Recent studies of $e^+e^$ annihilation into hadrons at low energies via ISR at *BABAR*



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Motivation: $(g - 2)_{\mu}$



Muon anomalous magnetic moment

$$a_{\mu} = \frac{g_{\mu} - 2}{2}$$

- Precise test of the Standard Model $a_{\mu}^{SM} = a_{\mu}^{QED} + a_{\mu}^{EW} + a_{\mu}^{Had}$
- Long-standing discrepancy (4.2σ) between theory and experiment
- New tensions between dispersive and Lattice QCD
- Main uncertainty of rising from hadronic loop contribution

source	contribution $(\times 10^{-11})$	
	value	error
a_{μ}^{QED}	116 584 718.93	±0.10
$a_{\mu}^{\rm EW}$	53.6	±1.0
$a_{\mu}^{\text{Had}}[\text{LO}]$	6931	±40
$a_{\mu}^{\text{Had}}[\text{NLO}]$	-98.3	±0.7
$a_{\mu}^{\text{LBL}}[\text{NLO}]$	92	±18
$a_{\mu}^{\rm SM}$	116 591 810.1	±43.9
a_{μ}^{\exp}	116 592 061	±41
a_{μ}^{\exp} - a_{μ}^{SM}	4.2 σ 251	±59

Hadronic Vacuum Polarization

Dominated by the Hadronic Vacuum Polarization LO Can be calculated by using dispersion relations $a_{\mu}^{Had} = \frac{\alpha^2}{3\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \frac{K(s)}{s} R(s)$ $R(s) = \frac{\sigma^0 [e^+e^- \rightarrow \text{hadrons}]}{\sigma^0 [e^+e^- \rightarrow \mu^+\mu^-]}$

 $\sigma^0 [e^+e^- \rightarrow hadrons]$ can be measured experimentally as a sum over exclusive channels



The BABAR experiment

The BABAR detector was located at the interaction point of PEP II at SLAC Asymmetric e^+e^- collider, mostly at $\sqrt{s} \sim 10.58$ GeV



 $\int L dt \sim 469 \text{ fb}^{-1}$ close to the $\Upsilon(4S)$ peak

Initial State Radiation





$$\frac{d\sigma_{e^+e^- \to f\gamma}(s, m_f)}{dm_f \, d\cos\theta_{\gamma}^*} = \frac{2m_f}{s} W(s, x, \theta_{\gamma}^*) \cdot \sigma_{e^+e^- \to f}(m_f)$$

- The hadronic cross section $e^+e^- \rightarrow f$ can be extracted from the ISR cross section $e^+e^- \rightarrow \gamma f$.
- The radiator function *W*(*s*,*x*) is calculated in QED with accuracy better than 1% level

Common ISR analysis strategy

- Tagged analysis (E_{γ}^* >3 GeV)
- Back-to-back topology btw ISR γ and the rest of the event
- $\pi/K/p$ discrimination based on dE/dx e Cherenkov angle
- Kinematic fit for 4-momentum conservation
- Fitted χ^2 used for signal selection and background subtraction
- Detector acceptances and selection efficiencies estimated with MC simulation

The BaBar ISR program for light hadrons

- 30 publications for more than 40 final states studied
- Almost any channel from 2 to 7 light hadrons in the final state
- Many first measurements and significant precision improvement in most cases
- Discoveries (e.g. $\phi(2170)$ in $e^+e^- \rightarrow \phi(1020)f_0(980)$)
- Most precise measurement of $e^+e^- \rightarrow \pi^+\pi^-$



Study of the process $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ using initial state radiation with BaBar

BaBar coll., PRD104, 112003 (2021)

The process $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ at BaBar

The 2nd largest contribution mode, $e^+e^- \rightarrow 3\pi$, ~7% to a_{μ}^{Had} but ~19% to its uncertainty

The relative uncertainty on the 3π channel is over 3%, almost 5 times larger than the 2π channel



The 3π channel was analyzed with the full BaBar dataset, 469 fb⁻¹,

- ISR photon: with highest energy over 3 GeV,
- Require only 2 "good" tracks with opposite charge,
- At least 2 additional good photons, with $m_{\gamma\gamma} \in (0.10, 0.17)$ GeV
- Kinematic fit, and further cuts to suppress backgrounds

Fit to $\pi^+\pi^-\pi^0$

The fit to the measured mass spectrum is based on the VMD model with $\omega(782) + \omega(1420) + \omega(1680) + \phi(1020) + \rho(770)$

The true spectrum is smeared to account for data-MC difference in the mass resolution, and then multiplied by the transfer matrix obtained from simulation for the unfolding



• Parameters fitted for ω and ϕ in good agreement with world average





• The $\rho \rightarrow 3\pi$ decay needed to describe the data

- The significance of $\rho \rightarrow 3\pi$ is greater than 6σ
- In agreement with SND PRD68, 052006 (2003)

	$BF(\rho\to 3\pi)\times 10^4$	φ (•)
BABAR	$0.88 \pm 0.23 \pm 0.30$	$-(99 \pm 9 \pm 15)$
SND	$1.01^{+0.54}_{-0.36} \pm 0.34$	$-(135^{+17}_{-13} \pm 9)$

