High precision calculation of the hadronic vacuum polarisation contribution to the muon anomaly

Fabian J. Frech for the BMW collaboration

Workshop at 1GeV scale: From mesons to axions
Jagiellonian University, Krakow

September 20, 2024

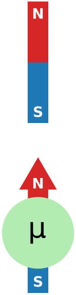
Magnetic moment of the muon

- Muons are charged particles with spin
- Interaction with external magnetic fields via

$$U = -\vec{\mu} \cdot \vec{B}$$

- Magnetic moment

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$



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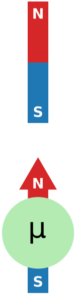

Classical: $g \leq 1$

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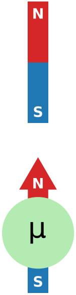

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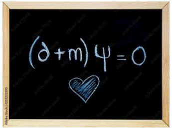
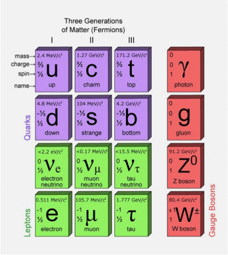


Dirac: $g = 2$

Magnetic moment of the muon

QED: $2 + \frac{\alpha}{\pi} + \dots$

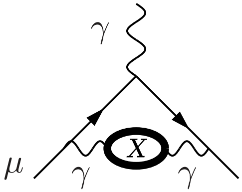


SM: $g = ???$

- Particle creation and annihilation effects by QFT
- Perturbative expansion $\Rightarrow$ desired precision
- Standard Model effects
- Even more?

$$a_\mu = \frac{g_\mu - 2}{2}$$
$$= a_\mu^{qed} + a_\mu^{HVP} + \dots$$

Beyond Standard Model?

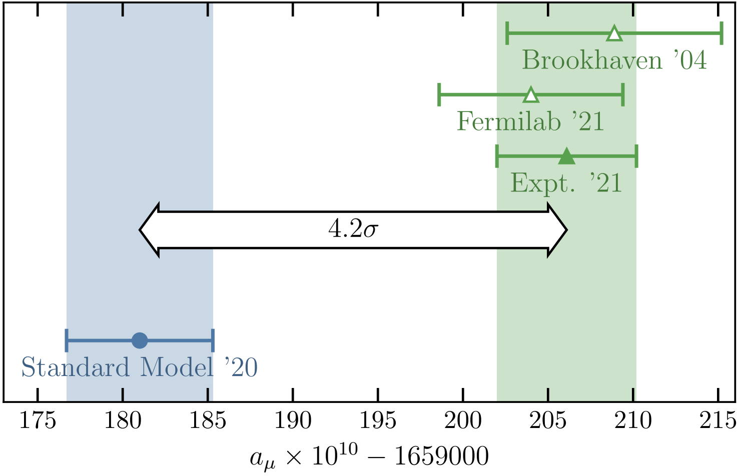


Corrections to a_μ from X:

$$a_\mu^X = \frac{1}{45}\left(\frac{m_\mu}{m_X}\right)^2\left(\frac{\alpha}{\pi}\right)^2 + \ldots$$

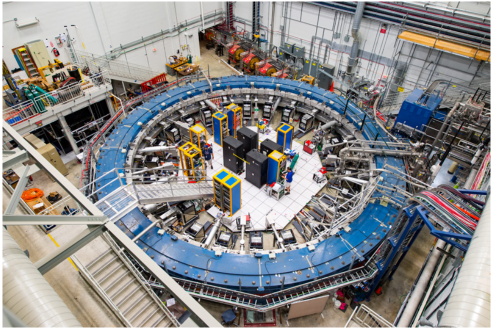
- If theory and experiment do not agree it is a hint for beyond Standard Model effects
- Muon 40,000 times more sensitive than electron

So what is the muon anomaly?



