Hadron spectroscopy on the Lattice

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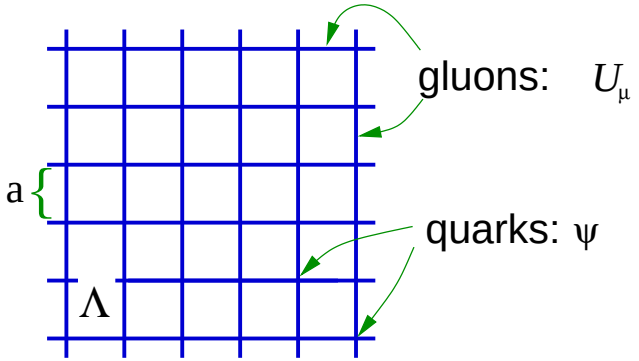
Outline

My method of choice: Lattice QCD

- Lattice QCD: Regularization of QCD by a 4-d Euclidean space-time lattice. Provides a calculational method.



Euclidean correlator of two Hilbert-space operators $\hat{O}_1$ and $\hat{O}_2$.

$$\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle = \sum_n e^{-t\Delta E_n} \langle 0|\hat{O}_2|n\rangle\langle n|\hat{O}_1|0\rangle$$
$$= \frac{1}{Z}\int \mathcal{D}[\psi,\bar{\psi},U] e^{-S_E} O_2[\psi,\bar{\psi},U] O_1[\psi,\bar{\psi},U]$$

- Path integral over the Euclidean action $S_{E,QCD}[\psi,\bar{\psi},U]$; (a sum over quantum fluctuations)
- Can be evaluated with *Markov Chain Monte Carlo* (using methods well established in statistical physics)

Lattice QCD and quark-model puzzles

- Various kind of exotic/unconventional states (examples)
  - light scalar resonances (σ and κ)
  - $D_{s0}^*(2317)$, $D_{s1}(2460)$ and b-quark cousins
  - XYZ states
  - Roper resonance; $\Lambda(1405)$
  - Pentaquark states
- Various possible structures: regular mesons/baryons; molecules; tetraquarks/pentaquarks; hybrid hadrons; glueballs

- Simple Lattice QCD calculations with $\bar{q}q$ and qqq interpolating fields struggle to make contact to experiment
  $\rightarrow$ Interpolating fields for multi-hadron states needed
- Lattice QCD has moved from naive finite-volume energies using $\bar{q}q$ and qqq structures to the determination of hadronic scattering amplitudes

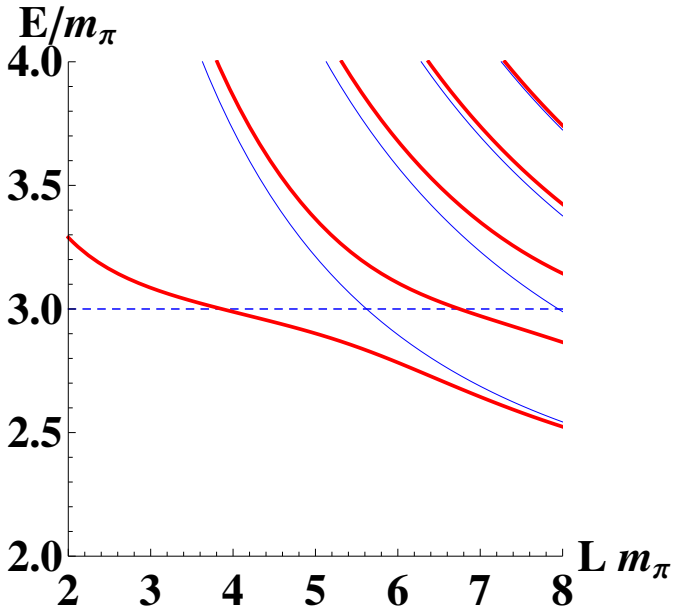
Progress from an old idea: Lüscher's finite-volume method

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Basic observation: Finite-volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

$$E = E(p_1) + E(p_2) + \Delta_E$$

- Energy shifts encode scattering amplitude(s)
- Original method: Elastic scattering in the rest-frame in multiple spatial volumes L^3
- Coupled 2-hadron channels well understood
- $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ transitions well understood (example $\pi\pi \to \pi\gamma^*$)
- Significant progress for 3-particle scattering



Lattice QCD challenges

- Hierarchy of difficulties
  - Meson systems are simpler than baryons
    (exponentially degrading signal to noise)
  - Cost of correlation functions much larger for systems with baryons
  - Complicated scattering amplitudes need many data points
    (volumes, frames)
    1 two-hadron channel; coupled two-hadron channels; three-hadron scattering
- Hierarchy of projects:
  - Proof of principle (often single ensemble)
  - Explore quark mass dependence
  - Full spectroscopy calculation including continuum limit
  - Structure observables (transitions, form factors, . . .)
- Hierarchy of difficulties not the same as in experiment

Systematic calculations and gauge field ensembles

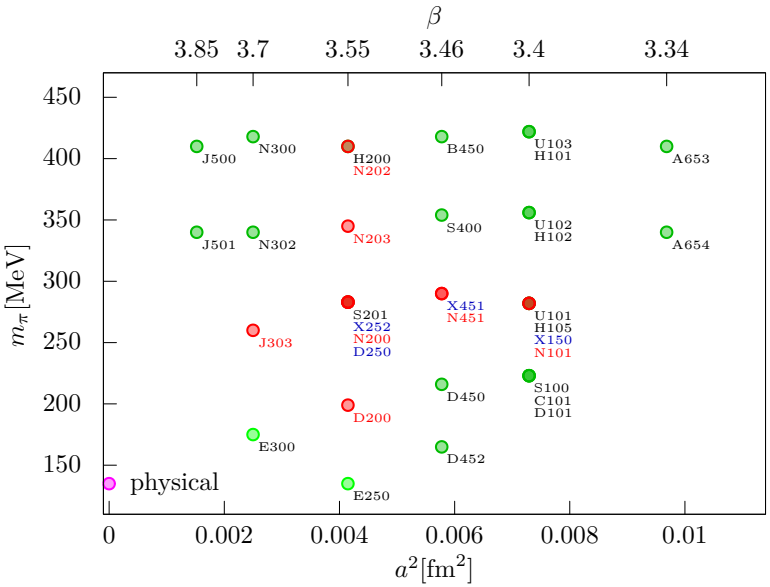
Important lattice systematics from

- Taking the *continuum limit*: $a(g, m) \to 0$
- Taking the *infinite volume limit*: $L \to \infty$
- Calculation at (or extrapolation to) physical quark masses

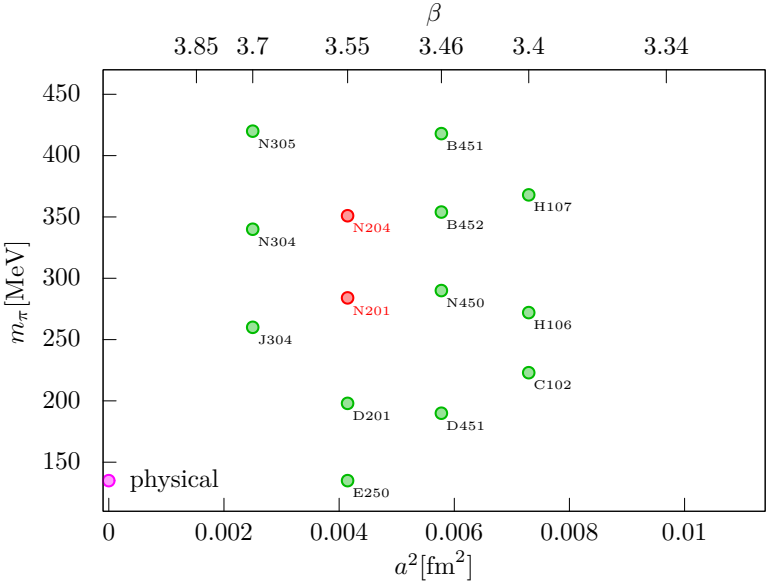
Example: CLS gauge-field library

`Bruno et al. JHEP 1502 043 (2015); Bali et al. PRD 94 074501 (2016)`

$Tr(M) = \text{const.}$



$m_s = \text{const.}$



`plot style by Jakob Simeth, RQCD`

Systematic calculations and gauge field ensembles

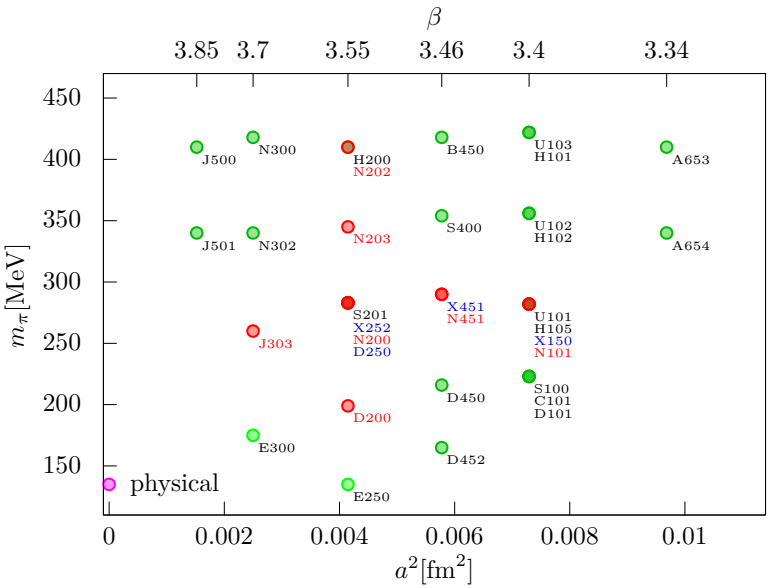
Important lattice systematics from

- Taking the *continuum limit*: $a(g, m) \to 0$
- Want to exploit (power law) finite volume effects (keeping exponential effects small)
- Calculation at (or extrapolation to) physical quark masses

Example: CLS gauge-field library

`Bruno et al. JHEP 1502 043 (2015); Bali et al. PRD 94 074501 (2016)`

$Tr(M) = \text{const.}$



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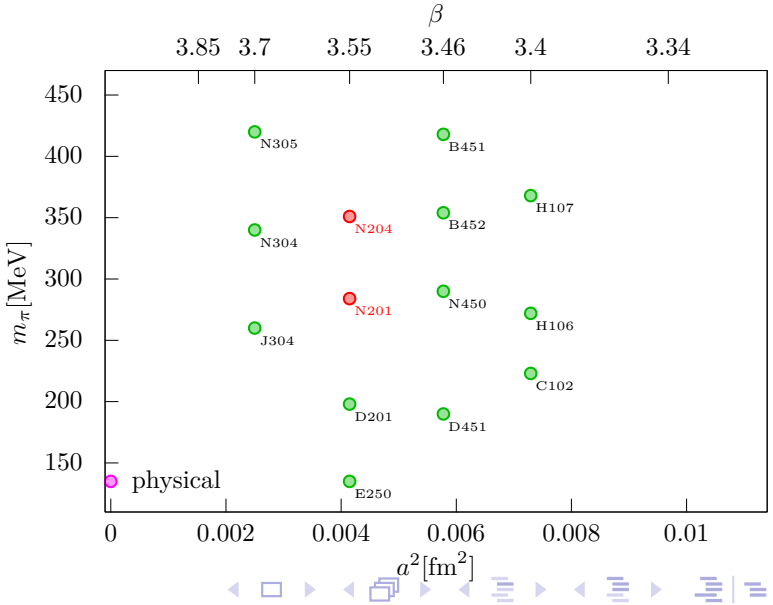
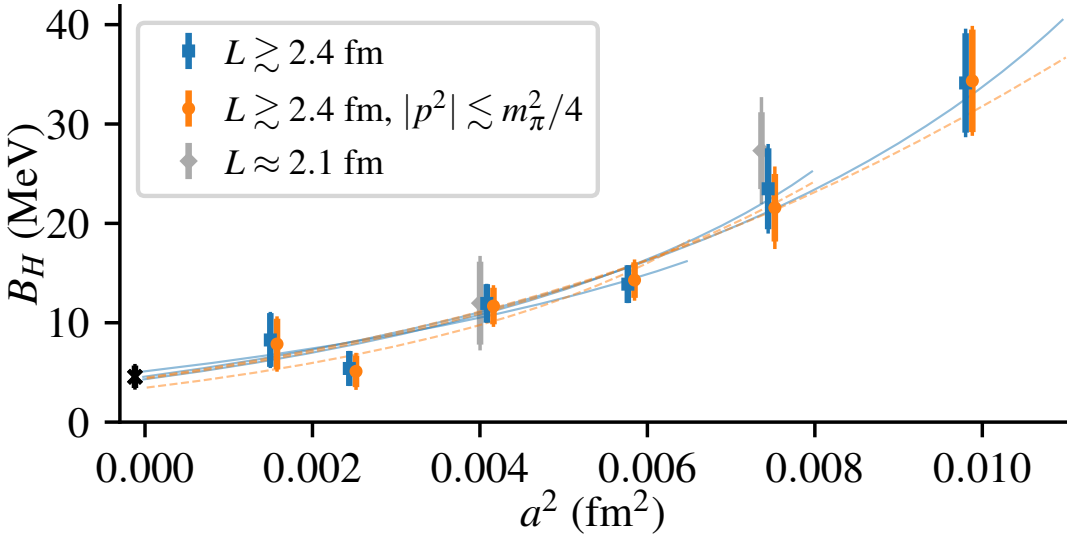


Cautionary tale: The H-Dibaryon and discretization effects

Green, Hanlon, Junnarkar, Wittig, PRL 127 242003 (2021)



- First study of baryon-baryon scattering in the continuum limit
- Strategy: Global fits to the energy levels with parameterizations that account for discretization effects
- Binding energy at $SU(3)$ point with $m_\pi = 420$ MeV

$$B_H^{SU(3)_f} = 4.56 \pm 1.13 \pm 0.63 \text{ MeV}$$

- Very large discretization effects in the binding energy!

