Anomalous interactions for mesons with J=1,2 and glueballs

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in collab. with:

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Based (mostly) on
Phys.Rev.D 109 (2024) 7, L071502
e-Print: 2309.00086 [hep-ph]

Workshop at 1GeV scale: From mesons to axions
19-20/9/2014, Jagiellonian University, Krakow, Poland

Motivation

Chiral (or axial) anomaly: a classical symmetry of QCD broken by quantum fluctuations

Chiral anomaly important for η and η'. What about other mesons?

Classification of mesons in heterochiral and homochiral multiplets.

What about the pseudoscalar glueball

Summary

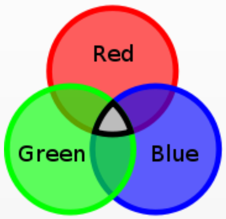
QCD Lagrangian: symmetries and anomalies

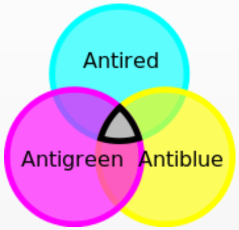
Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B



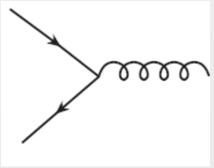


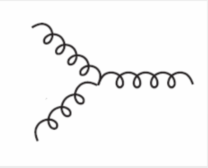
$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix}; \; i = u,d,s,...$$

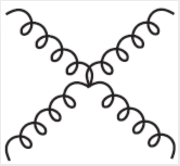
8 type of gluons ($R\bar{G}$, $B\bar{G}$,…)

$$A_\mu^a \; ; \; a = 1,...,8$$

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}$$

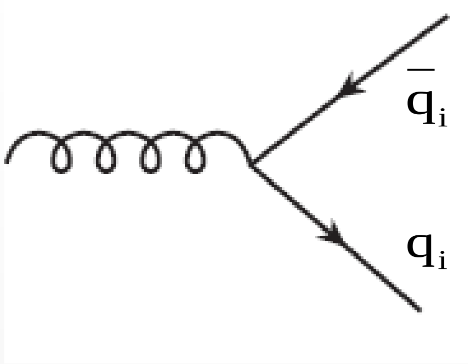






Btw: **where are glueballs?**

Flavor symmetry



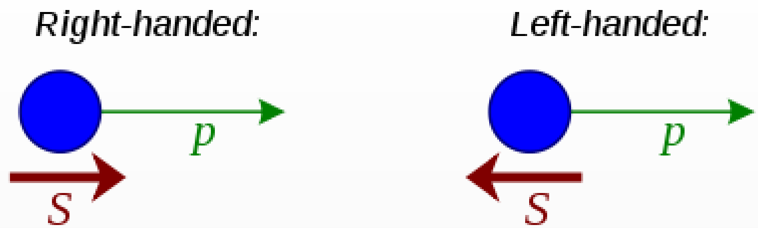
Gluon-quark-antiquark vertex.

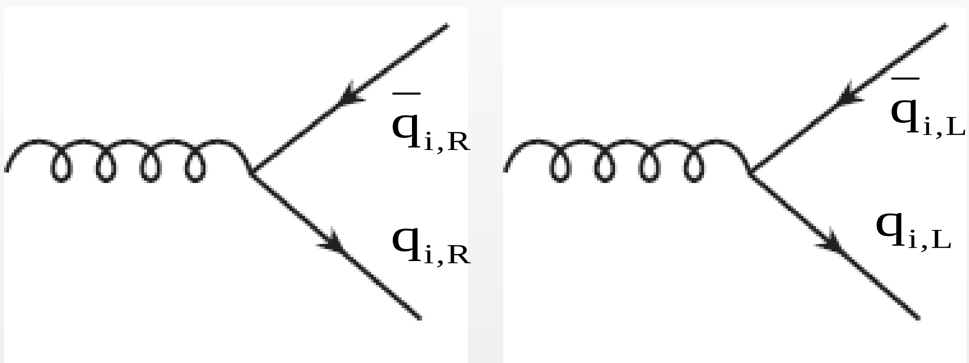
It is democratic! The gluon couples to each flavor with the same strength

$$q_i \to U_{ij} q_j$$

$$U \in U(3)_V \to U^+ U = 1$$

Chiral symmetry





$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow U^R_{ij} q_{j,R} + U^L_{ij} q_{j,L}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

In the chiral limit (m_i=0) chiral symmetry is exact

Chiral transformations and axial anomaly

$$SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times U(1)_{\mathrm{A}}$$

$$q_{\mathrm{L,R}} \longrightarrow \mathrm{e}^{\mp \mathrm{i}\alpha/2} U_{\mathrm{L,R}} q_{\mathrm{L,R}}$$

U(1)A Chiral

Axial anomaly:

$$\partial^{\mu}(\bar{q}^{i}\gamma_{\mu}\gamma_{5}q^{i}) = \frac{3g^{2}}{16\pi^{2}}\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}(G_{\mu\nu}G_{\rho\sigma})$$

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are white and are called hadrons.

Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Chiral partners

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d},\ d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s},\ d\bar{s}$ $s\bar{d},\ s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Chiral partners

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

TABLE I. Chiral multiplets, their currents, and transformations up to $J = 3$. [* and/or $f_0(1500)$; **a mix of.] The first two columns correspond to the assignment suggested in the Quark Model review of the PDG [8], to which we refer for further details and references (see also the discussion in the text).

J^{PC}, $^{2S+1}L_J$	$I=1(\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}})$; $I=1(-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d)$; $I=0(\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{**}$	Microscopic currents	Chiral multiplet	Transformation under $SU(3)_{\rm L} \times SU(3)_{\rm R} \times \times U(1)_{\rm A}$	
0^{-+}, $^1S_0$	π; K; $\eta, \eta'(958)$	$P^{ij} = \frac{1}{2}\bar{q}^j \mathrm{i}\gamma^5 q^i$	$\Phi = S + \mathrm{i}P$ $(\Phi^{ij} = \bar{q}^j_{\rm R} q^i_{\rm L})$	$\Phi \to e^{-2i\alpha} U_{\rm L} \Phi U^\dagger_{\rm R}$	**Heterochiral**
0^{++}, $^3P_0$	$a_0(1450)$; $K_0^*(1430)$; $f_0(1370), f_0(1710)^*$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$			
1^{--}, $^1S_1$	$\rho(770)$; $K^*(892)$; $\omega(782), \phi(1020)$	$V^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L^{ij}_\mu = \bar{q}^j_{\rm L}\gamma_\mu q^i_{\rm L})$	$L_\mu \to U_{\rm L} L_\mu U^\dagger_{\rm L}$	**Homochiral**
1^{++}, $^3P_1$	$a_1(1260)$; $K_{1,A}$; $f_1(1285), f_1(1420)$	$A^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma^5 \gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R^{ij}_\mu = \bar{q}^j_{\rm R}\gamma_\mu q^i_{\rm R})$	$R_\mu \to U_{\rm R} R_\mu U^\dagger_{\rm R}$	
1^{+-}, $^1P_1$	$b_1(1235)$; $K_{1,B}$; $h_1(1170), h_1(1380)$	$P^{ij}_\mu = -\frac{1}{2}\bar{q}^j \gamma^5 \overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + \mathrm{i}P_\mu$ $(\Phi^{ij}_\mu = \bar{q}^j_{\rm R} \mathrm{i}\overleftrightarrow{D}_\mu q^i_{\rm L})$	$\Phi_\mu \to e^{-2i\alpha} U_{\rm L} \Phi_\mu U^\dagger_{\rm R}$	**Heterochiral**
1^{--}, $^3D_1$	$\rho(1700)$; $K^*(1680)$; $\omega(1650), \phi(?)$	$S^{ij}_\mu = \frac{1}{2}\bar{q}^j \mathrm{i}\overleftrightarrow{D}_\mu q^i$			
2^{++}, $^3P_2$	$a_2(1320)$; $K_2^*(1430)$; $f_2(1270), f_2'(1525)$	$V^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j(\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L^{ij}_{\mu\nu} = \bar{q}^j_{\rm L}(\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots)q^i_{\rm L})$	$L_{\mu\nu} \to U_{\rm L} L_{\mu\nu} U^\dagger_{\rm L}$	**Homochiral**
2^{--}, $^3D_2$	$\rho_2(?)$; $K_2(1820)$; $\omega_2(?), \phi_2(?)$	$A^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R^{ij}_{\mu\nu} = \bar{q}^j_{\rm R}(\gamma_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_{\rm R})$	$R_{\mu\nu} \to U_{\rm R} R_{\mu\nu} U^\dagger_{\rm R}$	
2^{-+}, $^1D_2$	$\pi_2(1670)$; $K_2(1770)$; $\eta_2(1645), \eta_2(1870)$	$P^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(\mathrm{i}\gamma^5 \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + \mathrm{i}P_{\mu\nu}$ $(\Phi^{ij}_{\mu\nu} = \bar{q}^j_{\rm R}(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_{\rm L})$	$\Phi_{\mu\nu} \to e^{-2i\alpha} U_{\rm L} \Phi_{\mu\nu} U^\dagger_{\rm R}$	
2^{++}, $^3F_2$	$a_2(?)$; $K_2^*(?)$; $f_2(?), f_2'(?)$	$S^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$			
3^{--}, $^3D_3$	$\rho_3(1690)$; $K_3^*(1780)$; $\omega_3(1670), \phi_3(1850)$	⋮	⋮	⋮	

