# Decays of the tensor glueball in a chiral approach 

Arthur Vereijken ${ }^{1, *}$<br>${ }^{1}$ Institute of Physics, Jan Kochanowski University, ul. Uniwersytecka 7, 25-406, Kielce, Poland


#### Abstract

Glueballs remain an experimentally undiscovered prediction of QCD. Lattice QCD predicts a spectrum of glueballs, with the tensor ( $J^{P C}=2^{++}$) glueball being the second lightest, behind the scalar glueball. From an effective hadronic model based on spontaneous and explicit chiral symmetry breaking, we compute decay ratios of the tensor glueball into various meson decay channels. We find the tensor glueball to primarily decay into 2 vector mesons, dominated by $\rho \rho$ and $K^{*} K^{*}$. These results are compared to experimental data of decay rates of spin 2 mesons. Based on this comparison we make statements on the eligibility of these mesons as potential tensor glueball candidates.


## 1 Introduction

The experimental verification of glueballs has been, and still is, a long-standing open issue in QCD [1]. Numerous theoretical [2, 3] and experimental [4] approaches have made headway, yet the situation is still not completely clear [5-8]. The different theoretical methods agree on the mass hierarchy of the lowest lying glueball states, with the scalar $\left(J^{P C}=0^{++}\right)$being the lightest and the tensor $\left(J^{P C}=2^{++}\right)$the second lightest glueball. In this work we will focus on the tensor glueball, for which there are many experimentally observed isoscalar-tensor candidate resonances. We will present results on the tensor glueball [10] in the extended Linear Sigma Model [11], which is an extension of earlier work on axial-tensor mesons in the same model [12]. Different glueballs have been studied before in the same type of model, such as the scalar [13] and the pseudoscalar glueball [14].

## 2 Chiral model

The meson resonances are gathered into the nonets $V^{\mu}\left(J^{P C}=1^{--}\right)$containing vector mesons, $A_{1}^{\mu}\left(J^{P C}=1^{++}\right)$containing axial-vector mesons, $P\left(J^{P C}=0^{-+}\right)$containing pseudoscalar mesons, $S\left(J^{P C}=0^{++}\right)$containing scalar mesons, $T^{\mu \nu}\left(J^{P C}=2^{++}\right)$containing tensor mesons, and $A_{2}^{\mu \nu}\left(J^{P C}=2^{--}\right)$containing axial-tensor mesons. For details on the resonance assignment of the nonets see $[10,12]$. The tensor glueball itself is a flavor blind object $G_{2, \mu v}$.

The chiral invariant Lagrangians relevant to us are as follows

$$
\begin{equation*}
\mathcal{L}_{\lambda}=\frac{\lambda}{\sqrt{6}} G_{2, \mu v}\left(\operatorname{Tr}\left[\left\{L^{\mu}, L^{\nu}\right\}\right]+\operatorname{Tr}\left[\left\{R^{\mu}, R^{\nu}\right\}\right]\right), \tag{1}
\end{equation*}
$$

[^0]| Decay Ratio | theory | Decay Ratio | theory | Decay Ratio | theory |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{G_{2}(2369) \rightarrow \bar{K} K}{G_{2}(2369) \rightarrow \pi \pi}$ | 0.4 | $\frac{G_{2}(2369) \rightarrow \rho(770) \rho(770)}{G_{2}(2369) \rightarrow \pi \pi}$ | 51 | $\frac{G_{2}(2369) \rightarrow a_{1}(1260) \pi}{G_{2}(2369) \rightarrow \pi \pi}$ | 0.26 |
| $\frac{G_{2}(2369) \rightarrow \eta \eta}{G_{2}(2369) \rightarrow \pi \pi}$ | 0.1 | $\frac{G_{2}(2369) \rightarrow \bar{K}^{*}(892) \bar{K}^{*}(892)}{G_{2}(2369) \rightarrow \pi \pi}$ | 44 | $\frac{G_{2}(2369) \rightarrow K_{1, A} K}{G_{2}(2369) \rightarrow \pi \pi}$ | 0.12 |
| $\frac{G_{2}(2369) \rightarrow \eta \eta^{\prime}}{G_{2}(2369) \rightarrow \pi \pi}$ | 0.005 | $\frac{G_{2}(2369) \rightarrow \omega(782) \omega(782)}{G_{2}(2369) \rightarrow \pi \pi}$ | 17 | $\frac{G_{2}(2369) \rightarrow f_{1}(1285) \eta}{G_{2}(2369) \rightarrow \pi \pi}$ | 0.03 |
| $\frac{G_{2}(2369) \rightarrow \eta^{\prime} \eta^{\prime}}{G_{2}(2369) \rightarrow \pi \pi}$ | 0.01 | $\frac{G_{2}(2369) \rightarrow \phi(1020) \phi(1020)}{G_{2}(2369) \rightarrow \pi \pi}$ | 7 | $\frac{G_{2}(2369) \rightarrow f_{1}(1420) \eta}{G_{2}(2369) \rightarrow \pi \pi}$ | 0.008 |