$g-2$ EXPERIMENT

Experiment



[Fermi National Accelerator Laboratory 2017]

- BNL (2004) results inconsistent with theory

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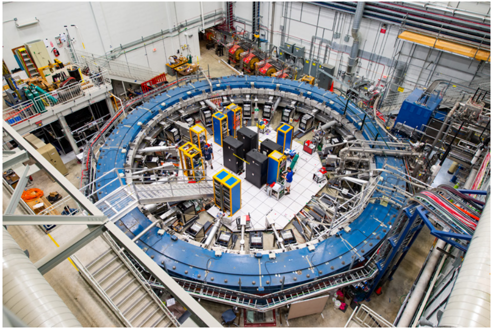

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- Brought experiment to Fermilab
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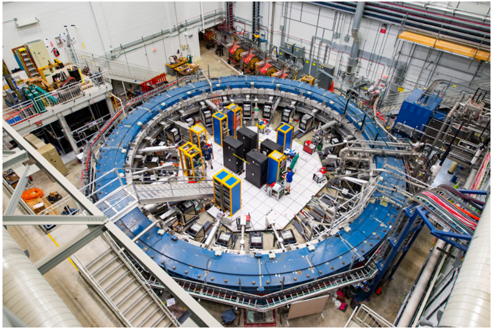

[Fermi National Accelerator Laboratory 2017]

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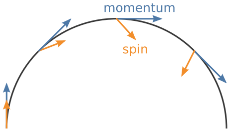
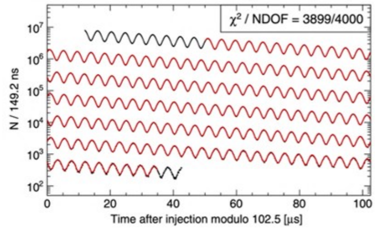
- Brought experiment to Fermilab $\Rightarrow$ cleaner muon beam
- Started runs in 2017
- Finished in 2023
- Results not yet fully published

Experiment

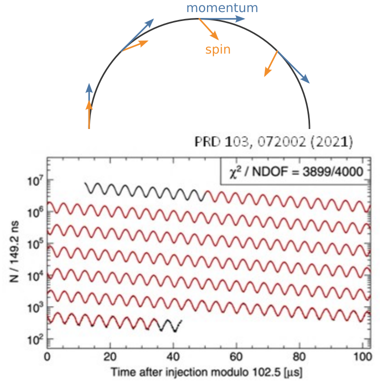


PRD 103, 072002 (2021)



- Polarized (spin momentum parallel) muon on storage ring

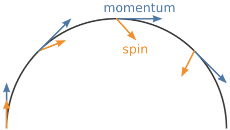
Experiment



- Polarized (spin momentum parallel) muon on storage ring
- Spin and momentum rotate differently around the magnetic field

$$\Delta\omega = \omega_s - \omega_r = a_\mu \frac{eB}{m_\mu}$$

Experiment



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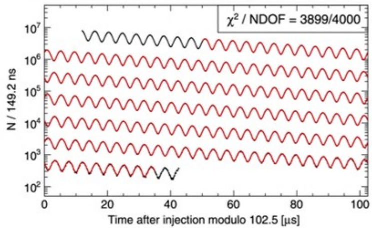


- Polarized (spin momentum parallel) muon on storage ring
- Spin and momentum rotate differently around the magnetic field

$$\Delta\omega = \omega_s - \omega_r = a_\mu \frac{eB}{m_\mu}$$

- Precise measurement of $\Delta\omega$ and B leads to a_μ

Precision?

Experiment

Precision?

Experiment

g_μ

Precision?

Experiment

g_μ

a_μ

Precision?

Experiment

g_μ

a_μ^{HVP}

a_μ

Precision?