Table from:

F.G., R. Pisarski,
A. Koenigstein
Phys.Rev.D 97 (2018) 9, 091901
e-Print: 1709.07454

Extended Linear Sigma Model: eLSM

eLSM: Chiral model with all previous fields + glueballs, hybrids…

(since 2008 up to now, for spectroscopy and medium properties)

Recent review paper in:

Ordinary and exotic mesons in the extended Linear Sigma Model

F.G., S. Jafarzade, P. Kovacs

arXiv: 2407.18348 [hep-ph].

(Pseudo)scalar mesons: heterochiral scalars

Pseudoscalar mesons: {π, K, η(547), η'(958)}
Scalar mesons: {a0(1450), K0*(1430),f0(1370),f0(1500)}

$$\Phi = S + iP = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_0^{*+} \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_0^{*0} \\ K_0^{*-} & K_0^{*0} & \sigma_S \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

f0(1710) mostly glueball
See 1408.4921

$$q_{\mathrm{L,R}} \longrightarrow \mathrm{e}^{\mp \mathrm{i}\alpha/2} U_{\mathrm{L,R}}\, q_{\mathrm{L,R}}$$

$$\Phi \longrightarrow \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi U_{\mathrm{R}}^{\dagger}$$

Chirally invariant terms

We call the transformation of the matrix Φ **heterochiral**!
We thus have heterochiral scalars or heteroscalars.

$$\mathrm{tr}(\Phi^\dagger \Phi),\ \mathrm{tr}(\Phi^\dagger \Phi)^2$$

clearly invariant; typical terms for a chiral model.

$$\det(\Phi)$$

interesting, since it breaks only $U(1)_A$ axial anomaly

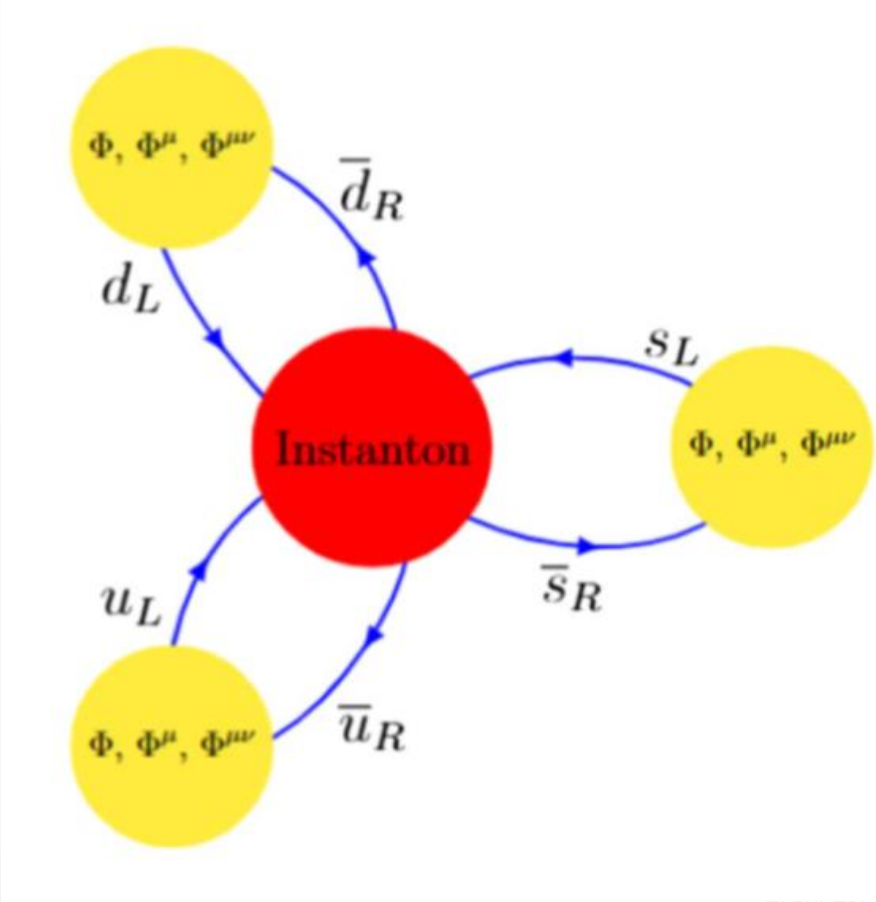
$$\det(\Phi) = \frac{1}{6}\varepsilon^{ijk}\varepsilon^{i'j'k'}\Phi^{ii'}\Phi^{jj'}\Phi^{kk'} \to e^{-3i\alpha}\det(\Phi)$$

Anomalous interactions between mesons with nonzero spin and glueballs

Phys.Rev.D 109 (2024) 7, L071502

2309.00086 [hep-ph]

$$\mathcal{L}_{\text{eff}}^{J=0} = -a_0(\det\Phi + \det\Phi^\dagger)$$



The constant a0 can be calculated as an overage over instanton denisty

Average over instantons

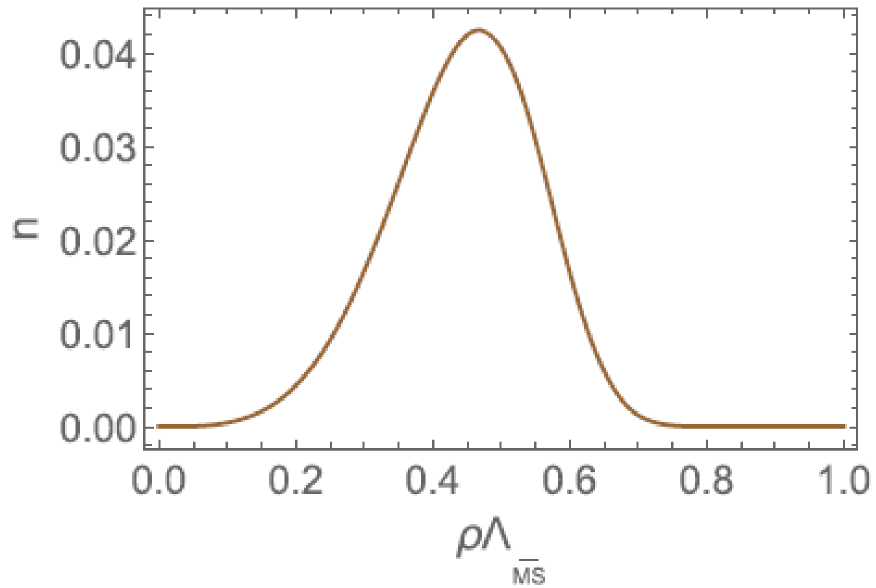


FIG. 1. The density of instantons for $N_c = N_f = 3$.

$$\mathcal{L}_{\text{eff}}^{J=0} = -a_0(\det\Phi + \det\Phi^\dagger)$$

$$k_J = (8\pi^2)^3 \int_0^{\Lambda_{\overline{\text{MS}}}^{-1}} d\rho n(\rho)\rho^{9+2J},$$

$J = 0$

$$a_0 = k_0 M_0^6/48 > 0.$$

$$a_0 = 1.3\ \text{GeV}$$

$$M_0 = 170\ \text{MeV}$$

The chiral anomaly in mesons

There are 8 but not 9 Goldstone bosons:
3 pions, 4 kaons, and one η(547) meson.

The η'(958) meson has a mass of almost 1 GeV.

$$m^2_{\eta'} \sim 1/N_c$$

E. Witten, Current Algebra Theorems for the U(1) Goldstone Boson, Nucl. Phys. B 156 (1979), 269-283

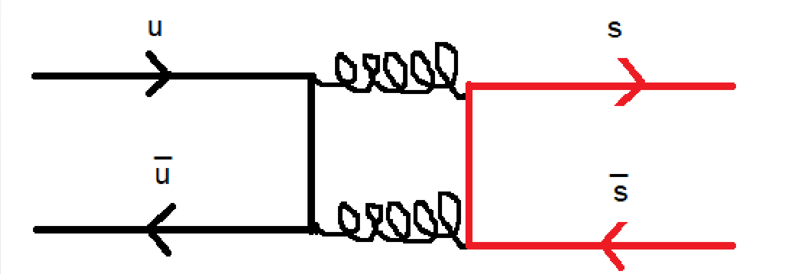
G. 't Hooft, Computation of the quantum effects due to a four-dimensional pseudoparticle, Phys. Rev. D 14, 3432 (1976).

