

Compositeness of near-threshold exotic hadrons with decay and coupled-channel effects

Tomona Kinugawa^{1,*} and Tetsuo Hyodo^{1,**}

¹Department of Physics, Tokyo Metropolitan University, Hachioji 192-0397, Japan

Abstract. The near-threshold exotic hadrons such as T_{cc} and $X(3872)$ are naively considered as the hadronic molecular state from the viewpoint of the low-energy universality. However, it is also known that the elementary dominant state is not completely excluded as the internal structure of the near-threshold states. Furthermore, the dominance of molecules is expected to be modified by the decay or coupled channels. We discuss these features of the near-threshold bound states by calculating the compositeness with the effective field theory.

1 Introduction

The exotic hadrons have been of interest in hadron physics. It is considered that the exotic hadrons have a different internal structure from ordinary hadrons (qqq or $\bar{q}q$), such as the multi-quarks or hadronic molecules [1, 2]. Recent experiments reported the observation of exotic hadrons, including $X(3872)$ and T_{cc} by the Belle collaboration [3] and by the LHCb collaboration [4], respectively. One of the characteristic features of T_{cc} and $X(3872)$ is the fact that these are observed in the near-threshold energy region.

The low-energy universality is known as an important notion to constrain the property of the near-threshold states [5, 6]. The universality holds when the magnitude of the scattering length $|a_0|$ is sufficiently larger than other length scales in the system. In this case, the microscopic features of the interaction are irrelevant, and all physical quantities are scaled only by $|a_0|$. For example, the binding energy B is given by $B = 1/(2\mu a_0^2)$ with the reduced mass μ . The universality indicates that the binding energy is smaller than other energy scales, because $|a_0|$ is much larger than other length scales. As a consequence, the low-energy universality holds when the binding energy is sufficiently small.

The internal structure of the near-threshold states can be quantitatively expressed by the compositeness X [7, 8]. The compositeness X is schematically written as the weight of the hadronic molecular component $|\text{molecule}\rangle$ in the bound state $|\psi\rangle$:

$$X = |\langle \text{molecule} | \psi \rangle|^2. \quad (1)$$

For the bound states, X can be regarded as the probability because it is a real value and $0 \leq X \leq 1$. Therefore, we can quantitatively classify the bound states whether they are composite dominant ($X > 0.5$) or elementary dominant ($X < 0.5$).

*e-mail: kinugawa-tomona@ed.tmu.ac.jp

**e-mail: hyodo@tmu.ac.jp

In the weak-binding limit $B \rightarrow 0$, it is shown that the compositeness X goes to unity from the viewpoint of the low-energy universality [9, 10]. In this sense, it is naively expected that the compositeness of the near-threshold states with $B \neq 0$ but $B \sim 0$ is close to unity, $X \sim 1$, and the states are composite dominant. However, the state can be elementary dominant with a fine tuning of the parameters, no matter how small the binding energy is [11]. In this study, we aim to show the features of the near-threshold bound states from the viewpoint of the fine tuning and the low-energy universality. More detailed discussions can be found in Ref. [12].

2 Universality of near-threshold states

In this section, we focus on the weakly bound state in the single-channel $\psi_1\psi_2$ scattering. For this purpose, we introduce the effective field theory model with the following Hamiltonian:

$$\mathcal{H} = \frac{1}{2m_1} \nabla\psi_1^\dagger \cdot \nabla\psi_1 + \frac{1}{2m_2} \nabla\psi_2^\dagger \cdot \nabla\psi_2 + \frac{1}{2M} \nabla\phi^\dagger \cdot \nabla\phi + \nu_0\phi^\dagger\phi + g_0(\phi^\dagger\psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger\phi). \quad (2)$$

