Revealing violations of macrorealism in flavor oscillations: Leggett-Garg inequalities and no-signaling-in-time conditions

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Abstract. We briefly review recent developments in the study of the quantum nature of flavor mixing; in particular, the attention will be devoted to neutrino and neutral meson oscillations. We employ Leggett–Garg type inequalities and no-signaling-in-time conditions to probe the intrinsic quantumness of such a physical manifestation, showing how the analysis is not affected by the wave-packet spreading (for neutrinos) and the intrinsic particle instability (for mesons).

1 Introduction

The notion of macrorealism has been proposed in order to formalize the everyday intuition of macroscopic world, and it can be summarized in two postulates: *macrorealism per se* (given a set of available macroscopically distinct states, a macroscopic object is in one of them at any given time) and *non-invasive measurability* (the state of the macroscopic object can be determined without affecting either itself or its dynamical evolution) [1]. Similarly to Bell inequalities (which constitute a necessary and sufficient condition for local realism), a set of inequalities, known as *Leggett-Garg inequalities* (LGIs), can be derived as a condition necessarily satisfied by a macrorealistic system. Given a dichotomous observable O with outcomes ± 1, its measurement performed by an observer at fixed time points $\{t_0, ..., t_{N-1}\}$ results in a set of outcomes $\{O_0, ..., O_{N-1}\}$. Assuming for simplicity $N = 3$, four LGIs can be established [2, 3]:

$$\mathcal{L}_{ab}(t_0, t_1, t_2) \equiv 1 + (-1)^b C_{01} + (-1)^{a+b} C_{12} + (-1)^a C_{02} \geq 0\,, \tag{1}$$

where $a, b \in \{0, 1\}$ and $C_{ij} = \langle O_i O_j \rangle$ being the 2-time correlation functions.

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Table 1. The set of NSIT/AoT conditions for $N = 3$.

NSIT conditions	AoT conditions
$P(O_2) = \sum_{O_1} P(O_1, O_2)$	$P(O_0, O_1) = \sum_{O_2} P(O_0, O_1, O_2)$
$P(O_0, O_2) = \sum_{O_1} P(O_0, O_1, O_2)$	$P(O_0) = \sum_{O_1} P(O_0, O_1)$
$P(O_1, O_2) = \sum_{O_0} P(O_0, O_1, O_2)$	$P(O_1) = \sum_{O_2} P(O_1, O_2)$

LGIs provide a device-independent tool to probe macroscopic coherence and verify quantum mechanical laws at different scales. Oscillating systems are promising candidates for such tests, e.g., neutrinos can exhibit coherence even at macroscopic scales. Indeed, it has been proven that LGIs are violated in neutrino oscillations experiments [4], and similar considerations were carried out also for neutral mesons [5]. However, in contrast to Bell inequalities, LGIs are not sufficient as a condition for macrorealism, meaning that the quantumness of the analyzed system is not excluded even assuming their fulfillment [6, 7]. On the other hand, a necessary and sufficient condition for macrorealism can still be found as a set of equalities, known as no-signaling-in-time (NSIT) and arrow-of-time (AoT) conditions [6], summarized in Table 1. In Ref. [8], we have studied NSIT/AoT for neutrinos and confirmed that these are stronger than LGIs. In the following, we review the results and present a preliminary generalization to the case of neutral meson oscillations.

2 Macrorealism and particle oscillations

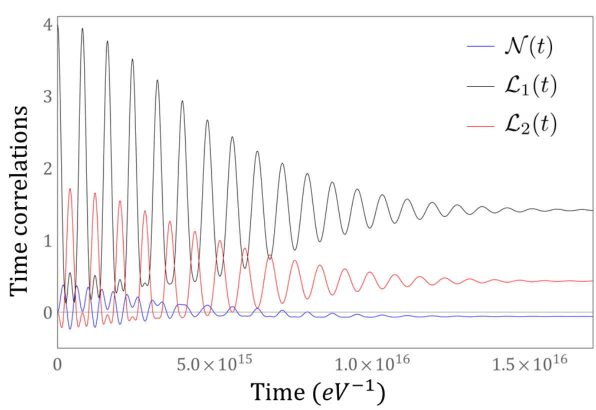

Figure 1. $\mathcal{N}(t)$ (blue) vs $\mathcal{L}_1(t)$ (black) and $\mathcal{L}_2(t)$ (red) as functions of time expressed in eV^{-1}. The values used to generate the plot have been taken from the MINOS experiment [9], with $\sin^2\theta = 0.314$, $\Delta m^2 = 7.92 \times 10^{-5}\ \mathrm{eV}^2$, $E = 10$ GeV and $\sigma_x = 0.5\ \mathrm{GeV}^{-1}$.