Mixing in the isoscalar sector

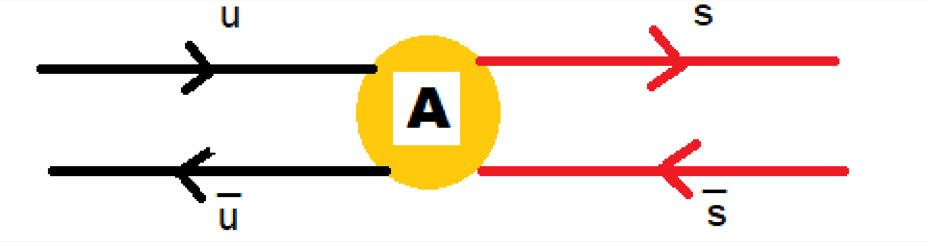
$$\begin{pmatrix} \eta \equiv \eta(547) \\ \eta' \equiv \eta(958) \end{pmatrix} = \begin{pmatrix} \cos\theta_P & \sin\theta_P \\ -\sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

$$\theta_P \simeq -42^\circ$$

Such a mixing is suppressed...



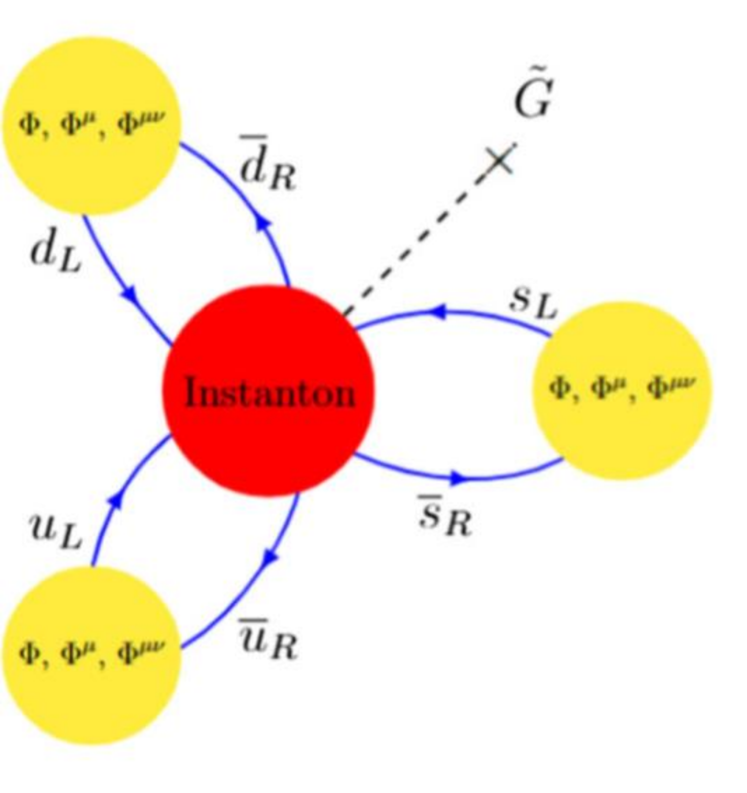
But this can be large



The numerical value can be correctly described, S. D. Bass and A. W. Thomas, Phys. Lett. B **634** (2006) 368 doi:10.1016/j.physletb.2006.01.071 [hep-ph/0507024].

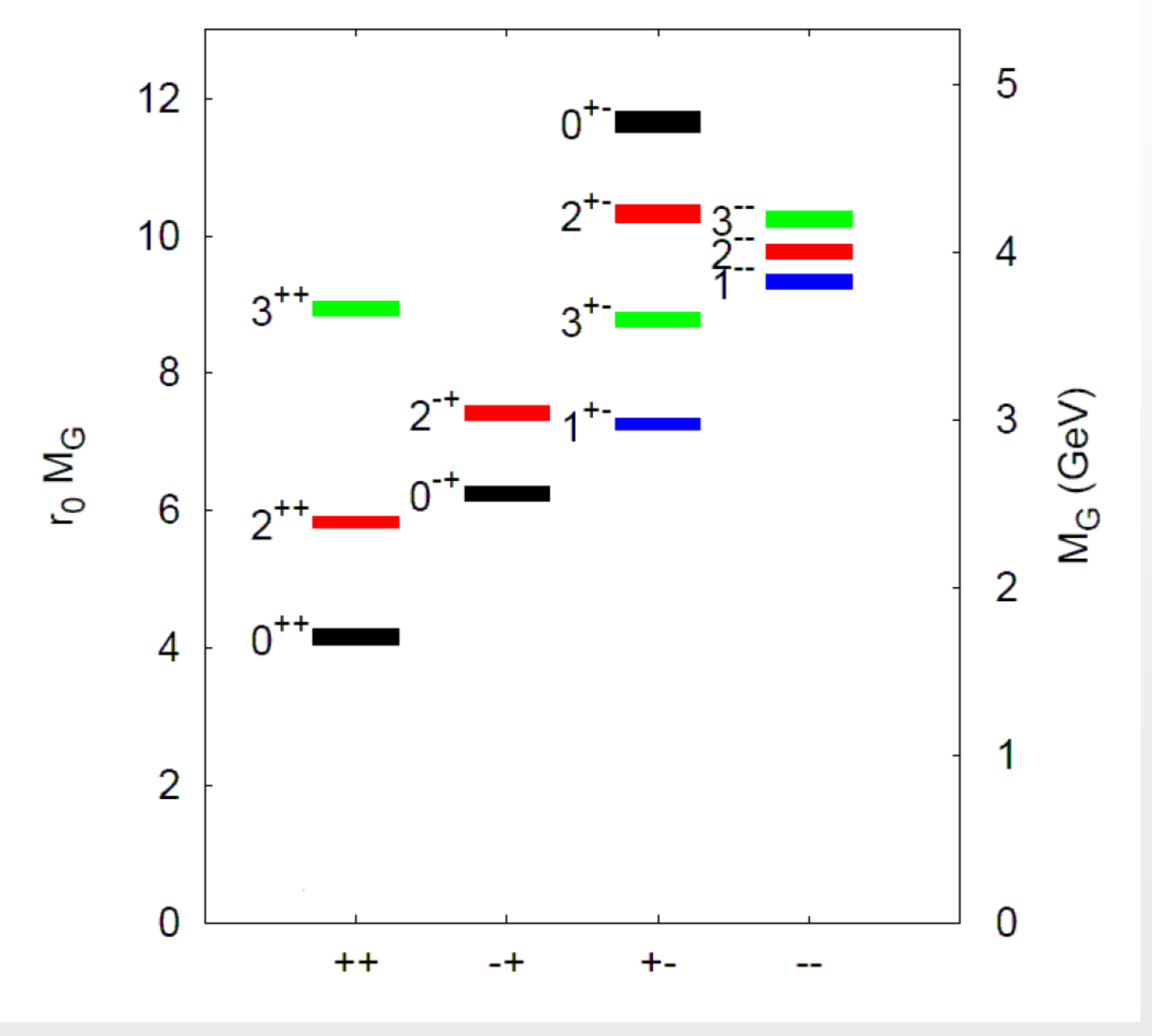
Going further: pseudoscalar glueball

$$\mathcal{L}_{c_g} = -ic_g\tilde{G}_0(\det\Phi - \det\Phi^{\dagger}).$$



2309.00086

Glueball spectrum from lattice



The pseudoscalar glueball: predictions from the eLSM

$$\mathcal{L}^{int}_{\tilde{G}\text{-mesons}} = i c_{\tilde{G}\Phi} \tilde{G} \left(\det\Phi - \det\Phi^{\dagger} \right)$$

M_G = 2.6 GeV as been used as an input.

Quantity	Value
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma^{tot}_{\tilde{G}}$	0.049
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.019
$\Gamma_{\tilde{G}\to\eta\eta\eta}/\Gamma^{tot}_{\tilde{G}}$	0.016
$\Gamma_{\tilde{G}\to\eta\eta\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.0017
$\Gamma_{\tilde{G}\to\eta\eta'\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.00013
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma^{tot}_{\tilde{G}}$	0.46
$\Gamma_{\tilde{G}\to\eta\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.16
$\Gamma_{\tilde{G}\to\eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.094

Quantity	Value
$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}}$	0.059
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}}$	0.083
$\Gamma_{\tilde{G}\to\eta\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.028
$\Gamma_{\tilde{G}\to\eta\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0.012
$\Gamma_{\tilde{G}\to\eta'\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.019

$$\Gamma_{\tilde{G}\to\pi\pi\pi} = 0$$

X(2370)and X(2600) found at BESIII possible candidate.