Here m_i (M) is the mass of ψ_i (the bare state ϕ), ν_0 is the energy of the bare state which is measured from the threshold of the $\psi_1\psi_2$ scattering, and g_0 is the coupling constant of the contact three-point interaction. From the Lippmann-Schwinger equation, the amplitude of the $\psi_1\psi_2$ scattering is obtained as

$$f(k) = -\frac{\mu}{2\pi} \left[\frac{\frac{k^2}{2\mu} - \nu_0}{g_0^2} + \frac{\mu}{\pi^2} \left[\Lambda + ik \arctan\left(-\frac{\Lambda}{ik}\right) \right] \right]^{-1}, \quad (3)$$

where μ is the reduced mass, and the cutoff Λ is introduced to avoid the divergence of the momentum integral in the loop function. We consider that the system has a bound state with the binding energy B . We can derive the exact expression of the compositeness X of the bound state in this model by applying the method in Ref. [13]:

$$X = \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2} \left(\arctan\left(\frac{\Lambda}{\kappa}\right) - \frac{\frac{\Lambda}{\kappa}}{1 + \left(\frac{\Lambda}{\kappa}\right)^2} \right) \right]^{-1}^{-1}, \quad \kappa = \sqrt{2\mu B}. \quad (4)$$

There are three model parameters, the coupling constant g_0 , bare state energy ν_0 , and cutoff Λ . Here we fix the binding energy B to discuss the compositeness of the weakly bound state. From the bound state condition $f^{-1}(ik) = 0$, the square of the coupling constant g_0^2 is written by B , ν_0 , and Λ :

$$g_0^2(B; \nu_0, \Lambda) = \frac{\pi^2}{\mu} (B + \nu_0) \left[\Lambda - \kappa \arctan\left(\frac{\Lambda}{\kappa}\right) \right]^{-1}. \quad (5)$$

We can further eliminate one more degree of freedom by using the dimensionless quantities by Λ . Eventually, the energy of the bare state ν_0 remains as a parameter. In general, the compositeness X has the model dependence because it is not an observable. The choice of the value of ν_0 represents the choice of the model. Therefore, we can investigate the model dependence of X by examining its ν_0 dependence. In this work, we vary ν_0 in the region $-B \leq \nu_0 \leq E_{\text{typ}}$, with $E_{\text{typ}} = \Lambda^2/(2\mu)$ as the typical energy scale in the model. The lower boundary of ν_0 region is determined to have a real coupling constant ($g_0^2 \geq 0$), and the upper is given not to exceed the applicable limit of the effective field theory.

In Fig. 1, we plot the compositeness X of the weakly bound state with $B = 0.01E_{\text{typ}}$ as a function of the model parameter ν_0 . We see that $X > 0.5$ for 88 % of the ν_0 region (shaded

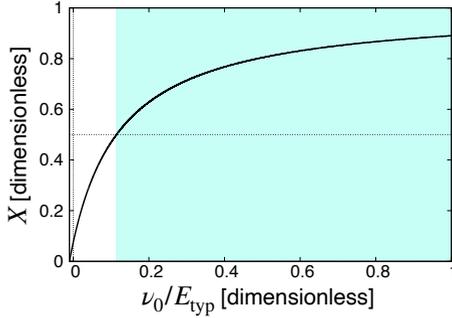


Figure 1. The compositeness X of the weakly bound state ($B = 0.01E_{\text{typ}}$) as a function of the model parameter ν_0 . The shaded region is the parameter region with $X(\nu_0) > 0.5$.

region). From this result, it is quantitatively shown that the weakly bound state is composite dominant in most of the parameter region, and the realization of the low-energy universality is demonstrated with a simple model. At the same time, even for the weakly bound state, we show that the elementary dominant state with $X < 0.5$ can be realized with a fine tuning of ν_0 to 12 % of the parameter region.