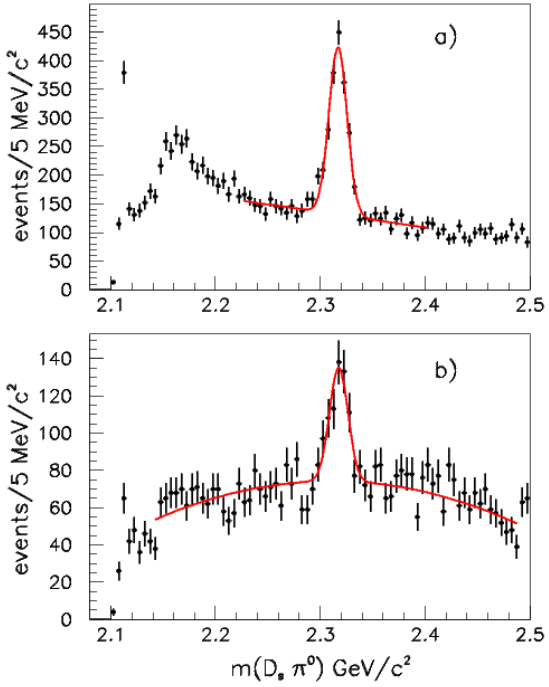
Exotic D_s and B_s candidates?

Established s and p-wave hadrons:

D_s ($J^P = 0^-$) and D_s^* (1^-)
$D_{s0}^*(2317)$ (0^+), $D_{s1}(2460)$ (1^+),
$D_{s1}(2536)$ (1^+), $D_{s2}^*(2573)$ (2^+)

B_s ($J^P = 0^-$) and B_s^* (1^-)
not yet seen
$B_{s1}(5830)$ (1^+), $B_{s2}^*(5840)$ (2^+)

$D_{s0}^*(2317)$:
PRL 90 242001 (2003)



- Corresponding $D_0^*(2400)$ and $D_1(2430)$ are broad resonances
- Perceived peculiarity: $M_{c\bar{s}} \approx M_{c\bar{d}}$ (an old dispute; likely not the case)
- Additional exotic states are expected (in the sextet representation)

See for example Kolomeitsev, Lutz, PLB 582, 39 (2004)

Positive-parity heavy-light mesons: Some older calculations

- $D_{s0}^*(2317)$ and $D_{s1}(2460)$ using finite-volume methods

  DM *et al.* PRL 111 222001 (2013)
  Lang, DM *et al.* PRD 90 034510 (2014)

  - Combined basis of quark-antiquark and $D^{(*)}K$ interpolating fields
  - Spectrum qualitatively agrees with experiment (unlike earlier studies)
  - Very few energy levels; only 2 pion masses; single lattice spacing;
- $D_{s0}^*(2317)$ and $D_{s1}(2460)$ in multiple volumes

  Bali, Collins, Cox, Schäfer, PRD 96 074501 (2017)

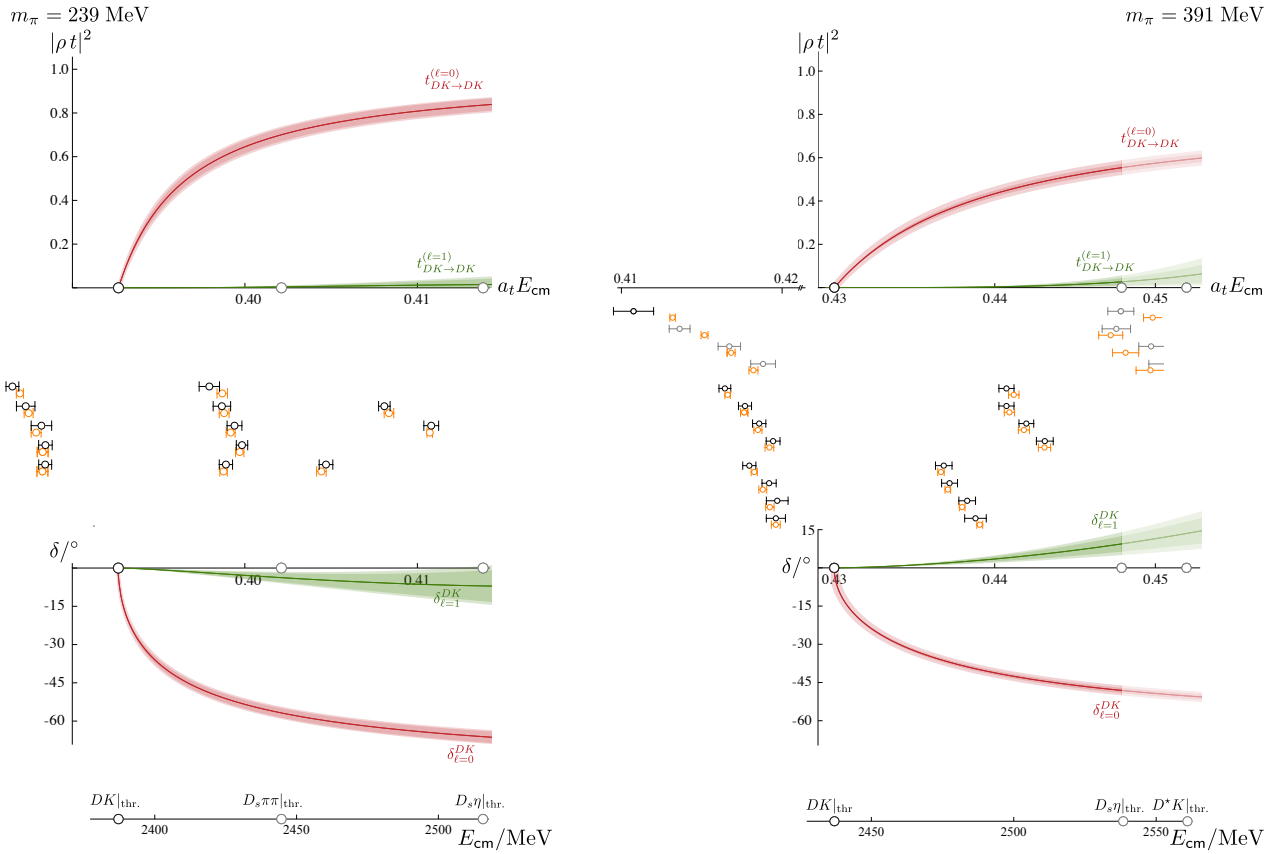
  - Allows to test effective-range approximation (more levels)
  - Checks for neglected finite-volume effects
- P-wave B_{s0}^* and B_s1 states

  Lang, DM, Prelovsek, Woloshyn PLB 750 17 (2015)

  - Prediction for physical states from Lattice QCD
  - Systematic uncertainties somewhat crudely estimated

DK and $D\bar{K}$ scattering and the $D_{s0}^*(2317)$

Hadron spectrum collaboration, Cheung *et al.* JHEP 02 100 (2021)



- Use of moving frames results in large number of energy levels; allows for exploring various amplitude parameterizations

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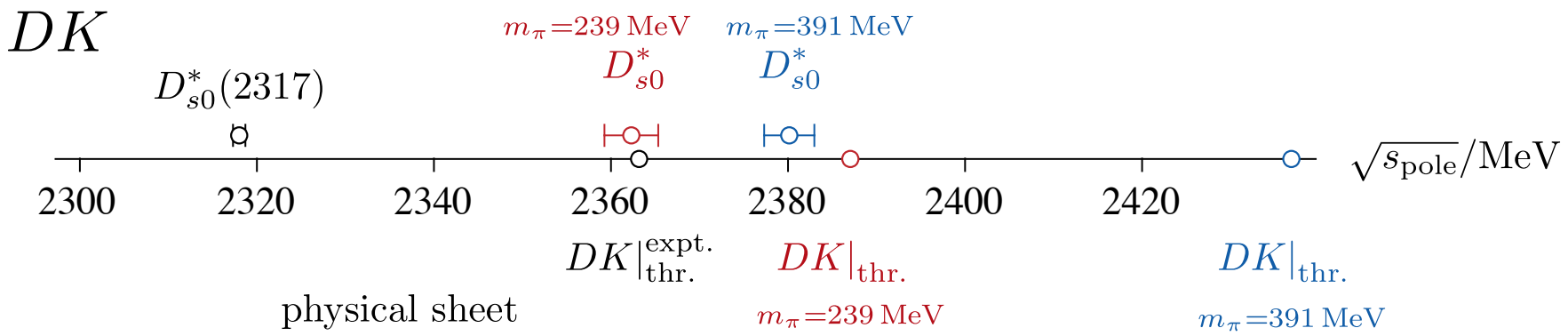

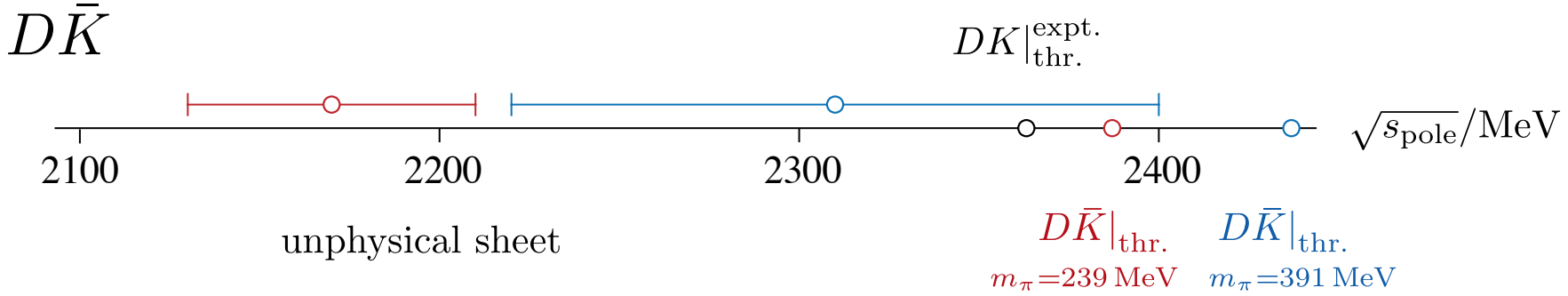

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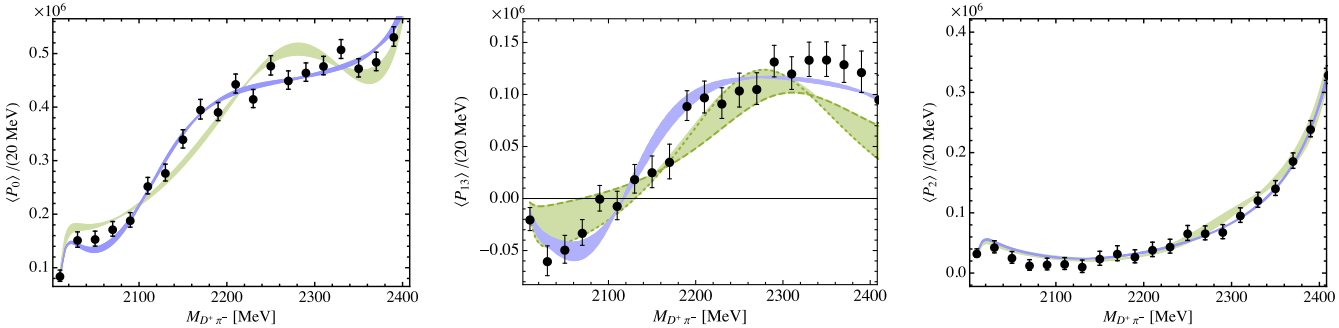
The lightest $J^P = 0^+$ mesons

$\boxed{D_0^*(2300)}$ $I(J^P) = \frac{1}{2}(0^+)$ M.-L. Du *et al.*, PRL 126 192001 (2021)

