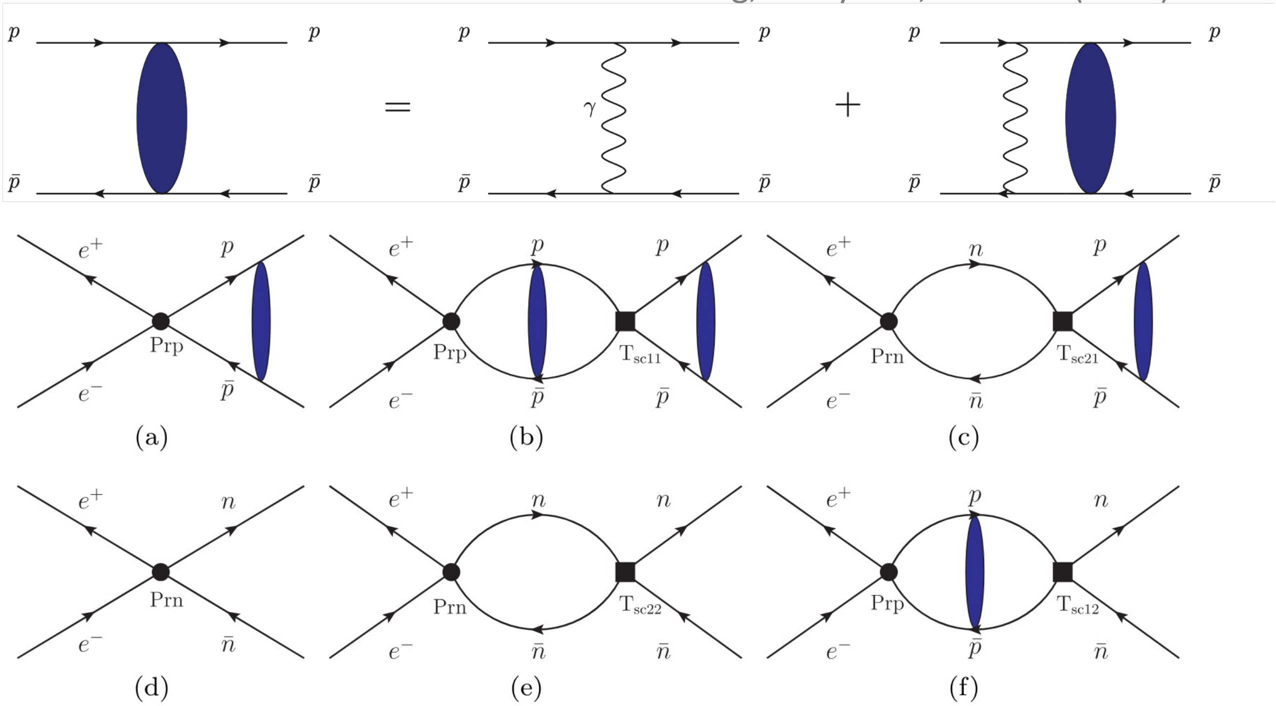


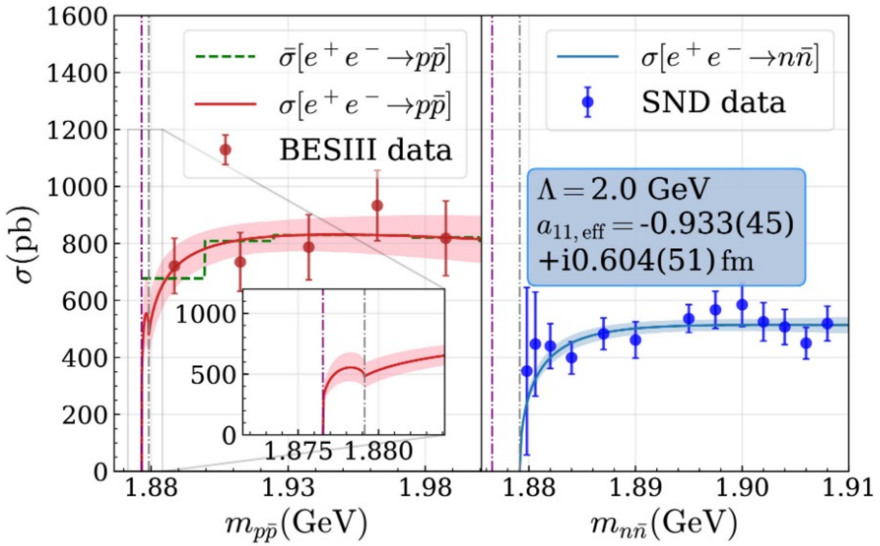
# Threshold cusp: $N\bar{N}$

Z.-S. Jia, Z.-H. Zhang, FKG, G. Li, PRD 111 (2025) 054014

- $e^+e^- \to p\bar{p}$ in the near-threshold region: $p\bar{p} + n\bar{n}$ coupled channels
  - ☐ + Coulomb interaction for $p\bar{p}$ in two-potential formalism

