

# A coupled-channel system with anomalous thresholds and unitarity

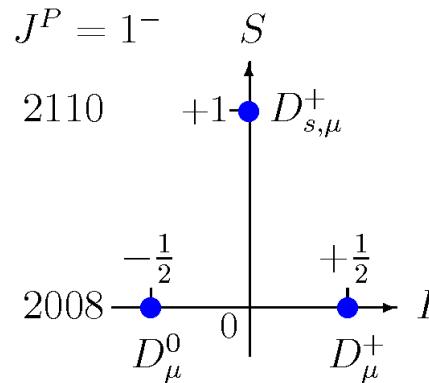
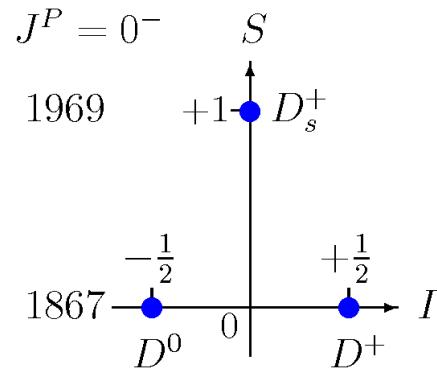
Matthias F.M. Lutz

*GSI Helmholtzzentrum für Schwerionenforschung GmbH*

- ✓ Chiral SU(3) expansions in QCD
- ✓ Chiral extrapolation for charmed meson masses
- ✓ Coupled-channel dynamics for charmed mesons (GPA)
- ✓ Summary and outlook

# Chiral Lagrangian for charmed mesons

## ✓ Heavy-light mesons $(c\bar{q})$ SU(3) anti-triplet [3]



$$\mathcal{L} = (\partial_\mu D)(\partial^\mu \bar{D}) - M^2 D \bar{D} - (\partial_\mu D^{\mu\alpha})(\partial^\nu \bar{D}_{\nu\alpha}) + \frac{1}{2} \tilde{M}^2 D^{\mu\alpha} \bar{D}_{\mu\alpha}$$

$$+ 2 g_P \left\{ D_{\mu\nu} U^\mu (\partial^\nu \bar{D}) - (\partial^\nu D) U^\mu \bar{D}_{\mu\nu} \right\}$$

$$- \frac{i}{2} \tilde{g}_P \epsilon^{\mu\nu\alpha\beta} \left\{ D_{\mu\nu} U_\alpha (\partial^\tau \bar{D}_{\tau\beta}) + (\partial^\tau D_{\tau\beta}) U_\alpha \bar{D}_{\mu\nu} \right\}$$

covariant derivative

$$\partial_\mu \rightarrow \partial_\mu + \frac{1}{2} e^{-i \frac{\Phi}{2f}} \partial_\mu e^{+i \frac{\Phi}{2f}} + \frac{1}{2} e^{+i \frac{\Phi}{2f}} \partial_\mu e^{-i \frac{\Phi}{2f}}$$

- chiral symmetry :  $f \sim 90$  MeV chiral SU(3) limit value of  $f_\pi$
- hadronic decay of  $D^* \rightarrow D\pi$  implies  $|g_P| = 0.57 \pm 0.07$
- heavy-quark spin symmetry :  $\tilde{g}_P = g_P$  and  $M = \tilde{M}$  as  $m_c \rightarrow \infty$

# Chiral SU(3) for heavy-light meson resonance

## ✓ Coupled-channel interaction from chiral SU(3) symmetry

S \ I	0	1/2	1	3/2
+2		$(D_s K)$		
+1	$(DK, D_s \eta)$		$(D_s \pi, DK)$	
0		$(D\pi, D\eta, D_s K)$		$(D\pi)$
-1	$(\bar{K}D)$		$(\bar{K}D)$	

$$\mathbf{8} \otimes \overline{\mathbf{3}} = \overline{\mathbf{15}} \oplus \mathbf{6} \oplus \overline{\mathbf{3}}$$