Experiment

g_μ

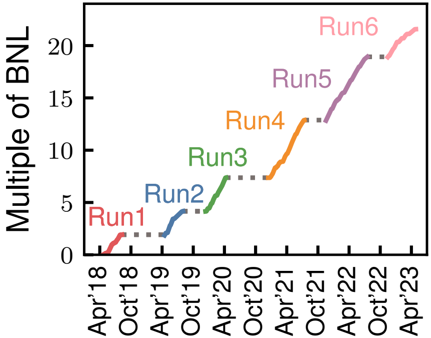
a_μ^{HVP}

a_μ

δa_μ

Experiment

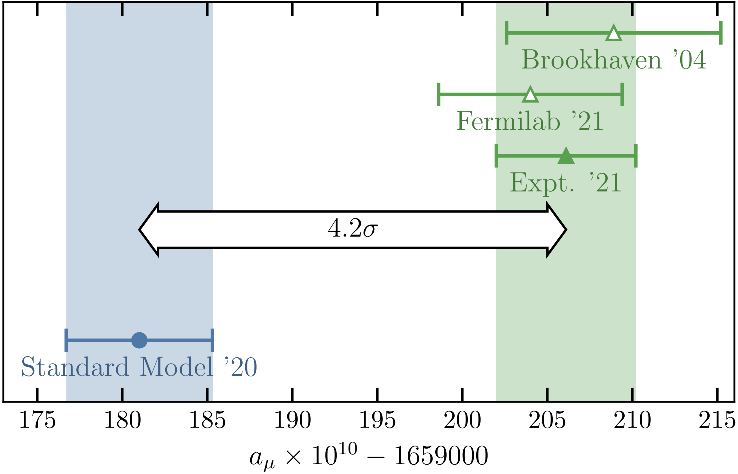
- All six runs done
- More than 20 times the statistics of BNL
- First three runs published
- Further reduction of error by a factor of two expected
  $\Rightarrow$ Goal for theorists



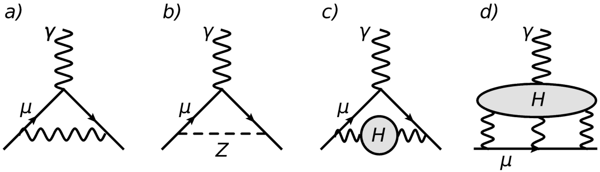
[Fermilab '21]

Data-driven theory prediction (R-ratio)

Muon anomaly



Contributions to a_μ

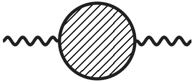
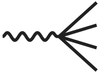


[2104.03281]

a)	$a_\mu^{QED} \times 10^{10}$	11658471.8931	$\pm$	0.0104
b)	$a_\mu^{EW} \times 10^{10}$	15.36	$\pm$	0.1
c)	$a_\mu^{HVP} \times 10^{10}$	684.6	$\pm$	4.0
d)	$a_\mu^{HLbL} \times 10^{10}$	9.2	$\pm$	1.8

R-Ratio (optical theorem)

- The unitarity of the S-matrix implies:

$\mathrm{Im}\Big[$  $\Big] \sim \Big|$  $\Big\}\ \text{hadrons}\ \Big|^2 \sim R(s)$

R-Ratio (optical theorem)

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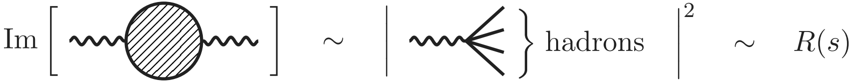
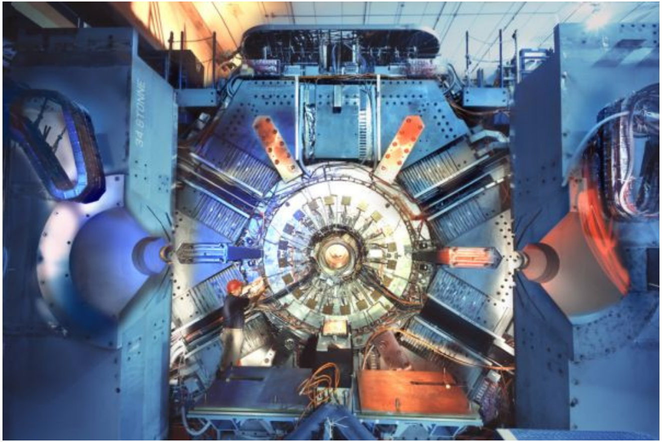


- a_μ from R-Ratio:

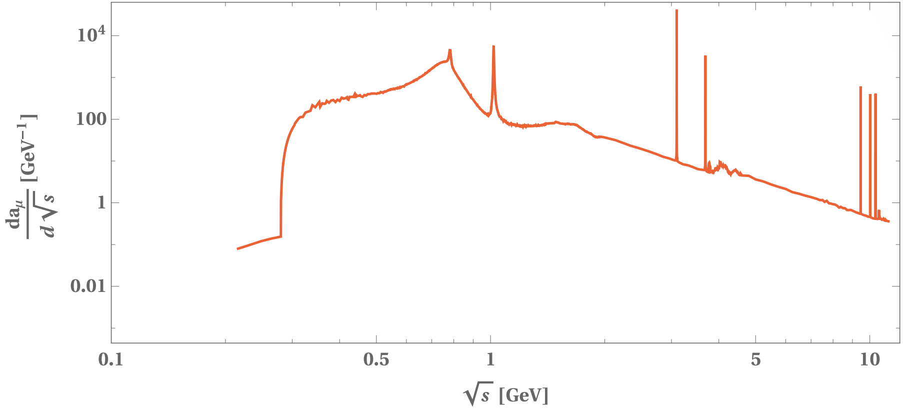
$$a_\mu^{HVP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_0^\infty \frac{\mathrm{d}s}{s^2} \underbrace{\hat{K}(s)}_{\text{th.}} \underbrace{R(s)}_{\text{exp.}}$$

R-Ratio/Experimental input

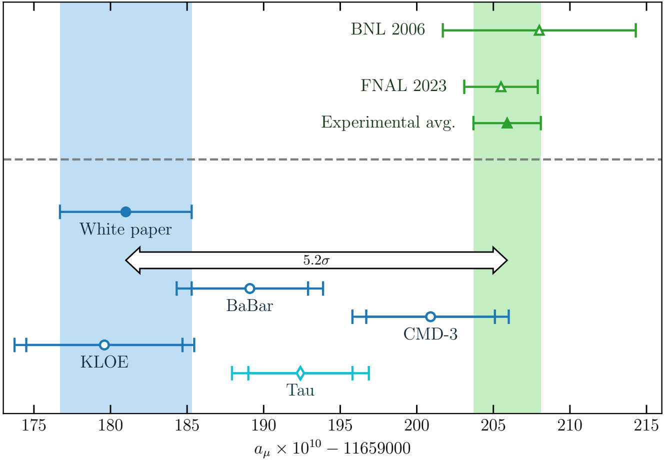
- Input from from electron-positron scattering
- $R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$
- BaBar (picture) and KLOE for 2020 value
- Tau and CMD-3



R-Ratio (integrand)



Results



BNL 2006
FNAL 2023
Experimental avg.
White paper
5.2σ
BaBar
CMD-3
KLOE
Tau
175
180
185
190
195
200
205
210
215
$a_\mu \times 10^{10} - 11659000$

Lattice computation

Lattice QCD

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(\mathrm{i}\not{D} - m)\psi \quad F_{\mu\nu} = [D_\mu, D_\nu]$$

Lattice QCD

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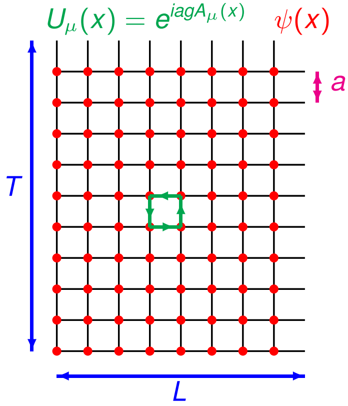


- Ab-initio calculations
- Simulate Path-Integral of QCD
- Replace space-time by a finite lattice
- Solve integral with Monte-Carlo methods