X. Kong, F. Rayndal, NPA 665 (2000) 137

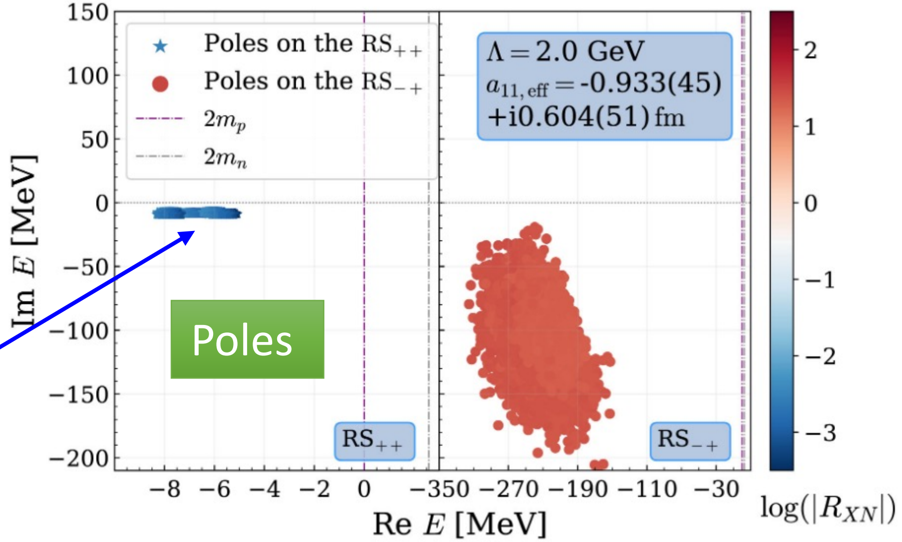



Data: BESIII, PLB 817 (2021) 136328; SND, Phys. Atom. Nucl. 87 (2024) 604

  - ☐ Coulomb modified $p\bar{p}$ s.l. from pionic hydrogen

D. Gotta et al., NPA 660 (1999) 283



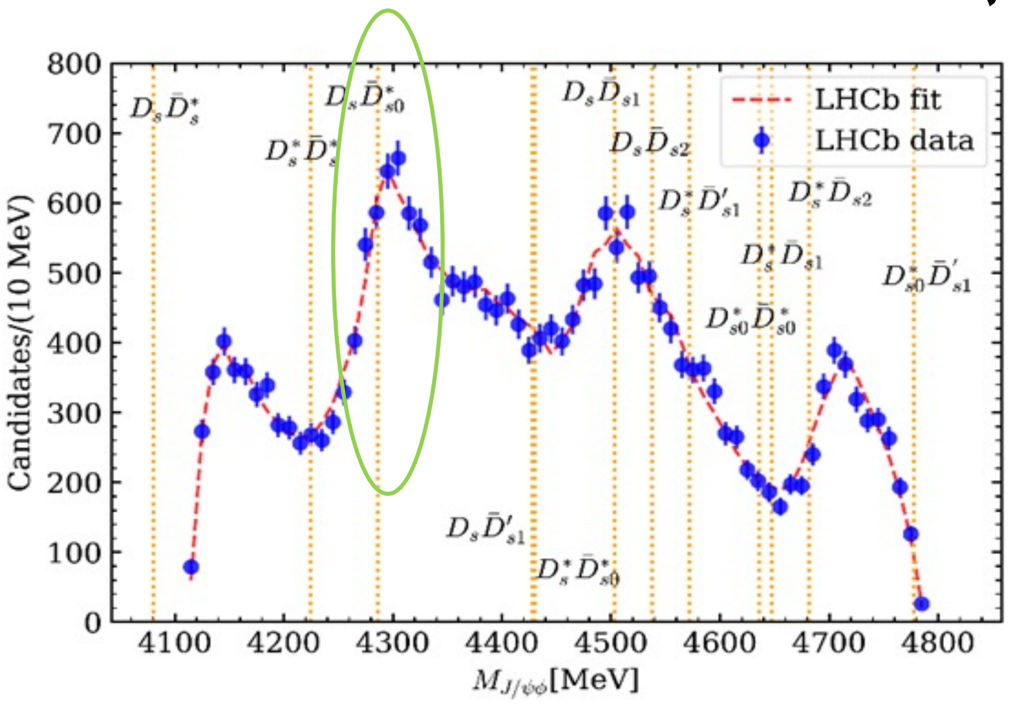

- Scattering lengths extracted; well constrained isoscalar-vector pole below $p\bar{p}$ threshold

# Threshold cusp as a mass meter

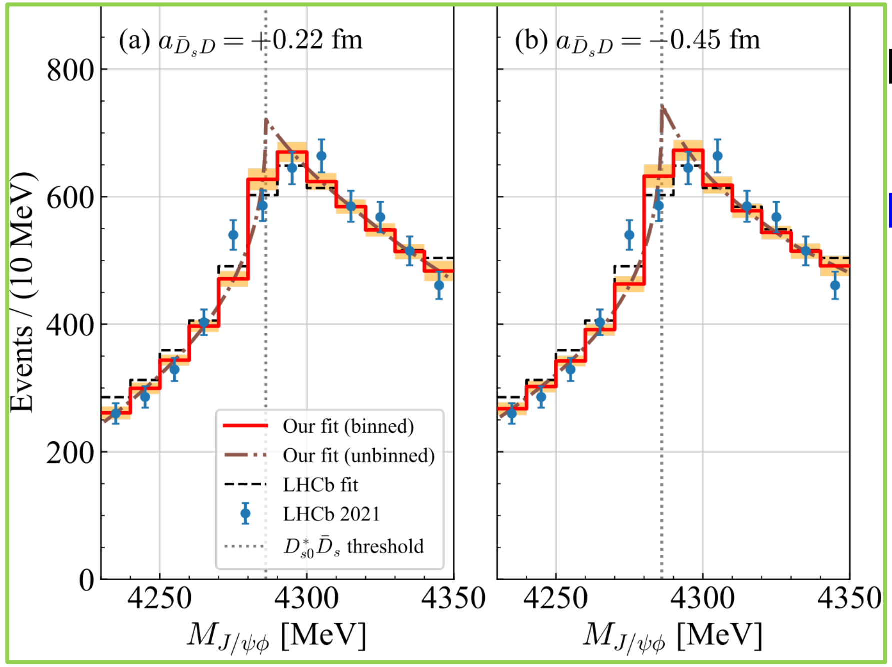
H.-L. Fu, X. Zhang, FKG, C. Hahart, U.-G. Meißner, M.-J. Yan, arXiv:2606.16976

- Some hadrons decay mostly radiatively, e.g., bottom partners of $D_{s0}^*(2317), D_{s1}(2460)$, unobserved
  - $B_{s0}^* \to B_s\pi^0(\to \gamma\gamma),\ B_s^*(\to B_s\gamma)\gamma$
  - $B_{s1} \to B_s^*\pi^0, B_s\gamma, B_s^*\gamma, B_{s0}^*\gamma$ (hadronic decay → $B_s\pi^+\pi^-$ expected to have a tiny partial width ~ 3 keV) M.-N. Tang et al., Commun. Theor. Phys. 75 (2023) 055203
  - Challenging to be observed (at LHC)
- Cusp at threshold with a well-measured hadron ⇒ mass measurement

✓ data for X(4274) well described by $D_{s0}^*\bar{D}_s$ thr. cusp ($DK\bar{D}_s$ 3-body calc.)




