Mathematical Ambiguities in Partial Wave Analysis

WYATT A. SMITH

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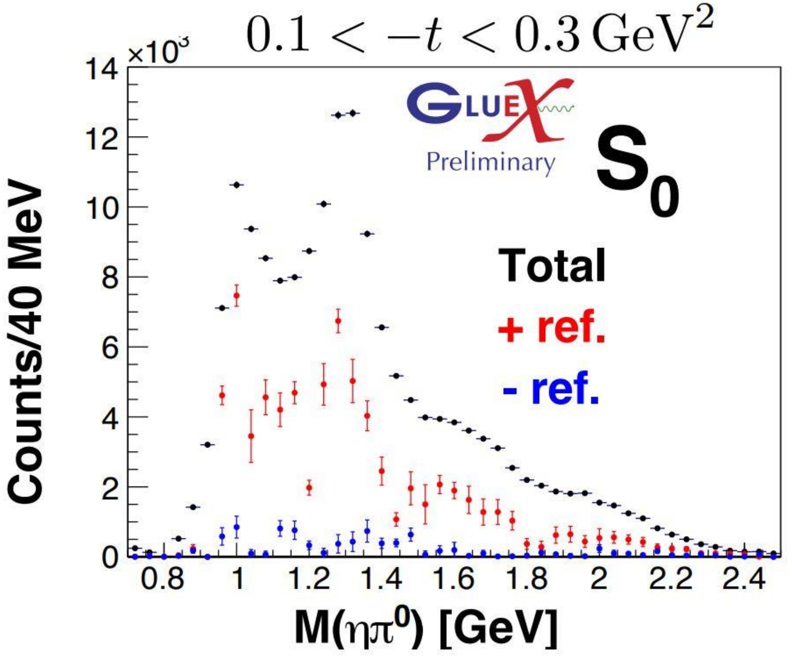
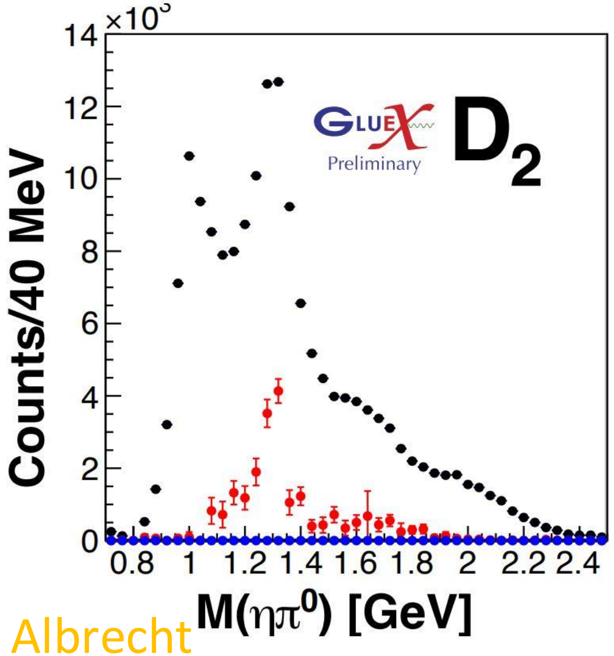
Partial Wave Analysis



Partial waves are widely used to characterize resonances, exotic contributions (P-wave $a_2(1320)$)

$$f(s,z) = \frac{1}{q}\sum_{\ell=0}^{\ell_m}(2\ell+1)a_\ell(s)P_\ell(z)$$

Ambiguities in PWA = multiple sets of partial waves, which describe data *equally well*



From expt: Multiple solutions close together in likelihood = possible ambiguities?

Plots courtesy of M. Albrecht

Ambiguities from Barrelet Zeroes

First noted by Barrelet, studied extensively for pion-beams

E. Barrelet, Nuovo Cim. A 8, 331 (1972)
S. U. Chung, Phys. Rev. D 56, 7299 (1997)

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Which one should we pick?

