Quark mass dependence of $D_{s0}^{*}(2317)$ and $D_{s1}(2460)$ states.

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Abstract. We investigate the dependence of both light and heavy quark masses on the properties of low-lying charmed mesons within the framework of one-loop HHχ PT. Determination of the low energy constants is accomplished through an analysis of lattice data obtained from various Lattice Quantum Chromodynamics (LQCD) simulations. Model selection tools are employed to identify the pertinent parameters needed to achieve higher precision alignment with the data. Our study extends to the analysis of HSC energy levels for DK scattering in $I = 0$, considering different boosts and two pion masses. A comprehensive global fit is performed, incorporating HSC energy levels along with those from DK and D^*K scattering obtained from RQCD and Prelovsek et al. Finally, we extract the dependence of $D_{s0}^{*}(2317)$ and $D_{s1}(2460)$ resonances on the pion mass.

1 Introduction

The identification of exotic hadrons within the heavy quark sector, unexplainable by conventional $q\bar{q}$ mesons or qqq baryons, such as those featuring tetraquark or pentaquark structures, underscores the significance of hadronic loops in elucidating the masses and properties of numerous states in the hadron spectrum [1]. This leads to the formation of a pair of bound states with a binding energy of approximately 40 MeV relative to the DK and D^*K thresholds. These states, namely $D_{s0}^{*}(2317)$ and $D_{s1}(2460)$, exhibit a composition probability of 70% associated with DK and D^*K components, respectively. Due to the complexity arising from the numerous parameters in one-loop HMχPT, precise determination of theory parameters remains elusive. Recent advancements in lattice QCD, however, motivate a reexamination of this matter.

The first section delves into the quark mass dependence of ground state charmed mesons (D, D^*, D_s, and D_s^*) within one-loop HMχPT [2], leveraging available lattice data. In the second section, the dataset from [3] is analyzed, explicitly incorporating the coupling of DK components to $c\bar{s}$ states, with the mass of the latter considered as a fitting parameter. The meson-meson interaction is described by the hidden gauge formalism. At leading order, this interaction is of the same type as the one of HMχPT. The quark mass dependence of the $D_{s0}^{*}(2317)$ properties in both the light and charm quarks is extracted [4]. In order to perform the extrapolation, we use the analysis of charmed meson masses [5] from the previous section. Finally, a global fit of LQCD data from [3, 6–9] is performed in order to extract the quark mass dependence of $D_{s0}^{*}(2317)$ and $D_{s1}(2460)$.

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2 Quark mass dependence of the low-lying charmed mesons at one loop in HHχPT

The formulas for the low-lying charmed meson masses (D, D^*, D_s and D_s^*) are calculated at one loop in HHχPT in the Appendices (A.1) and (A.10) of [2]. These are,

$$\frac{1}{4}(M_{D_a}+3M_{D_a^*}) = m_H+\alpha_a-\sum_{X=\pi,K,\eta}\beta_a^{(X)}\frac{M_X^3}{16\pi f^2}+\sum_{X=\pi,K,\eta}\left(\gamma_a^{(X)}-\lambda_a^{(X)}\alpha_a\right)\frac{M_X^2}{16\pi^2 f^2}\log\left(M_X^2/\mu^2\right)+c_a \quad (1)$$

and

$$(M_{D_a^*}-M_{D_a}) = \Delta+\sum_{X=\pi,K,\eta}\left(\gamma_a^{(X)}-\lambda_a^{(X)}\Delta\right)\frac{M_X^2}{16\pi^2 f^2}\log\left(M_X^2/\mu^2\right)+\delta c_a. \quad (2)$$

The meson masses are fitted to the collected lattice data. Due to the substantial number of parameters involved, we employ the Least Absolute Shrinkage and Selection Operator (LASSO) method [10] to eliminate less relevant model parameters. To achieve this, a penalty term is introduced into the χ^2 in the form: $P = \frac{\lambda}{10}\sum_i^n |p_i|$. As the penalty coefficient λ increases, zeroes out less significant parameters to minimize the fully penalized $\chi_F^2 = \chi^2 + P$.