Data: LHCb, PRL 127 (2021) 082001




- X(4274) as a virtual state, $J^{PC} = 0^{-+}$
- Measure $\Upsilon\phi$ inv. mass distribution in the 11.0-11.2 GeV region

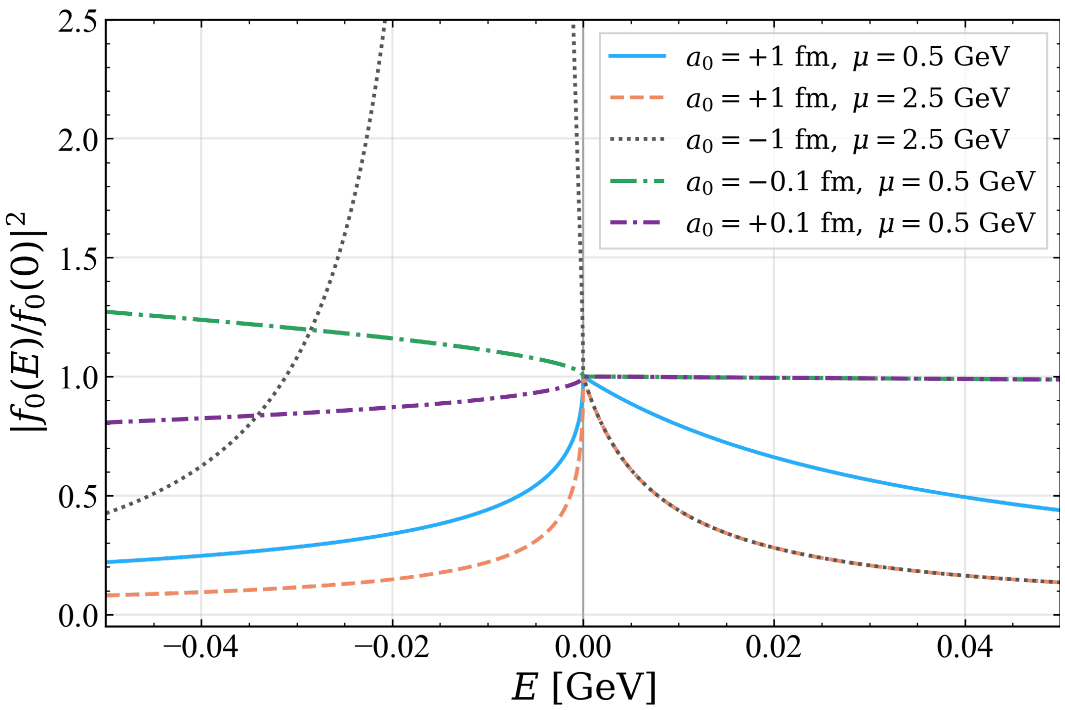
$M_{B_{s0}^*} \approx M_{\text{peak}} - M_{B_s}$

# Threshold cusp is not always easily visible, even for S-wave

- S-wave scattering amplitude in the near-threshold region: $f_0^{-1}(k) = \frac{1}{a_0} - ik + \mathcal{O}\left(\frac{k^2}{\beta^2}\right), \quad k = \sqrt{2\mu E}$
  - ☐ Positive $a$: attractive interaction (virtual state pole)
  - ☐ Negative $a$: repulsion (small $|a|$) or attraction with a near-threshold bound state (large $|a|$)
- Line shape of $|f_0|^2$

$$|f_0(E)|^2 = \begin{cases} \frac{1}{1/a_0^2 + 2\mu E} & \text{for } E \geq 0 \\ \frac{1}{\left(1/a_0 + \sqrt{-2\mu E}\right)^2} & \text{for } E < 0 \end{cases}$$

- ☐ always a cusp
- ☐ positive $a_0$ (attraction): maximal at threshold
  - ➢ Half-maximum width: $2/(\mu a_0^2)$
- ☐ negative $a_0$:
  - ➢ large $|a_0|$: bound state peak below threshold
- ☐ small $|a_0|$: cusp is mild, needs high statistics



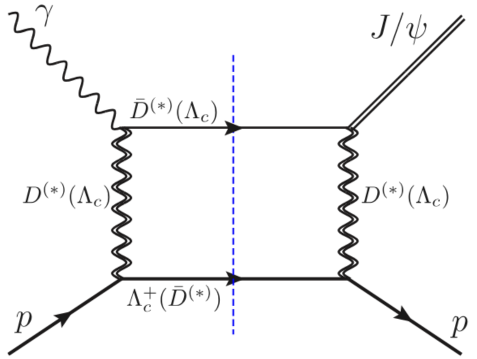

For coupled-channel discussion, see

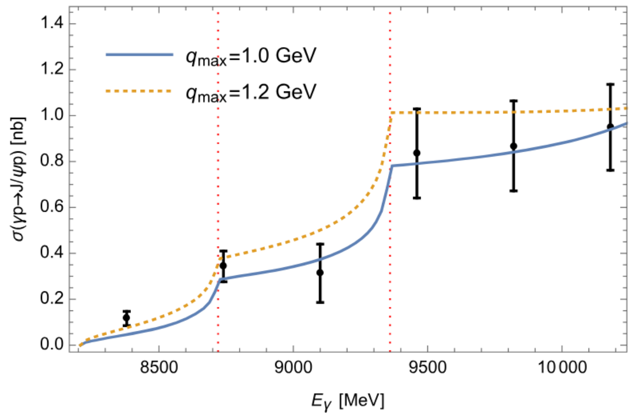
X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001;
Z.-H. Zhang, FKG, PLB 863 (2025) 139387

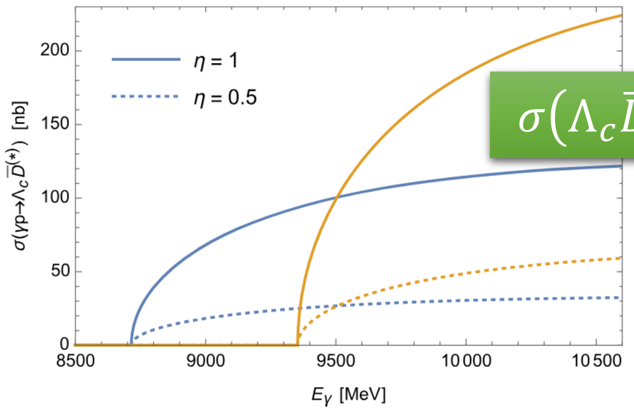
# $J/\psi p$ near-threshold photoproduction: cusp visible?