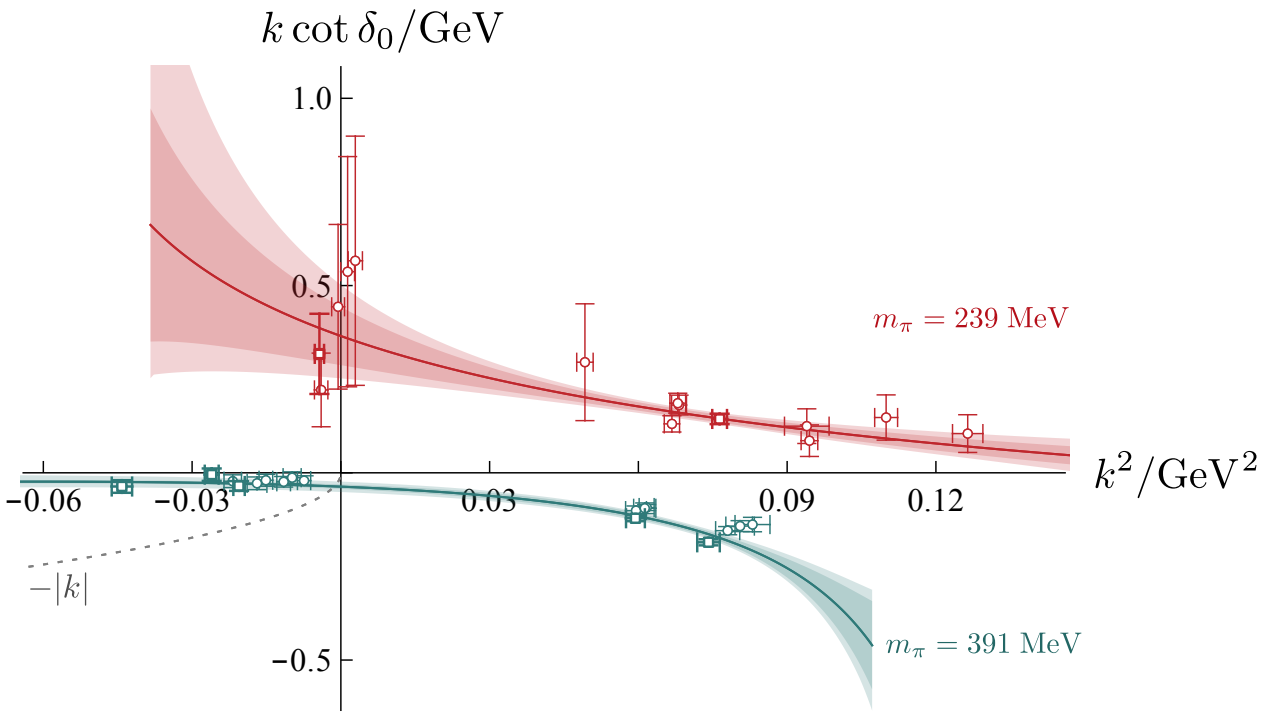
- Unitarized ChiPT leads to a much lower mass than indicated by the PDG
- Authors compare data from LHCb to PDG (Breit Wigner) and Unitarized ChiPT scenarios



- Recent Lattice QCD results from HSC also obtain a much lighter state
  HSC L. Gayer *et al.*, JHEP 07 (2021) 123

Isospin $\frac{1}{2}$ $D\pi$ scattering amplitude at $m_\pi = 239$ MeV

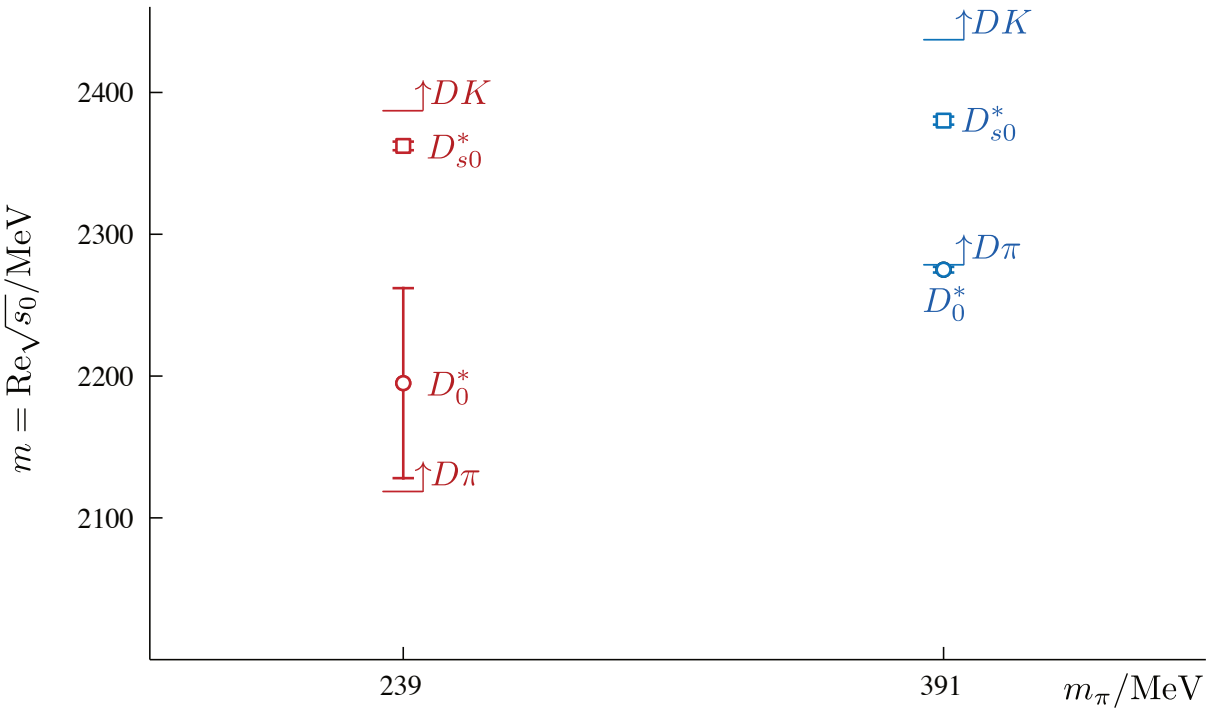
Hadron Spectrum collaboration, Gayer *et al.*, JHEP 07 123 (2021)



- Extracts pole position from various simple parameterizations
- Perceived peculiarity: $M_{c\bar{s}} \approx M_{c\bar{d}}$ not present!

Isospin $\frac{1}{2}$ $D\pi$ scattering amplitude at $m_\pi = 239$ MeV

Hadron Spectrum collaboration, Gayer *et al.*, JHEP 07 123 (2021)

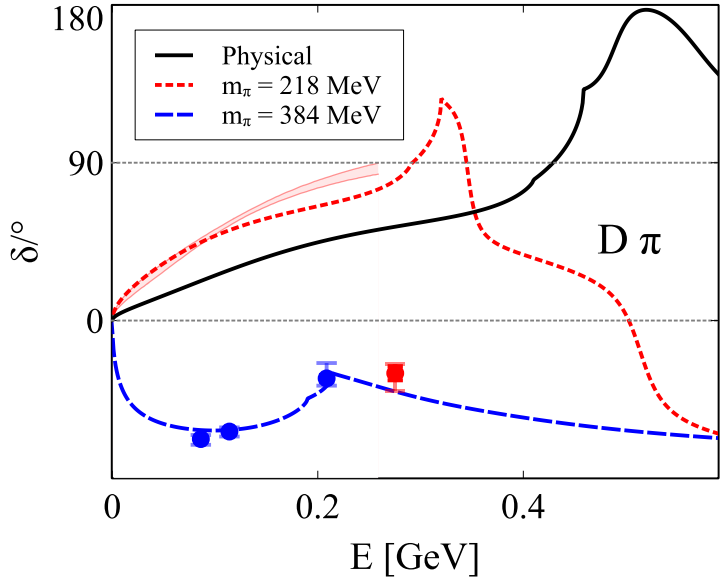


- Extracts pole position from various simple parameterizations
- Perceived peculiarity: $M_{c\bar{s}} \approx M_{c\bar{d}}$ not present!

Physical predictions from EFT fits to lattice data

$D_0^*(2300)$ $I(J^P) = \frac{1}{2}(0^+)$

Guo, Heo, Lutz, PRD 98 014510 (2018)
also PRD 106 114038 (2022)



- Low energy constants from fits to heavy-light ground-state masses and elastic phase-shift from Lattice QCD
- Chiral EFT bridges the gap between lattice data at unphysical pion masses and physical (coupled-channel) system
- Future: Chiral EFT/Lattice/results from femtoscopy at physical m_π

B_s: Chiral – infinite volume extrapolation

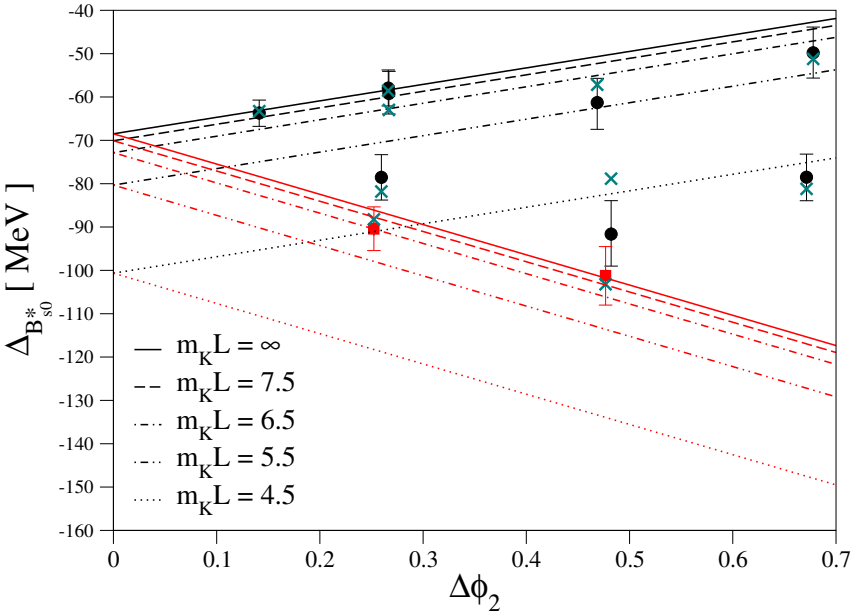
R.J. Hudspith, DM, PRD 107, 114510 (2023)

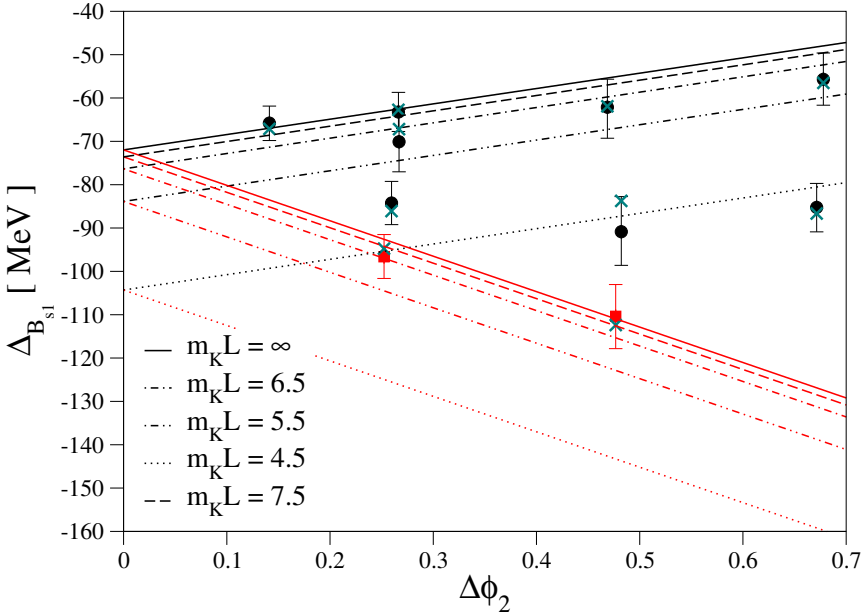
- We explore the previously predicted $J^P = 0^+$ and 1^+ bound states
- Mainly the CLS $\mathrm{Tr}M = \mathrm{const}$ trajectory and $2\,m_S = \mathrm{const}$ ensembles

Combined extrapolation:

$$\Delta_{B^*_{s0}/B_{s1}}(\Delta\phi_2, m_K L, a) = \Delta_{B^*_{s0}/B_{s1}}(0, \infty, a)\left(1 + A\Delta\phi_2 + Be^{-m_K L}\right)$$

$$\Delta\phi_2 = \phi_2^{\mathrm{Lat}} - \phi_2^{\mathrm{Phys}} \quad ; \quad \phi_2 = 8t_0 m_\pi^2$$