- Weinberg- Tomozawa interaction

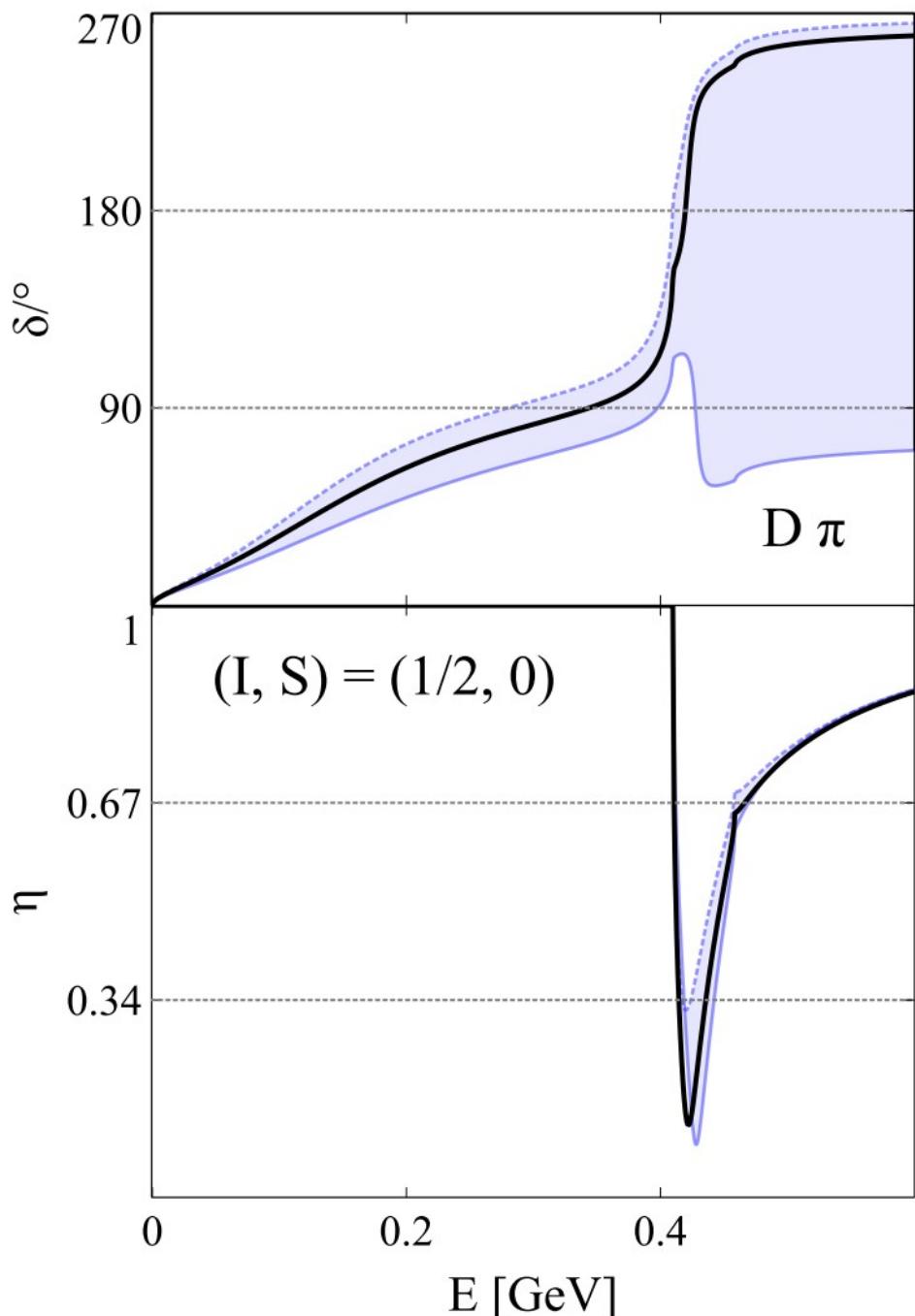
$$\mathcal{L}_{WT} = \frac{1}{8f^2} D [\Phi, (\partial_\mu \Phi)] (\partial^\mu \bar{D}) + \dots$$

$$V_{WT}^{[15]} : V_{WT}^{[6]} : V_{WT}^{[3]} = -1 : \mathbf{1} : \mathbf{3}$$

- strong attraction in anti-triplet and sextet
- dynamically generated  $J^P = 0^+$  and  $J^P = 1^+$  meson resonances  
in particular  $D_{s0}^*(2317)$  and  $D_{s1}^*(2460)$