Table 1. Decay ratios of $G_{2}$ w.r.t. $\pi \pi$ for a mass of 2369 MeV . The columns are sorted as $P P$ on the left, $V V$ in the middle, and $A_{1} P$ on the right.

$$
\begin{equation*}
\mathcal{L}_{\alpha}=\frac{\alpha}{\sqrt{6}} G_{2, \mu v}\left(\operatorname{Tr}\left[\Phi \mathbf{R}^{\mu v} \Phi^{\dagger}\right]+\operatorname{Tr}\left[\Phi^{\dagger} \mathbf{L}^{\mu v} \Phi\right]\right) \tag{2}
\end{equation*}
$$

These are the leading terms in large- $N_{c}$ expansion, where $N_{c}$ is the number of colors of underlying the gauge group. The nonets of chiral partners are grouped together and are given by:

$$
\begin{align*}
L^{\mu} & :=V^{\mu}+A_{1}^{\mu}, R^{\mu}:=V^{\mu}-A_{1}^{\mu}, \Phi=S+i P \\
\mathbf{L}^{\mu \nu} & =T^{\mu \nu}+A_{2}^{\mu \nu}, \mathbf{R}^{\mu \nu}=T^{\mu \nu}-A_{2}^{\mu \nu}, \tag{3}
\end{align*}
$$

such that they obey the transformation rules $L^{\mu} \rightarrow U_{L} L^{\mu} U_{L}^{\dagger}, R^{\mu} \rightarrow U_{R} R^{\mu} U_{R}^{\dagger}, \Phi \rightarrow$ $U_{L} \Phi U_{R}^{\dagger}, \mathbf{R}^{\mu \nu} \rightarrow U_{R} \mathbf{R}^{\mu \nu} U_{R}^{\dagger}, \mathbf{L}^{\mu \nu} \rightarrow U_{L} \mathbf{L}^{\mu \nu} U_{L}^{\dagger}$ under the chiral transformations of $U_{L}(3) \times$ $U_{R}(3)$. The first Lagrangian (1) models the two-body decays of the tensor glueball into 2 vector mesons, into 2 pseudoscalar mesons, and into an axial-vector and a pseudoscalar meson. The second Lagrangian (2) leads to the decay into a tensor and a pseudoscalar meson. Since the coupling constants $\alpha$ and $\lambda$ are not known a priori and cannot be fitted to experimental data, we are limited to computing decay ratios, seperately for each Lagrangian. Lattice calculations in [15] find a tensor glueball mass of 2369 MeV . The decay ratios of the first Lagrangian with respect to $\pi \pi$ for this mass are shown in table 1 . As evident from the results in table 1 , the 2 -vector decay channel is dominant, in particular the decays into $\rho \rho$ and $K^{*} \bar{K}^{*}$. A similar dominance of the 2-vector channel was recently found in [16] with the holographic Witten-Sakai-Sugimoto model.

## 3 Results \& Data Comparison

We compare results to available data of spin-2 isoscalar resonances $\left(J^{P C}=\right.$ $2^{++}$, $\mathrm{I}=0$ ) with masses of 1.9 GeV and upwards. These are the $f_{2}(1910), f_{2}(1950), f_{2}(2010), f_{2}(2150), f_{J}(2220), f_{2}(2300)$, and the $f_{2}(2340)$. In table 2 decay ratios are computed and compared with PDG data [9] where available, revealing how well they fit as glueball candidates. We see that every glueball candidate other than the $f_{2}(1950)$ has disagreement with experimental data, and that the $f_{2}(1950)$ fits reasonably well, given uncertainties on both sides. Therefore we interpret $f_{2}(1950)$ to be the best candidate for the lightest tensor glueball, which has not been the first time it has been proposed as the tensor glueball, see e.g. [17].

| Resonances | Decay Ratios | PDG [9] | Model Prediction |
| :---: | :---: | :---: | :---: |
| $f_{2}(1910)$ | $\rho(770) \rho(770) / \omega(782) \omega(782)$ | $2.6 \pm 0.4$ | 3.1 |
| $f_{2}(1910)$ | $f_{2}(1270) \eta / a_{2}(1320) \pi$ | $0.09 \pm 0.05$ | 0.07 |
| $f_{2}(1910)$ | $\eta \eta / \eta \eta^{\prime}(958)$ | $<0.05$ | $\sim 8$ |
| $f_{2}(1910)$ | $\omega(782) \omega(782) / \eta \eta \prime(958)$ | $2.6 \pm 0.6$ | $\sim 200$ |
| $f_{2}(1950)$ | $\eta \eta / \pi \pi$ | $0.14 \pm 0.05$ | 0.081 |
| $f_{2}(1950)$ | $K \bar{K} / \pi \pi$ | $\sim 0.8$ | 0.32 |
| $f_{2}(1950)$ | $4 \pi / \eta \eta$ | $>200$ | $>700$ |
| $f_{2}(2150)$ | $f_{2}(1270) \eta / a_{2}(1320) \pi$ | $0.79 \pm 0.11$ | 0.1 |
| $f_{2}(2150)$ | $K \bar{K} / \eta \eta$ | $1.28 \pm 0.23$ | $\sim 4$ |
| $f_{2}(2150)$ | $\pi \pi / \eta \eta$ | $<0.33$ | $\sim 10$ |
| $f_{J}(2220)$ | $\pi \pi / K \bar{K}$ | $1.0 \pm 0.5$ | $\sim 2.5$ |

Table 2. Decay ratios for the decay channels with available data.