Systematic uncertainties and final result

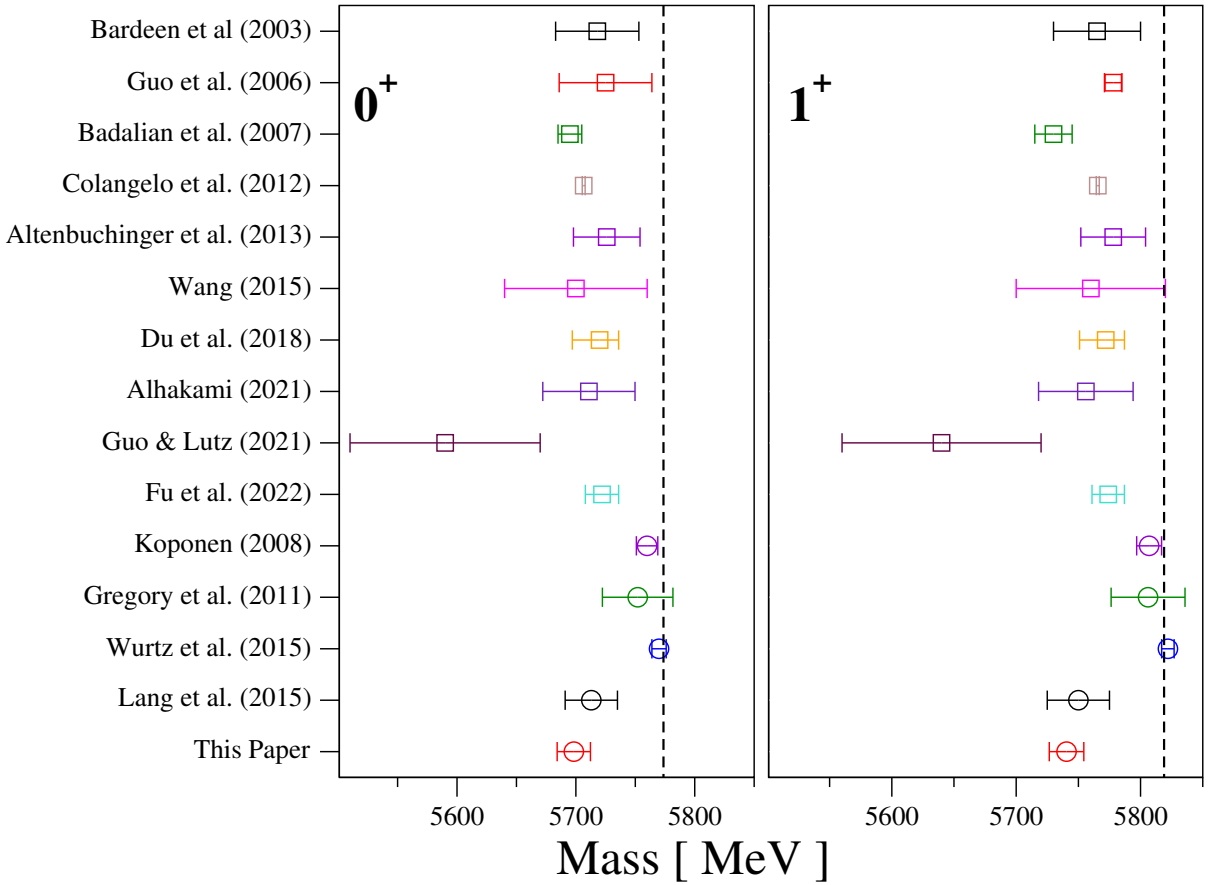
R.J. Hudspith, DM, PRD 107, 114510 (2023)

Resulting binding energies:

$$\Delta_{B^*_{s0}}(0,\infty,0) = -75.4(3.0)_{\text{Stat.}}(13.7)_{\text{a}}\ [\text{MeV}],$$
$$\Delta_{B_{s1}}(0,\infty,0) = -78.7(3.7)_{\text{Stat.}}(13.4)_{\text{a}}\ [\text{MeV}].$$

- Small uncertainty from statistics + combined extrapolation
- Largest systematics from usage of NRQCD/discretization effects
- Central value shifted by applying half the mass difference between two different lattice-spacings
- All other explored uncertainties (finite volume shapes, modified quark-mass dependence, etc.) small

Comparison to the literature



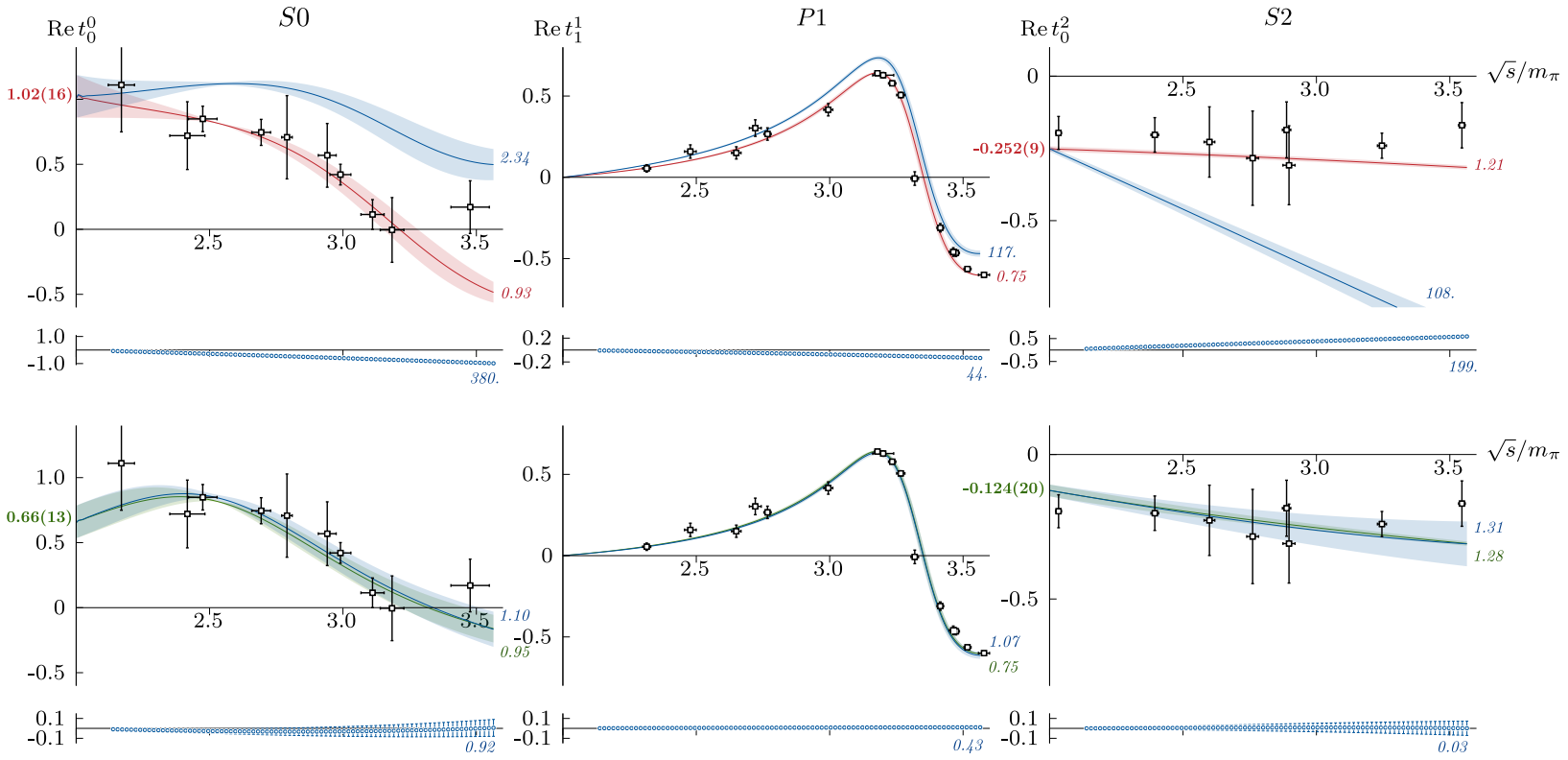
- Results agree well with models based on unitarized χPT
- Improved uncertainty estimate over older Lattice calculations

Dispersive determination of the σ-resonance

Rodas, Dudek, Edwards, arXiv:2304.03762

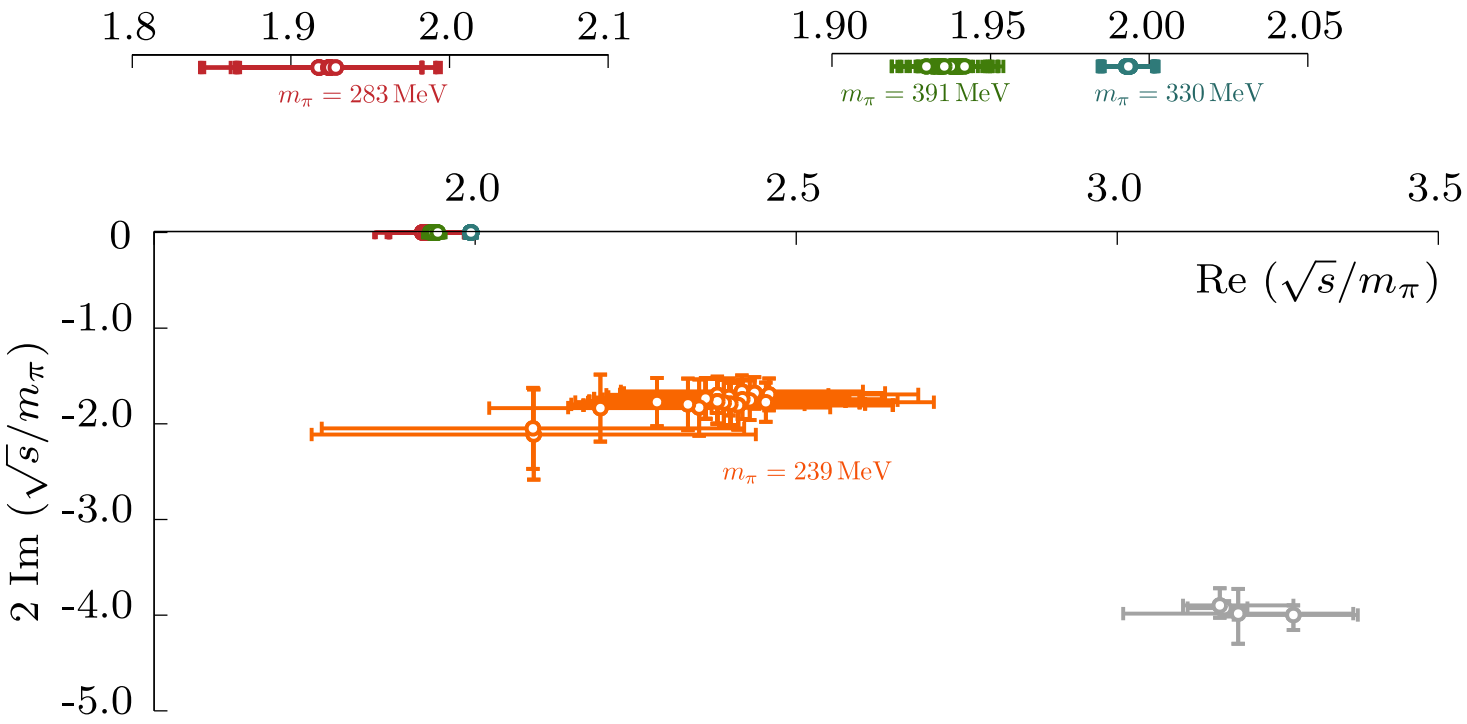
- Based on $\pi\pi$ partial-wave scattering amplitudes with Isospins $0, 1, 2$
- Four different pion masses from 391 MeV down to 239 MeV
- Based on a various K-Matrix fit models for each of the scattering amplitudes (ensures unitarity)
- Amplitudes are run through a dispersive framework
- Demanding consistency of input fits and dispersion relation output effectively selects amplitudes respecting analyticity/crossing symmetry
- Yields significantly reduced spread in pole position (with slightly increases statistical uncertainty)

Illustration of the method (example)



- Red combination of input amplitudes (top) yields incompatible output (blue)
- Green combination of input amplitudes (bottom) yields acceptable output

Resulting pole positions



- Orange results at $m_\pi = 239$ MeV show substantially reduced scatter
- Grey points show the results of dispersive extractions from experimental data

An old puzzle: $\Lambda(1405)$, $J^P = \frac{1}{2}^-$

- PDG (4 star resonance)

$$M_\Lambda = 1405^{+1.3}_{-1.0} MeV \qquad \Gamma_\Lambda = 50.5 \pm 2.0$$

  (Some) quark models struggled to accommodate this state.
- However
  - Unitarized χPT + Model input yields 2 poles with $\Re \approx 1400$ MeV
    $\rightarrow$ Now new PDG state $\Lambda(1380)$
  - CLAS observes different line shapes for $\Sigma^-\pi^+$, $\Sigma^+\pi^-$ and $\Sigma^0\pi^0$
    Interference between $I = 0$ and $I = 1$ amplitudes is the likely reason
  - Even the $\Sigma^0\pi^0$ is badly described by a single Breit-Wigner
  - CLAS data consistent with popular 2-pole picture
  - No satisfactory lattice results (although claims exist)
- Relevant channels: $\Sigma\pi$, $N\bar{K}$ (and maybe $\Lambda\eta$); simulation in isospin limit
- Goal: Explore coupled-channel problem and extract scattering amplitudes from the low-lying energy spectrum

