Exclusive production of η and η' mesons in proton-proton collisions at FAIR energies

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Introduction

 Exclusive production of axial-vector f₁(1285) meson (J^{PC} = 1⁺⁺) in proton-(anti)proton collisions for energy ranges available at the GSI-FAIR with HADES and PANDA *P. Lebiedowicz, O. Nachtmann, P. Salabura, A. Szczurek,* PRD 104 (2021) 034031



We shall learn from f_1 production about the $\rho \rho f_1$ and $\omega \omega f_1$ coupling strengths.



The narrow width of the η meson allows to set a mass cut on the $\pi^+\pi^-\pi^0$ invariant mass and suppresses the multi-pion background efficiently.

	Contribution	Cross section (μb)	[1] G. Alexander et al., Phys. F	Rev. 154 (1967) 1284
1	$pp \rightarrow pp\pi^+\pi^-\pi^+\pi^-\pi^0$	88	$\sigma = (88 \pm 14) \ \mu b \ [1], P = 5.5 \ \text{GeV}$ [2] S. Danieli et al., Nucl. Phys	. B27 (1971) 157
			$\sigma = (90 \pm 30) \ \mu b \ [2] \text{ for } pp \rightarrow pp\pi^+\pi^-\omega \text{ at } P = 6.92 \text{ GeV}$	
2	$pp \rightarrow pp\pi^+\pi^-\eta (\rightarrow \pi^+\pi^-\pi^0)$	0.18	estimates via double N^* production (via π^0 exchange)	
			$pp \to N(1440)N(1535)$ and $pp \to N(1535)N(1535)$	
3	$pp \rightarrow ppf_1[\rightarrow \pi^+\pi^-\eta(\rightarrow \pi^+\pi^-\pi^0)]$	0.012	$\sigma = 3.2 - 12.4$ nb, see (C7)–(C10), from $VV \rightarrow f_1$ fusion mechanism	

• We have estimated that HADES should allow the identification of $f_1(1285)$ in the $\pi^+\pi^-\eta$ channel.

- $\eta'(958) \rightarrow \pi^+\pi^-\eta$ and can also be visible, this requires a careful analysis of the production mechanism
- In this talk I will discuss exclusive production of η and η' mesons

We study exclusive production of pseudoscalar meson in the reaction

 $p(p_a, \lambda_a) + p(p_b, \lambda_b) \rightarrow p(p_1, \lambda_1) + p(p_2, \lambda_2) + \eta(k)$

where $p_{a,b}$, $p_{1,2}$ and $\lambda_{a,b}$, $\lambda_{1,2} = \pm \frac{1}{2}$ denote the four-momenta and helicities of the nucleons, respectively, and k denotes the four-momentum of the η meson.

The cross section is as follows

$$\sigma(pp \to pp\eta) = \frac{1}{2} \frac{1}{2\sqrt{s(s-4m_p^2)}} \int \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3p_1}{(2\pi)^3 2p_1^0} \frac{d^3p_2}{(2\pi)^3 2p_2^0} \times (2\pi)^4 \delta^{(4)}(p_1+p_2+k-p_a-p_b) \frac{1}{4} \sum_{p \text{ spins}} |\mathcal{M}_{pp \to pp\eta}|^2$$

including a statistics factor 1/2 due to identical particles appearing in the final state. The complete amplitude is

$$\mathcal{M}_{pp \to pp\eta} = \mathcal{M}_{pp \to pp\eta}(p_1, p_2) - \mathcal{M}_{pp \to pp\eta}(p_2, p_1)$$

The relative minus sign here is due to the Fermi statistics, which requires the amplitude to be antisymmetric under interchange of the two final protons.

Particle	J^P	overall	$N\gamma$	$N\pi$	$N\sigma$	$N\eta$		$N\rho$	$N\omega$	$N\eta\prime$	
N	$1/2^{+}$	****									
N(1440)	$1/2^{+}$	****	****	****	***						
N(1520)	$3/2^{-}$	****	****	****	**	****					
N(1535)	$1/2^{-}$	****	****	****	*	****	4	?	?		
N(1650)	$1/2^{-}$	****	****	****	*	****			· ·		
N(1675)	$5/2^{-}$	****	****	****	***	*					
N(1680)	$5/2^{+}$	****	****	****	***	*					
N(1700)	$3/2^{-}$	***	**	***	*	*		*			
N(1710)	$1/2^{+}$	****	****	****		***		*	*		
N(1720)	$3/2^+$	****	****	****	*	*		*	*		
N(1860)	$5/2^{+}$	**	*	**	*	*					
N(1875)	$3/2^{-}$	***	**	**	**	*		*	*		
N(1880)	$1/2^{+}$	***	**	*	*	*			**		
N(1895)	$1/2^{-}$	****	****	*	*	****		*	*	****	
N(1900)	$3/2^{+}$	****	****	**	*	*			*	**	
N(1990)	$7/2^{+}$	**	**	**		*					
N(2000)	$5/2^{+}$	**	**	*	*	*			*		
N(2040)	$3/2^{+}$	*		*							
N(2060)	$5/2^{-}$	***	***	**	*	*		*	*		
N(2100)	$1/2^{+}$	***	**	***	**	*		*	*	**	
N(2120)	$3/2^{-}$	***	***	**	**				*	*	,
N(2190)	$7/2^{-}$	****	****	****	**	*		*	*		
N(2220)	$9/2^{+}$	****	**	****		*					
N(2250)	$9/2^{-}$	****	**	****		*					
N(2300)	$1/2^{+}$	**		**							
N(2570)	$5/2^{-}$	**		**							
N(2600)	$11/2^{-}$	***		***							
N(2700)	$13/2^+$	**		**							
****	**** Existence is certain. \leftarrow status of the N resonances (N*)										
***	Existen	ce is very	likely.	a	nd the	ir de	cavs fro	om PD	G	,	
**	Evidence of existence is fair.								5		
*	Evidence of existence is poor.										