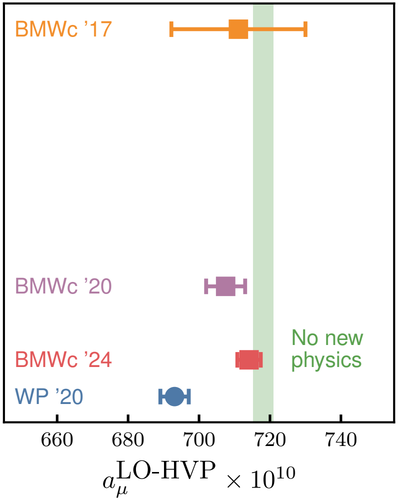
Lattice computing

HAWK at HPCC Stuttgart

10^{10}-dimensional integrals

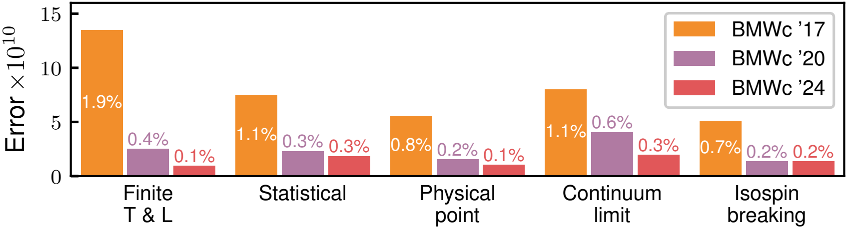
100,000 years on a laptop corresponds to 1 year on a supercomputer

Seven years of progress



- 2020: $3.4\times$ increase in precision compared to 2017
- 2024: $1.7\times$ increase in precision compared to 2020
- Many improvements needed to attain this precision
- Made possible thanks to the work of many groups around the world

Error improvement



- Finite volume/spacing
- Statistical evaluation of path integral
- Matching parameters

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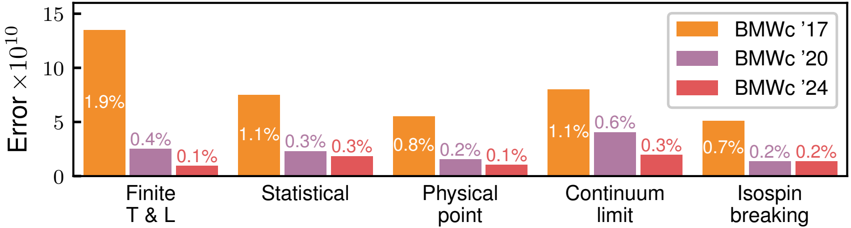
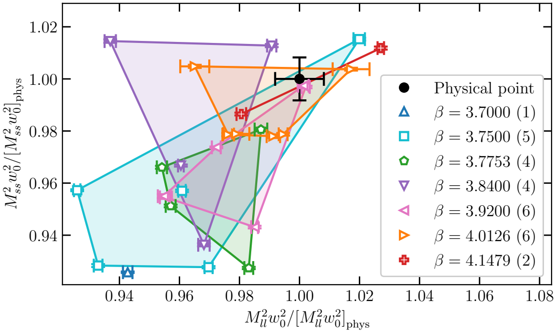


- Finite volume/spacing
- Statistical evaluation of path integral
- Matching parameters

- Adding larger volumes and finer lattices
- Algorithmic improvements
- Separation of window contributions
- Use perturbation theory and data-driven methods

Lattice Setup



β	a
3.7000	0.134 fm
3.7500	0.118 fm
3.7753	0.111 fm
3.8400	0.095 fm
3.9200	0.078 fm
4.1206	0.064 fm
4.1479	0.048 fm

- 28 ensembles with 7 different lattice spacings
- Meson masses from lattice are matched to those from experiment
- Physical volume is fixed to $L^3 \times T = 6^3 \times 9\,\text{fm}^4$

Observables

$$G(t) = -\frac{1}{3e^2}\sum_{\mu=1}^{3}\int \mathrm{d}^3x \,\langle J_\mu(\vec{x},t) J_\mu(0)\rangle$$

$$J_\mu/e = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c - \frac{1}{3}\bar{b}\gamma_\mu b + \frac{2}{3}\bar{t}\gamma_\mu t$$

$$a_\mu^{HVP} = \alpha^2 \int_0^\infty \mathrm{d}t\, K(t m_\mu) G_{1\gamma I}(t)$$

