A dispersive estimate of the $a_0(980)$ contribution to hadronic light-by-light scattering in $(g-2)_\mu$

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Abstract. A dispersive implementation of the $a_0(980)$ resonance to $(g-2)_\mu$ requires the knowledge of the double-virtual S-wave $\gamma^*\gamma^* \to \pi\eta/K\bar{K}_{I=1}$ amplitudes. To obtain these amplitudes we used a modified coupled-channel Muskhelishvili–Omnès formalism, with the input from the left-hand cuts and the hadronic Omnès function. The latter were obtained using a data-driven N/D method in which the fits were performed to the different sets of experimental data on two-photon fusion processes with $\pi\eta$ and $K\bar{K}$ final states. This yields the preliminary dispersive estimate $a_\mu^{\text{HLbL}}[a_0(980)]_{\text{resc.}} = -0.46(2) \times 10^{-11}$.

1 Introduction

The tension between the presently ultra-precise measurements of the anomalous magnetic moment of the muon $(g-2)_\mu$ and the theoretical calculations amounts to around $5.0\,\sigma$ difference [1] when compared to the theoretical value from the 2020 White Paper [2]. The source of the current theoretical error solely arises from contributions from hadronic vacuum polarization (HVP) and hadronic light-by-light scattering (HLbL). Apart from the pseudo-scalar pole contributions, further nontrivial contributions to HLbL arise from the two-particle intermediate states such as $\pi\pi$, $\pi\eta$, and $K\bar{K}$. Currently, only the contributions from the $\pi\pi_{I=0,2}$ and $K\bar{K}_{I=0}$ channels have been considered in a dispersive manner [3, 4]. The isospin-0 part of this result can be understood as a model-independent implementation of the contribution from the $f_0(500)$ and $f_0(980)$ resonances. The contribution from the $a_0(980)$ resonance arises from the rescattering of the $\pi\eta/K\bar{K}_{I=1}$ states and necessitates knowledge of the double-virtual processes $\gamma^*\gamma^* \to \pi\eta/K\bar{K}_{I=1}$. On the experimental side, currently, data is only available for the real photon case from the Belle Collaboration [5, 6]. The measurement of the photon-fusion processes with a single tagged photon is a part of the two-photon physics program of the BESIII Collaboration [7]. To describe the currently available data and provide theoretical predictions for the single- and double-virtual processes, we opt for the dispersive approach, which adheres to the fundamental properties of the S-matrix, namely, analyticity and coupled-channel unitarity.

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2 Formalism

To compute the HLbL contribution of $a_0(980)$ to $(g-2)_\mu$, we adopt the formalism outlined in [3]. This approach yields the following master formula:

$$a_\mu^{HLbL} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2}\, Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1,Q_2,Q_3)\,\bar{\Pi}_i(Q_1,Q_2,Q_3)\,, \quad (1)$$

where $\bar{\Pi}_i$ are scalar functions containing the dynamics of the HLbL amplitude, T_i denote known kernel functions, and τ is defined as $Q_3^2 = Q_1^2 + 2Q_1Q_2\tau + Q_2^2$. For the S-wave, the only contributing scalar functions can be written as

$$\bar{\Pi}_3^{J=0} = \frac{1}{\pi}\int_{s_{th}}^\infty ds' \frac{-2}{\lambda_{12}(s')(s'+Q_3^2)^2}\left(4s'\mathrm{Im}\bar{h}^{(0)}_{++,++}(s') - (s'-Q_1^2+Q_2^2)(s'+Q_1^2-Q_2^2)\mathrm{Im}\bar{h}^{(0)}_{00,++}(s')\right),$$

$$\bar{\Pi}_9^{J=0} = \frac{1}{\pi}\int_{s_{th}}^\infty ds' \frac{4}{\lambda_{12}(s')(s'+Q_3^2)^2}\left(2\,\mathrm{Im}\bar{h}^{(0)}_{++,++}(s') - (s'+Q_1^2+Q_2^2)\,\mathrm{Im}\bar{h}^{(0)}_{00,++}(s')\right), \quad (2)$$

plus crossed versions. Here $\lambda_{12}(s) \equiv \lambda(s, Q_1^2, Q_2^2)$ is a Källén triangle function.