Basic production mechanisms for $p_1p_2 \rightarrow p_1'p_2'\eta$ with $M = \pi$, σ , η , ρ^0 , ω , ...

 p'_1

 p_2



In the calculations we can consider the amplitude given by the sum of the contributions with the intermediate protons, nucleon resonances and the vector-meson exchanges (VV-fusion processes).



where $p_{1f} = p_a + q_2 = p_1 + k$, $q_2 = p_b - p_2$, $t_2 = q_2^2$

The amplitude (b) with the ρ^0 -meson exchange is obtained by making the replacement

$$\begin{aligned} \Delta^{(\pi)}(q_2) &\to \quad \Delta^{(\rho)}_{\mu\nu}(q_2) \,, \\ \Gamma^{(\pi N N_{1/2}^*)}(p_{1f}, p_a) &\to \quad \Gamma^{(\rho N N_{1/2}^*)\mu}(p_{1f}, p_a) \,, \\ \Gamma^{(\pi N N)}(p_2, p_b) &\to \quad \Gamma^{(\rho N N)\nu}(p_2, p_b) \end{aligned}$$

The pseudoscalar-meson–nucleon coupling Lagrangians can be written as

$$\mathcal{L}_{\pi NN} = -\frac{g_{\pi NN}}{2m_N} \bar{N} \gamma_5 \gamma_\mu \partial^\mu (\boldsymbol{\tau} \boldsymbol{\Phi}_\pi) N$$
$$\mathcal{L}_{\eta NN} = -g_{\eta NN} \bar{N} \left(i\gamma_5 \lambda + (1-\lambda) \frac{1}{2m_N} \gamma_5 \gamma_\mu \partial^\mu \right) \Phi_\eta N$$

where N and Φ denote the nucleon and meson fields, respectively. The parameter λ controls the admixture of the two types of couplings: psedoscalar (PS) ($\lambda = 1$) and pseudovector (PV) ($\lambda = 0$). We take $g_{\pi NN}^2/4\pi = 14.0$. For the ηNN coupling we take $\lambda = 0.504$ and $g_{\eta NN} \rightarrow g_{\eta} = g_{\eta NN}/\lambda = f_{\eta NN}/(1-\lambda) = 4.03$ [Kirchbach]. For the $\eta' NN$ case we take $\lambda = 1$ (PS) or 0 (PV) and $g_{\eta' NN} = 2$.

The MNN^* vertices involving spin-1/2 nucleon resonances are obtained from the effective Lagrangians

$$\mathcal{L}_{\pi N N_{1/2^{\mp}}^{\mathrm{PS}}}^{\mathrm{PS}} = \pm i g_{\pi N N^{*}} \bar{N}^{*} \begin{pmatrix} 1\\ \gamma_{5} \end{pmatrix} (\boldsymbol{\tau} \boldsymbol{\Phi}_{\pi}) N + \mathrm{h.c.},$$

$$\mathcal{L}_{\eta N N_{1/2^{\mp}}}^{\mathrm{PS}} = \pm i g_{\eta N N^{*}} \bar{N}^{*} \begin{pmatrix} 1\\ \gamma_{5} \end{pmatrix} \boldsymbol{\Phi}_{\eta} N + \mathrm{h.c.},$$

$$\mathcal{L}_{\pi N N_{1/2^{\mp}}}^{\mathrm{PV}} = \pm \frac{g_{\pi N N^{*}}}{m_{N^{*}} \mp m_{N}} \bar{N}^{*} \begin{pmatrix} \gamma_{\mu}\\ \gamma_{5} \gamma_{\mu} \end{pmatrix} \partial^{\mu} (\boldsymbol{\tau} \boldsymbol{\Phi}_{\pi}) N + \mathrm{h.c.},$$

$$\mathcal{L}_{\eta N N_{1/2^{\mp}}}^{\mathrm{PV}} = \pm \frac{g_{\eta N N^{*}}}{m_{N^{*}} \mp m_{N}} \bar{N}^{*} \begin{pmatrix} \gamma_{\mu}\\ \gamma_{5} \gamma_{\mu} \end{pmatrix} \partial^{\mu} \boldsymbol{\Phi}_{\eta} N + \mathrm{h.c.},$$

M. Kirchbach and L. Tiator, On the coupling of the η meson to the nucleon, Nucl. Phys. A 604 (1996) 385

K. Nakayama, J. Speth, and T.-S. H. Lee, η meson production in NN collisions, Phys.Rev. C65 (2002) 045210

K. Nakayama, J. Haidenbauer, C. Hanhart, and J. Speth, Analysis of the reaction $pp \rightarrow pp\eta$ near threshold, Phys. Rev. C 68 (2003) 045201

L. P. Kaptari and B. Kämpfer, Di-electrons from η -meson Dalitz decay in proton-proton collisions, Eur. Phys. J. A 33 (2007) 157

R. Shyam, η-meson production in nucleonnucleon collisions within an effective Lagrangian model, Phys. Rev. C 75 (2007) 055201

where the upper (lower) sign and factor in bracket correspond to negative (positive)-parity resonances.

The $MNN^*_{3/2}$ vertices involving spin-3/2 nucleon resonances can be written

as

$$\mathcal{L}_{\pi N N_{3/2^{\mp}}^{*}} = \frac{g_{\pi N N^{*}}}{m_{\pi}} \bar{N}^{*\mu} \Theta_{\mu\nu}(z) \begin{pmatrix} \gamma_{5} \\ 1 \end{pmatrix} \partial^{\nu}(\boldsymbol{\tau}\boldsymbol{\Phi}_{\pi})N + \text{h.c.},$$
$$\mathcal{L}_{\eta N N_{3/2^{\mp}}^{*}} = \frac{g_{\eta N N^{*}}}{m_{\eta}} \bar{N}^{*\mu} \Theta_{\mu\nu}(z) \begin{pmatrix} \gamma_{5} \\ 1 \end{pmatrix} \partial^{\nu} \Phi_{\eta} N + \text{h.c.},$$

where $\Theta_{\mu\nu}(z) = g_{\mu\nu} - (A(1+4z)/2+z)\gamma_{\mu}\gamma_{\nu}$. The choice of the so-called "off-shell parameter" z is arbitrary and it is treated as a free parameter to be determined by fitting to the data. We take A = -1 and z = -1/2 for simplicity.