- $\gamma p \to J/\psi p$
  - Unitarity: $J/\psi p \to J/\psi p$ enters coupled-channel scattering ($J/\psi p$, $\Lambda_c \bar{D}^{(*)}$, $\Sigma_c^{(*)} \bar{D}^{(*)}$) w/o VMD
  - Simple model estimate: consider $\Lambda_c \bar{D}^{(*)}$ channels   M.-L. Du et al., EPJC 80 (2020) 1053
    - Charmed meson/baryon exchanges at 1-loop (so no unitarity)

Measure the open-charm production!



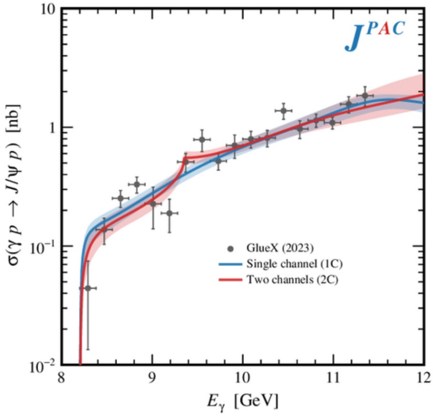



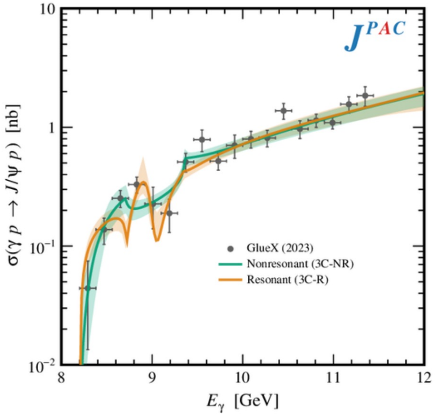




$\sigma(\Lambda_c \bar{D}^{(*)}) \gg \sigma(J/\psi p)$

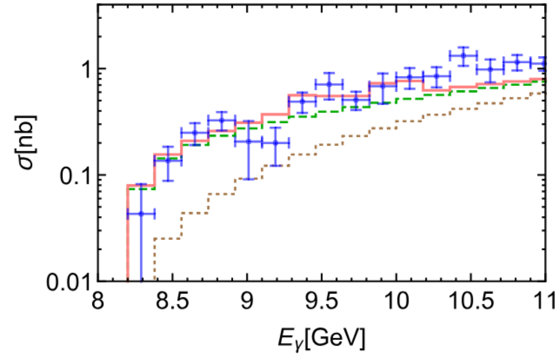
- Results with unitarity
  - ✧ with cusp(s)

JPAC, PRD 108 (2023) 054018;
talks by D. Winney
& R. Tyson




✧ without an obvious cusp




X. Zhang, EPJC 85 (2025) 1120

# Scattering length extractions

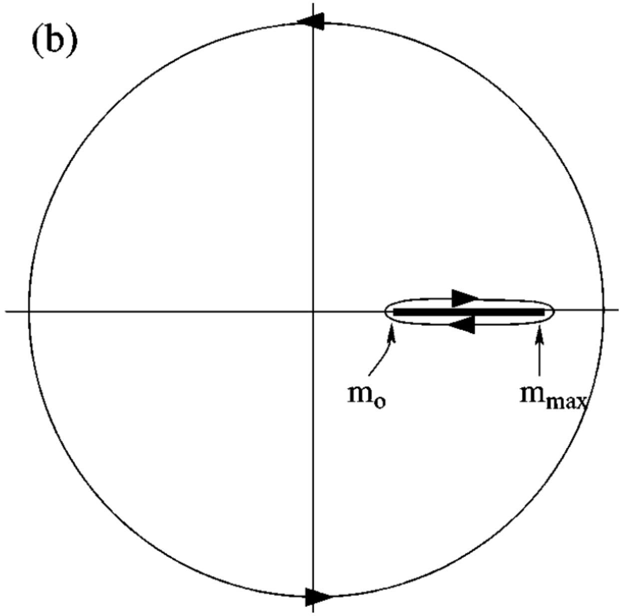
- From a dispersion integral of cross section — A. Gasparyan, J, Haidenbauer, C. Hanhart, PRC 69 (2004) 034006
  - From reactions with a large momentum transfer, Omnes repr. for production amplitude — R. Omnes (1958); Talk by I. Danikin

$$A_S(s,t,m^2) = \exp\left[\frac{1}{\pi}\int_{m_0^2}^{m_{\max}^2} \frac{\delta_S(m'^2)}{m'^2 - m^2 - i0} dm'^2\right]\Phi(s,t,m^2)$$

$\delta_S$: phase shift for a given partial wave

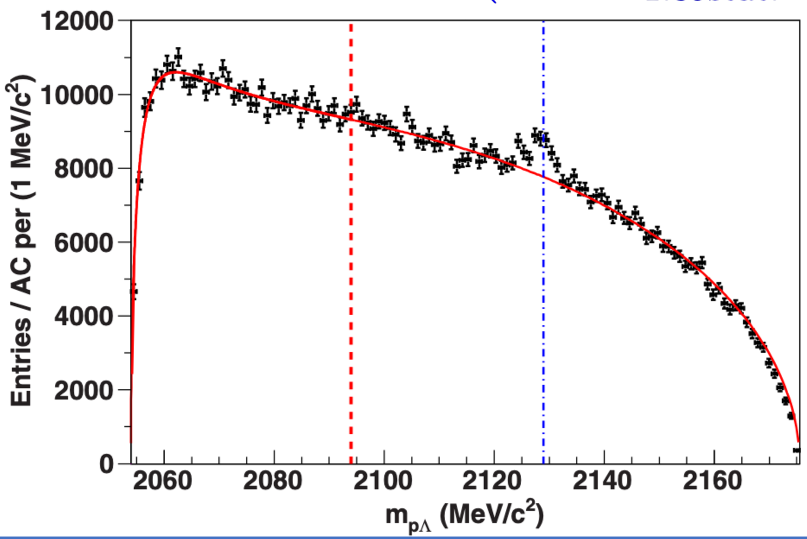
  - Truncate the integral below inelastic threshold:

$$a_S = \lim_{m^2\to m_0^2} \frac{1}{2\pi}\left(\frac{m_1+m_2}{\sqrt{m_1 m_2}}\right) \fint_{m_0^2}^{m_{\max}^2} dm'^2 \frac{\sqrt{(m_{\max}^2 - m^2)(m_{\max}^2 - m'^2)}}{\sqrt{m'^2 - m_0^2}\,(m'^2 - m^2)} \ln\left[\frac{1}{k}\left(\frac{d^2\sigma_S}{dm'^2 dt}\right)\right]$$




- $\Lambda p$ spin-1 scattering length extracted from $\vec{p}p \to K^+ p\Lambda$

$$\left(-2.55^{+0.72}_{-1.39\text{stat.}} \pm 0.6_{\text{syst.}} \pm 0.3_{\text{theo.}}\right)\ \text{fm}$$




COSY-TOF, PRC 95 (2017) 034001

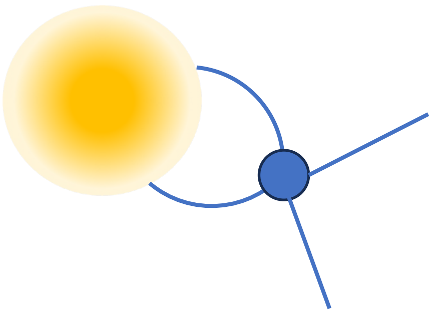
# Scattering length extractions

- Femtoscopy method — For recent reviews, see L. Fabbietti et al., ARNPS 71 (2021) 377; M.-Z. Liu et al., Phys.Rept. 1108 (2025) 1

Talks by R. Del Grande

- Koonin-Pratt formula — S. Koonin (1977); S. Pratt (1986)

$$C(\boldsymbol{q}) = \int d^3r S(\boldsymbol{r}) \left|\Psi_{\boldsymbol{q}}(\boldsymbol{r})\right|^2$$



  - In practice, a potential is assumed to calculate the wave function (w.f.), strength adjusted to get $a_0$
- Lednický-Lyuboshitz model — R. Lednický, V. Lyuboshitz (1981)
  - Assuming Gaussian source
  - S-wave dominance at small momenta
  - Wave function taking the asymptotic form ($r \to \infty$), thus $\propto f_0(k) = \left(\frac{1}{a_0} + \frac{1}{2} r_0 k^2 - ik\right)^{-1}$
    - Only applicable for large source size $R \gg 1/\beta$
- Challenge to get model-independent measurements:

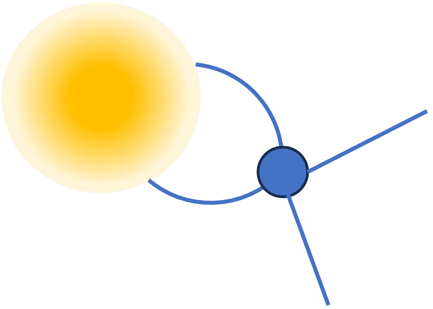
Talks by A. Feijoo

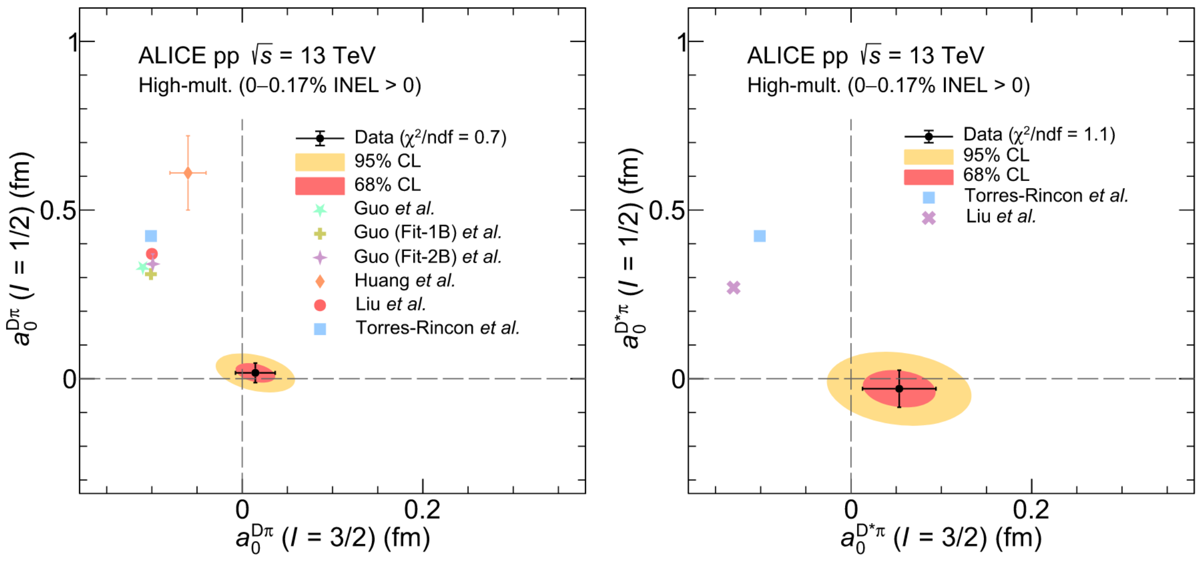
  - short-distance w.f. is not uniquely defined, w.f. can change without modifying observables
  - potential not uniquely defined
  - source profile dependent

For recent discussions, J. Chen et al., NST 36 (2025) 55; E. Epelbaum, U.-G. Meißner, A. Tscherwon, PRL 136 (2026) 212301; R. Molina, E. Oset, PRD 112 (2025) 096006; M. Albaladejo et al., PLB 866 (2025) 139552; …