Since $a_0(980)$ is known to have a dynamical coupled-channel $\pi\eta/K\bar{K}$ origin, the inclusion of $K\bar{K}$ intermediate states is necessary. In this case, the unitarity relation implies

$$\mathrm{Im}\bar{h}^{(0)}_{1,\lambda_1\lambda_2,\lambda_3\lambda_4}(s) = \bar{h}^{(0)}_{1,\lambda_1\lambda_2}(s)\,\rho_{\pi\eta}(s)\,\bar{h}^{(0)*}_{1,\lambda_3\lambda_4}(s) + \bar{k}^{(0)}_{1,\lambda_1\lambda_2}(s)\,\rho_{KK}(s)\,\bar{k}^{(0)*}_{1,\lambda_3\lambda_4}(s)\,, \quad (3)$$

where $\rho_{\pi\eta}(\rho_{KK})$ is the phase space factor of $\pi\eta\,(K\bar{K})$ system, and $\bar{h}^{(0)}_{1,\lambda\lambda'}$ ($\bar{k}^{(0)}_{1,\lambda\lambda'}$) denotes the $I=1$, $J=0$ Born subtracted (e.g. $\bar{k} \equiv k - k^{\,\mathrm{Born}}$) partial-wave (p.w.) amplitude of the $\gamma^*(Q_1^2)\gamma^*(Q_2^2) \to \pi\eta\,(K\bar{K})$ process. These p.w. amplitudes contain kinematic constraints and therefore it is important to find a transformation to a new basis of amplitudes which can be used in a modified Muskhelishvili-Omnès (MO) method [8]. For the S-wave, the amplitudes which are free from kinematic constraints can be written as [3] [1]

$$\bar{h}^{(0)}_{i=1,2} = \frac{\bar{h}^{(0)}_{++} \mp Q_1Q_2\bar{h}^{(0)}_{00}}{s - s_{\mathrm{kin}}^{(\mp)}}\,, \quad s_{\mathrm{kin}}^{(\pm)} \equiv -(Q_1 \pm Q_2)^2\,, \quad (4)$$

with $Q_i \equiv \sqrt{Q_i^2}$. In Eq.(4) we omitted the isospin index for simplicity. In the case of a single virtual or real photons, this constraint arises from the requirement of the soft-photon theorem. Similarly to $\gamma^*\gamma^* \to \pi\pi/K\bar{K}$ process [9, 10], the coupled-channel dispersion relation for the $\gamma^*\gamma^* \to \pi\eta/K\bar{K}$ process with $J=0$, $I=1$ can be written as follows

$$\begin{pmatrix} h_i^{(0)}(s) \\ k_i^{(0)}(s) \end{pmatrix} = \begin{pmatrix} 0 \\ k_i^{(0),\,\mathrm{Born}}(s) \end{pmatrix} + \Omega^{(0)}(s)\left[-\int_{s_{th}}^\infty \frac{ds'}{\pi}\,\frac{\mathrm{Disc}(\Omega^{(0)}(s'))^{-1}}{s'-s}\begin{pmatrix} 0 \\ k_i^{(0),\,\mathrm{Born}}(s') \end{pmatrix}\right], \quad (5)$$

where only kaon-pole left-hand cut is currently taken into account. The generalization of the kaon-pole left-hand contribution $k_i^{(0),\mathrm{Born}}$ to the case involving off-shell photons is achieved by the product of the scalar QED result with the electromagnetic kaon form factors [11]. The latter is parameterized using the VMD model. We have verified that within the $Q^2 \lesssim 1$ GeV^2

[1] To maintain consistency with Eq.(2) we follow the conventions from [3], which slightly differ from those in [9].

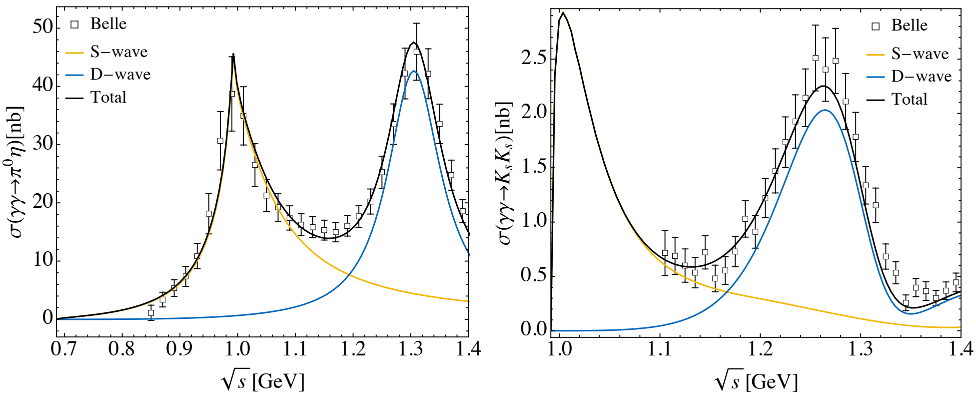

Figure 1. Total cross sections ($|\cos\theta| < 0.8$) for the $\gamma\gamma \to \pi^0\eta$ (left) and $\gamma\gamma \to K_sK_s$ (right) processes compared to the fit results. The data are taken from [5, 6].

range, which is crucial for the a_μ calculation, VMD is consistent with a simple monopole fit to the existing data and the dispersive estimation from [12].