The partial decay widths of nucleon resonances with J = 1/2, 3/2 could be calculated by the Lagrangian couplings, as following

$$\Gamma(N_{1/2^{\mp}}^* \to NM) = f_{\rm ISO} \frac{g_{MNN^*}^2}{4\pi} p_N \frac{E_N \pm m_N}{m_{N^*}},$$

$$\Gamma(N_{3/2^{\mp}}^* \to NM) = f_{\rm ISO} \frac{g_{MNN^*}^2}{12\pi} \frac{p_N^3}{m_M^2} \frac{E_N \mp m_N}{m_{N^*}},$$

where $p_N = |\mathbf{p}_N|$ and E_N denote the absolute value of the three-momentum and energy of the nucleon in the rest frame of N^* , respectively. The isospin factor $f_{\rm ISO}$ is equal to 3 for decays into mesons with isospin one (π) , 1 otherwise (η) . The absolute value of coupling constants g_{MNN^*} $(M = \pi, \eta)$, could be determined by the experimental decay widths of $\Gamma(N^* \to NM)$ in the compilation of PDG.

Table 1: Coupling constants for the MNN^* $(M = \pi, \eta, \eta')$ vertices. The coupling constants g_{MNN^*} are dimensionless. The symbol "(-)" indicates the negative sign of the g_{MNN^*} coupling constant. The hadronic Breit-Wigner parameters for N^* resonances and the branching ratios (\mathcal{B}) are taken from PDG.

$N^* J^P$	Mass (MeV)	Width (MeV)	Decay channel	\mathcal{B} (%) [PDG]	$g_{MNN^*}^2/4\pi$
$N(1520) 3/2^{-}$	1515	110	πN	$65 \ [60 \pm 5]$	0.204
			ηN	$0.08[0.08\pm 0.01]$	3.945
			$\pi\pi N$	$34.92 \ [30 \pm 5]$	
$N(1535) 1/2^-$	1530	150	πN	$50 \ [42 \pm 10]$	0.042
			ηN	$42 \ [42.5 \pm 12.5]$	0.290
			$\pi\pi N$	$8 [17.5 \pm 13.5]$	
$N(1650) 1/2^{-}$	1650	125	πN	$60 \ [60 \pm 10]$	0.037
			ηN	$25 [25 \pm 10]$	$(-) \ 0.076$
			$\pi\pi N$	$15 \; [39 \pm 19]$	
$N(1700) 3/2^{-}$	1720	200	πN	$12 \ [12 \pm 5]$	0.021
			ηN	2 [seen]	0.916
			$\pi\pi N$	86 [> 89]	
$N(1710) 1/2^+$	1710	140	πN	$12.5 \cdots 20 \ [12.5 \pm 7.5]$	$0.101 \cdots 0.161$
			ηN	$30 \cdots 50 \ [30 \pm 20]$	$2.021 \cdots 3.368$
			$\pi\pi N$	$57.5 \cdots 30 \ [31 \pm 17]$	
$N(1720) 3/2^+$	1720	250	πN	$11 \ [11 \pm 3]$	0.002
			ηN	$3 [3 \pm 2]$	0.079
			$\pi\pi N$	86 [> 50]	
$N(1895) 1/2^{-}$	1900	120	πN	$5 [10 \pm 8]$	0.0025
			ηN	$30 \; [30 \pm 15]$	0.058
			$\eta' N$	$25 [25 \pm 15]$	0.510
			$\pi\pi N$	$40 \ [45.5 \pm 28.5]$	
$N(1900) 3/2^+$	1920	200	πN	$10.5 \ [10.5 \pm 9.5]$	0.001
			ηN	$8 \ [8 \pm 6]$	0.064
			$\eta' N$	$6 \ [6 \pm 2]$	9.862
			$\pi\pi N$	75.5 [> 56]	
$N(2100) 1/2^+$	2100	260	πN	$20 \ [20 \pm 12]$	0.138
			ηN	$25 \ [25 \pm 20]$	0.750
			$\eta' N$	$8 [8 \pm 3]$	0.942
			$\pi\pi N$	47 [> 55]	

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Each vertex obtained from the interaction Lagrangian is multiplied by a phenomenological cutoff function

$$F_{MNN^*}(q^2, p^2, p'^2) = F_M(q^2)F_B(p^2)F_B(p'^2)$$

where q denotes the four-momentum of the meson $M = \pi^0, \eta, \eta', p'$ and p are the four-momenta of the two baryons.

$$F_M(q^2) = \frac{\Lambda_{MNN^*}^2 - m_M^2}{\Lambda_{MNN^*}^2 - q^2}, \qquad \Lambda_{MNN} = 1.0 \text{ GeV}, \quad \Lambda_{MNN^*} = 1.3 \text{ GeV}$$
$$F_B(p^2) = \frac{\Lambda_B^4}{(p^2 - m_B^2)^2 + \Lambda_B^4}, \quad B = N, N_{1/2}^*, \qquad \Lambda_N = 1.0 \text{ GeV}, \quad \Lambda_{N^*} = 1.2 \text{ GeV}$$

For spin-3/2 nucleon resonances $(B = N_{3/2}^*)$ we use the multidipole form:

$$F_B(p^2) = \frac{\Lambda_B^4}{(p^2 - m_B^2)^2 + \Lambda_B^4} \left(\frac{m_B^2 \tilde{\Gamma}_B^2}{(p^2 - m_B^2)^2 + m_B^2 \tilde{\Gamma}_B^2} \right)^2 \text{ where } \tilde{\Gamma}_B = \Gamma_B / \sqrt{2^{1/3} - 1}$$

T. Vrancx, L. De Cruz, J. Ryckebusch, and P. Vancraeyveld, Consistent interactions for high-spin fermion fields, Phys. Rev. C 84 (2011) 045201, arXiv:1105.2688 [nucl-th]

σ (mb)