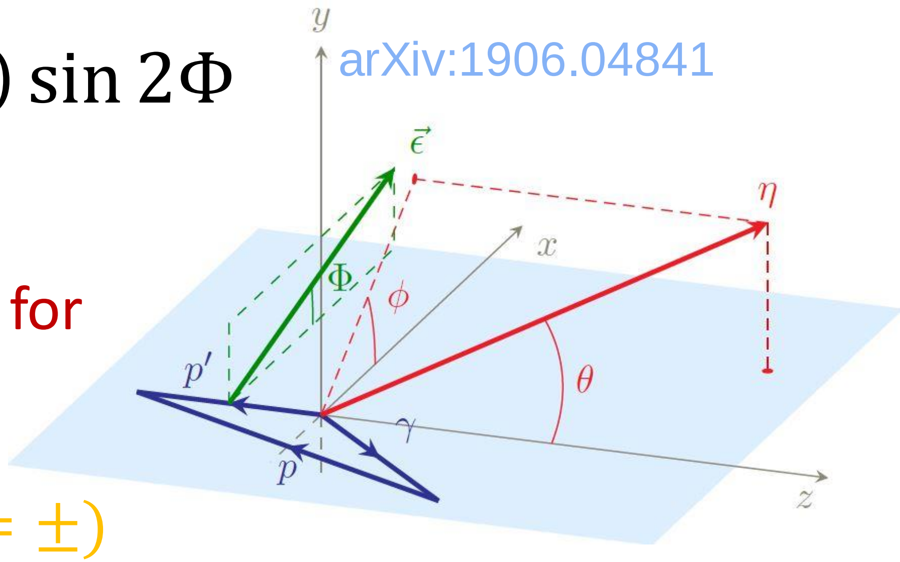
$$\underbrace{a_\ell(z_0, z_1, \dots, z_{\ell_m}) \;,\; a_\ell(z_0^*, z_1, \dots, z_{\ell_m}) \;,\; a_\ell(z_0, z_1^*, \dots, z_{\ell_m}) \;, \dots ,\; a_\ell(z_0^*, z_1, \dots, z_{\ell_m}^*), \dots}$$

2^{ℓ_m+1} choices of partial waves!

$\eta\pi$ −photoproduction at GlueX

$$I(\Omega,\Phi) = I^0(\Omega) - P_\gamma I^1(\Omega)\cos 2\Phi - P_\gamma I^2(\Omega)\sin 2\Phi$$

arXiv:1906.04841



Polarized intensities provide additional information for determination of partial waves

Parity conservation is dealt with via reflectivity ($\epsilon = \pm$)

Formalism is applicable to other experiments (MesonEx)

Provides additional constraints on ambiguities compared to pion-beams

$$f_m^{(\epsilon)}(\theta) = \sum_\ell \sqrt{2\ell+1}\,[\ell]_m^{(\epsilon)} d_{m0}^{l}(\theta)$$

Constraints on Ambiguities

$$I^0(\Omega) = \frac{1}{2\pi} \sum_{\epsilon m m'} f_m^{(\epsilon)}(\theta) f_{m'}^{(\epsilon)*}(\theta) \cos[(m-m')\phi],$$
$$I^1(\Omega) = \frac{-1}{2\pi} \sum_{\epsilon m m'} \epsilon f_m^{(\epsilon)}(\theta) f_{m'}^{(\epsilon)*}(\theta) \cos[(m+m')\phi],$$
$$I^2(\Omega) = \frac{-1}{2\pi} \sum_{\epsilon m m'} \epsilon f_m^{(\epsilon)}(\theta) f_{m'}^{(\epsilon)*}(\theta) \sin[(m+m')\phi].$$

$\longleftrightarrow$

$$I^0(\Omega) = \frac{1}{2\pi}\left[\frac{1}{2} h_0^0(\theta) + h_1^0(\theta)\cos(\phi) + \ldots \right]$$
$$I^1(\Omega) = -\frac{1}{2\pi}\left[\frac{1}{2} h_0^1(\theta) + h_1^1(\theta)\cos(\phi) + \ldots \right]$$
$$I^2(\Omega) = -\frac{1}{2\pi}\left[0 + h_1^2(\theta)\sin(\phi) + \ldots \right]$$