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- Short dist. $(0.0 - 0.4)\,\mathrm{fm}$

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- Short dist. $(0.0 - 0.4)\,\mathrm{fm}$
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- Long dist. $(1.0 - 2.8)\,\mathrm{fm}$

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- Tail $(2.8 - \infty)\,\mathrm{fm}$

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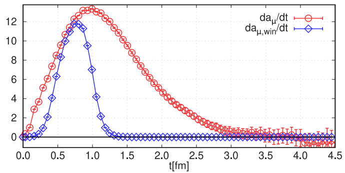
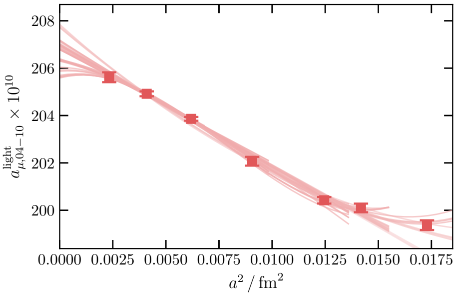
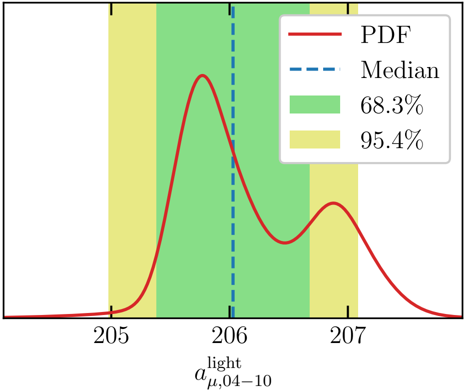


Cont. extrapolations (intermediate dist.)





Tail contributions

- Problem: Lattice QCD results are very noisy for $t > 2.8\,\mathrm{fm}$

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Tail contributions

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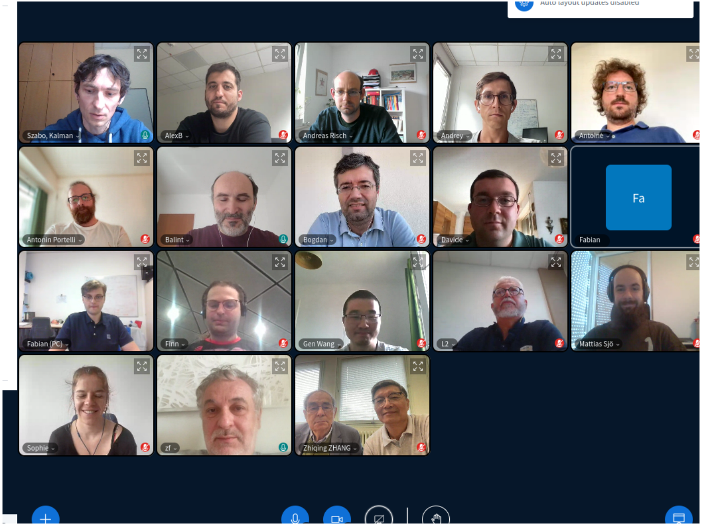
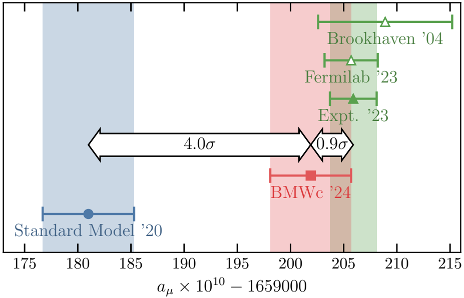
- Solution: Use R-Ratio in this regime ($< 5\%$ of total result)
- What about problems with data-driven input?
- Experimental disagreements do not appear in this region

Unblinding


Auto layout updates disabled
Szabo, Kalman
AlexB
Andreas Risch
Andrey
Antoine
Antonin Portelli
Balint
Bogdan
Davide
Fa
Fabian
Fabian (PC)
Finn
Gen Wang
L2
Mattias Sjö
Sophie
zf
Zhiqing ZHANG