$\Lambda(1405)$ – Experimental developments

- Angular analysis of the process $\gamma + p \to K^+ + \Sigma + \pi$ by CLAS strongly favors the assignment of quantum numbers $J^P = \frac{1}{2}^-$

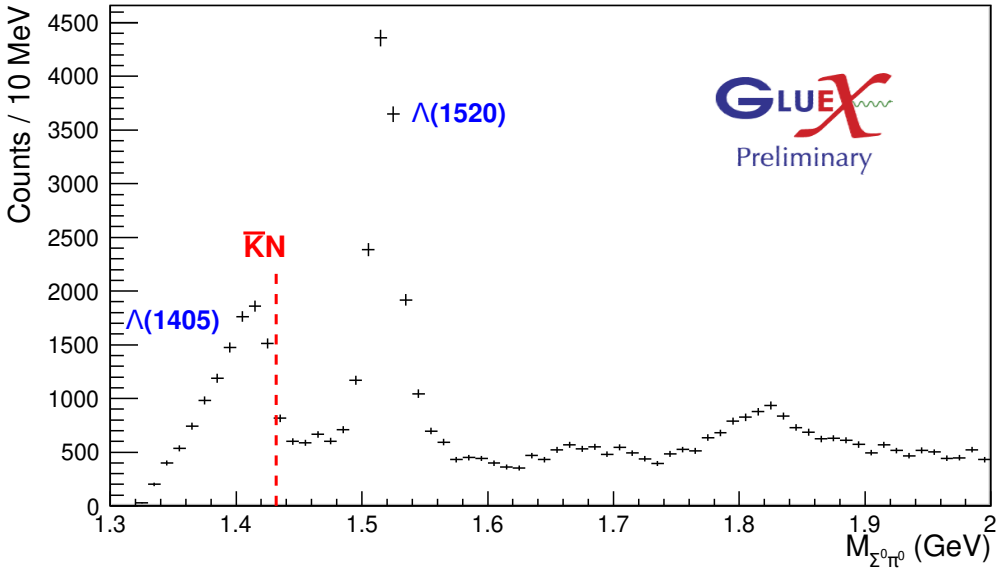
  Moriya *et al.*, PRC 87 035206 (2013)

- $K^- p$ scattering length determined by the SIDDHARTHA collaboration

  Bazzi *et al.*,PLB 704 (2011) 113

- A glimpse of the future: Preliminary analysis at GlueX

  Wickramaarachchi *et al.*, arXiv:2209.06230



Gauge-field ensemble and rotational symmetry breaking

Current data on CLS gauge-field ensemble

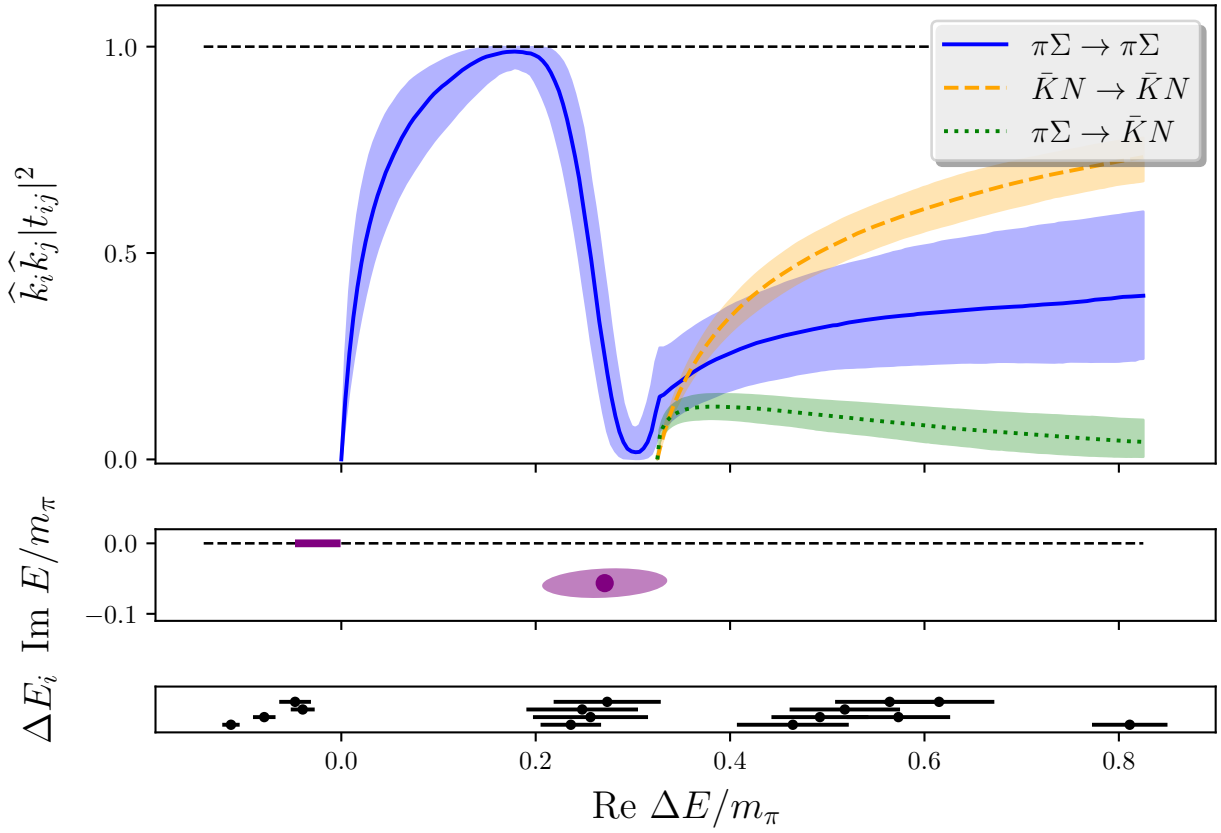
a [fm]	$T \times L^3$	m_π [MeV]	m_K [MeV]	$m_\pi L$	N_{cnfg}
0.0633(4)(6)	128×64^3	200	480	4.3	2000

Lattice irreducible representations for a given J^P

see Morningstar *et al.* arXiv:1303.6816

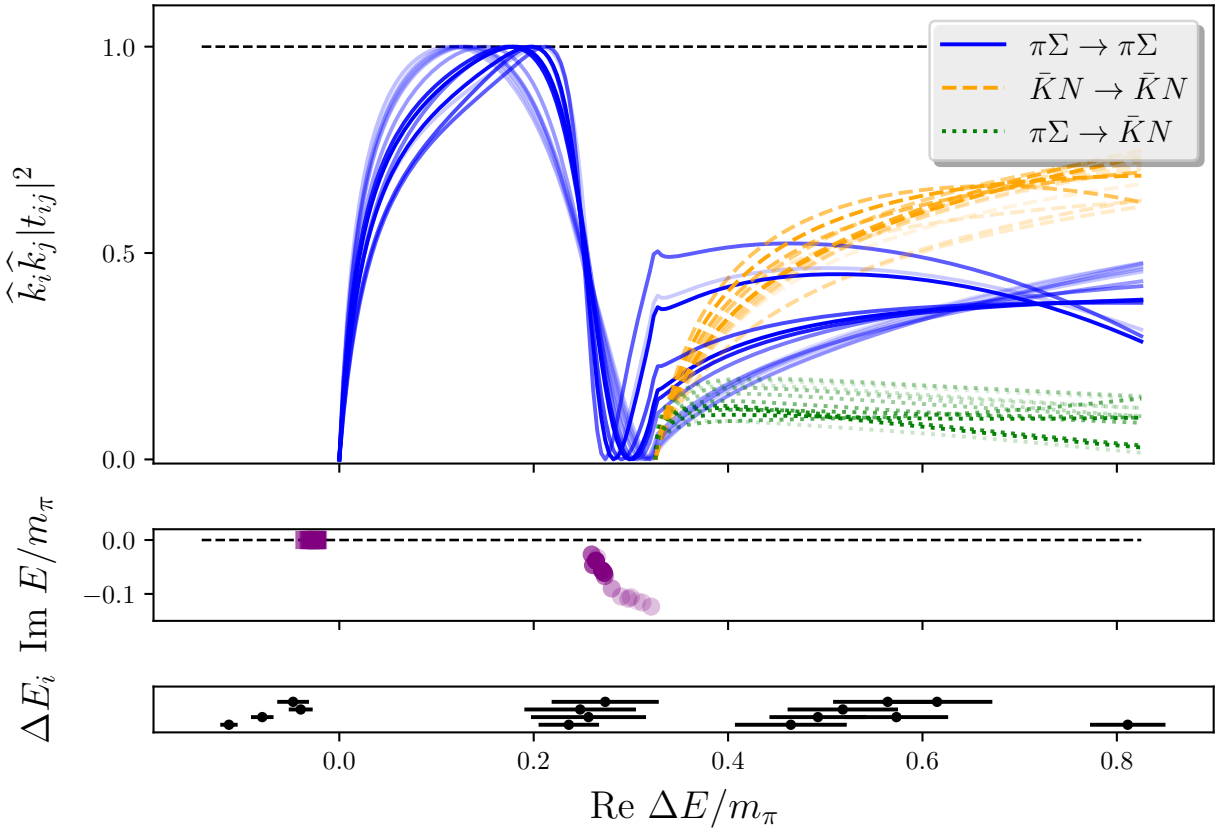
J^P	$[000]$	$[00n]$	$[0nn]$	$[nnn]$	
$\frac{1}{2}^+$	G_{1g}	G_1	G	G	Λ, $\Lambda(1600)$
$\frac{1}{2}^-$	G_{1u}	G_1	G	G	$\Lambda(1405)$, $\Lambda(1670)$
$\frac{3}{2}^+$	H_g	G_1, G_2	$2G$	F_1, F_2, G	$\Lambda(1690)$
$\frac{3}{2}^-$	H_u	G_1, G_2	$2G$	F_1, F_2, G	$\Lambda(1520)$, $\Lambda(1690)$

Preferred amplitude and resulting poles (preliminary)



- Amplitudes evaluated with Akaike Information Criterion: $AIC = \chi^2 - 2\text{dof}$
- Sub-threshold levels pose strong constraints on the amplitude
- Limited data and therefore limited possibility to vary parameterizations

Some Variations of the used amplitude



- Results from varying parameterization/ omitting highest data point
- Amplitudes agnostic to the number of poles lead all yield 2 poles
- We also explored simple constraints for higher partial waves (negligible effect in range used)

Pole positions and expectations from the literature

- Poles labeled as $(\pm, \pm)$
  depending on the signs of the imagianary part of $(k_{KN}, k_{\pi\Sigma})$
- Two poles are found on the $(-,+)$ sheet,
  the closest to physical scattering between the thresholds
- Our (preliminary) result for the poles is

$$\begin{aligned} \text{Pole II} \quad & 1395(9)_{\text{stat}}(2)_{\text{model}}(16)_{\text{a}}\ \text{MeV} \\ \text{Pole I} \quad & 1456(14)_{\text{stat}}(2)_{\text{model}}(16)_{\text{a}}\ \text{MeV} \\ & -i \times 11.7(4.3)_{\text{stat}}(4)_{\text{model}}(0.1)_{\text{a}}\ \text{MeV} \end{aligned}$$

- Examples from the PDG review

approach	pole 1 [MeV]	pole 2 [MeV]
Refs. [14, 15], NLO	$1424^{+7}_{-23} - i\ 26^{+3}_{-14}$	$1381^{+18}_{-6} - i\ 81^{+19}_{-8}$
Ref. [17], Fit II	$1421^{+3}_{-2} - i\ 19^{+8}_{-5}$	$1388^{+9}_{-9} - i\ 114^{+24}_{-25}$
Ref. [18], solution #2	$1434^{+2}_{-2} - i\ 10^{+2}_{-1}$	$1330^{+4}_{-5} - i\ 56^{+17}_{-11}$
Ref. [18], solution #4	$1429^{+8}_{-7} - i\ 12^{+2}_{-3}$	$1325^{+15}_{-15} - i\ 90^{+12}_{-18}$

Status and prospects for Lattice QCD spectroscopy

Summary:

- Lattice calculations of scattering amplitudes are starting to mature
- Simple coupled channel systems are already feasible (also for baryons)
- We are starting to address some of the quark-model puzzles
- Once spectroscopy gets settled, we can start addressing structure (transitions, form factors, etc.)