Each function $h_M^J(\theta)$ can be associated with a sum of polynomials in $u = \tan(\frac{\theta}{2})$

$$h_M^J(u) = \sum_i |g_i(u)|^2$$

$\Longrightarrow$ 2^{ℓ_m+1} ambiguities to consider *for each g!*

General procedure

1. From experimental data, find the partial waves for a mass bin by fitting to the intensity profile
2. Find all the roots of every function $g_i(u)$
3. Generate the (up to 2^{ℓ_m+1}) ambiguous wave sets for each $g_i(u)$
4. Select the 'right' wave set by enforcing continuity across mass bins/other physics

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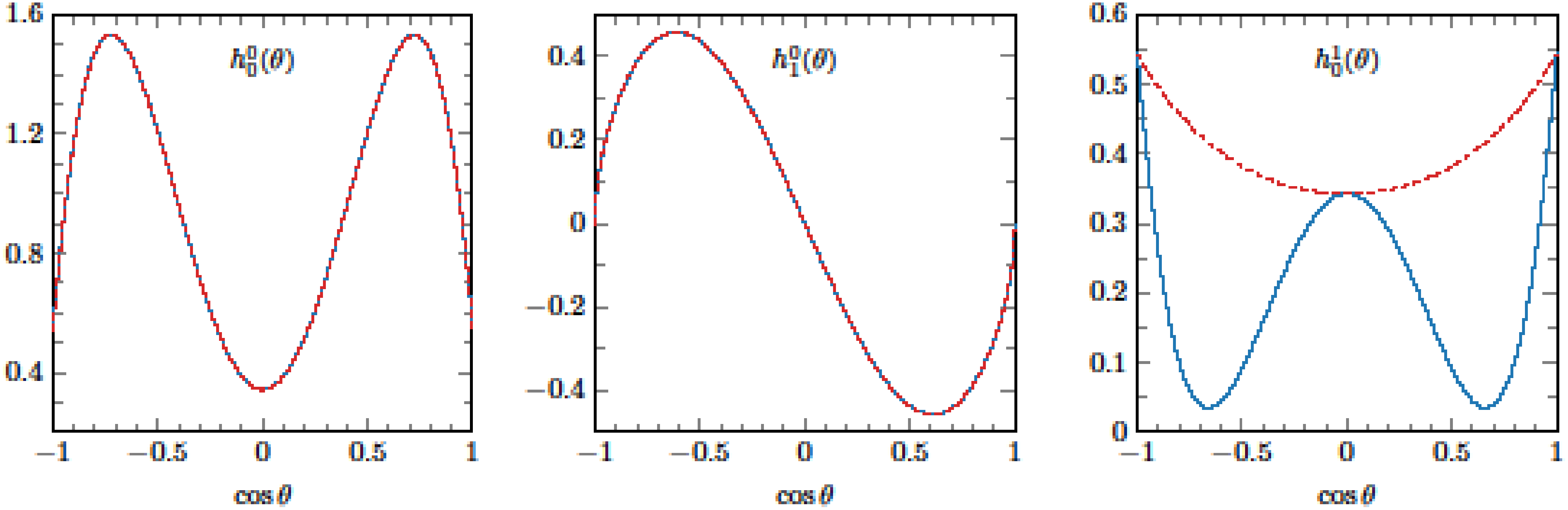
Key observation: any ambiguous set of partial waves must leave *every* $|g_i(u)|^2$ unchanged

Example wave set: S, D waves with $m = 0,1$

$[\ell]_m$	original	ambiguous
S_0	0.139	0.240
D_0	$0.871 + i0.586$	$0.539 - i0.892$
D_1	$0.337 + i0.031$	$0.195 - i0.231$



For pion-beam meson production this wave set has mathematical ambiguities

E. Barrelet, Nuovo Cim. A 8, 331 (1972)

Information from the polarized intensities breaks the ambiguity!