# Scattering length extractions

- Femtoscopy method
  - To show the challenge, $D\pi$ as an example  ALICE, PRD 110 (2024) 032004
    extracted by ALICE in $pp$ collisions using KP formula under model assumptions






- In conflict with all theoretical predictions, including both lattice QCD and unitarized ChPT results
- Significantly deviate from chiral symmetry prediction, parameter-free at leading order:

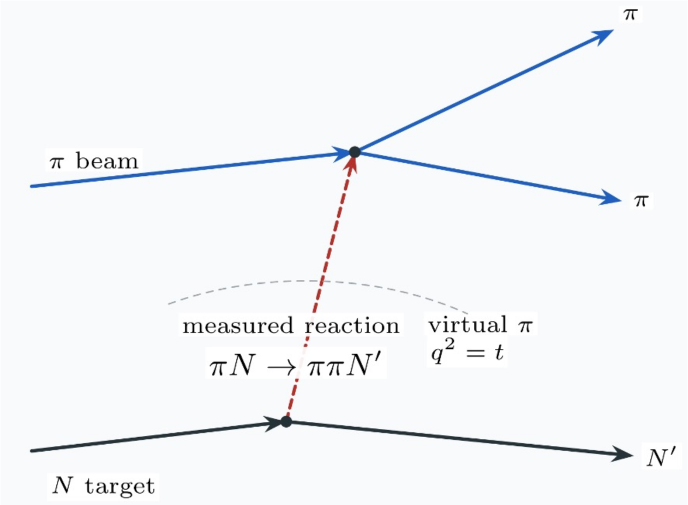
$$a_{0,D\pi}^{I=1/2} = \frac{\mu_{D\pi}}{4\pi F_\pi^2} \approx 0.24\ \text{fm}, \quad a_{0,D\pi}^{I=3/2} = -\frac{\mu_{D\pi}}{8\pi F_\pi^2} \approx -0.12\ \text{fm}$$

FKG, C. Hanhart, U.-G. Meißner, EPJA 40 (2011) 179;

M.-L. Du, FKG, C. Hanhart, F. Herren, B. Kubis, R. van Tonder, EPJC 85 (2025) 1289

# Summary

- In most cases, hadron-hadron scattering information needs to be extracted from production experiments
- ERE parameters, in particular, scattering length extractions
  - Threshold cusp for S-wave scattering
    - shape determined by masses & scattering length
    - as a mass meter ⇒ measure $\Upsilon\phi$ in 11.0~11.2 GeV to extract the $B_{s0}^*$ mass
    - for weak interaction, high statistics required
  - Dispersive approach
  - Femtoscopy method
- Phase shifts not covered
  - Chew-Low method: use off-shell exchanged pion — G.F. Chew, F.E. Low (1959)
  - Cabibbo-Maksymowicz method: $K \to \pi\pi\ell\nu \Rightarrow \delta_0^0 - \delta_1^1$ — N. Cabibbo, A. Maksymowicz (1968)
  - Dispersive (Roy, GKPY, Roy-Steiner)
  - PWA — Talk by D. Rönchen



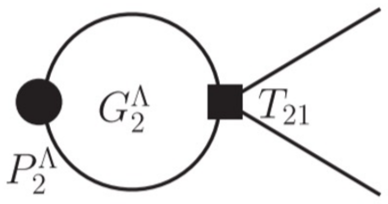

**Thank you for your attention!**

# Distinct line shapes of the same pole

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

Line shapes of the same pole depend on the production mechanism. Consider production of particles in ch-1

- Dominated by ch-2 $T_{21}(E) = \frac{-8\pi\Sigma_2}{a_{12}(1/a_{11} - ik_1)}\left[\frac{1}{a_{22,\rm eff}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}.$




$|T_{21}(E)|^2 \propto |T_{22}(E)|^2 \propto$ ≤ 0 due to unitarity

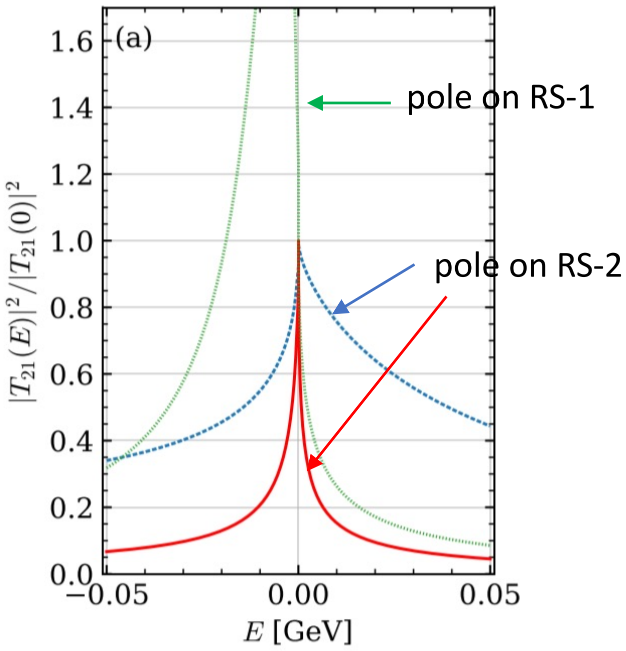
$$\begin{cases} \left[\left(\mathrm{Re}\,\frac{1}{a_{22,\rm eff}}\right)^2 + \left(\mathrm{Im}\,\frac{1}{a_{22,\rm eff}} - \sqrt{2\mu E}\right)^2\right]^{-1} & \text{for } E \geq 0 \\ \left[\left(\mathrm{Im}\,\frac{1}{a_{22,\rm eff}}\right)^2 + \left(\mathrm{Re}\,\frac{1}{a_{22,\rm eff}} + \sqrt{-2\mu E}\right)^2\right]^{-1} & \text{for } E < 0 \end{cases}$$

- Maximal at threshold for positive $\mathrm{Re}(a_{22,\rm eff})$ (attraction), FWHM $\propto 1/\mu$
  - Quasi-virtual state
  - more pronounced for heavier hadrons
  - and for stronger interactions

$$\frac{1}{\mu}\left(\frac{4}{|a_0|^2} - \sum_x x\sqrt{\frac{3}{|a_0|^2} + x^2}\right),$$