For error propagation evaluation, we utilize automatic differentiation [11] with the Julia ADerrors package. The final outcome of the D and D^* meson masses fit to lattice data is presented in Figure 1.

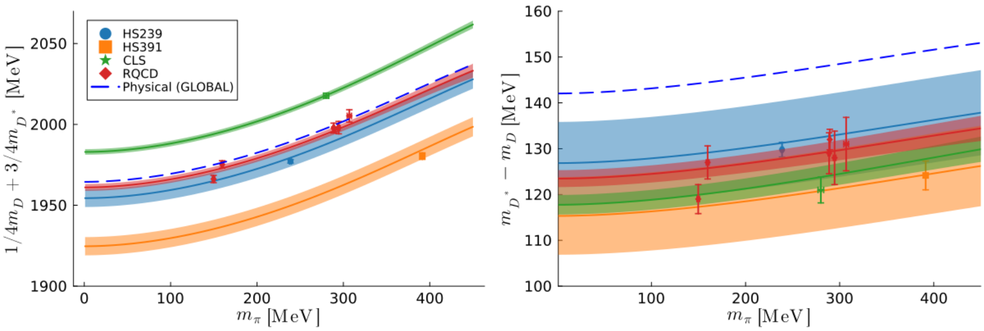

Figure 1. Results of the D and D^* meson masses dependence with the pion mass. Solid lines are the result of the global analysis with several lattice collaboration data. The dashed line is the trajectory with physical charm and strange quark masses that we extrapolate as a result.

3 Quark mass dependence of $D_{s0}^*(2317)$ and $D_{s1}(2460)$

To extrapolate the $D_{s0}(2317)$ pole, the initial step involves fitting the undetermined parameters of the $DK \to DK$ amplitude. The scattering amplitude in infinite volume is expressed through the Bethe-Salpeter equation as follows:

$$T = \frac{V}{1-VG}, \quad (3)$$

Here, V represents the scattering potential matrix, and G denotes the loop matrix. To compute the V potential for $DK \to DK$, the Hidden Gauge Lagrangian is employed, corresponding to the hidden gauge formalism [12, 13], along with the lowest order of HMχPT. Specifically, for the $DK \to DK$ scattering, only the Lagrangian term of pseudoscalar-pseudoscalar-vector is necessary, denoted as $\mathcal{L}^{PPV} = ig' Tr\left[[\partial_i \Phi, \Phi]\mathcal{V}^i\right]$, where g' is defined as $m_\rho/(2f_\pi)$, Φ represents the $SU(4)$ pseudoscalar meson matrix as given in [13], and $\mathcal{V}^i$ stands for the vector meson matrix. Subsequently, the amplitude computed just before the projection to partial waves is

$$V_{DK} = -\frac{s-u}{2f^2}. \tag{4}$$

Note that one arrives also to this interaction through the LO HMχPT [14, 15]. A term associated with the interaction of DK with a bare $c\bar{s}$ states of the $J_l^P = \frac{1}{2}^+$ HQSS doublet can be added. At LO in the heavy quark expansion this gives [15],

$$V_{\rm ex} = \frac{V_{c\bar{s}}^2}{s - m_{c\bar{s}}^2}\,, \qquad \text{with} \qquad V_{c\bar{s}}(s) = -\frac{c}{f}\sqrt{M_D m_{c\bar{s}}}\frac{s + m_K^2 - M_D^2}{\sqrt{s}}\,, \tag{5}$$