## 4 Conclusion

In this note we have computed decays of the tensor glueball using a chiral hadronic model. We find that the decay to two vectors, in particular $\rho \rho$ and $K^{*} \bar{K}^{*}$, is dominant. Upon comparing with experimental data, we find that the $f_{2}(1950)$ is the most suitable candidate (even if some deviations are present) for the tensor glueball based on the known decay ratios. In the future, one should investigate why there is up to a 400 MeV mass difference between this resonance and predicted masses from lattice methods. One possible explanation could be the role of mesonic loops and/or the mixing with nearby quark-antiquark states.

## Acknowledgements

We thank Francesco Giacosa and Shahriyar Jafarzade for useful discussions. We also acknowledge financial support from the Polish National Science Centre (NCN) via the OPUS project 2019/33/B/ST2/00613.

## References

[1] F. Gross, E. Klempt, S. J. Brodsky, A. J. Buras, V. D. Burkert, G. Heinrich, K. Jakobs, C. A. Meyer, K. Orginos and M. Strickland, et al. [arXiv:2212.11107 [hep-ph]].
[2] F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Phys. Rev. D 72, 114021 (2005) doi:10.1103/PhysRevD. 72.114021 [arXiv:hep-ph/0511171 [hep-ph]].
[3] F. Brünner, D. Parganlija and A. Rebhan, Phys. Rev. D 91, no.10, 106002 (2015) [erratum: Phys. Rev. D 93, no.10, 109903 (2016)] doi:10.1103/PhysRevD.91.106002 [arXiv:1501.07906 [hep-ph]].
[4] E. Klempt, K. V. Nikonov, A. V. Sarantsev and I. Denisenko, Phys. Lett. B 830, 137171 (2022) doi:10.1016/j.physletb.2022.137171 [arXiv:2205.07239 [hep-ph]].
[5] V. Crede and C. A. Meyer, Prog. Part. Nucl. Phys. 63, 74-116 (2009) doi:10.1016/j.ppnp.2009.03.001 [arXiv:0812.0600 [hep-ex]].
[6] V. Mathieu, N. Kochelev and V. Vento, Int. J. Mod. Phys. E 18, 1-49 (2009) doi:10.1142/S0218301309012124 [arXiv:0810.4453 [hep-ph]].
[7] F. J. Llanes-Estrada, Eur. Phys. J. ST 230, no.6, 1575-1592 (2021) doi:10.1140/epjs/s11734-021-00143-8 [arXiv:2101.05366 [hep-ph]].
[8] H. X. Chen, W. Chen, X. Liu, Y. R. Liu and S. L. Zhu, Rept. Prog. Phys. 86, no.2, 026201 (2023) doi:10.1088/1361-6633/aca3b6 [arXiv:2204.02649 [hep-ph]].
[9] R. L. Workman et al. [Particle Data Group], PTEP 2022, 083C01 (2022) and 2023 update doi:10.1093/ptep/ptac097
[10] A. Vereijken, S. Jafarzade, M. Piotrowska and F. Giacosa, Phys. Rev. D 108, no.1, 014023 (2023) doi:10.1103/PhysRevD.108.014023 [arXiv:2304.05225 [hep-ph]].
[11] D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87, no.1, 014011 (2013) doi:10.1103/PhysRevD. 87.014011 [arXiv:1208.0585 [hep-ph]].
[12] S. Jafarzade, A. Vereijken, M. Piotrowska and F. Giacosa, Phys. Rev. D 106, no.3, 036008 (2022) doi:10.1103/PhysRevD.106.036008 [arXiv:2203.16585 [hep-ph]].
[13] S. Janowski, F. Giacosa and D. H. Rischke, Phys. Rev. D 90, no.11, 114005 (2014) doi:10.1103/PhysRevD. 90.114005 [arXiv:1408.4921 [hep-ph]].
[14] W. I. Eshraim, S. Janowski, F. Giacosa and D. H. Rischke, Phys. Rev. D 87, no.5, 054036 (2013) doi:10.1103/PhysRevD. 87.054036 [arXiv:1208.6474 [hep-ph]].
[15] A. Athenodorou and M. Teper, JHEP 11, 172 (2020) doi:10.1007/JHEP11(2020)172 [arXiv:2007.06422 [hep-lat]].
[16] F. Hechenberger, J. Leutgeb and A. Rebhan, Phys. Rev. D 107, no.11, 114020 (2023) doi:10.1103/PhysRevD.107.114020 [arXiv:2302.13379 [hep-ph]].
[17] A. A. Godizov, Eur. Phys. J. C 76, no.7, 361 (2016) doi:10.1140/epjc/s10052-016-4229z [arXiv:1604.01689 [hep-ph]].


[^0]:    *arthur.vereijken@gmail.com