$x = \mathrm{Im}(1/a_0)$ and $\mathrm{Re}(1/a_0)$

- Peaking at pole for negative $\mathrm{Re}(a_{22,\rm eff})$:
  - Quasi-bound state



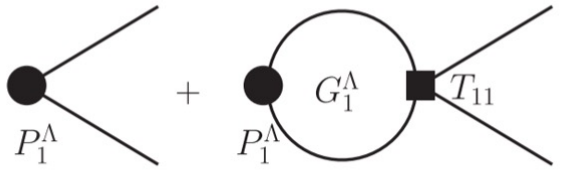
# Distinct line shapes of the same pole

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

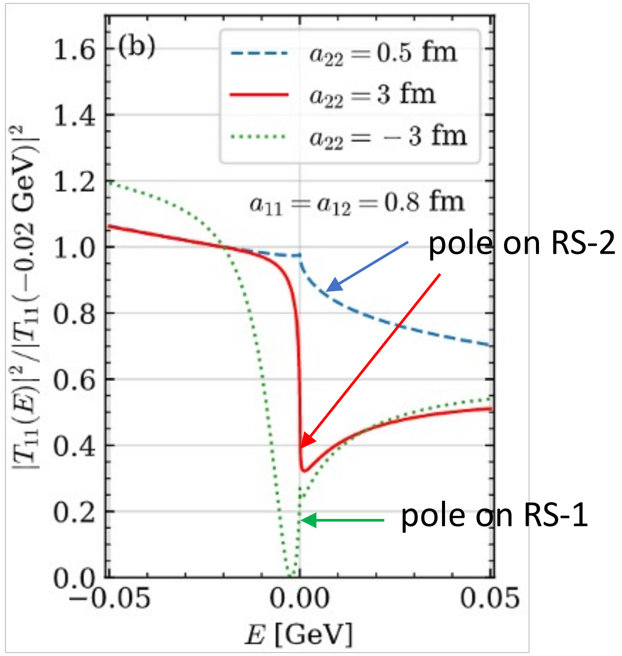
Line shapes of the same pole depend on the production mechanism. Consider production of particles in ch-1

- Dominated by ch-1

$$T_{11}(E) = \frac{-8\pi\Sigma_2\left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E}\right)}{\left(\frac{1}{a_{11}} - i\,k_1\right)\left[\frac{1}{a_{22,\rm eff}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]}$$




- One pole and one zero
- Universality for large scattering length:
  - Unitarity limit for ch-2 ($|a_{22}| = \infty$): there must be a zero at the threshold of ch-2!
  - For large $|a_{22}|$, a dip around threshold
    - Zero below threshold: single-ch. bound state
    - Dip but not zero: single-ch. virtual state




- Rewritten in an interference form:

V. Baru, FKG, C. Hanhart, A. Nefediev, PRD 109, L111501 (2024)

$$T_{11}(E) = -8\pi E_2^{\rm thr}\left(\frac{1}{a_{11}^{-1} - ik_1} + \frac{a_{12}^{-2}(a_{11}^{-1} - ik_1)^{-2}}{a_{22,\rm eff}^{-1} - ik_2}\right)$$

background — pole term

interfering phase is fixed by unitarity!

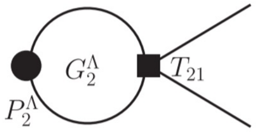
Process-dependent line shapes known since long; see, e.g., J. Taylor, *Scattering Theory*

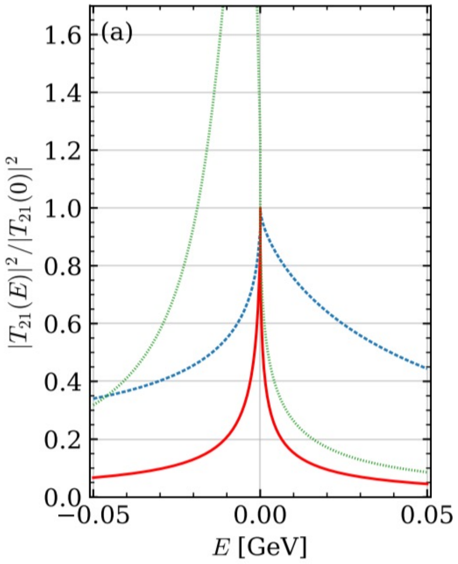
# Distinct line shapes of the same pole

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

## Summary of the differences:

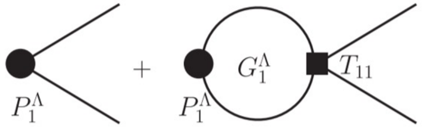
- Dominated by ch-2
  - Maximal at threshold for positive $\mathrm{Re}(a_{22,\mathrm{eff}})$ (attraction), FWHM $\propto 1/\mu$
    - more pronounced for heavier hadrons and stronger interactions
  - Peaking at pole for negative $\mathrm{Re}(a_{22,\mathrm{eff}})$

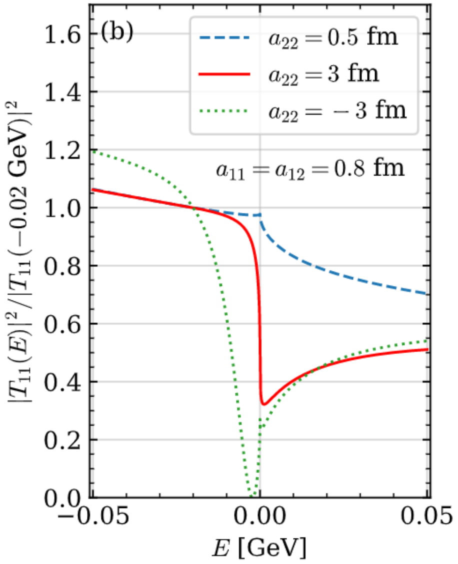



$$T_{21} \propto \frac{1}{a_{22,\mathrm{eff}}^{-1} - i\sqrt{2\mu_2 E}}$$

- Dominated by ch-1
  - One pole and one zero
  - Universality for large scattering length: for large $|a_{22}|$, there must be a dip around threshold (zero close to threshold)