Future experimental search, e.g. at BES and PANDA

Details in:

W. Eshraim, S. Janowski, F.G., D. Rischke, **Phys.Rev. D87 (2013) 054036.** **arxiv: 1208.6474** .

W. Eschraim, S. Janowski, K. Neuschwander, A. Peters, F.G., **Acta Phys. Pol. B**, Prc. Suppl. 5/4, **arxiv: 1209.3976**

Thanks to DIG it was possible to estimate the coupling constant, see 2309.00086
Then not only ratio possible, but actual widths!

$$\Gamma(\tilde{G}_0 \to K\bar{K}\pi) \approx 0.24\ \text{GeV} \ \text{ and } \ \Gamma(\tilde{G}_0 \to \pi\pi\eta') \approx 0.05\ \text{GeV}$$

Recent experimental results:

PHYSICAL REVIEW LETTERS **129**, 042001 (2022)

Observation of a State $X(2600)$ in the $\pi^+\pi^-\eta'$ System in the Process $J/\psi \to \gamma\pi^+\pi^-\eta'$

PHYSICAL REVIEW LETTERS **132**, 181901 (2024)

Editors' Suggestion

Determination of Spin-Parity Quantum Numbers of $X(2370)$ as 0^{-+} from $J/\psi \to \gamma K_S^0 K_S^0 \eta'$

M. Ablikim *et al.*[*]
(BESIII Collaboration)

(Axial-)vector mesons: homochiral vectors: homochiral multiplet

Vector mesons: {ρ(770), K*(892), ω(782), φ(1020)}
Axial-vector mesons: {a1(1230), K1A,, f1(1285), f1(1420)}

$$V^{\mu}=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^{\mu-} & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star0} \\ K^{\star-} & \bar{K}^{\star0} & \omega_S \end{pmatrix}^{\mu}$$

$$A^{\mu}=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_{1,A}^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_{1,A}^0 \\ K_{1,A}^- & \bar{K}_{1,A}^0 & f_{1S} \end{pmatrix}^{\mu}$$

$$R^{\mu}=V^{\mu}-A^{\mu} \text{ and } L^{\mu}=V^{\mu}+A^{\mu}$$

Chiral transformations

$$q_{\mathrm{L,R}} \longrightarrow \mathrm{e}^{\mp \mathrm{i}\alpha/2}\, U_{\mathrm{L,R}}\, q_{\mathrm{L,R}}$$

$$L_{\mu} \longrightarrow U_{\mathrm{L}} L_{\mu} U_{\mathrm{L}}^{\dagger}$$

$$R_{\mu} \longrightarrow U_{\mathrm{R}} R_{\mu} U_{\mathrm{R}}^{\dagger}$$

We have here a **homochiral** multiplet.

Mixing among vector mesons

$$\begin{pmatrix} \omega(782) \\ \phi(1020) \end{pmatrix} = \begin{pmatrix} \cos\theta_V & \sin\theta_V \\ -\sin\theta_V & \cos\theta_V \end{pmatrix} \begin{pmatrix} \omega_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

$$\theta_V \simeq -3.2^\circ$$

The mixing is very small.
This is understandable: there is no term analogous to the determinant.
Namely, anomaly-driven terms are more complicated, involve derivatives and do not affect isoscalar mixing, e.g. Wess-Zumino like terms.

Same situation for tensor mesons. Indeed isoscalar mixing is small.

A novel mathematical object? Extension of det

Extend the determinant

$$\det[\Phi] = \frac{1}{N!}\varepsilon^{i_1 i_2 \ldots i_N}\varepsilon^{j_1 j_2 \ldots j_N}\Phi^{i_1 j_1}\Phi^{i_2 j_2}\ldots\Phi^{i_N j_N}$$

to the following new object:

$$\varepsilon[\Phi_1, \Phi_2, ..., \Phi_N] = \frac{1}{N!}\varepsilon^{i_1 i_2 \ldots i_N}\varepsilon^{j_1 j_2 \ldots j_N}\Phi_1^{i_1 j_1}\Phi_2^{i_2 j_2}\ldots\Phi_N^{i_N j_N}$$

$$\varepsilon[\Phi_1, \Phi_2, ..., \Phi_i, ..., \Phi_j, ...\Phi_N] = \varepsilon[\Phi_1, \Phi_2, ..., \Phi_j, ..., \Phi_i, ...\Phi_N]$$

$$\varepsilon[\Phi_1, \Phi_1, ...\Phi_1] = \det\Phi_1$$

Definition in GPJ in arxiv: 2309.00086, see also review 2407.18348 .
(Implicit def by GKP 1709.07454)

Pseudovectors and orbitally excited vectors: Heterochiral vectors

Pseudovextor mesons: {$b_1(1230)$, K_{1B}, $h_1(1170)$, $h_1(1380)$}
Excited vector mesons: {ρ(1700), K*(1680) , ω(1650), φ(???)}

$$B^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{h_{1,N}+b_1^0}{\sqrt{2}} & b_1^+ & K_{1,B}^{\star+} \\ b_1^- & \frac{h_{1,N}+b_1^0}{\sqrt{2}} & K_{1,B}^{\star 0} \\ K_{1,B}^{\star-} & \bar{K}_{1,B}^{\star 0} & h_{1,S} \end{pmatrix}^\mu$$

$$E_{\text{ang}}^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_{\text{ang},N}+\rho_{\text{ang}}^0}{\sqrt{2}} & \rho_{\text{ang}}^+ & K_{\text{ang}}^{\star+} \\ \rho_{\text{ang}}^- & \frac{\omega_{\text{ang},N}-\rho_{\text{ang}}^0}{\sqrt{2}} & K_{\text{ang}}^{\star 0} \\ K_{\text{ang}}^{\star-} & \bar{K}_{\text{ang}}^{\star 0} & \omega_{\text{ang},S} \end{pmatrix}^\mu$$

$$\tilde{\Phi}^\mu = E_{\text{ang}}^\mu - iB^\mu$$

Chiral transformations

$$q_{\mathrm{L,R}} \longrightarrow \mathrm{e}^{\mp \mathrm{i}\alpha/2}\, U_{\mathrm{L,R}}\, q_{\mathrm{L,R}}$$

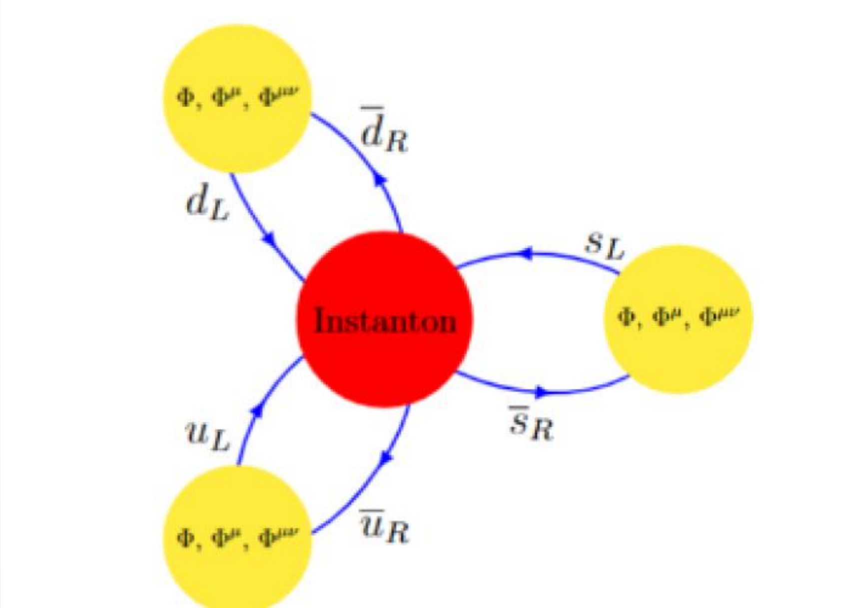
$$\Phi_\mu \longrightarrow \mathrm{e}^{-\mathrm{i}\alpha}\, U_{\mathrm{L}}\, \Phi_\mu\, U_{\mathrm{R}}^\dagger$$

The pseudovector mesons and the excited vector mesons form a **heterochiral** multiplet. We thus call them heterochiral vectors.