π p \rightarrow nn reaction



The differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\boldsymbol{k}_{\eta}|}{|\boldsymbol{k}_{\pi}|} \frac{1}{2} \sum_{\text{spins}} |\mathcal{M}_{\pi^- p \to \eta n}|^2,$$

where k_{π} and k_{n} are the c.m. three-momenta of the initial π^{-} and the final η mesons, respectively.

$$\mathcal{M}_{\pi^- p \to \eta n} = \mathcal{M}_s^{(n)} + \mathcal{M}_u^{(p)} + \sum_{N_{1/2}^*} \mathcal{M}_s^{(N_{1/2}^*)} + \sum_{N_{3/2}^*} \mathcal{M}_s^{(N_{3/2}^*)}$$

"Model 1" uses the PS-type couplings. $\pi p \rightarrow \eta n$ $\pi p \rightarrow \eta n$ "Model 2" assumes PV-type couplings total, model 1model 1 for the N(1710) resonance. model 1 w/o N(1520) N(1520) ----- model 1 w/o N(1650) N(1535 model 1 w/o N(1710) The N(1535) resonance gives a major N(1650) N(1700)- model 2 contribution. N(1710)..... N(1720) Complete result indicates a large interference effect between a different contributions, for instance, between N(1535) and N(1650). For W > 1.65 GeV a large contribution is possible from N(1710). However, there the reaction mechanism is under debate. 1.7 1.9 1.5 1.8 1.9 1.5 1.6 1.8 2 1.6 1.7 2 $W_{\pi \bar{} p} \left(GeV \right)$ $W_{\pi \bar{p}} (GeV)$



"Model 1" uses the PS-type couplings. "Model 2" assumes PV-type couplings for the N(1710) resonance.

Data are from:

S. Prakhov et al., (Crystal Ball Collaboration), Measurement of π p \rightarrow η n from threshold to $p\pi^2 = 747$ MeV/c, Phys. Rev. C 72 (2005) 015203,

R. M. Brown et al., Differential cross sections for the reaction $\pi p \rightarrow \eta n$ between 724 and 2723 MeV/c, Nucl. Phys. B 153 (1979) 89,

and at backward scattering region from:

N. C. Debenham et al., Backward π p reactions between 0.6 and 1.0 GeV/c, Phys. Rev. D 12 (1975) 2545

The Lagrangians for vector meson-nucleon interactions are

$$\mathcal{L}_{\rho NN} = -g_{\rho NN} \bar{N} \Big[\gamma_{\mu} - \kappa_{\rho} \frac{\sigma_{\mu\nu} \partial^{\nu}}{2m_{N}} \Big] (\boldsymbol{\tau} \boldsymbol{\Phi}^{\mu}_{\rho}) N$$
$$\mathcal{L}_{\omega NN} = -g_{\omega NN} \bar{N} \Big[\gamma_{\mu} - \kappa_{\omega} \frac{\sigma_{\mu\nu} \partial^{\nu}}{2m_{N}} \Big] \boldsymbol{\Phi}^{\mu}_{\omega} N$$

 κ_V : tensor-to-vector coupling ratio, $\kappa_V = f_{VNN}/g_{VNN}$

We use $g_{\rho pp} = 3.0$, $\kappa_{\rho} = 6.1$ $g_{\omega pp} = 9.0$, $\kappa_{\omega} = 0$

The effective Lagrangians describing the interactions of the nucleon resonance of $J^P = 1/2^-$, e.g. N(1535), with the nucleon and the meson ρ^0 :

$$\mathcal{L}_{\rho N N_{1/2^{-}}^{*}} = -\frac{1}{2m_{N}} \bar{N}^{*} \gamma_{5} \Big[g_{\rho N N^{*}} \Big(\frac{\gamma_{\mu} \partial^{2}}{m_{N^{*}} + m_{N}} - i \partial_{\mu} \Big) - f_{\rho N N^{*}} \sigma_{\mu\nu} \partial^{\nu} \Big] (\boldsymbol{\tau} \boldsymbol{\Phi}_{\rho}^{\mu}) N + \text{h.c.}$$



The VV-fusion amplitude $(VV = \rho^0 \rho^0, \omega \omega)$ is

$$\mathcal{M}_{pp \to pp\eta} = \mathcal{M}_{pp \to pp\eta}^{(\omega\omega \text{ fusion})} + \mathcal{M}_{pp \to pp\eta}^{(\rho\rho \text{ fusion})} - (p_1, \lambda_1 \leftrightarrow p_2, \lambda_2)$$

$$\mathcal{M}_{\lambda_a \lambda_b \to \lambda_1 \lambda_2 \eta}^{(VV \text{ fusion})} = (-i)\bar{u}(p_1, \lambda_1)i\Gamma_{\mu_1}^{(Vpp)}(p_1, p_a)u(p_a, \lambda_a)$$

$$\times i\tilde{\Delta}^{(V) \,\mu_1 \nu_1}(s_1, t_1) i\Gamma_{\nu_1 \nu_2}^{(VV\eta)}(q_1, q_2) i\tilde{\Delta}^{(V) \,\nu_2 \mu_2}(s_2, t_2)$$

$$\times \bar{u}(p_2, \lambda_2)i\Gamma_{\mu_2}^{(Vpp)}(p_2, p_b)u(p_b, \lambda_b)$$

Here $\Gamma^{(VV\eta)}$ and $\Gamma^{(Vpp)}$ are the $VV\eta$ and Vpp vertex functions, respectively, and $\tilde{\Delta}^{(V)}$ is the propagator for the reggeized vector meson V.