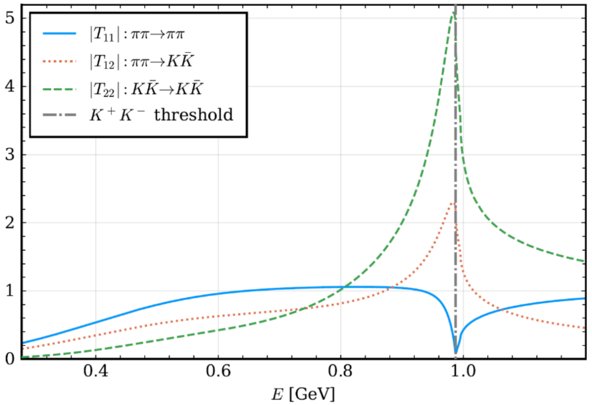

$$T_{11} \propto \frac{a_{22}^{-1} - i\sqrt{2\mu_2 E}}{a_{22,\mathrm{eff}}^{-1} - i\sqrt{2\mu_2 E}}$$

✓ Classification of near-threshold structures and pole trajectories for two heavy channels: Z.-H. Zhang, FKG, PLB 863 (2025) 139387

# Distinct line shapes of the same pole

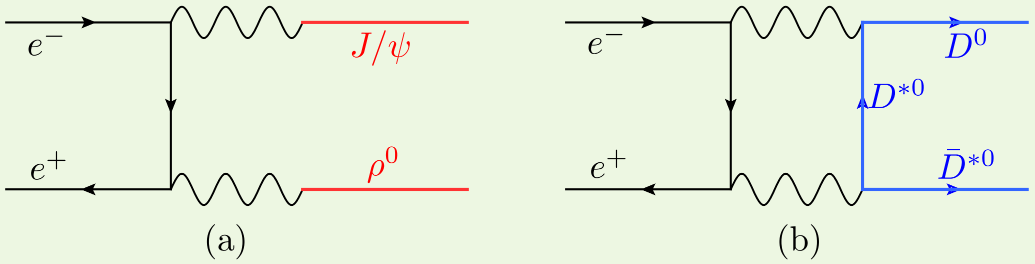
- Example-1: $f_0(980)$
  - $T$-matrix for $\pi\pi$ and $K\bar{K}$ coupled channels

    with the T-matrix from L.-Y. Dai, M. R. Pennington, PRD 90 (2014) 036004




- Example-2: direct production of $X(3872)$ in $e^+e^-$

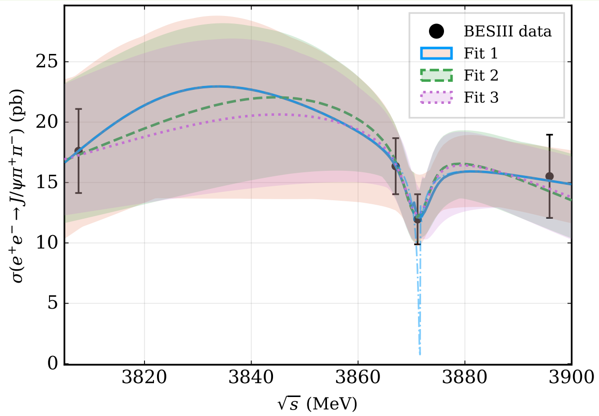
V. Baru, FKG, C. Hanhart, A. Nefediev, PRD 109 (2024) L111501



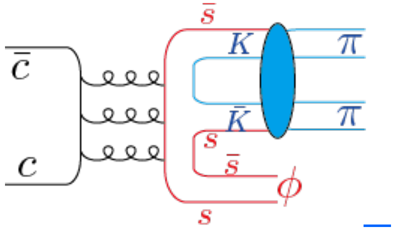

- Driving channel:

  $J/\psi$ +light vector

- Prediction: dip around $D^*\bar{D}^*$ threshold

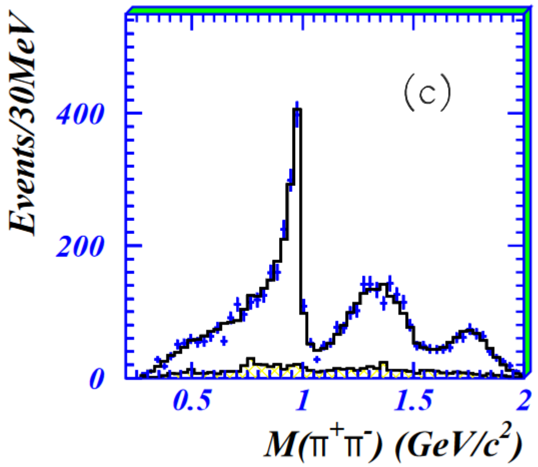



- $J/\psi \to \phi\pi^+\pi^-$



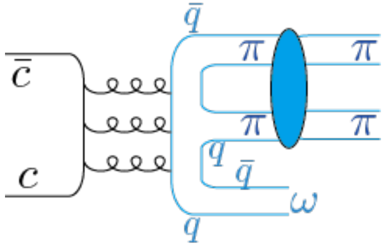
Driving channel: $K\bar{K}$

$J/\psi \to \phi K\bar{K} \to \phi\pi^+\pi^-$



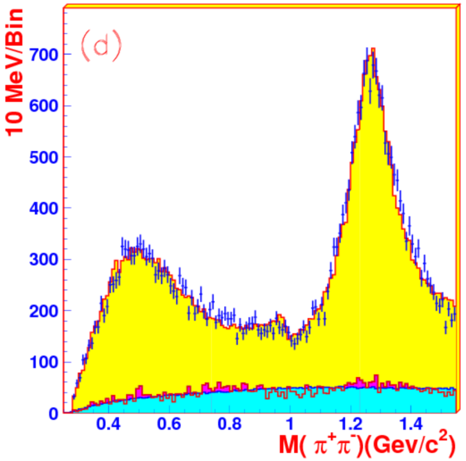

BES, PLB 607 (2005) 243

- $J/\psi \to \omega\pi^+\pi^-$



Driving channel: $\pi\pi$

$J/\psi \to \omega\pi\pi \to \omega\pi^+\pi^-$




BES, PLB 598 (2004) 149