Excited vector mesons: φ(1930) predicted to be the missing state, see M. Piotrowska, C. Reisinger and FG., arXiv:1708.02593 [hep-ph]

Anomalous Lagrangian for pseudovector

$$\mathcal{L}_{\text{eff}}^{J=1} = a_1\Big(\epsilon[\Phi\,,\Phi_\mu\,,\Phi^\mu] + \text{c.c.}\Big)$$



$$\mathcal{L}_{\text{eff}}^{J=0} = -a_0\Big(\det\Phi + \det\Phi^\dagger\Big)$$

Generalized determinant for 3x3 matrices

Determinant of a 3×3 Matrix

$$\det\Phi = \frac{1}{3!}\epsilon^{ijk}\epsilon^{i'j'k'}\Phi^{ii'}\Phi^{jj'}\Phi^{kk'}$$

One can write determinant like a product for matrices $\Phi_{1,2,3}$

$$\epsilon[\Phi_1, \Phi_2, \Phi_3] := \frac{1}{3!}\epsilon^{ijk}\epsilon^{i'j'k'}\Phi_1^{ii'}\Phi_2^{jj'}\Phi_3^{kk'}$$

It has the following properties

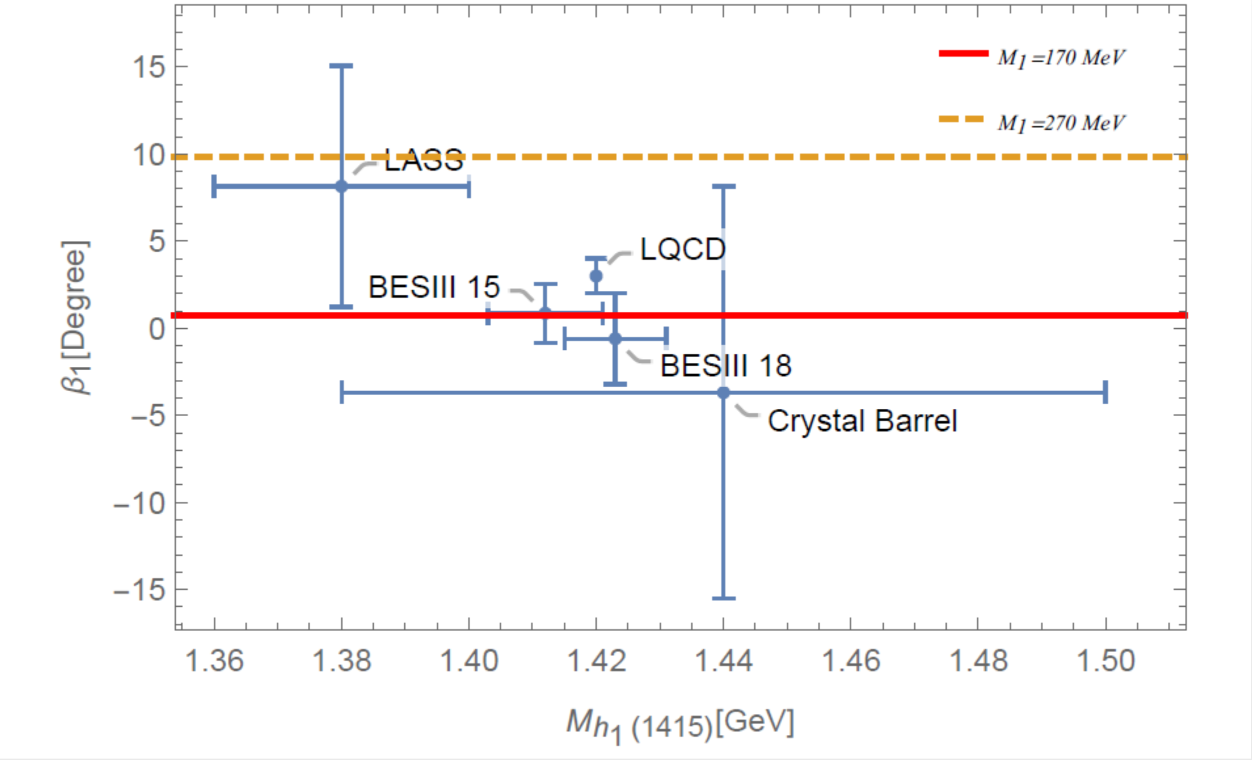
$$\epsilon[\Phi_1, \Phi_1, \Phi_1] = \det\Phi_1\,, \quad \epsilon[1, 1, \Phi_1] = \frac{1}{3}\mathrm{Tr}[\Phi_1]$$

By using the epsilon product, we can construct anomalous lagrangians

Isoscalar mixing angle for h1 mesons

$$\begin{pmatrix} h_1(1170) \\ h_1(1380) \end{pmatrix} = \begin{pmatrix} \cos\beta_{AV} & \sin\beta_{AV} \\ -\sin\beta_{AV} & \cos\beta_{AV} \end{pmatrix} \begin{pmatrix} h_{1,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ h_{1,S} = \bar{s}s \end{pmatrix}$$

Angle between 0-10 degrees: between red and yellow lines



Pseudotensor mesons (and their chiral partners): heterochiral tensors

Pseudotensor mesons: {π2(1670), K2(1770),η2(1645),η2(1870)}
Chiral partners: {a_2(???), K_2*(???) , f_2(???), f_2(???)}

Chiral transformations

$$q_{\mathrm{L,R}} \longrightarrow \mathrm{e}^{\mp \mathrm{i}\alpha/2}\, U_{\mathrm{L,R}}\, q_{\mathrm{L,R}}$$

$$\Phi_{\mu\nu} \longrightarrow \mathrm{e}^{-\mathrm{i}\alpha}\, U_{\mathrm{L}}\, \Phi_{\mu\nu}\, U_{\mathrm{R}}^{\dagger}$$

Thus, we have **heterochiral** tensor states.
Transformation just as heterochiral scalars.
Mixing between strange-nonstrange possible.

Mixing angle for pseudotensor mesons

$$\begin{pmatrix} \eta_2(1645) \\ \eta_2(1870) \end{pmatrix} = \begin{pmatrix} \cos\beta_2 & \sin\beta_2 \\ -\sin\beta_2 & \cos\beta_2 \end{pmatrix} \begin{pmatrix} \eta_{2,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_{2,S} = \bar{s}s \end{pmatrix}$$

$$\beta_2 \approx -(1^\circ, 10^\circ) < 0$$

Instanton-based result,
GPJ 2309.00086

$$\beta_2 \approx -40^\circ$$

Phenomenology results,
FG & A. Koenigstein 1608.08777
V. Shastry, E. Trotti, FG: 2107.13501

Concluding remarks

- Concept of homochirality and heterochirality.
- For heterochiral multiplets an axial-anomalous strange-nonstrange mixing is possible.

  (η-η', but possibly η_2(1645)-η_2(1870) and evt h1 states)
- For homochiral multiplets no anomalous mixing.

  (ω-phi(1020), f_2(1270)-f_2'(1525),..., are nonstrange and strange, resp.)
- Pseudoscalar glueball: anomalous coupling to mesons and baryons.
- Interesting mathematical object as a viable extension of the determinant
- Outlook:

Thanks

eLSM Lagrangian: 2407.18348

$$\mathcal{L} = \mathcal{L}_{\rm dil} + \mathcal{L}_{\Phi} + \mathcal{L}_{U(1)_A} + \mathcal{L}_{LR} + \mathcal{L}_{\Phi LR} \ ,$$

with

$$\mathcal{L}_{\rm dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 \ln\frac{G^2}{\Lambda_G^2} - \frac{G^4}{4}\right) \ ,$$

$$\mathcal{L}_{\Phi} = \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\left(\frac{G}{G_0}\right)^2 \mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2 + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] \ ,$$

$$\mathcal{L}_{U(1)_A} = c_2(\det\Phi - \det\Phi^\dagger)^2 \ ,$$

$$\mathcal{L}_{LR} = -\frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\left(\frac{G}{G_0}\right)^2 + \Delta\right)\frac{m_1^2}{2}(L_\mu^2 + R_\mu^2)\right] + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}$$

$$+ g_3[\mathrm{Tr}(L_\mu L_\nu L^\mu L^\nu) + \mathrm{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\mathrm{Tr}\,(L_\mu L^\mu L_\nu L^\nu) + \mathrm{Tr}\,(R_\mu R^\mu R_\nu R^\nu)]$$

$$+ g_5\,\mathrm{Tr}\,(L_\mu L^\mu)\,\mathrm{Tr}\,(R_\nu R^\nu) + g_6[\mathrm{Tr}(L_\mu L^\mu)\ \mathrm{Tr}(L_\nu L^\nu) + \mathrm{Tr}(R_\mu R^\mu)\ \mathrm{Tr}(R_\nu R^\nu)] \ ,$$