where $m_{c\bar{s}}$ is the mass of the bare $c\bar{s}$ component, and c is a dimensionless constant that provides the strength of the coupling of this component to the DK channel. In this work we will consider this coupling as a free parameter, together with $m_{c\bar{s}}$. Note that $m_{c\bar{s}}$ can vary for every LQCD simulation with a different setup. The potential $V(s)$ consistent with HQSS is then given by $V(s) = V_{DK}(s) + V_{\rm ex}(s)$. To project to partial waves we use

$$V_l(s) = \frac{1}{2}\int_{-1}^{1} V(s,\theta)\mathcal{P}_l(\cos(\theta))d\cos(\theta), \tag{6}$$

where $\mathcal{P}_l(x)$ are the Legendre polynomials. However, as we are going to fit to lattice data we need to discretize the momenta $q = (2\pi/L)\vec{n}$ with $\vec{n} \in \mathbb{Z}^3$. In the finite volume the scattering amplitude is given by $\widetilde{T} = V/(1 - V\widetilde{G})$ The final $\widetilde{G}$ that we insert to the Bethe-Salpeter equation is constructed with the elements

$$\widetilde{G}^{(i)co}_{fin} = G^{(i)DR}_{inf} + \lim_{q_{co}\to\infty}\left(G^{(i)}_{fin} - G^{(i)co}_{inf}\right). \tag{7}$$

where superscripts DR and co means that we have used the dimensional and the cutoff regularization method and the inf, fin subscripts means infinite or finite volume respectively. With this, the value of the cutoff is cancelled between the last two terms of Eq. (7). Fixing the dimensional regularization scale $\mu = 1000$ MeV we only have the subtraction constant a as a free parameter. To extrapolate to the physical point, we adopt the assumption that the subtraction constant can be expressed as a first-order Taylor expansion of the squared pion mass, represented as $a = a_1 + a_2 m_\pi^2$. A global fit is conducted by analyzing the energy levels from [3] for DK scattering in $I = 0$, along with the DK and D^*K scattering levels sourced from [6, 7, 9]. The fitting parameters are c, $m_{c\bar{s}}$ and a_1, a_2 as the subtraction constants. Two scenarios are considered: one where the bare $c\bar{s}$ component is absent, i.e., $V_{\rm ex} = 0$, and another where $V_{\rm ex} \neq 0$.

To conclude, in Figure 2 we illustrate the binding energies and mass splitting of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ states relative to the pion mass in both fits. The binding energies of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ align well with experimental data in Fit II. However, for Fit I, we predict the $D_{s1}(2460)$ state to be less than 20 MeV from its physical value. Additionally, these energies exhibit a quadratic trend, increasing with pion masses exceeding 200 MeV. Figure 2 is generated using an extrapolation of the bare mass to the physical point.

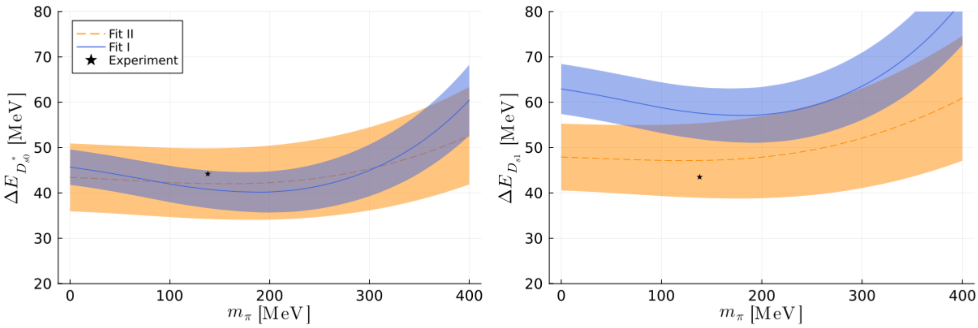

Figure 2. Binding energies of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ for physical charm and strange quark masses. Solid lines are the result of the Fit I with just the subtraction constant as a free parameter and dashed lines correspond to Fit II where we include two additional parameters to describe the coupling to the bare $c\bar{s}$ state.

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