To obtain the Omnès matrix $\Omega^{(0)}(s)$, which encodes the hadronic $\pi\eta/K\bar{K}$ rescattering effects, we utilize the coupled-channel dispersion relation for the partial wave amplitude. The latter is numerically solved using the N/D ansatz [13], with input from the left-hand cuts. When bound states or Castillejo-Dalitz-Dyson (CDD) poles are absent, the Omnès matrix is the inverse of the D-matrix. We parameterize the left-hand cuts in a model-independent manner, expressing them as an expansion in a suitably constructed conformal mapping variable [14, 15], which is chosen to map the left-hand cut plane onto the unit circle. In the absence of experimental $\pi\eta/K\bar{K}$ data, the coefficients of this conformal expansion can be estimated theoretically from χPT, as demonstrated in [16–18]. However, for the $\pi\eta/K\bar{K}$ system, it is necessary to rely on the slowly convergent $SU(3)$ χPT. Instead, we directly determine the unknown coefficients by fitting to $\gamma\gamma \to \pi\eta/K_SK_S$ data [5, 6] and use χPT predictions only as additional constraints. Particularly, for the $\pi\eta \to K\bar{K}$ channel, we impose an Adler zero and ensure that the $\pi\eta \to K\bar{K}$ amplitude remains consistent with χPT at $s_{th} = (m_\pi + m_\eta)^2$. Furthermore, for the $\pi\eta \to \pi\eta$ channel, we employ the χPT scattering length as a constraint. In all cases, the NLO result with low-energy coefficients from [19] is considered as the central value, with an error range defined by the spread between LO and NLO results.

3 Results and Outlook

To reconstruct the physical $\gamma\gamma \to K_SK_S$ cross section, the input for $I = 0$, S-wave amplitude $k^{(0)}_{0,++}(s)$ is taken from the coupled-channel $\pi\pi/K\bar{K}_{I=0}$ analysis [20]. Since we are aiming to describe $\gamma\gamma \to \pi\eta/K_SK_S$ data in the region from threshold up to 1.4 GeV, we also incorporate the D-wave resonances $f_2(1270)$ and $a_2(1320)$ using the Breit-Wigner parametrization, similar to the approach in [21]. We find that with as few as $(2, 2, 2)$ S-wave parameters in $(11, 12, 22)$ channels ($1 = \pi\eta, 2 = K\bar{K}$) we obtain the fit with $\chi^2/\text{d.o.f.} = 0.83$. The resulting total cross sections for $\gamma\gamma \to \pi\eta/K_SK_S$ processes are illustrated in Fig. 1. Through analytical continuation into the complex plane we find the pole on the Riemann sheet II, corresponding to the $a_0(980)$ resonance with $\sqrt{s_{a_0(980)}} = 1.06 - i0.058$ GeV.

With the obtained $\gamma^*\gamma^* \to \pi\eta/K\bar{K}$ amplitudes in hand, we can now proceed to calculate the $a_0(980)$ contribution to the HLbL in $(g-2)$. The preliminary result is

$$a_\mu^{\rm HLbL}[a_0(980)]_{\rm rescatering} = -0.46(2) \times 10^{-11}\,, \tag{6}$$

where the uncertainty currently covers only the sum-rule violation (reflecting the choice of the HLbL basis [3]). It is useful to compare the obtained dispersive result with the outcome from the narrow width approximation $a_\mu^{\rm HLbL}[a_0(980)]_{\rm NWA} = -\left([0.3, 0.6]^{+0.2}_{-0.1}\right) \times 10^{-11}$ [4], where the range reflects the variation in the scale of transition form factor parametrisation taken from the quark model [22].

It is planned to further add new experimental data into the current analysis, in particular, $\gamma\gamma \to K^+K^-$ data from BESIII [23]. In addition, the hadronic $\pi\eta/K\bar{K}$ rescattering will be further constrained by including the existing data for the $\phi \to \gamma\pi\eta$ [24] and $\eta' \to \pi\pi\eta$ [25] decays.

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