$$\mathcal{L}_{\Phi LR} = \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\,\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\,\mathrm{Tr}[|L_\mu\Phi|^2 + |\Phi R_\mu|^2] + 2h_3\,\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) \ ,$$

where

$$D^{\mu}\Phi \equiv \partial^{\mu}\Phi - ig_1(L^{\mu}\Phi - \Phi R^{\mu}) - ieA^{\mu}[t_3, \Phi] \ ,$$

$$L^{\mu\nu} \equiv \partial^{\mu}L^{\nu} - ieA^{\mu}[t_3, L^{\nu}] - \{\partial^{\nu}L^{\mu} - ieA^{\nu}[t_3, L^{\mu}]\} \ ,$$

$$R^{\mu\nu} \equiv \partial^{\mu}R^{\nu} - ieA^{\mu}[t_3, R^{\nu}] - \{\partial^{\nu}R^{\mu} - ieA^{\nu}[t_3, R^{\mu}]\} \ ,$$

and

$$H = H_0 t_0 + H_8 t_8 = \begin{pmatrix} \frac{h_{0N}}{2} & 0 & 0 \\ 0 & \frac{h_{0N}}{2} & 0 \\ 0 & 0 & \frac{h_{0S}}{\sqrt{2}} \end{pmatrix} \ ,$$

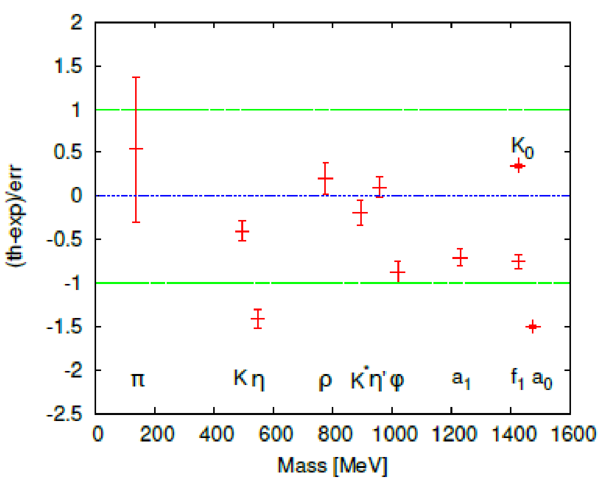
$$\Delta = \Delta_0 t_0 + \Delta_8 t_8 = \begin{pmatrix} \frac{\tilde{\delta}_N}{2} & 0 & 0 \\ 0 & \frac{\tilde{\delta}_N}{2} & 0 \\ 0 & 0 & \frac{\tilde{\delta}_S}{\sqrt{2}} \end{pmatrix} \equiv \begin{pmatrix} \delta_N & 0 & 0 \\ 0 & \delta_N & 0 \\ 0 & 0 & \delta_S \end{pmatrix} \ .$$

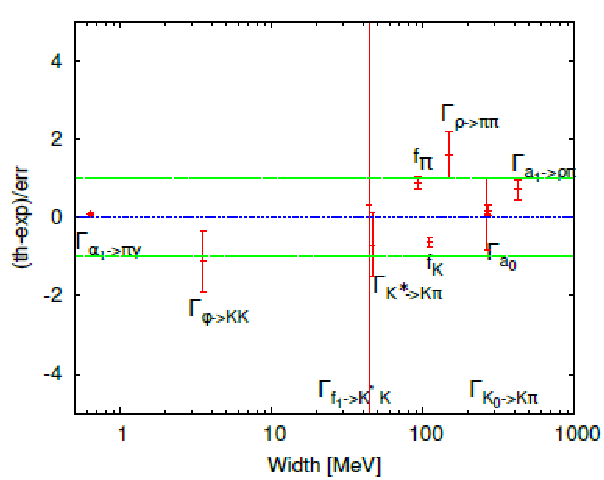
Mass squares	Analytical expressions
m_π^2	$Z_\pi^2\left[m_0^2+\left(\lambda_1+\frac{\lambda_2}{2}\right)\phi_N^2+\lambda_1\phi_S^2\right]\equiv\frac{Z_\pi^2 h_{0N}}{\phi_N}$
m_K^2	$Z_K^2\left[m_0^2+\left(\lambda_1+\frac{\lambda_2}{2}\right)\phi_N^2-\frac{\lambda_2}{\sqrt{2}}\phi_N\phi_S+\left(\lambda_1+\lambda_2\right)\phi_S^2\right]$
$m_{\eta_N}^2$	$Z_\pi^2\left[m_0^2+\left(\lambda_1+\frac{\lambda_2}{2}\right)\phi_N^2+\lambda_1\phi_S^2+c_2\,\phi_N^2\phi_S^2\right]\equiv Z_\pi^2\left(\frac{h_{0N}}{\phi_N}+c_2\,\phi_N^2\phi_S^2\right)$
$m_{\eta_S}^2$	$Z_{\eta_S}^2\left[m_0^2+\lambda_1\phi_N^2+\left(\lambda_1+\lambda_2\right)\phi_S^2+\frac{c_2}{4}\phi_N^4\right]\equiv Z_{\eta_S}^2\left(\frac{h_{0S}}{\phi_S}+\frac{c_2}{4}\phi_N^4\right)$
$m_{\eta_{NS}}^2$	$Z_{\eta_N}Z_{\eta_S}\frac{c_2}{2}\phi_N^3\phi_S$
$m_{a_0}^2$	$m_0^2+\left(\lambda_1+\frac{3}{2}\lambda_2\right)\phi_N^2+\lambda_1\phi_S^2$
$m_{K_0^\star}^2$	$Z_{K_0^\star}^2\left[m_0^2+\left(\lambda_1+\frac{\lambda_2}{2}\right)\phi_N^2+\frac{\lambda_2}{\sqrt{2}}\phi_N\phi_S+\left(\lambda_1+\lambda_2\right)\phi_S^2\right]$
$m_{\sigma_N}^2$	$m_0^2+3\left(\lambda_1+\frac{\lambda_2}{2}\right)\phi_N^2+\lambda_1\phi_S^2$
$m_{\sigma_S}^2$	$m_0^2+\lambda_1\phi_N^2+3\left(\lambda_1+\lambda_2\right)\phi_S^2$
$m_{\sigma_{NS}}^2$	$2\lambda_1\phi_N\phi_S$

Table 3.2: Mass expressions of spin-0 mesons (scalars and pseudoscalars) within the eLSM.

Observable	Fit [MeV]	Experiment [MeV]	Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6	f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	138 ± 6.9	m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4	$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8	$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0	m_{a_1}	1186 ± 6.0	1230 ± 62
$m_{f_1(1420)}$	1372.4 ± 5.3	1426 ± 71	m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71	$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3	$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175	$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2	Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80			

Table 3.4: An example of fit results from [6], together with the experimental values taken from [13].





Example: extension to pseudovector

$$\mathcal{L}^{\Phi_\mu}_{\text{mass}} = \text{Tr}\left[\left(\frac{m_1^2 G^2}{2G_0^2} + \Delta^{\text{pv}}\right)\left(\Phi_\mu^\dagger \Phi^\mu\right)\right] + \frac{\lambda_{\Phi_\mu,1}}{2}\text{Tr}\left[\Phi^\dagger\Phi\right]\text{Tr}\left[\Phi_\mu^\dagger\Phi^\mu\right] + \lambda_{\Phi_\mu,2}\text{Tr}\left[\Phi_\mu^\dagger\Phi\Phi^{\mu\dagger}\Phi + \Phi_\mu\Phi^\dagger\Phi^\mu\Phi^\dagger\right]$$
$$+ \lambda_{\Phi_\mu,3}\text{Tr}\left[\Phi_\mu\Phi^\dagger\Phi\Phi^{\mu\,\dagger} + \Phi_\mu^\dagger\Phi\Phi^\dagger\Phi^\mu\right] ,$$

$$\mathcal{L}^{\text{int}}_{\Phi^\mu} = g_{\Phi^\mu\Phi\Phi}\text{Tr}\left[\Phi^\mu\,\Phi\partial_\mu\Phi + \text{c.c}\right] + g_{\Phi^\mu LR}\text{Tr}\left[\Phi_\alpha^\dagger L_\beta L^\alpha\partial^\beta\Phi + R_\alpha\Phi_\beta^\dagger\partial^\alpha\Phi R^\beta + L_\alpha\partial^\beta\Phi\Phi_\alpha^\dagger L^\beta + \partial_\alpha\Phi^\dagger R_\beta R^\alpha\Phi^\beta\right] .$$