The $VV\eta$ vertices are derived from an effective Lagrangians

$$\mathcal{L}_{\rho\rho\eta} = \frac{g_{\rho\rho\eta}}{2m_{\rho}} \varepsilon_{\mu\nu\alpha\beta} \left(\partial^{\mu} \Phi^{\nu}_{\rho} \partial^{\alpha} \Phi^{\beta}_{\rho}\right) \Phi_{\eta}$$
$$\mathcal{L}_{\omega\omega\eta} = \frac{g_{\omega\omega\eta}}{2m_{\omega}} \varepsilon_{\mu\nu\alpha\beta} \left(\partial^{\mu} \Phi^{\nu}_{\omega} \partial^{\alpha} \Phi^{\beta}_{\omega}\right) \Phi_{\eta}$$

$$q_1 = p_a - p_1, \quad q_2 = p_b - p_2, \quad k = q_1 + q_2$$

$$t_1 = q_1^2, \quad t_2 = q_2^2, \quad m_\eta^2 = k^2$$

$$s = (p_a + p_b)^2 = (p_1 + p_2 + k)^2, \text{ c.m. energy squared}$$

$$s_1 = (p_1 + k)^2, \quad s_2 = (p_2 + k)^2$$

The $VV\eta$ vertex, including form factor, with q_1 , μ and q_2 , ν the momenta and vector indices of the incoming V mesons, is given by

$$i\Gamma^{(VV\eta)}_{\mu\nu}(q_1, q_2) = i\frac{g_{VV\eta}}{2m_V} \,\varepsilon_{\mu\nu\alpha\beta} \,q_1^{\alpha} q_2^{\beta} \,F^{(VV\eta)}(q_1^2, q_2^2, k^2) \qquad \qquad \text{Here}: \qquad \Gamma^{(VV\eta)}_{\mu\nu}(q_1, q_2) \,q_1^{\mu} = 0$$

In our case, the form factor should be normalised to 1 at $F^{(VV\eta)}(0, m_V^2, m_n^2)$ consistent with the kinematics at which the coupling constant g_{VVn} is determined; this is, from the radiative meson decays $V \to \eta \gamma$ in conjunction with the VMD assumption. We use, therefore, the form factor

$$F^{(VV\eta)}(t_1, t_2, m_{\eta}^2) = \frac{\Lambda_V^2}{\Lambda_V^2 - t_1} \frac{\Lambda_V^2 - m_V^2}{\Lambda_V^2 - t_2}$$

and $\Lambda_V = \Lambda_{V, \text{mon}} = 1.3 \text{ GeV}$ motivated by analysis of the $\gamma p \to \eta p$ reaction.

The form factor $F_{VNN}(t)$ describing the t-dependence of the V-proton coupling can be parametrised as

$$F_{VNN}(t) = \frac{\Lambda_{VNN}^2 - m_V^2}{\Lambda_{VNN}^2 - t}$$

where $\Lambda_{VNN} > m_V$ and t < 0. We take $\Lambda_{VNN} = 1.4$ GeV for both ρ^0 - and ω proton coupling. From the Bonn potential model [Machleidt] $\Lambda_{\rho NN} = 1.4 \text{ GeV}$ and $\Lambda_{\omega NN} = 1.5$ GeV are required for a fit to NN scattering data.

0 $\Gamma^{(VV\eta)}_{\mu\nu}(q_1,q_2) q_2^{\nu} = 0$

The standard form of the vector-meson propagator:

$$i\Delta_{\mu\nu}^{(V)}(q) = i\left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2 + i\epsilon}\right)\Delta_T^{(V)}(q^2) - i\frac{q_{\mu}q_{\nu}}{q^2 + i\epsilon}\Delta_L^{(V)}(q^2)$$
$$\Delta_T^{(V)}(t) = (t - m_V^2)^{-1}$$

For higher values of s_1 and s_2 we must take into account reggeization effect.

$$\Delta_T^{(V)}(t_i) \to \tilde{\Delta}_T^{(V)}(s_i, t_i) = \Delta_T^{(V)}(t_i) \left(\exp(i\phi(s_i)) \frac{s_i}{s_{\text{thr}}} \right)^{\alpha_V(t_i) - 1}$$
$$\phi(s_i) = \frac{\pi}{2} \exp\left(\frac{s_{\text{thr}} - s_i}{s_{\text{thr}}}\right) - \frac{\pi}{2}$$

see e.g.

Lebiedowicz, Nachtmann, Szczurek, Central exclusive diffractive production of K+K-K+K- via the intermediate $\phi\phi$ state in proton-proton collisions, Phys.Rev.D 99 (2019) 9, 094034

where s_{thr} is the lowest value of s_i possible in the MN system: $s_{\text{thr}} = (m_p + m_\eta)^2$

We use the linear form for the vector meson Regge trajectories :

$$\alpha_V(t) = \alpha_V(0) + \alpha'_V t$$
, $\alpha_V(0) = 0.5$, $\alpha'_V = 0.9 \text{ GeV}^{-2}$

Results $\gamma p \rightarrow \eta p$ and $\gamma p \rightarrow \eta' (958) p$ reactions



CBELSA/TAPS Collaboration, V. crede et al., PRC 80 (2009) 055202

 $\gamma (q) \qquad \rho^{0}; \omega \qquad \eta, \eta'(k)$ $q_{t} \qquad 1 \rho^{0}; \omega$ $p (p_{b}) \qquad p (p_{2})$

Coupling constants obtained from radiative decay rates $\eta' \rightarrow V \gamma$ and $V \rightarrow \eta \gamma$

$$\Gamma^{(\gamma V\widetilde{M})}_{\mu\nu}(q,q_t) = -ie \frac{g_{\gamma V\widetilde{M}}}{m_V} \varepsilon_{\mu\nu\alpha\beta} q^{\alpha} q_t^{\beta} F^{(\gamma V\widetilde{M})}(q^2,q_t^2,k^2)$$
$$F^{(\gamma V\widetilde{M})}(0,q_t^2,m_{\widetilde{M}}^2) = \left(\frac{\Lambda_V^2 - m_V^2}{\Lambda_V^2 - q_t^2}\right)^n$$

where
$$n = 1$$
 for $\widetilde{M} = \eta$ and $n = 2$ for $\widetilde{M} = \eta'$.

Reggeized V-meson exchange mechanism is dominant at higher energies and forward angles.