Powerful QCD tools:

- Map out the quark mass dependence of amplitudes
- Investigate properties of short-lived excitations
- Investigate states hard to produce/detect at current/future facilities

Examples presented

- Prediction of $J^P = 0^+$ B^*_{s0} and $J^p = 1^+$ B_{s1} QCD bound states
- Results for the quark-mass dependence of the $D^*_0(2300)$ pole
- Dispersive analysis of the σ resonance from Lattice QCD data
- The $\Lambda(1405)$ from coupled-channel $\pi\Sigma$–$\bar{K}N$-scattering

Thank you!

Credits

- Heavy-quark exotics ($ud\bar{b}\bar{b}$, $us\bar{b}\bar{b}$) and positive-parity heavy-light mesons
  R.J. Hudspith, DM, PRD 107, 114510 (2023)
  - TU Darmstadt/GSI: Jamie Hudspith, Daniel Mohler
- $\Lambda(1405)$ and meson-baryon scattering:
  - DESY Zeuthen $\rightarrow$ Bochum: John Bulava
  - BNL: Andrew Hanlon
  - Intel: Ben Hörz
  - North Carolina: Amy Nicholson, Joseph Moscoso
  - TU Darmstadt/GSI: Daniel Mohler, Barbara Cid Mora
  - CMU: Colin Morningstar, Sarah Skinner
  - MIT: Fernando Romero-López
  - LBNL: André Walker-Loud

Backup slides

$D_{s0}^*(2317)$: D-meson – Kaon s-wave scattering

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Charm-light hadrons

$D_{s0}^*(2317)^\pm$

$I(J^P) = 0(0^+)$
J, P need confirmation.

J^P is natural, low mass consistent with 0^+.

Mass $m = 2317.7 \pm 0.6$ MeV
$m_{D_{s0}^*(2317)^\pm} - m_{D_s^\pm} = 349.4 \pm 0.6$ MeV
Full width $\Gamma < 3.8$ MeV, CL = 95%

$$p \cot \delta_0(p) = \frac{2}{\sqrt{\pi} L} Z_{00}\left(1; \left(\frac{L}{2\pi} p\right)^2\right)$$
$$\approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$D^*_{s0}(2317)$: D-meson – Kaon s-wave scattering

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Charm-light hadrons

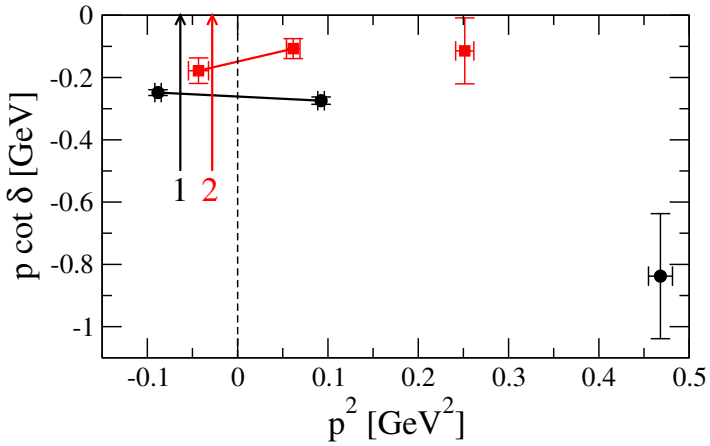
$D^*_{s0}(2317)^\pm$

$I(J^P) = 0(0^+)$
J, P need confirmation.
J^P is natural, low mass consistent with 0^+.
Mass $m = 2317.7 \pm 0.6$ MeV
$m_{D^*_{s0}(2317)^\pm} - m_{D_s^\pm} = 349.4 \pm 0.6$ MeV
Full width $\Gamma < 3.8$ MeV, CL = 95%

$$p \cot \delta_0(p) = \frac{2}{\sqrt{\pi} L} Z_{00}\left(1; \left(\frac{L}{2\pi} p\right)^2\right)$$
$$\approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

DM *et al.* PRL 111 222001 (2013)
Lang, DM *et al.* PRD 90 034510 (2014)

Results for ensembles (1) and (2)

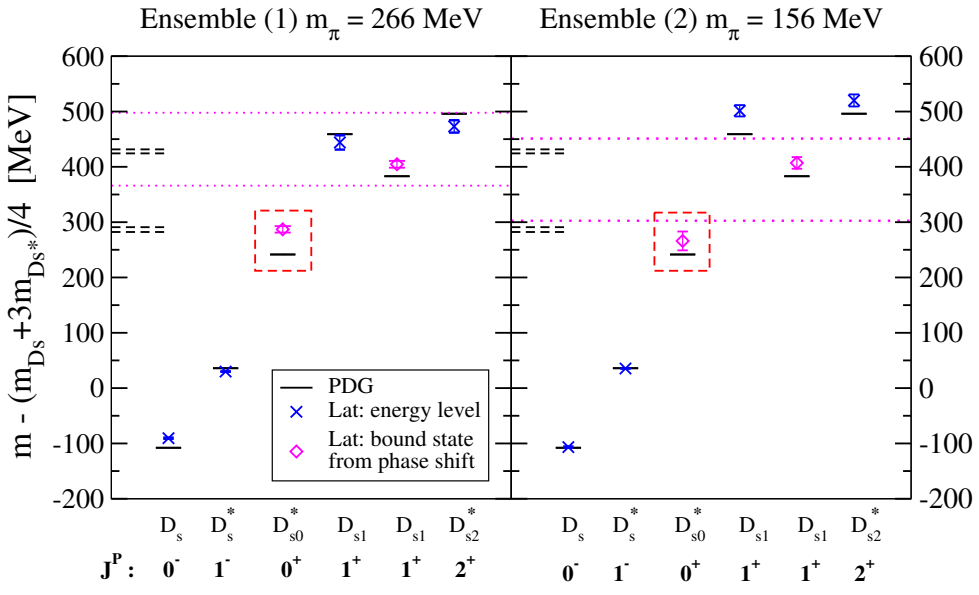


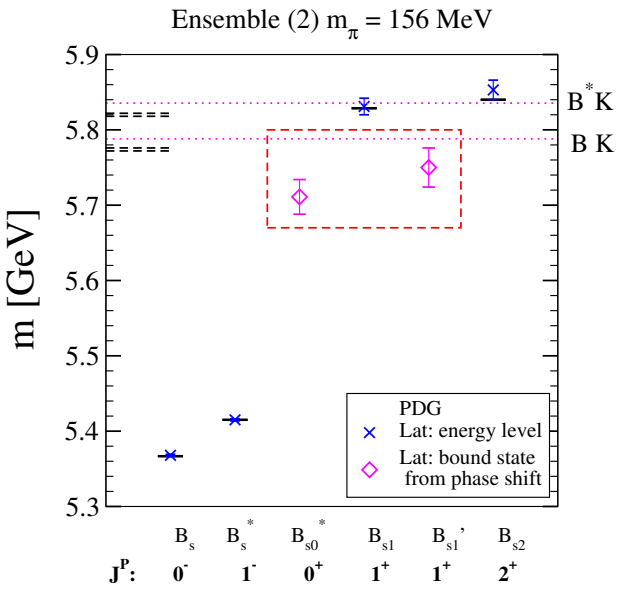
$$a_0 = -0.756 \pm 0.025\text{fm} \qquad (1)$$
$$r_0 = -0.056 \pm 0.031\text{fm}$$
$$a_0 = -1.33 \pm 0.20\text{fm} \qquad (2)$$
$$r_0 = 0.27 \pm 0.17\text{fm}$$

Positive-parity states in the D_s and B_s spectrum

DM *et al.* PRL 111 222001 (2013)
Lang, DM *et al.* PRD 90 034510 (2014)

Lang, DM, Prelovsek, Woloshyn PLB 750 17 (2015)



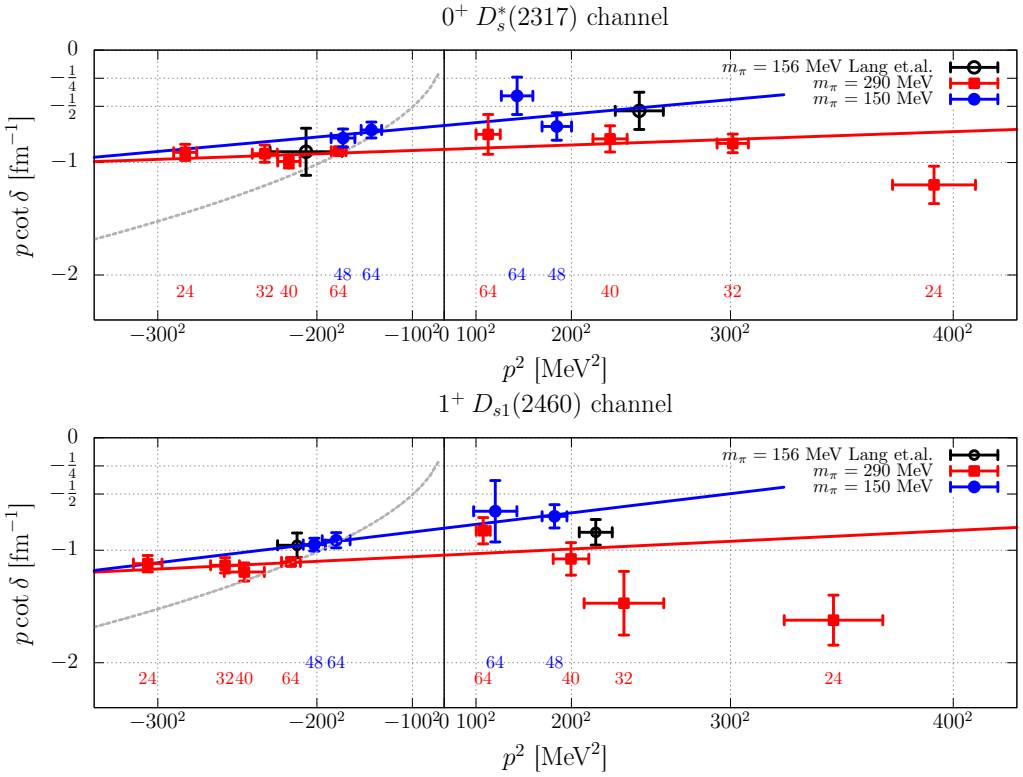


- Spectrum reliably extracted and agrees qualitatively with experiment
- Uncontrolled systematics sizable for the D_s states

- Full uncertainty estimate only for magenta B_s states
- Prediction of exotic states from Lattice QCD!

D_s results in multiple volumes from RQCD

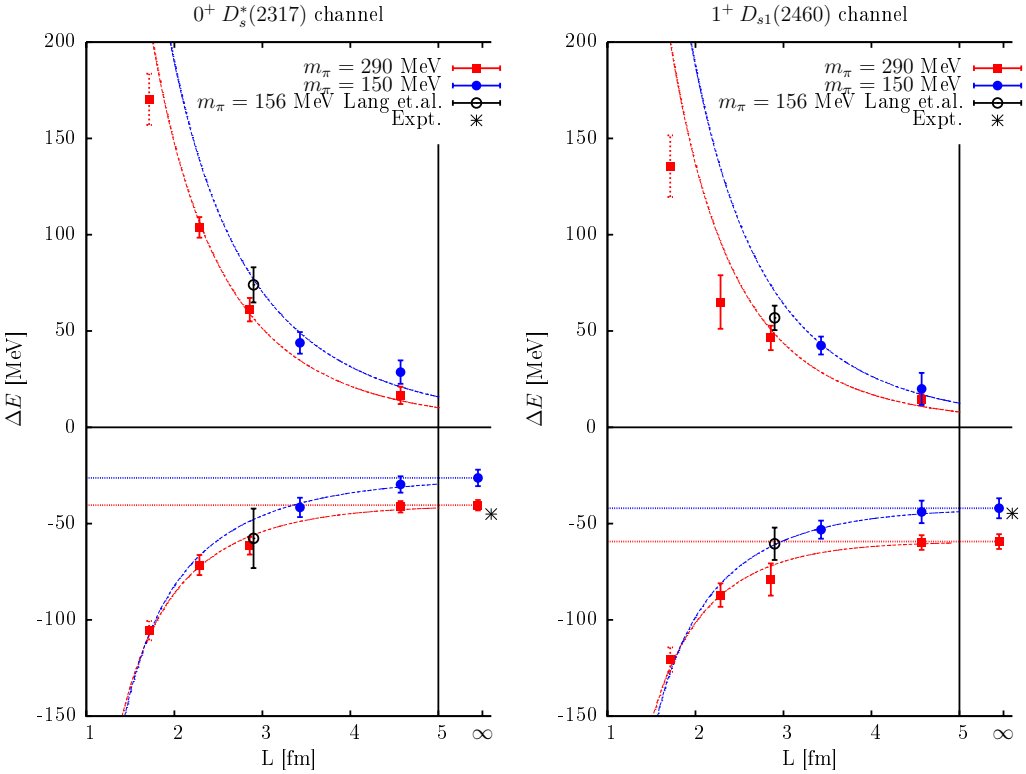
Bali, Collins, Cox, Schäfer, PRD 96 074501 (2017)