Decay process	Width (MeV)	Decay process	Width (MeV)
$\rho(1700) \to \overline{K}K$	40 ± 11	$\rho(1700) \to \pi\pi$	140 ± 37
$K^*(1680) \to K\pi$	82 ± 22	$K^*(1680) \to K\eta$	52 ± 14
$\omega(1650) \to \overline{K}K$	37 ± 10	$\rho(1700) \to \omega\pi$	140 ± 59
$\rho(1700) \to K^*(892)K$	56 ± 23	$\rho(1700) \to \rho\eta$	41 ± 17
$K^*(1680) \to K\rho$	64 ± 27	$K^*(1680) \to K\phi$	13 ± 6
$K^*(1680) \to K\omega$	21 ± 9	$K^*(1680) \to K^*(892)\pi$	81 ± 34
$K^*(1680) \to K^*(892)\eta$	0.5 ± 0.2	$\omega(1650) \to \rho\pi$	370 ± 156
$\omega(1650) \to K^*(892)K$	42 ± 18	$\omega(1650) \to \omega(782)\eta$	32 ± 13
$\phi(1930) \to K\bar{K}^*(892)$	260 ± 109	$\phi(1930) \to \phi(1020)\eta$	67 ± 28

Strange-nonstrange mixing in the isoscalar sector: recall and the strange case of pseudotensor mesons

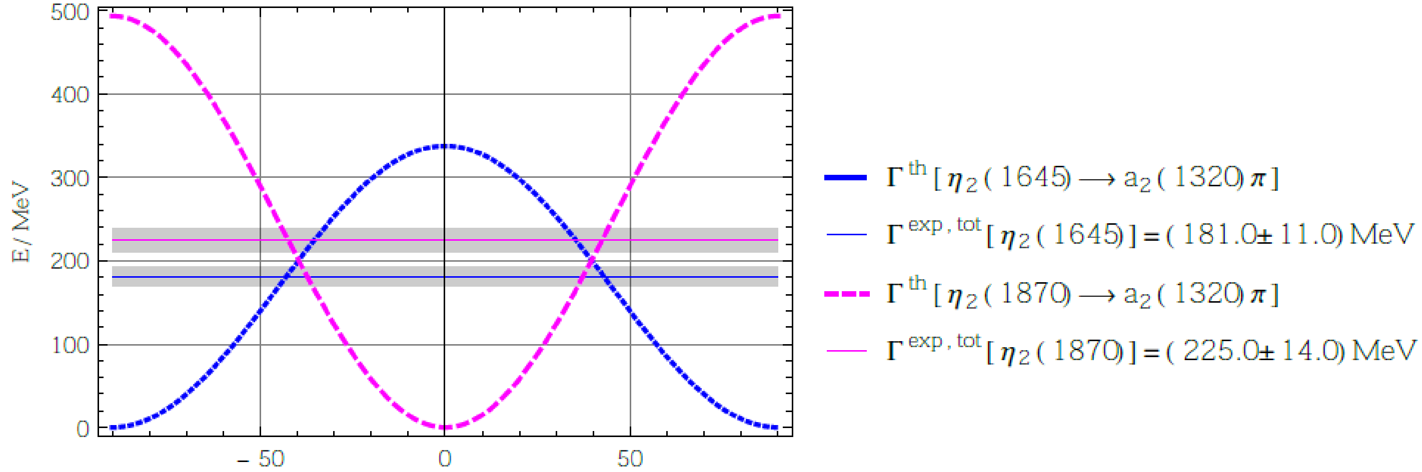
based on

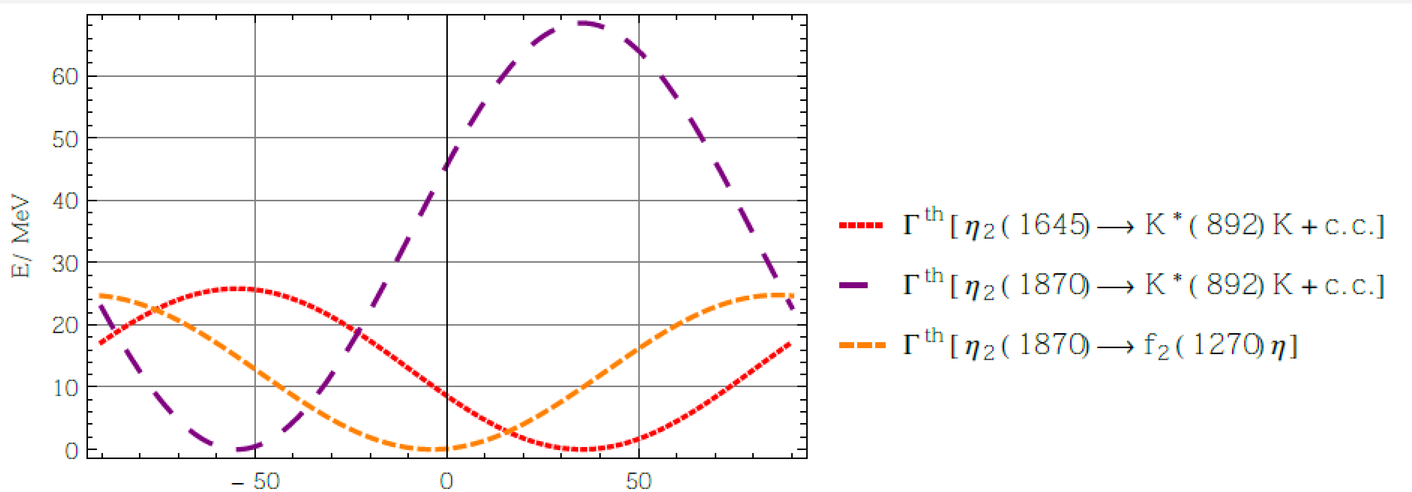
A . Koenigstein and F.G.

Eur. Phys.J. A52 (2016) no.12, 356, arXiv: 1608.8777

η2(1645) and η2(1870)

Only a large mixing angle Θmix = -40° is compatible with present experimental data.





Θmix

(Pseudo)scalar mesons: heterochiral scalars

Uniwersytet Jana Kochanowskiego w Kielcach

$$q_{\mathrm{L,R}} \longrightarrow e^{\mp i\alpha/2} U_{\mathrm{L,R}} \, q_{\mathrm{L,R}}$$

J^{PC}, ${}^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
0^{-+}, ${}^1S_0$	$\begin{cases} \pi \\ K \\ \eta, \eta'(958) \end{cases}$	$P^{ij} = \frac{1}{2}\bar{q}^j i\gamma^5 q^i$	$\Phi = S + iP$ $(\Phi^{ij} = \bar{q}^j_{\mathrm{R}} q^i_{\mathrm{L}})$	$\Phi \longrightarrow e^{-2i\alpha} U_{\mathrm{L}} \Phi U_{\mathrm{R}}^\dagger$
0^{++}, ${}^3P_0$	$\begin{cases} a_0(1450) \\ K_0^*(1430) \\ f_0(1370), f_0(1710)^\star \end{cases}$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		

$$\Phi \longrightarrow e^{-2i\alpha} U_{\mathrm{L}} \Phi U_{\mathrm{R}}^\dagger$$

We call the transformation of the matrix Φ **heterochiral**!
We thus have heterochiral scalars.

$\mathrm{tr}(\Phi^\dagger\Phi),\ \mathrm{tr}(\Phi^\dagger\Phi)^2$ are clearly invariant; typical terms for a chiral model.

$\det(\Phi)$ is interesting, since it breaks only U(1)A axial anomaly $\det\Phi \rightarrow e^{-i6\alpha}\det\Phi$

(Axial-)vector mesons: homochiral vectors

J^{PC}, $^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
1^{--}, $^1S_1$	$\begin{cases} \rho(770) \\ K^*(892) \\ \omega(782), \phi(1020) \end{cases}$	$V_\mu^{ij} = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L_\mu^{ij} = \bar{q}_{\mathrm{L}}^j \gamma_\mu q_{\mathrm{L}}^i)$	$L_\mu \longrightarrow U_{\mathrm{L}} L_\mu U_{\mathrm{L}}^\dagger$
1^{++}, $^3P_1$	$\begin{cases} a_1(1260) \\ K_{1,A} \\ f_1(1285), f_1(1420) \end{cases}$	$A_\mu^{ij} = \frac{1}{2}\bar{q}^j \gamma^5 \gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R_\mu^{ij} = \bar{q}_{\mathrm{R}}^j \gamma_\mu q_{\mathrm{R}}^i)$	$R_\mu \longrightarrow U_{\mathrm{R}} R_\mu U_{\mathrm{R}}^\dagger$

$$L_\mu \longrightarrow U_{\mathrm{L}} L_\mu U_{\mathrm{L}}^\dagger$$

$$R_\mu \longrightarrow U_{\mathrm{R}} R_\mu U_{\mathrm{R}}^\dagger$$

We have here a **homochiral** multiplet.
We call these states as homochiral vectors.