Cross section below 1.1 GeV

Sharp structure of mass spectrum from resonances, over 4 orders of magnitude below 1.1 GeV. It is essential to unfold the detect efficiency effects with the (iterative, dynamically stabilized) method B. Malaescu, arXiv:0907.3791



Cross section above 1.1 GeV



No narrow structures Bin size 25 MeV (100 MeV for $m_{3\pi} > 2.7$ GeV) => no need for unfolding

Systematic uncertainties (4-15%) dominated by background subtraction

- SND 2020: EPJC80, 993 (2020)
- Sizable difference between SND and BABAR data near 1.25 and 1.5 GeV, general agreement elsewhere
- SND systematic uncertainties are 4.4%

Result for $a_{\mu}^{3\pi}$

TABLE VI. Values of $a_{\mu}^{3\pi}$ for different mass intervals. The first three rows represent the *BABAR* result, while the last three are the calculations [1,49–51] based on previous $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ measurements.

$M_{3\pi}$ GeV/ c^2	$a_\mu^{3\pi} imes 10^{10}$	
0.62-1.10	$42.91 \pm 0.14 \pm 0.55 \pm 0.09$	
1.10-2.00	$2.95 \pm 0.03 \pm 0.16$	
<2.00	$45.86 \pm 0.14 \pm 0.58$	
<1.8 [1]	$46.21 \pm 0.40 \pm 1.40$	
<1.97 [49]	46.74 ± 0.94	
<2 [50]	44.32 ± 1.48	
<1.8 [51]	$46.2 \pm 0.6 \pm 0.6$	

TABLE VII. Contributions to the systematic uncertainty in $a_{\mu}^{3\pi}(0.62 < M_{3\pi} < 1.1 \text{ GeV}/c^2)$ from different effects.

Effect	Uncertainty (%)
Luminosity	0.4
Radiative correction	0.5
Detection efficiency	1.1
MC statistics	0.15
Background subtraction	0.073
Gaussian smearing	0.0007
Lorentzian smearing	0.003
Unfolding procedure	0.045
Total	(1.3)

 $a_{\mu}^{3\pi} = (45.86 \pm 0.14 \pm 0.58) \times 10^{-10}$

- LO hadronic contribution to muon anomaly magnetic moment is calculated with the measured cross section below 2.0 GeV
- Agreement with calculation based on previous measurements, more precise by a factor of ~2
- Previous relative uncertainty on the 3π mode is over 3%.

Study of the reactions $e^+e^- \rightarrow K^+K^-\pi^0\pi^0\pi^0$, $e^+e^- \rightarrow K_S^0K^{\pm}\pi^{\mp}\pi^0\pi^0$, and $e^+e^- \rightarrow K_S^0K^{\pm}\pi^{\mp}\pi^+\pi^$ at center-of-mass energies from threshold to 4.5 GeV using initial-state radiation

BaBar coll., PRD107, 072001 (2023)

The process $e^+e^- \rightarrow 2K3\pi$ at BaBar



- Three channels for the $2K3\pi$ mode measured at BaBar for the first time: $K^+ K^- \pi^0 \pi^0 \pi^0$, $K_S^0 K^\pm \pi^\mp \pi^0 \pi^0$, and $K_S^0 K^\pm \pi^\mp \pi^+$
- Main motivations:
 - systematic deviation seen exclusive cross sections near 2 GeV and pQCD
 - direct measurement of the final states reduces the need of isospin relations for a_{μ}
 - study of intermediate states, look for new states or decay modes
- Only $K^+ K^- \pi^+ \pi^- \pi^0$ has been studied previously (PRD76 (2007) 092005),
- Similar technique used as above

Cross sections



Intermediate resonances in $\phi \eta \rightarrow K^+ K^- 3\pi^0$



The cross section is dominated by the channel

- $e^+ e^- \rightarrow \phi \eta$. To extract the cross section:
- select events with $m_{KK} < 1.05 \text{ GeV}$
- divide the $2K3\pi^0$ mass spectrum in 50 MeV wide bins
- fit the 3π⁰ mass distribution to get the events in the η peak
- account for $\phi \to K^+K^-$ and $\eta \to 3\pi^0$ branching fractions



Other intermediate resonances



Resonances in $2K3\pi$



- X(2400) and $\phi(2170)$ decay to $K_S K 3\pi$ final states,
- Clear J/ψ and $\psi(2S)$ peaks in the $2K3\pi$ final states.

Summary

- The muon *g*-2 puzzle is important for SM precise test, even a potential gateway to New Physics
- Precision studies of e^+e^- annihilation into hadrons in low energy at BaBar with ISR technique, which provide important experimental input for a_{μ}^{Had} calculation
- The new measurement of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ improved the precision $a_{\mu}^{3\pi}$ calculation by a factor of ~2

PRD104, 112003 (2021)

• Measurements of the cross section and substructure associated with $e^+e^- \rightarrow 2K3\pi$ modes help to improve knowledge of the hadronic contributions to a_{μ}^{Had}

PRD107, 072001 (2023)

Thank you!



BACKUP