- Study with different volumes at pion masses of 150, 290 MeV
- Results confirm basic behavior seen in a single volume
- Discretization effects remain unexplored

D_s results in multiple volumes from RQCD

Bali, Collins, Cox, Schäfer, PRD 96 074501 (2017)



- Study with different volumes at pion masses of 150, 290 MeV
- Results confirm basic behavior seen in a single volume
- Discretization effects remain unexplored

CLS ensembles used for heavy-light mesons

R.J. Hudspith, DM, PRD 107, 114510 (2023)

Ensemble	Mass trajectory	$L^3 \times L_T$	$N_{\text{Conf}} \times N_{\text{Prop}}$
U103	$\text{Tr}[M] = C$	$24^3 \times 128$	1000×23
H101	$\text{Tr}[M] = C$	$32^3 \times 96$	500×12
U102	$\text{Tr}[M] = C$	$24^3 \times 128$	732×18
H102	$\text{Tr}[M] = C$	$32^3 \times 96$	500×16
U101	$\text{Tr}[M] = C$	$24^3 \times 128$	600×18
H105	$\text{Tr}[M] = C$	$32^3 \times 96$	500×16
N101	$\text{Tr}[M] = C$	$48^3 \times 128$	537×18
C101	$\text{Tr}[M] = C$	$48^3 \times 96$	400×16
H107	$\widetilde{m_s} = \widetilde{m_s}^{\text{Phys.}}$	$32^3 \times 96$	500×16
H106	$\widetilde{m_s} = \widetilde{m_s}^{\text{Phys.}}$	$32^3 \times 96$	500×16
H200	$\text{Tr}[M] = C$	$32^3 \times 96$	500×28

NRQCD action

Typical tadpole-improved NRQCD action (here we will use n=4)

Lepage *et al.*, PRD 46, 4052-4067 (1992)

$$H_0 = -\frac{1}{2aM_0}\Delta^2,$$

$$H_I = \left(-c_1\frac{1}{8(aM_0)^2} - c_6\frac{1}{16n(aM_0)^2}\right)\left(\Delta^2\right)^2 + c_2\frac{i}{8(aM_0)^2}(\tilde{\Delta}\cdot\tilde{E} - \tilde{E}\cdot\tilde{\Delta}) + c_5\frac{\Delta^4}{24(aM_0)}$$

$$H_D = -c_3\frac{1}{8(aM_0)^2}\sigma\cdot\left(\tilde{\Delta}\times\tilde{E} - \tilde{E}\times\tilde{\Delta}\right) - c_4\frac{1}{8(aM_0)}\sigma\cdot\tilde{B}$$

$$\delta H = H_I + H_D.$$

Propagators generated through symmetric evolution equation

$$G(x,t+1) = \left(1-\frac{\delta H}{2}\right)\left(1-\frac{H_0}{2n}\right)^n \tilde{U}_t(x,t_0)^\dagger \left(1-\frac{H_0}{2n}\right)^n \left(1-\frac{\delta H}{2}\right) G(x,t).$$

- We also tune a $\mathcal{O}(v^6)$ action with tree-level coefficients for the higher order terms

Neural net NRQCD tuning and setup

R.J. Hudspith, DM, PRD 106, 034508 (2022)
R.J. Hudspith, DM, PRD 107, 114510 (2023)

- Calculate runs with a random distribution for the action parameters
- Let the neural network make parameter predictions
- Due to additive mass we must only consider splittings $\rightarrow$ we subtract the η_B from all states
- Perform tuning at $SU(3)_f$-symmetric point
- Gauge-fixed wall sources
- Tuning precision is about 1%

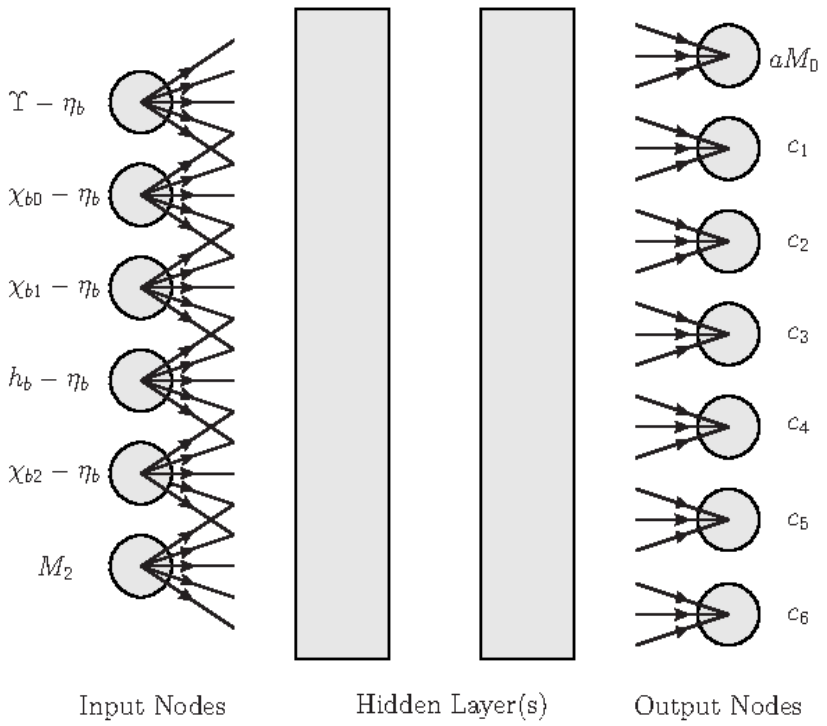


Figure: Schematic picture of our NRQCD setup

Input used for the tuning

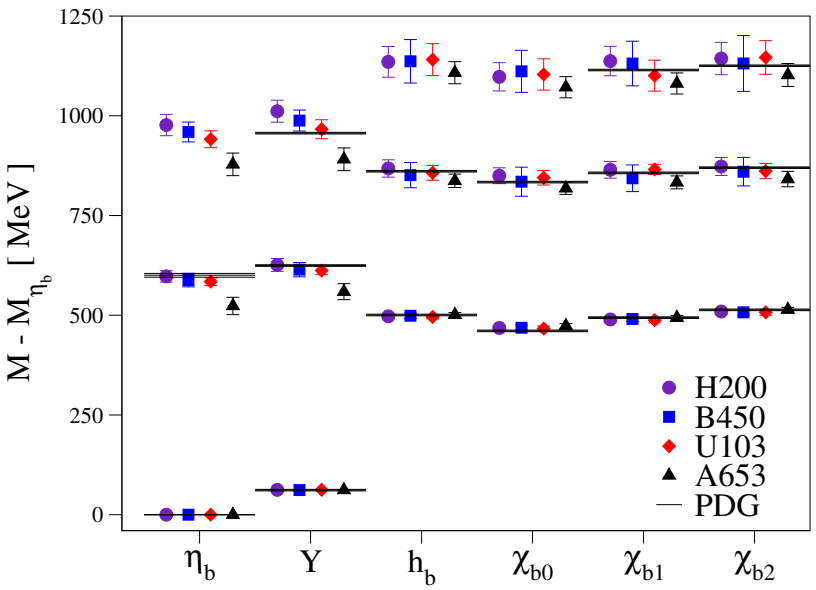
Consider only quark-line connected parts of simple meson operators

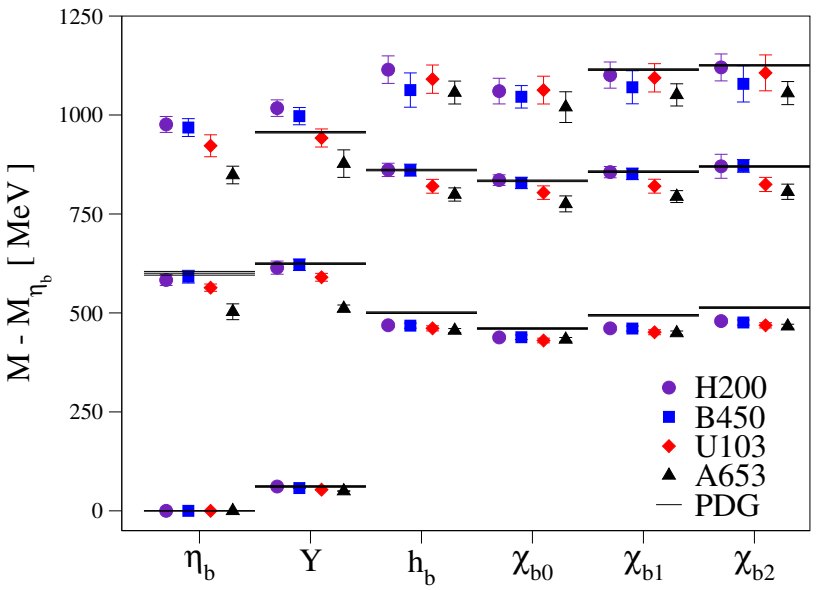
$$O(x) = (\bar{b}\Gamma(x)b)(x),$$

State	PDG mass [GeV]	$\Gamma(x)$
$\eta_b(1S)$	9.3987(20)	γ_5
$\Upsilon(1S)$	9.4603(3)	γ_i
$\chi_{b0}(1P)$	9.8594(5)	$\sigma \cdot \Delta$
$\chi_{b1}(1P)$	9.8928(4)	$\sigma_j\Delta_i - \sigma_i\Delta_j \ (i \neq j)$
$\chi_{b2}(1P)$	9.9122(4)	$\sigma_j\Delta_i + \sigma_i\Delta_j \ (i \neq j)$
$h_b(1P)$	9.8993(8)	Δ_i

Table: Table of lattice operators used and their continuum analogs.

NRQCD Neural Net Tuning: Stable s- and p-wave bottomonia



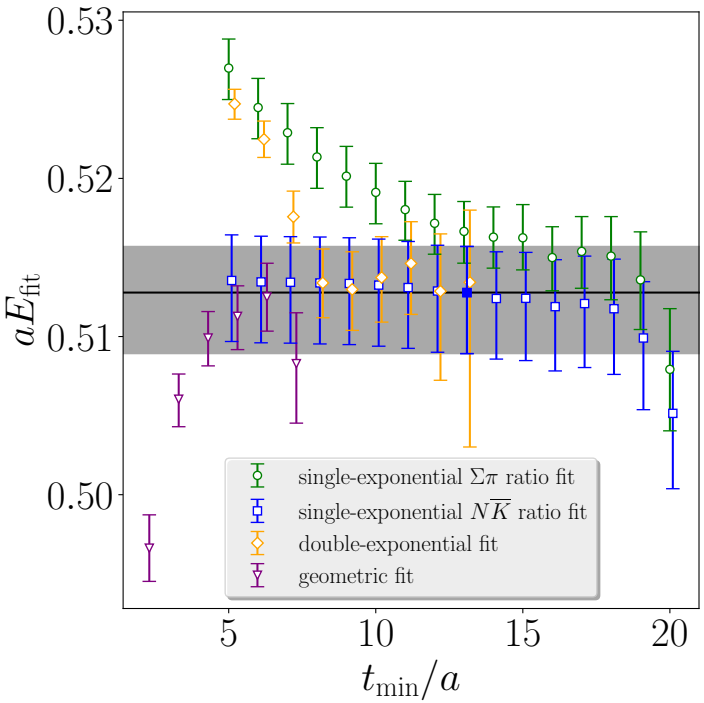


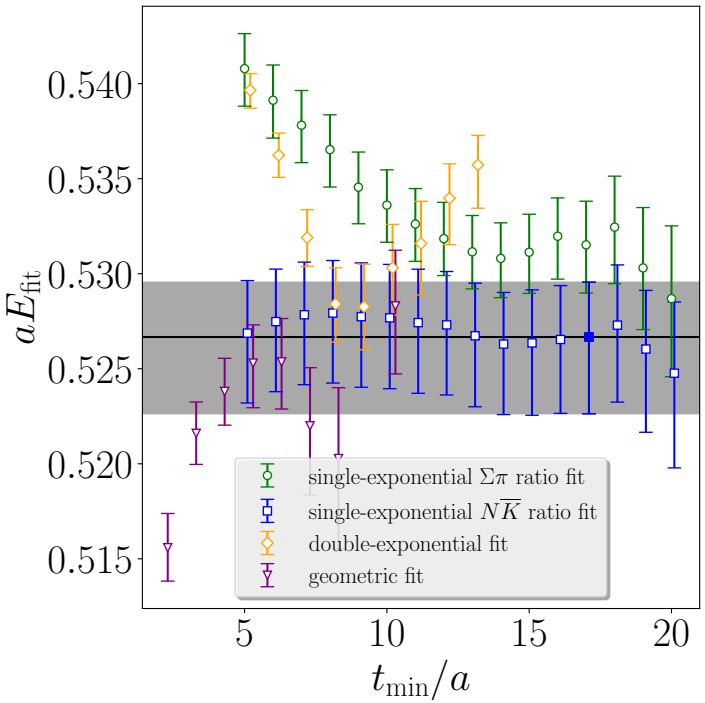
- Higher S- and P-wave states serve as a check whether our tuning leads to reasonable results
- Main results from the lattice spacing of U103; H200 used to estimate systematics