100 (nb/sr) $\gamma p \rightarrow \eta' p$, CLAS data $\gamma p \rightarrow \eta' p$, CLAS data 10^{2} $_{vn} = 2.05 \text{ GeV}$ $W_{yp} = 2.45 \text{ GeV}$ 80 = 2.55 GeV= 2.15 GeVdo/dΩ = 2.65 GeV= 2.25 GeV $r_{yp}^{(P)} = 2.75 \text{ GeV}$ = 2.35 GeV $\Lambda_{V, dip} = 1.4 \text{ GeV}$ 60 10 20 0.5 -0.50.5 -0.50 cosθ cosθ

CLAS Collaboration, M. Williams et al., PRC 80 (2009) 045213, T. Hu et al., PRC 102 (2020)065203; for η': R. Dickson et al., PRC 93 (2016) 065202¹⁶

Results $(p \ p \rightarrow p \ p \ \eta)$



To restore the energy dependence of the cross section for the N and N(1535) contributions via the ρ^0 -meson exchange, the relevant amplitudes were multiplied by the suppression function:

$$f(s) = \exp\left(-\frac{s - s_{\text{thr}}}{\Lambda^2}\right), \quad \text{here}: s_{\text{thr}} = (2m_p + m_\eta)^2$$

Results $(p \ p \rightarrow p \ p \ \eta)$



Data:

G. Agakishiev et al., (HADES Collaboration), Study of exclusive one-pion and oneeta production using hadron and dielectron channels in pp reactions at kinetic beam energies of 1.25 GeV and 2.2 GeV with HADES, Eur. Phys. J. A 48 (2012) 74

K. Teilab, (HADES Collaboration), ω and η meson production in p + p reactions at E_{kin} = 3.5 GeV, Int. J. Mod. Phys. A 26 (2011) 694

K. Teilab, Ph.D. thesis: The production of η and ω mesons in 3.5 GeV p+p interaction in HADES, Frankfurt U., 2011. Available at https://hades.gsi.de/node/4

F. Balestra et al., (DISTO Collaboration), Exclusive η production in proton-proton reactions, Phys. Rev. C 69 (2004) 064003

Model results:	$\sigma~(\mu{ m b})$					
$\sqrt{s} \; (\text{GeV})$	total	N(1535)	N	VV fusion		
2.748	101.48	117.74	4.36	4.96		
2.978	115.65	116.31	8.17	6.33		
3.46	99.96	83.58	11.92	6.77		
4.0	76.14	56.32	11.48	6.26		
5.0	35.92	22.34	7.61	4.96		
8.0	5.19	0.19	2.56	2.41		

 WA102, Phys.Lett. B 467 (1999) 165

 A. Kirk, Phys. Lett. B 489 (2000) 29
 $pp \rightarrow pp\eta$

 From WA102 at $\sqrt{s} = 29.1 \text{ GeV}$ $\sigma_{exp} = (3.86 \pm 0.37) \ \mu b$





Results $(p p \rightarrow p p \eta)$





Results ($p p \rightarrow p p \eta'(958)$)



• Data: Experiment SATURNE-213 F. Balestra et al. (DISTO Collaboration) PLB 491 (2000) 29

$$\sigma^{\rm DISTO}_{pp\to pp\eta'} = 1.12 \pm 0.15^{+0.42}_{-0.31} \ \mu {\rm b}$$

The ratio of total cross section for η' and η is $R = \sigma_{pp \to pp\eta'} / \sigma_{pp \to pp\eta} = (0.83 \pm 0.11^{+0.23}_{-0.18}) \times 10^{-2}$

•	Model results:		$g_{\rho NN^*(1895)}$	$f_{\rho NN^*(1895)}$
		$\lambda_{\eta'} = 1 \text{ (PS)}, g_{\eta'NN} = 2$	0.5~(a)	0.2 (c)
		$\lambda_{\eta'} = 0 \ \text{(PV)}, \ g_{\eta'NN} = 2$	0.6 (b)	0.23 (d)

Results ($p p \rightarrow p p \eta'(958)$)



Results ($p p \rightarrow p p \eta'(958)$)



Results ($p p \rightarrow p p \eta'(958)$)



Results ($p p \rightarrow p p \eta'(958)$)





Conclusions

- We have given predictions for experiments at FAIR energies.
- Comparison of the full model (including nucleonic contributions, N* resonances, VV-fusion processes and interferences between them) with ongoing experimental results from HADES, PANDA, SIS100 (both total cross-section and differential distributions) should help to learn more about the production mechanism of η, η', f₁. We shall learn about the coupling strengths ηNN, η'NN, ηNN*(1535), η'NN*(1895), π⁰NN*, ρ⁰NN*, ρρM, ωωM etc. The production of η and η' mesons in pp collisions is of particular importance because of their coupling to baryonic resonances.
- We are looking forward to first experimental results on the production of η , η' , and $f_1(1285)$ mesons in pp@4.5 GeV with HADES.

VV-fusion mechanism ($p p \rightarrow p p f_1$)