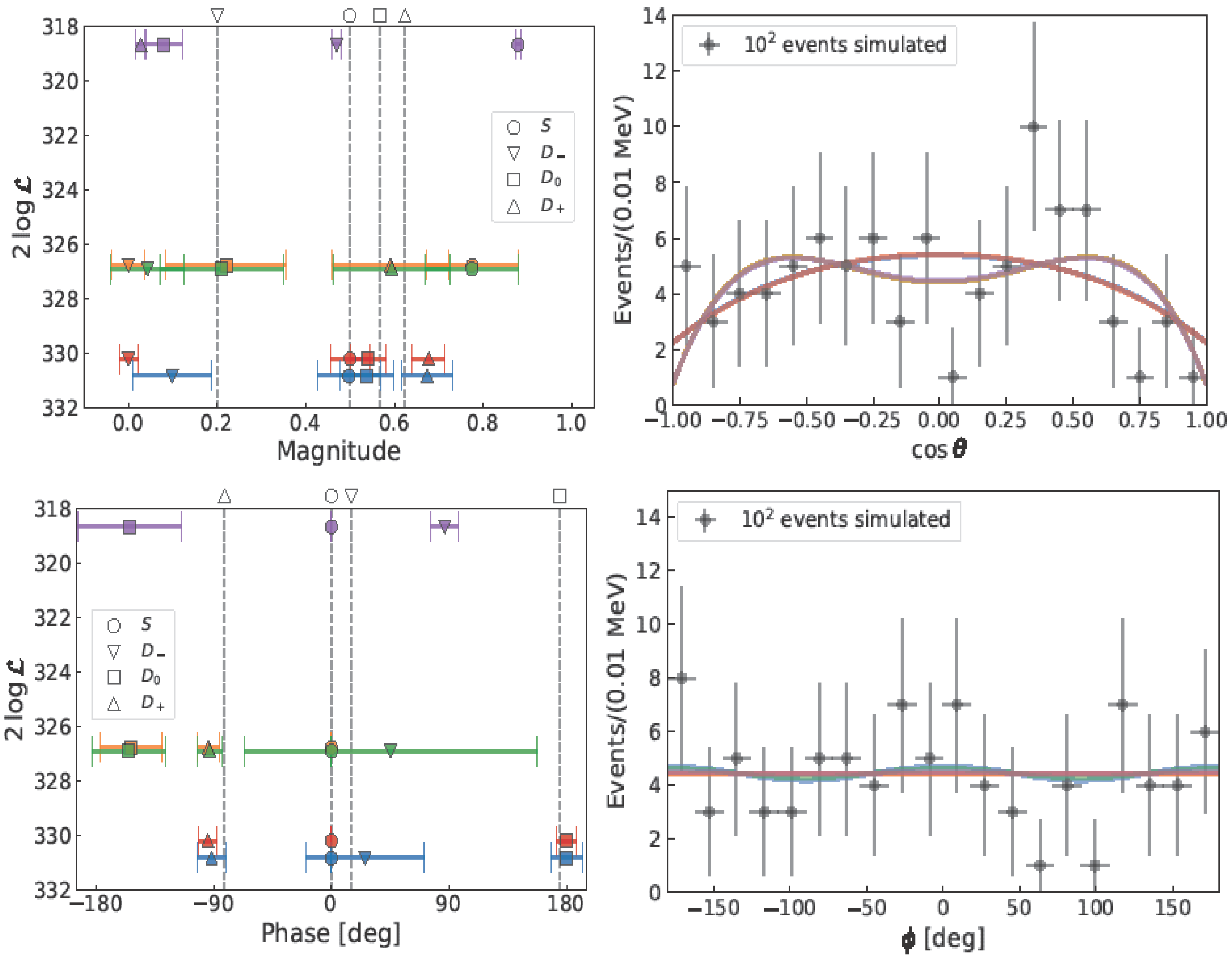
We don’t expect mathematical ambiguities to be present in any sensible wave set

Q: So where are the extra wave sets in fits to real data coming from?