## ✓ Are there exotic flavour sextet resonances?

# Leading order prediction for $\pi D$ phase shift in $I = 1/2$



Two resonances in phase shift

- a broad state from the anti-triplet
  - a narrow state from the sextet
  - a variation of the matching scale
- 
- s-wave phase shift required in unitarity studies of CKM matrix from  $B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-$  decays (LHCb and Belle)



How stable is the prediction?

- the broad state from the anti-triplet may move further into the complex plane
- the narrow state from the sextet always at  $E \sim 0.4$  GeV
- are higher order counter terms important?

# Chiral SU(3) correction terms for charmed mesons

$$\begin{aligned}\mathcal{L}^{(2)} = & - (4c_0 - 2c_1) D \bar{D} \text{tr } \chi_+ - 2c_1 D \chi_+ \bar{D} \\ & - 4(2c_2 + c_3) D \bar{D} \text{tr } (\mathbf{U}_\mu U^\mu) + 4c_3 D \mathbf{U}_\mu U^\mu \bar{D} \\ & - \frac{1}{M^2} (4c_4 + 2c_5) (\partial_\mu D)(\partial_\nu \bar{D}) \text{tr } [\mathbf{U}^\mu, \mathbf{U}^\nu]_+ + \frac{1}{M^2} 2c_5 (\partial_\mu D)[\mathbf{U}^\mu, \mathbf{U}^\nu]_+(\partial_\nu \bar{D}) \\ & - i c_6 \epsilon^{\mu\nu\rho\sigma} \left( D [\mathbf{U}_\mu, \mathbf{U}_\nu]_- \bar{D}_{\rho\sigma} - D_{\rho\sigma} [\mathbf{U}_\nu, \mathbf{U}_\mu]_- \bar{D} \right)\end{aligned}$$

covariant derivative

$$\partial_\mu \rightarrow \partial_\mu + \frac{1}{2} e^{-i \frac{\Phi}{2f}} \partial_\mu e^{+i \frac{\Phi}{2f}} + \frac{1}{2} e^{+i \frac{\Phi}{2f}} \partial_\mu e^{-i \frac{\Phi}{2f}},$$