Lattice results

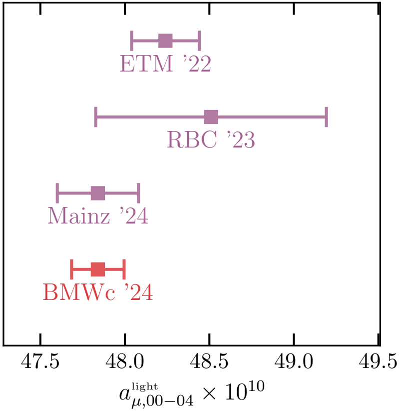


- QED, EW and QCD combined with a remarkably precision
- Reached a Standard Model prediction of $0.32\,\mathrm{ppm}$
- Found agreement within $1\,\sigma$ between theory and experiment

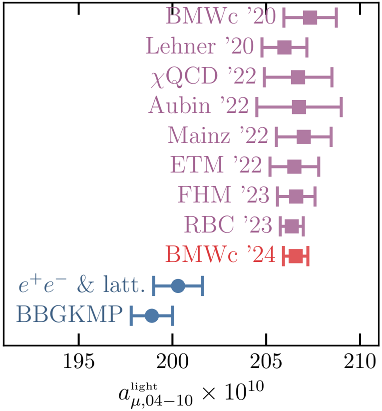
Thank you for your attention!

Any questions?

Window results

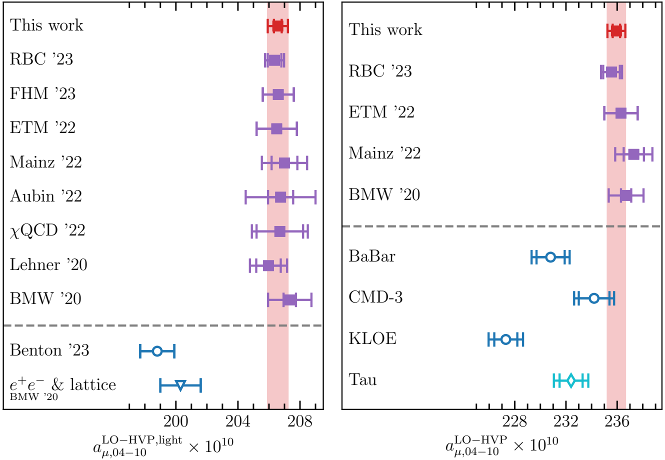


Short distance results

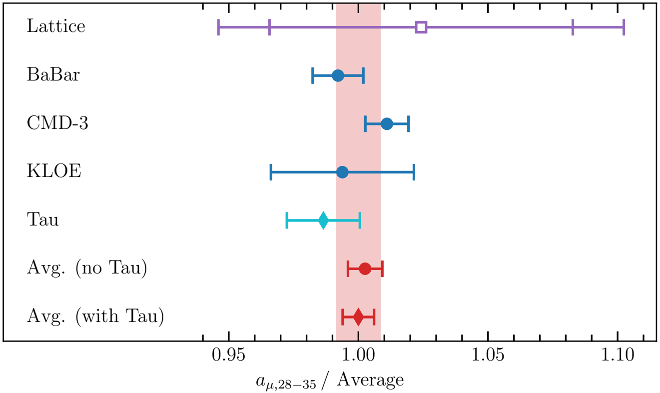


Intermediate distance results

Comparison of intermediate window



Comparison of tail



Lattice strategy

Free parameters:

$$T/a,\ L/a,\ \beta,\ \{am_f\}_{f\in l,s,c,..}$$

- β : different spacings
- m_f: scatter around the physical point
- $T, L =$ *const.*

$\mathcal{O}(10^3)$ configurations per ensemble

- measure observable on every configuration and ensemble
- estimate mean value and standard deviation for every ensemble
- extrapolate to physical masses and $a = 0$

Continuum extrapolations (short distance)