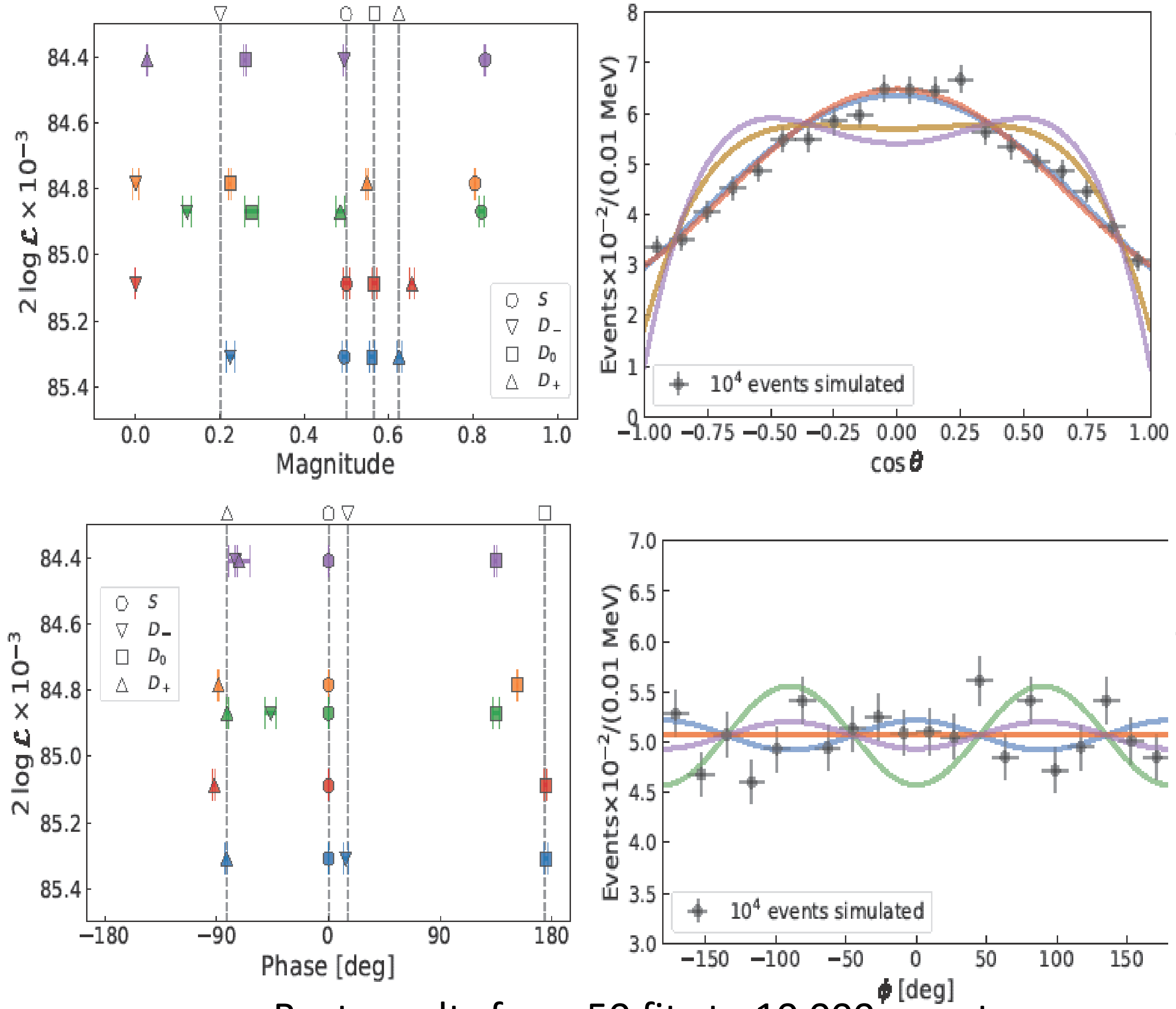
A: Local minima/maxima in likelihood functions

Starting with a 'true' solution, we simulated data with $10^2, 10^4, 10^6$ events