We consider particle oscillations between two flavors F_1 and F_2 focusing on two specific examples, namely neutrinos ($F_1 = \nu_e$, $F_2 = \nu_\mu$) and neutral kaons ($F_1 = K^0$, $F_2 = \bar{K}^0$). We choose the macroscopic dichotomous observable $O(t) = 2|F_1(t)\rangle\langle F_1(t)| - \mathbb{1}$, so that the value $+1$ corresponds to a particle of flavor F_1, and the value -1 corresponds to a particle *not* possessing the flavor F_1. For the sake of simplicity, we denote these values as $O = F_1$ and $O = \neg F_1$, respectively. In turn, we assume that the observable O is measured at $t_0 = 0$, $t_1 = t$ and $t_2 = 2t$. In the case of NSIT/AoT conditions (Table 1), the measurement outcome O_i is fixed, unless a summation over it is performed, and can be chosen arbitrarily. Without loss of generality, we choose $O_0 = F_1$, $O_1 = \neg F_1$, and $O_2 = \neg F_1$.

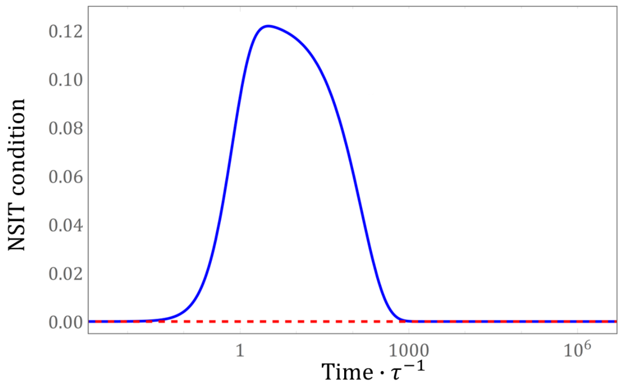

Figure 2. $\mathcal{N}_{K^0}(t)$ as function of time scaled by the proper mean lifetime $\tau = 8.954 \cdot 10^{-9}$ s of a neutral kaon. The values used to generate the plot have been taken from the summary published by the Particle Data Group [11], with $\Gamma = 5.5939 \times 10^9$ s^{-1}, $\Delta\Gamma = 1.1149 \times 10^{10}$ s^{-1}, and $\Delta m = 0.5293 \times 10^{10}\hbar$ s^{-1}.

We start with the case of neutrinos, assuming that an electronic neutrino ν_e is produced at t_0. For neutrinos, we represent the flavor eigenstates as a linear combination of Gaussian wave-packets [10], so that the oscillation probabilities are given by (setting $\hbar = c = 1$)

$$P_{\nu_e\to\nu_\mu}(t) = \frac{\sin^2(2\theta)}{2}\Big(1 - e^{-\left(\frac{t}{L^{coh}}\right)^2}\cos\Big(\frac{\Delta m^2}{E}t\Big)\Big), \tag{2}$$

$$P_{\nu_e\to\nu_e}(t) = 1 - P_{\nu_e\to\nu_\mu}(t), \tag{3}$$

where $L^{coh} = \frac{4\sqrt{2}E^2}{|\Delta m^2|}\sigma_x$ is the coherence length that defines the characteristic distance of oscillations' damping, σ_x is the wave-packet spread, $\Delta m^2 \equiv m_1^2 - m_2^2$ is the difference of neutrino squared-masses and E is the average energy of the neutrino wave-packets. In order to derive the LGIs and NSIT/AoT conditions, we take into account that, for two-flavor neutrino oscillations, the value $O = \neg\nu_e$ is equivalent to $O = \nu_\mu$. Therefore, we can choose the fixed measurement outcomes O_i of NSIT/AoT conditions as $O_0 = \nu_e$, $O_1 = \nu_\mu$, and $O_2 = \nu_\mu$. Bearing this in mind, the set of LGIs (1) reduces to two non-trivial inequalities [8]:

$$\mathcal{L}_1(t) \equiv \mathcal{L}_{00}(t) \equiv 2\Big(1 - P_{\nu_e\to\nu_\mu}(t)\Big) - P_{\nu_e\to\nu_\mu}(2t) \geq 0\,, \tag{4}$$

$$\mathcal{L}_2(t) \equiv \mathcal{L}_{01}(t) \equiv 2P_{\nu_e\to\nu_\mu}(t) - P_{\nu_e\to\nu_\mu}(2t) \geq 0\,. \tag{5}$$

On the other hand, the NSIT/AoT conditions reduce to a unique, non-trivial NSIT condition:

$$\mathcal{N}(t) \equiv P_{\nu_e\to\nu_\mu}(2t) - 2P_{\nu_e\to\nu_\mu}(t)\,P_{\nu_e\to\nu_e}(t) = 0. \tag{6}$$

In Figure 1, we compare LGIs (4)-(5) and NSIT/AoT condition (6). It can be seen that $\mathcal{N}(t)$ is non-zero even when LGI is fulfilled. Moreover, $\mathcal{N}(t)$ is also non-zero for time intervals much longer than the coherence length. This example confirms that NSIT/AoT conditions are stronger and contains more information than LGIs, as expected.

For neutral kaon oscillations, we require that a kaon K^0 is produced at time $t_0 = 0$ and study the decay by means of the Wigner-Weisskopf approach under the assumption that the $\mathcal{CP}$ symmetry is conserved. Therefore, the oscillation probabilities are given by (setting $\hbar = c = 1$):

$$P_{K^0\to K^0/\bar{K}^0}(t) = \frac{e^{-\Gamma t}}{2}\Big(\cosh\Big(\frac{\Delta\Gamma t}{2}\Big) \pm \cos(\Delta m t)\Big), \tag{7}$$

where $\Delta m = m_L - m_S$ is the difference of neutral kaon masses, $\Gamma = \frac{\Gamma_S+\Gamma_L}{2}$ and $\Delta\Gamma = \Gamma_S - \Gamma_L$, with $\Gamma_{S,L}$ being decay widths associated with the corresponding mass eigenstate. As NSIT/AoT conditions provide a stronger condition for macrorealism than LGIs, we focus only on the former. Similarly to the case of neutrino oscillations, there is a unique, non-trivial NSIT condition:

$$\mathcal{N}_{K^0}(t) \equiv P_{K^0\to K^0}(2t) - P^2_{K^0\to K^0}(t) - P^2_{K^0\to \bar{K}^0}(t) = 0. \tag{8}$$

The result is shown in Figure 2. The curve (blue solid line) evidently deviates from the macrorealistic value (red dashed line), confirming the intrinsic quantum nature of meson oscillations. A more detailed analysis will be presented elsewhere [12].

3 Conclusions

In this paper, we have addressed particle flavor mixing as a probe for testing the validity of macrorealism. We have revised the description of the quantumness of neutrino oscillations via NSIT/AoT conditions for macrorealism [8] and compared the results with the corresponding violation of LGIs. In turn, we sketched how this analysis can be generalized to oscillations of decaying particles such as neutral kaons, which are widely used to probe the validity of quantum theory. We have observed that flavor oscillations are a genuine quantum phenomenon, even in presence of decoherence mechanisms due to the wave-packet separation of oscillating particles.

A further development of our work consists in the analysis of NSIT/AoT conditions in the realm of quantum field theory (QFT). Indeed, a QFT treatment of neutrino oscillations [13] is known to lead to a stronger violation of LGIs compared to the quantum mechanical one [14]. This suggests that QFT could be less compatible with the macrorealistic interpretation with respect to quantum mechanics, thus agreeing with the known results for local realism via Bell inequalities. As macrorealism has a more complex structure than LGIs, exploring NSIT/AoT conditions in QFT particle oscillations will provide promising theoretical insights.

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