Ground-state tensors (and their chiral partners): Homochiral tensors

J^{PC}, ${}^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
2^{++}, ${}^3P_2$	$\begin{cases} a_2(1320) \\ K_2^*(1430) \\ f_2(1270), f_2'(1525) \end{cases}$	$V_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \ldots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L_{\mu\nu}^{ij} = \bar{q}_{\mathrm{L}}^j(\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \ldots)q_{\mathrm{L}}^i)$	$L_{\mu\nu} \longrightarrow U_{\mathrm{L}} L_{\mu\nu} U_{\mathrm{L}}^\dagger$
2^{--}, ${}^3D_2$	$\begin{cases} \rho_2(?) \\ K_2(1820) \\ \omega_2(?), \phi_2(?) \end{cases}$	$A_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \ldots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R_{\mu\nu}^{ij} = \bar{q}_{\mathrm{R}}^j(\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \ldots)q_{\mathrm{R}}^i)$	$R_{\mu\nu} \longrightarrow U_{\mathrm{R}} R_{\mu\nu} U_{\mathrm{R}}^\dagger$

$$L_{\mu\nu} \longrightarrow U_{\mathrm{L}} L_{\mu\nu} U_{\mathrm{L}}^\dagger$$

$$R_{\mu\nu} \longrightarrow U_{\mathrm{R}} R_{\mu\nu} U_{\mathrm{R}}^\dagger$$

Thus, we have **homochiral** tensors. We do not expect large mixing.

Pseudovectors and orbitally excited vectors: Heterochiral vectors

J^{PC}, ${}^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
1^{+-}, ${}^1P_1$	$\begin{cases} b_1(1235) \\ K_{1,B} \\ h_1(1170), h_1(1380) \end{cases}$	$P_\mu^{ij} = -\frac{1}{2}\bar{q}^j\gamma^5\overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + \mathrm{i}P_\mu$ $(\Phi_\mu^{ij} = \bar{q}_{\mathrm{R}}^j \mathrm{i}\overleftrightarrow{D}_\mu q_{\mathrm{L}}^i)$	$\Phi_\mu \longrightarrow \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi_\mu U_{\mathrm{R}}^\dagger$
1^{--}, ${}^3D_1$	$\begin{cases} \rho(1700) \\ K^*(1680) \\ \omega(1650), \phi(?) \end{cases}$	$S_\mu^{ij} = \frac{1}{2}\bar{q}^j \mathrm{i}\overleftrightarrow{D}_\mu q^i$		

$$\Phi_\mu \longrightarrow \mathrm{e}^{-\mathrm{i}\alpha}\, U_{\mathrm{L}}\, \Phi_\mu\, U_{\mathrm{R}}^\dagger$$

The pseudovector mesons and the excited vector mesons form a **heterochiral** multiplet. We thus call them heterochiral vectors.
The chiral transformation is just as the (pseudo)scalar mesons (which is also hetero).
Hence, an anomalous Lagrangian is possible for heterochiral vectors.

Excited vector mesons: phi(1930) predicted to be the missing state, see
M. Piotrowska, C. Reisinger and FG.,
``Strong and radiative decays of excited vector mesons and predictions for a new phi(1930)$ resonance,'' arXiv:1708.02593 [hep-ph], to appear in PRD.

Anomalous Lagrangian for heterochiral vectors

$$\mathcal{L}_{\Phi_\mu}^{\text{anomaly}} = - b_{\text{A}}^{(1)}[\text{tr}(\Phi \times \Phi_\mu \cdot \Phi^\mu) + \text{c.c.}] - b_{\text{A}}^{(2)}[\text{tr}(\Phi \times \partial_\mu \Phi \cdot \Phi^\mu) + \text{c.c.}] - b_{\text{A}}^{(3)}[\text{tr}(\Phi \times \Phi \cdot \Phi_\mu) - \text{c.c.}]^2 + \dots$$

$$(A \times B)^{ii'} = \frac{1}{3!}\epsilon^{ijk}\epsilon^{i'j'k'}A^{jj'}B^{kk'}$$

The first term contains objects as: $\varepsilon^{ijk}\varepsilon^{i'j'k'}\Phi^{ii'}\Phi_\mu^{jj'}\Phi_\mu^{kk'}$
So for the other terms. Such objects are $SU(3)_R \times SU(3)_L$ invariant but break $U(1)_A$.

The first term generates mixing among both nonets (pseudovector and excited vector).
The second term generates decay into (pseudo)scalar states (interesting for future works).
The third terms generates mixing for pseudovectors only.

Ground-state tensors (and their chiral partners): Homochiral tensors

Tensor mesons: {a2(1320), K_2*(1430), f2(1270), f2(1535)}
Axial-vector mesons: {ρ_2(???), K_2(1820), ω_2(???), φ2(???)}

Chiral transformations

$$q_{\mathrm{L,R}} \longrightarrow \mathrm{e}^{\mp \mathrm{i}\alpha/2}\, U_{\mathrm{L,R}}\, q_{\mathrm{L,R}}$$

$$L_{\mu\nu} \longrightarrow U_{\mathrm{L}} L_{\mu\nu} U_{\mathrm{L}}^{\dagger}$$

$$R_{\mu\nu} \longrightarrow U_{\mathrm{R}} R_{\mu\nu} U_{\mathrm{R}}^{\dagger}$$

Thus, we have **homochiral** tensors. We do not expect large mixing.

Tensor mixing

$$\begin{pmatrix} f_2(1270) \\ f_2'(1525) \end{pmatrix} = \begin{pmatrix} \cos\theta_T & \sin\theta_T \\ -\sin\theta_T & \cos\theta_T \end{pmatrix} \begin{pmatrix} f_{2,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ f_{2,S} = \bar{s}s \end{pmatrix}$$

$$\theta_T \simeq 3.2^\circ$$

As expected, the mixing is very small.

A small mixing is also expected for the (yet unknown) chiral partners of tensor mesons.

Extension to other mesons with higher spin

$$\mathcal{L}_{\text{eff}}^{J=1} = -\frac{k_1}{3!}\left(\epsilon\left[(\bar{q}_L q_R)(\bar{q}_L \overleftrightarrow{D}_\mu q_R)^2\right] + R \leftrightarrow L\right)$$
$$= a_1(\epsilon[\Phi\Phi_\mu\Phi^\mu] + \text{c.c.}),$$

where we introduce the symbol [44]

$$\epsilon[ABC] = \epsilon^{ijk}\epsilon^{i'j'k'}A_{ii'}B_{jj'}C_{kk'}/3!,$$

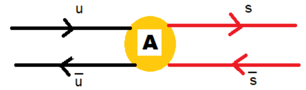
Indeed, it turns out that it the chiral anomaly effects for spin 1,2 mesons is Quite small…

Large Nc works!!!!

But…

Pseudotensor mixing

$$\mathcal{L}^{\text{anomaly}}_{\Phi_{\mu\nu}} = -\beta_A \left(\sqrt{2} h_{1,N} + h_{1,SS}\right)^2 + \ldots$$



$$\begin{pmatrix} \eta_2(1645) \\ \eta_2(1870) \end{pmatrix} = \begin{pmatrix} \cos\theta_{PT} & \sin\theta_{PT} \\ -\sin\theta_{PT} & \cos\theta_{PT} \end{pmatrix} \begin{pmatrix} \eta_{2,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_{2,S} = \bar{s}s \end{pmatrix}$$

$$\theta_{PV} \simeq -\frac{1}{2}\arctan\left[\frac{2\sqrt{2}\beta_A}{m^2_{K_{1,B}} - m^2_{b_1(1235)} - \beta_A}\right]$$

According to the phenomenological study in
A. Koenigstein, F.G., Eur.Phys.J. A**52** (2016) no.12, 356, arXiv: 1608.8777:

$$\theta_{PT} \approx -40^0$$

Anomalous Lagrangian for heterochiral tensors

$$\mathscr{L}^{\text{anomaly}}_{\Phi_{\mu\nu}} = c_{\text{A}}^{(3)} (\varepsilon^{ijk} \varepsilon^{i'j'k'} \Phi^{ii'} \Phi^{jj'} \Phi^{kk'}_{\mu\nu} - h.c.)^2 + \ldots,$$

Again, the various terms are $SU(3)_R \times SU(3)_L$ invariant but break $U(1)_A$.

First term generates mixing for pseudotensors and also for their chiral partners.
Second term generates decays of pseudotensor (and partners) into (pseudo)scalars.
Third term generates mixing for pseudotensors only.