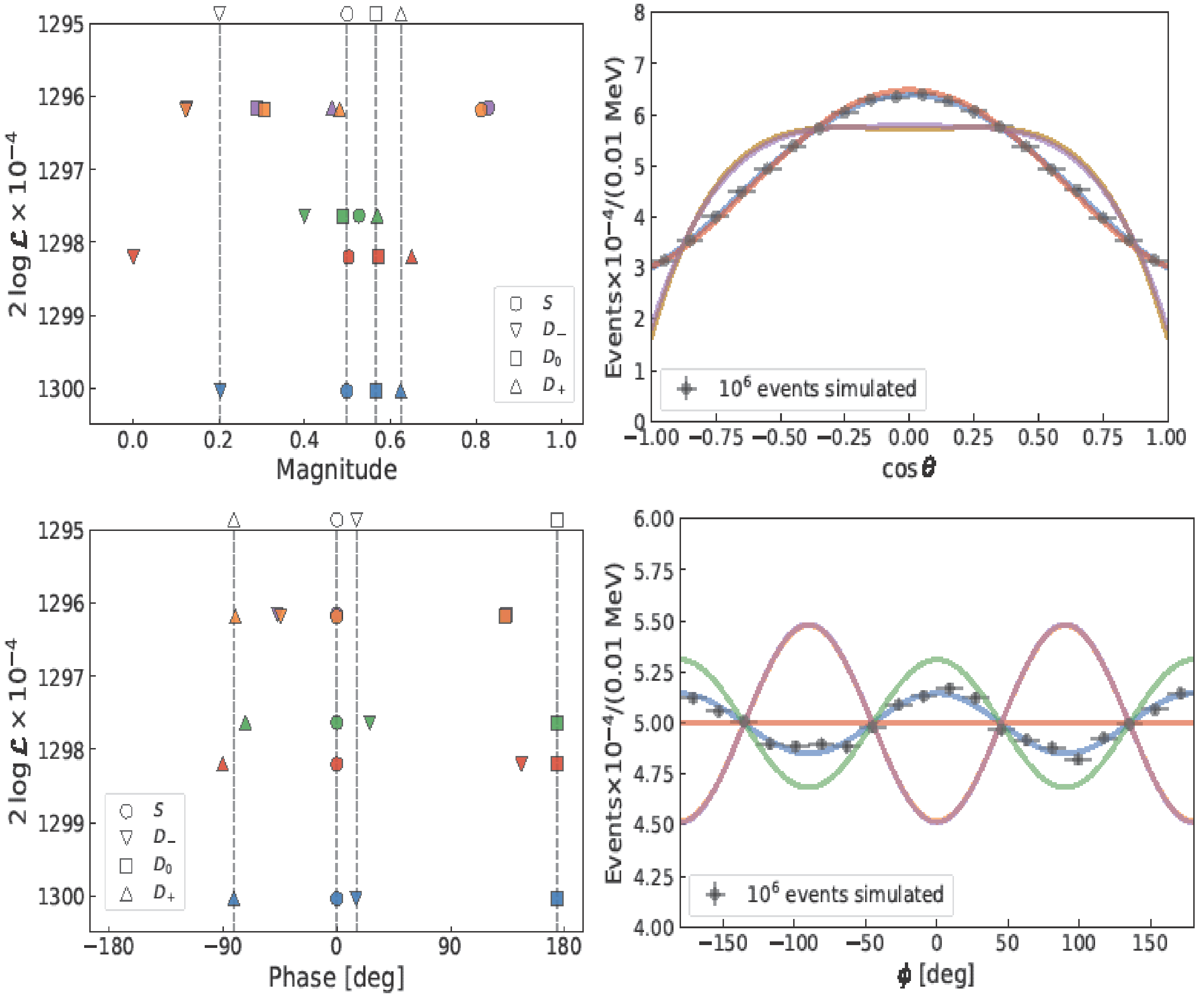
50 fits were performed with MINUIT for each case, similar false solutions were found



Best results from 50 fits to 100 events



Best results from 50 fits to 10,000 events



Best results from 50 fits to 1,000,000 events

Summary

We don’t expect mathematical ambiguities to be present in any sensible wave set

Polarization information helps to disambiguate the partial waves

Even without detector effects, systematics from experiment, fitting procedure can find false solutions

Our results apply to other linearly polarized photoproduction experiments

More work is needed to understand and address the false solutions which appear in fits