$\Lambda(1405)$: Specific setup on D200

- Combined basis of simple 3-quark structures and 2 hadron interpolators with the lowest few momentum combinations in each irrep
- Distillation setup:
  - $n_{ev} = 448$ eigenmodes of the Lattice Laplacian
  - Quark lines connecting source and sink:
    Noise dilution scheme with $(TF, SF, LI16)$ and 6 noises
  - Lines starting end ending on the same time slice:
    Noise dilution scheme with $(TI8, SF, LI16)$ and 2 noises
  - Four source time slices
  - Lattice Laplacian constructed on stout smeared links with $(\rho, n) = (0.1, 36)$

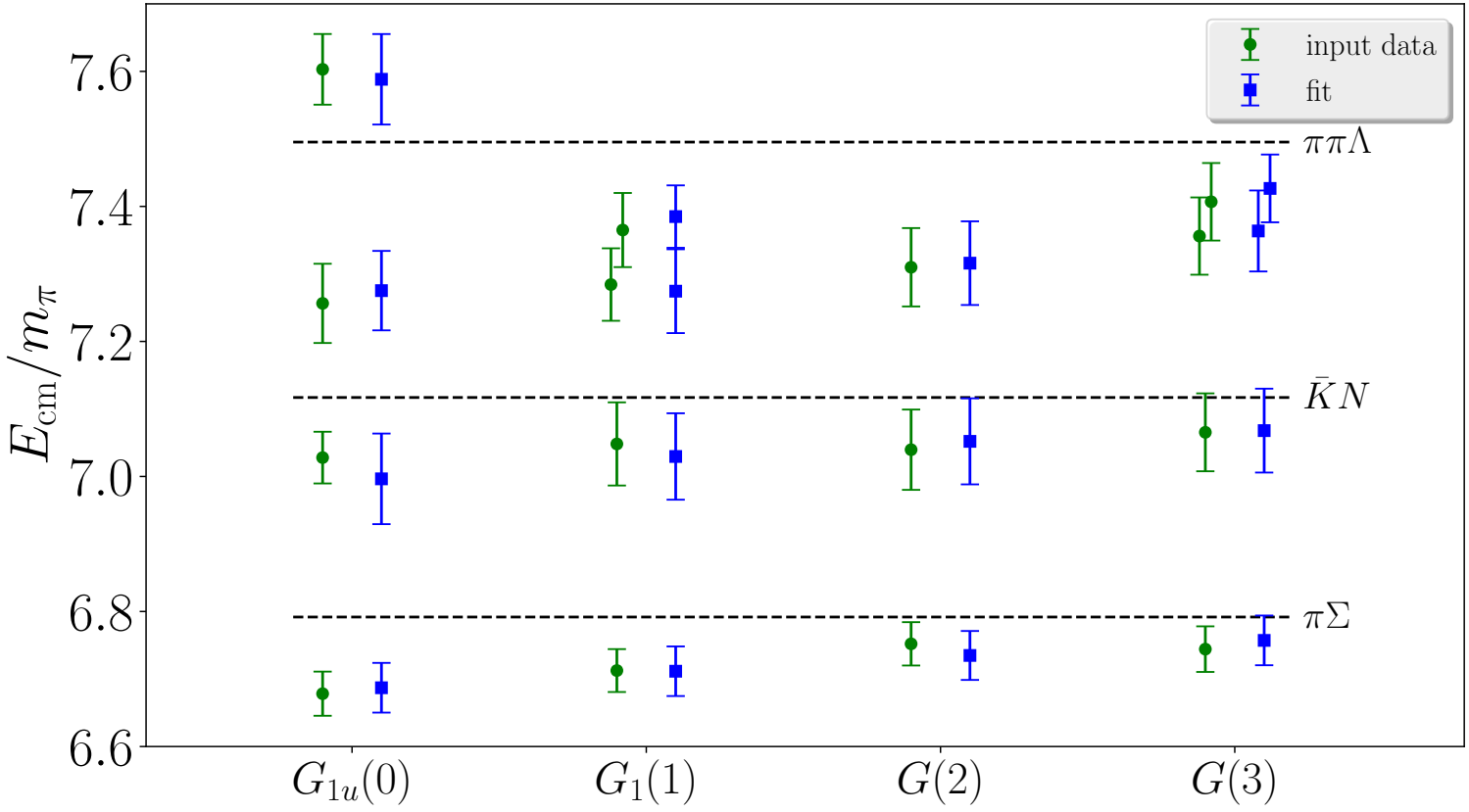
Extracting the spectrum (examples)





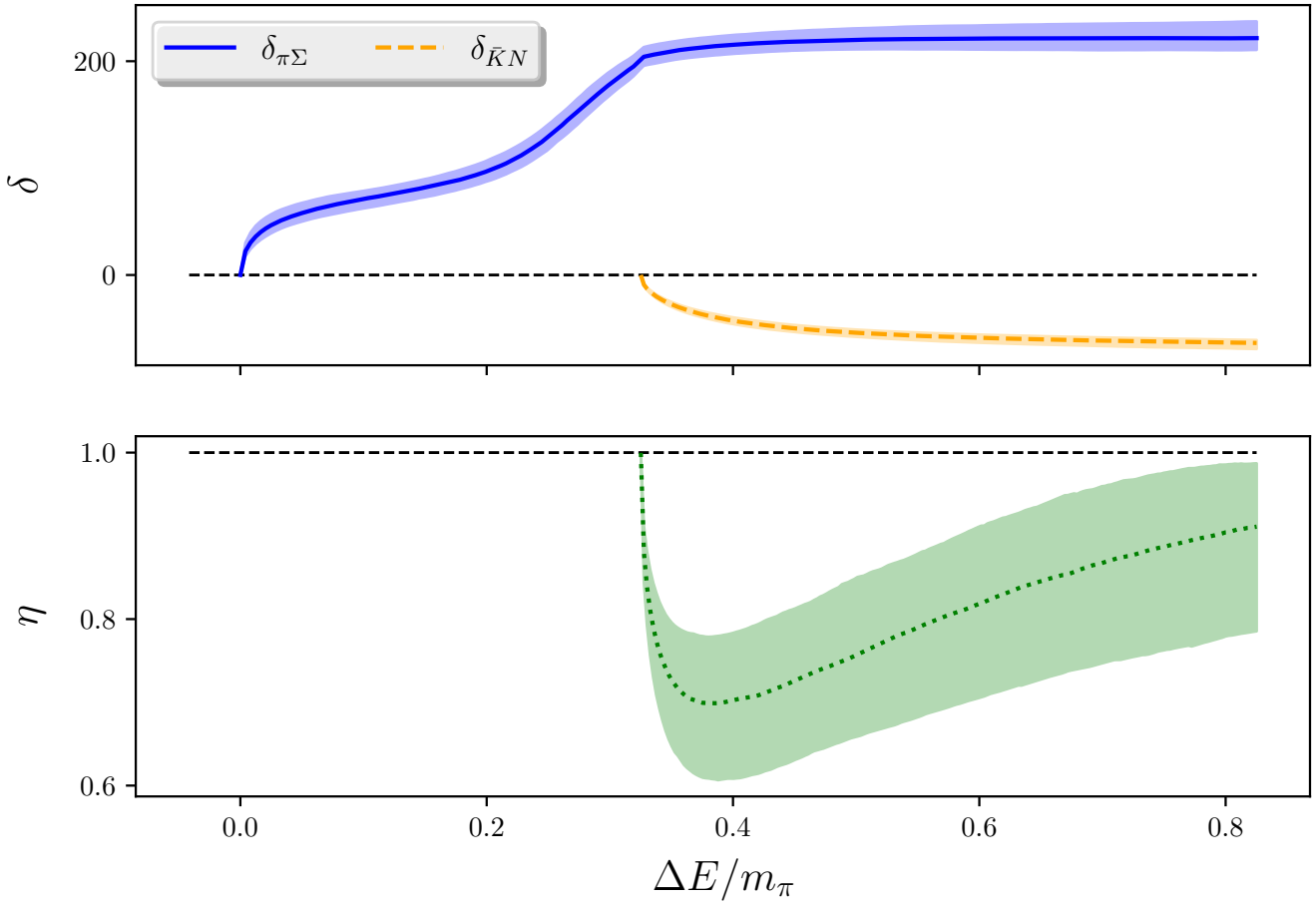
- We used various methods/cross checks
- Geometric series fit: $C(t) = \frac{Ae^{-E_0 t}}{1-Be^{-\Delta E t}}$
- Two students with two slightly different analysis methods

Finite-volume spectra



- Amplitude analysis uses ratios to extract energy differences with regard to non-interacting levels
- Blue squares indicate results from our preferred amplitude fit

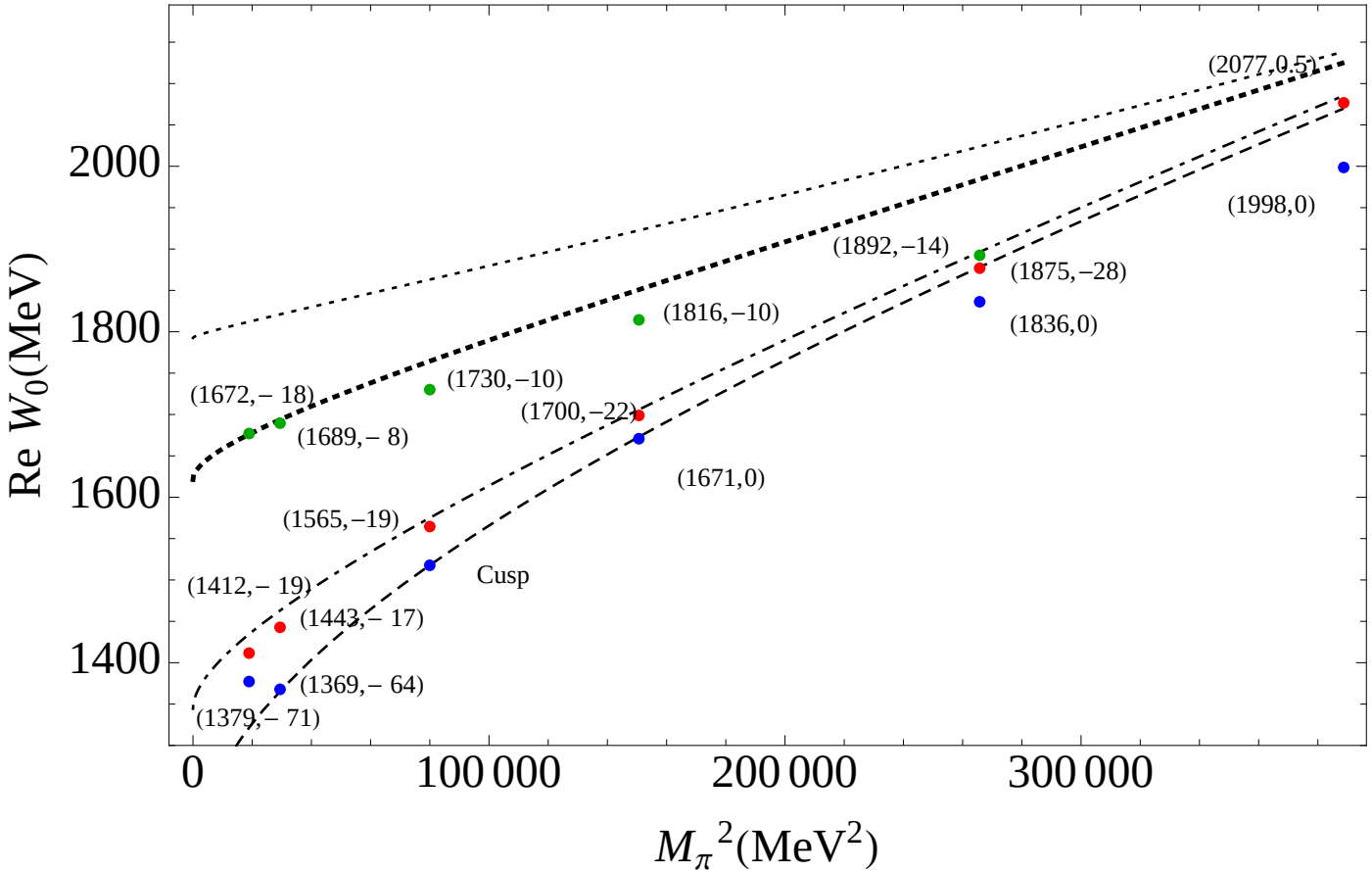
Same thing different: Phases and inelasticity



- Alternative way of showing our results: 2 phases and inelasticity η

Expected quark-mass dependence

Molina, Döring, PRD 94 056010 (2016)



- Plots shows expected behavior for PACS-CS ensembles
- Qualitative agreement with regard to expected behavior