 $p(p_a, \lambda_a) + p(p_b, \lambda_b) \rightarrow p(p_1, \lambda_1) + f_1(k, \lambda_{f_1}) + p(p_2, \lambda_2)$ $p_{a,b}, p_{1,2}$ and $\lambda_{a,b}, \lambda_{1,2} = \pm \frac{1}{2}$: the four-momenta and helicities of protons k and $\lambda_{f_1} = 0, \pm 1$: the four-momentum and helicity of the f_1 meson $\begin{array}{cccc} q_1 & p_a - p_1, & q_2 = p_b - p_2, & k = q_1 + q_2 \\ & & f_1(k) & f_1(k)$ $p(p_b)$ $p(p_2)$ $VV\text{-fusion amplitude: } \mathcal{M}_{pp \to ppf_1}^{(VV \text{ fusion})} = \mathcal{M}_{pp \to ppf_1}^{(\rho\rho \text{ fusion})} + \mathcal{M}_{pp \to ppf_1}^{(\omega\omega \text{ fusion})}$ $\mathcal{M}^{(VV \text{ fusion})}_{\lambda_a \lambda_b \to \lambda_1 \lambda_2 \lambda_{f_1}} = (-i) \left(\epsilon^{\alpha} (\lambda_{f_1}) \right)^* \bar{u}(p_1, \lambda_1) i \Gamma^{(Vpp)}_{\mu_1}(p_1, p_a) u(p_a, \lambda_a)$ $\times i \tilde{\Delta}^{(V) \mu_1 \nu_1}(s_1, t_1) i \Gamma^{(VV f_1)}_{\mu_1 \mu_2 \alpha}(q_1, q_2) i \tilde{\Delta}^{(V) \nu_2 \mu_2}(s_2, t_2)$ $\times \bar{u}(p_2,\lambda_2)i\Gamma^{(Vpp)}_{\mu_2}(p_2,p_b)u(p_b,\lambda_b)$ $i\Gamma_{\mu}^{(Vpp)}(p',p) = -i\Gamma_{\mu}^{(V\bar{p}\bar{p})}(p',p) = -ig_{Vpp} F_{VNN}(t) \left[\gamma_{\mu} - i\frac{\kappa_{V}}{2m_{\pi}}\sigma_{\mu\nu}(p-p')^{\nu}\right]$ $g_{opp} = 3.0$, $\kappa_o = 6.1$, $g_{\omega pp} = 9.0$, $\kappa_\omega = 0$ κ_V : tensor-to-vector coupling ratio, $\kappa_V = f_{VNN}/g_{VNN}$ $F_{VNN}(t) = \frac{\Lambda_{VNN}^2 - m_V^2}{\Lambda^2 - t}$

For the proton-antiproton collisions we have

$$\begin{split} \bar{u}(p_2,\lambda_2)i\Gamma^{(Vpp)}_{\mu_2}(p_2,p_b)u(p_b,\lambda_b) &\to \bar{v}(p_b,\lambda_b)i\Gamma^{(V\bar{p}\bar{p})}_{\mu_2}(p_2,p_b)v(p_2,\lambda_2) \\ &= -\bar{u}(p_2,\lambda_2)i\Gamma^{(Vpp)}_{\mu_2}(p_2,p_b)u(p_b,\lambda_b) \\ \mathcal{M}^{(VV\,\text{fusion})}_{p\bar{p}\to p\bar{p}M} &= -\mathcal{M}^{(VV\,\text{fusion})}_{pp\to ppM} \end{split}$$

The standard form of the vector-meson propagator:

$$i\Delta_{\mu\nu}^{(V)}(q) = i\left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2 + i\epsilon}\right)\Delta_T^{(V)}(q^2) - i\frac{q_{\mu}q_{\nu}}{q^2 + i\epsilon}\Delta_L^{(V)}(q^2)$$
$$\Delta_T^{(V)}(t) = (t - m_V^2)^{-1}$$

For higher values of
$$s_1$$
 and s_2 we must take into account reggeization:

$$\Delta_T^{(V)}(t_i) \to \tilde{\Delta}_T^{(V)}(s_i, t_i) = \Delta_T^{(V)}(t_i) \left(\exp(i\phi(s_i)) \frac{s_i}{s_{\text{thr}}} \right)^{\alpha_V(t_i)-1}$$

$$\phi(s_i) = \frac{\pi}{2} \exp\left(\frac{s_{\text{thr}} - s_i}{s_{\text{thr}}}\right) - \frac{\pi}{2}$$

where s_{thr} is the lowest value of s_i possible here: $s_{\text{thr}} = (m_p + m_{f_1})^2$ We use the linear form for the vector meson Regge trajectories :

$$\alpha_V(t) = \alpha_V(0) + \alpha'_V t$$
, $\alpha_V(0) = 0.5$, $\alpha'_V = 0.9 \text{ GeV}^{-2}$

Results



R. Dickson et al. (CLAS Collaboration), PRC 93 (2016) 065202

The $\rho\rho f_1$ coupling constant is extracted from the radiative decay rate $f_1 \rightarrow \rho^0 \gamma$ using the VMD approach.

from PDG : $\Gamma(f_1(1285) \to \gamma \rho^0) = 1384.7^{+305.1}_{-283.1} \text{ keV}$

from CLAS : $\Gamma(f_1(1285) \rightarrow \gamma \rho^0) = (453 \pm 177) \text{ keV}$ we use We consider decay $f_1 \rightarrow \rho^0 \gamma \rightarrow \pi^+ \pi \gamma$ taking ρ^0 mass distribution. We estimate the cotoff parameter Λ_ρ in the $f_1 \rho \rho$ form factor:

 $F_{\rho\rho f_1}(k_{\rho}^2, k_{\gamma}^2, k^2) = F_{\rho\rho f_1}(k_{\rho}^2, 0, m_{f_1}^2) = \tilde{F}_{\rho}(k_{\rho}^2)\tilde{F}_{\rho}(0)F(m_{f_1}^2) = \tilde{F}_{\rho}(k_{\rho}^2)\tilde{F}_{\rho}(0)$

Photoproduction process:



We assume $g_{\omega\omega f_1} = g_{\rho\rho f_1}$ based on arguments from the quark model and VMD. We assume $\Lambda_{\rho} = \Lambda_{\omega} = \Lambda_{V}$ and $\Lambda_{\rho NN} = \Lambda_{\omega NN} = \Lambda_{VNN}$. Reggeization effect included

The t-channel V-exchange mechanism play a crucial role in reproducing the forward-peaked angular distributions, especially at higher energies. From the comparison of differential cross sections to the CLAS data we estimate:

 $\begin{array}{l} ({\rm C7}): \Lambda_{VNN} = 1.35 \; {\rm GeV} \; {\rm for} \; \Lambda_V = 0.65 \; {\rm GeV} \; , |g_{VVf_1}| = 20.03 \\ ({\rm C9}): \Lambda_{VNN} = 1.01 \; {\rm GeV} \; {\rm for} \; \Lambda_V = 0.8 \; {\rm GeV} \; , |g_{VVf_1}| = 12.0 \\ ({\rm C10}): \Lambda_{VNN} = 0.9 \; {\rm GeV} \; {\rm for} \; \Lambda_V = 1.0 \; {\rm GeV} \; , |g_{VVf_1}| = 8.49 \\ ({\rm C11}): \Lambda_{VNN} = 0.834 \; {\rm GeV} \; {\rm for} \; \Lambda_V = 1.5 \; {\rm GeV} \; , |g_{VVf_1}| = 6.59 \end{array}$ $\begin{array}{l} \text{(C11)} : \text{ $\alpha_{VNN} = 0.834 \; {\rm GeV} \; {\rm for} \; \Lambda_V = 1.5 \; {\rm GeV} \; , |g_{VVf_1}| = 6.59 \\ \text{(C11)} : \text{ $\alpha_{VNN} = 0.834 \; {\rm GeV} \; {\rm for} \; \Lambda_V = 1.5 \; {\rm GeV} \; , |g_{VVf_1}| = 6.59 \\ \end{array}$

Missing N* resonances and s/u-channel proton exchange Possible N(2300) contribution

 \rightarrow postulated in Y.-Y. Wang et al., PRD 95 (2017) 096015

Results $(p p \rightarrow p p f_1)$



No data for the $pp \rightarrow pp f_1$ and $p\overline{p} \rightarrow p\overline{p} f_1$ reactions at low energies

 \leftarrow Integrated cross section for VV \rightarrow f1 fusion with different parameters.

In our procedure of extracting the model parameters from the CLAS data the dominant sensitivity of cross section is on coupling constants not on the cut-off parameters in form factors.

Reggeization effect must be included, it reduces cross section by a factor of 1.8 already for HADES

Diffractive contribution (IPIP fusion) is very small for the HADES and PANDA energy range \rightarrow IPIP-fusion contribution should be regarded as an upper limit [*PL, Leutgeb, Nachtmann, Rebhan, Szczurek, PRD 102 (2020) 114003*]. If at the WA102 c.m. energy (29.1 GeV) there are important contributions from subleading Reggeon exchanges (IP f_{2IR}, f_{2IR} IP, f_{2IR} f_{2IR}, a_{2IR} a_{2IR}, $\omega_{IR}\omega_{IR}$, $\rho_{IR}\rho_{IR}$, etc.) the IPIP contribution could be smaller (by a factor of up to 4).

Barberis et al. (WA102 Collaboration), PLB 440 (1998) 225:	$pp \to ppf_1(1285)$	$ x_{F,M} \leqslant 0.2$
	$\sqrt{s} = 12.7 \text{ GeV}$	$\sigma_{\rm exp} = (6.86 \pm 1.31) \ \mu {\rm b}$
	$\sqrt{s} = 29.1 \text{ GeV}$	$\sigma_{\rm exp} = (6.92 \pm 0.89) \ \mu {\rm b}$



The narrow width of the η meson allows to set a mass cut on the $\pi^+\pi^-\pi^0$ invariant mass and suppresses the multi-pion background efficiently.

and our predictions for $f_1(1285)$ signal at SIS100 ($\sqrt{s} = 7.61 \text{ GeV}$)

Experimental data for P = 24 GeV ($\sqrt{s} = 6.84$ GeV)

 $\sigma(pp \to ppf_1[\to \pi^+\pi^-\eta(\to \pi^+\pi^-\pi^0)]) = 0.03 - 0.15 \ \mu b$

[1] G. Alexander et al., Phys. Rev. 154 (1967) 1284 [2] S. Danieli et al., Nucl. Phys. B27 (1971) 157

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[3] Blobel et al., NPB 135 (1978) 379 $\sigma(pp \to pp\pi^+\pi^-\pi^+\pi^-\pi^0) = (660 \pm 130) \ \mu b \ [3]; \ \sigma(pp \to pp\pi^+\pi^-\omega) = (200 \pm 40) \ \mu b \ [4]$ [4] Blobel et al., NPB 111 (1976) 397 $\mathcal{BR}(\omega(782) \to \pi^+\pi^-\pi^0) = (89.3 \pm 0.6) \%$ $\mathcal{BR}(\eta \to \pi^+ \pi^- \pi^0) = (22.92 \pm 0.28) \%$ $\mathcal{BR}(f_1(1285) \to \pi^+\pi^-\eta) = (35 \pm 15) \%$

Results

 \sqrt{s} = 3.46 GeV (top) and 5.0 GeV (bottom)



- At near threshold energy (HADES) the values of small $|t_1|$ and $|t_2|$ are not accessible kinematically HADES and PANDA experiments have a good opportunity to study physics of large four-momentum transfer squared $|t_{12}| \rightarrow \text{probes}$ corresponding form factors at relatively large values of $|t_{12}|$ and far from their on mass-shell values $t_{1,2} = m_V^2$ at where they were normalised $\rho^{0}\rho^{0}$ - and $\omega\omega$ -fusion processes have different kinematic dependences. Both terms play similar role. With increasing c.m. energy the averages of $|t_{12}|$ decrease (damping by form factors) hence the $\omega\omega$ term becomes more important We predict a strong preference for
- We predict a strong preference for the outgoing nucleons to be produced with their transverse momenta being back-to-back,

 $d\sigma/d\phi_{pp}$ at $\phi_{pp} = \pi$

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Results

 \sqrt{s} = 3.46 GeV (top) and 5.0 GeV (bottom)



- Since f₁(1285) and η(1295) are close in mass and both decaying to π⁺π⁻η channel, care must be taken for potential overlap of these resonances with each other in the measurement
- $\eta(1295)$ has about 2 times larger total width than $f_1(1285)$
- In order to distinguish both resonances the distribution in azimuthal angle may be used



- With the couplings of V to protons we see that the helicity flipping tensor coupling of the ρ to the protons is large whereas the tensor coupling of the ω is small (taken to be zero)
- At higher energies, available in the future at PANDA and SIS100, $\omega\omega$ fusion giving $\eta(1295)$ should dominate over $\rho\rho$ fusion
- The distribution for $\eta(1295)$ should (nearly) vanish for $\phi_{pp} = 0$ and π