3 Application to T_{cc} and $X(3872)$

Most of the exotic hadrons have couplings to other channels in addition to the threshold channel. In this case, the universal nature of the near-threshold states shown in the previous section can be modified. Therefore, in this section, we calculate the compositeness of the actual exotic hadrons T_{cc} and $X(3872)$ with the effects of the decay and coupled channels, and discuss the modification of the universality implications by channel couplings. We use the complex coupling constant to effectively take into account the decay contribution, and introduce the coupling to the isospin partner channel above the threshold [12]. In this case, \tilde{X}_1 stands for the compositeness of the threshold channel, and \tilde{X}_2 for that of the coupled channel [12, 14]. For the calculation, we use the central value of the pole position in Ref. [4] for the eigenenergy of T_{cc} ($E = -0.36 - 0.024i$ MeV), and the value in Particle Data Group [15] for the eigenenergy of $X(3872)$ ($E = -0.04 - 0.595i$ MeV). We employ the cutoff $\Lambda = m_\pi = 140$ MeV by assuming the π exchange as the long-range interaction between D mesons.

In Fig. 2, we show the ν_0 dependence of the compositeness of T_{cc} and $X(3872)$. The dotted lines stand for the compositeness of the threshold channel \tilde{X}_1 , solid lines for the sum of the compositeness of threshold and coupled channels $\tilde{X}_1 + \tilde{X}_2$. To see the decay contribution, $\tilde{X}_1 + \tilde{X}_2$ in the bound state limit $\Gamma \rightarrow 0$ is plotted by the dashed lines (Γ is the decay width). In the left panel, we see that there is almost no difference between the solid and dashed lines. Therefore, the compositeness of T_{cc} is not affected very much by the decay contribution. This reflects the much smaller decay width of T_{cc} ($\Gamma = 0.048$ MeV) than the binding energy ($B = 0.36$ MeV). In contrast, for $X(3872)$, the difference between the solid and dashed lines is sizable. This is because of the larger decay width of $X(3872)$ ($\Gamma = 1.19$ MeV) than the binding energy ($B = 0.04$ MeV). By comparing \tilde{X}_2 of T_{cc} with that of $X(3872)$, the effect of the coupled channel contributes more to the compositeness of T_{cc} . The reason is that the threshold energy difference of T_{cc} (1.41 MeV) is smaller than that of $X(3872)$ (8.23 MeV). In both cases, we quantitatively show that the threshold-channel compositeness \tilde{X}_1 decreases

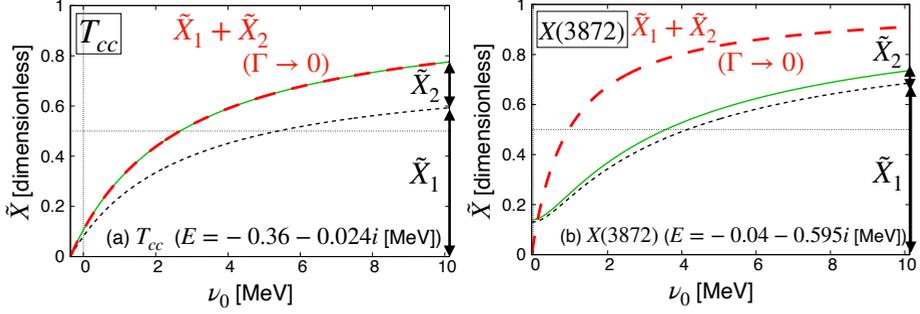


Figure 2. The compositeness of T_{cc} (left panel) and $X(3872)$ (right panel) as a function of the model parameter ν_0 . The dotted lines correspond to the compositeness of the threshold channel \tilde{X}_1 , the solid lines to the sum of the compositeness of threshold and coupled channels $\tilde{X}_1 + \tilde{X}_2$, and the dashed lines to the sum of the compositeness in the bound state limit $\Gamma \rightarrow 0$.

by introducing the decay and coupled channel effects. Furthermore, we find that \tilde{X}_1 largely decreases with the larger decay width or the smaller threshold energy difference.

4 Summary

In this study, we have studied the nature of the weakly-bound states with the compositeness. At first, in the single-channel scattering model, we have quantitatively demonstrated that the elementary dominant state is always realized even for the weakly bound states but with a significant fine tuning. Then, the decay effect and coupled channel are introduced to calculate the compositeness of T_{cc} and $X(3872)$. We have shown the importance of the coupled channel (decay effect) for T_{cc} [$X(3872)$] with the small threshold energy difference (large decay width) to consider their compositeness.

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