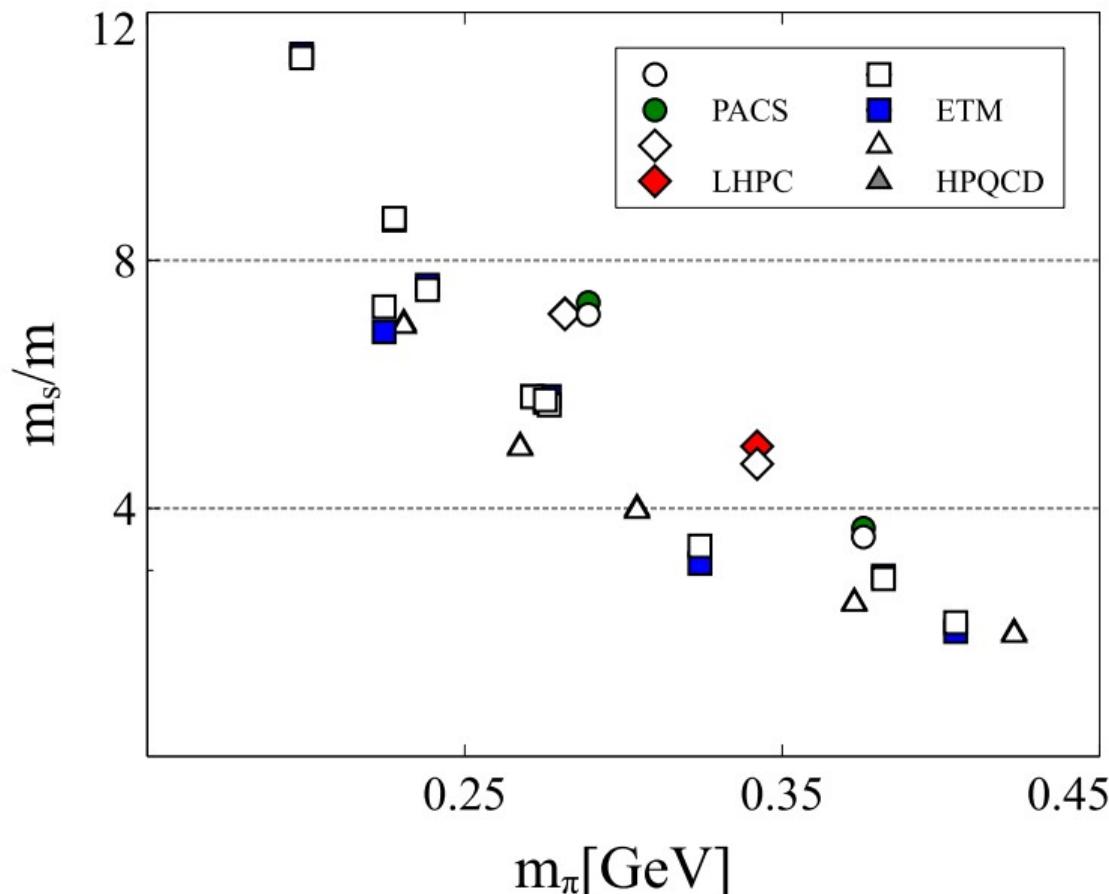
## ✓ How to determine the low-energy constants?

- additional 6 terms parameterized by  $\tilde{c}_n$  involving two  $J^P = 1^-$  fields ( $D_{\alpha\beta} \dots \bar{D}_{\mu\nu}$ )
  - from heavy-quark spin symmetry  $\tilde{c}_n = c_n$  at  $m_c \rightarrow \infty$
  - from large- $N_c$
- |                             |                              |                              |
|-----------------------------|------------------------------|------------------------------|
| $c_0 \simeq \frac{c_1}{2},$ | $c_2 \simeq -\frac{c_3}{2},$ | $c_4 \simeq -\frac{c_5}{2},$ |
|-----------------------------|------------------------------|------------------------------|
- use QCD lattice data on the quark-mass dependence of the D meson masses

# Predictions for quark-mass ratios in lattice ensembles

## ✓ How to fit the lattice data?

- take pion and kaon mass of the ensemble → compute quark masses
- this requires the low-energy constants  $L_4 - 2 L_6, L_5 - 2 L_8, L_8 + 3 L_7$
- we do not fit to the quark-mass ratios given by the lattice groups!



## ✓ A fit to the D meson masses

- renormalization scale  $\mu = 0.77$  GeV

$10^3 (L_4 - 2 L_6)$	-0.1575
$10^3 (L_5 - 2 L_8)$	-0.0370
$10^3 (L_8 + 3 L_7)$	-0.5207
$m_s/m$	26.600

- at physical quark masses our ratio compares well with lattice result

$m_s/m = 26.66(32)$       from ETMC  
in Nucl. Phys. B887, 19 (2014)

## Coupled-channel scattering with long range forces

$$T_{ab}^J(s) = U_{ab}^J(s) + \sum_{c,d} \int_{\mu_{thr}^2}^{\infty} \frac{d\bar{s}}{\pi} \frac{s - \mu_M^2}{\bar{s} - \mu_M^2} \frac{T_{ac}^J(\bar{s}) \rho_{cd}^J(\bar{s}) T_{db}^{J*}(\bar{s})}{\bar{s} - s - i\epsilon}$$

- ✓ Compute  $T_{ab}^J(s)$  from the Chiral Lagrangian (GPA)
- ✓  $T_{ab}^J(s)$  is computed in terms of non-linear integral equations
  - generalized potential approach (GPA)
  - use perturbation theory for  $U_{ab}^J(s)$  followed by a conformal expansion

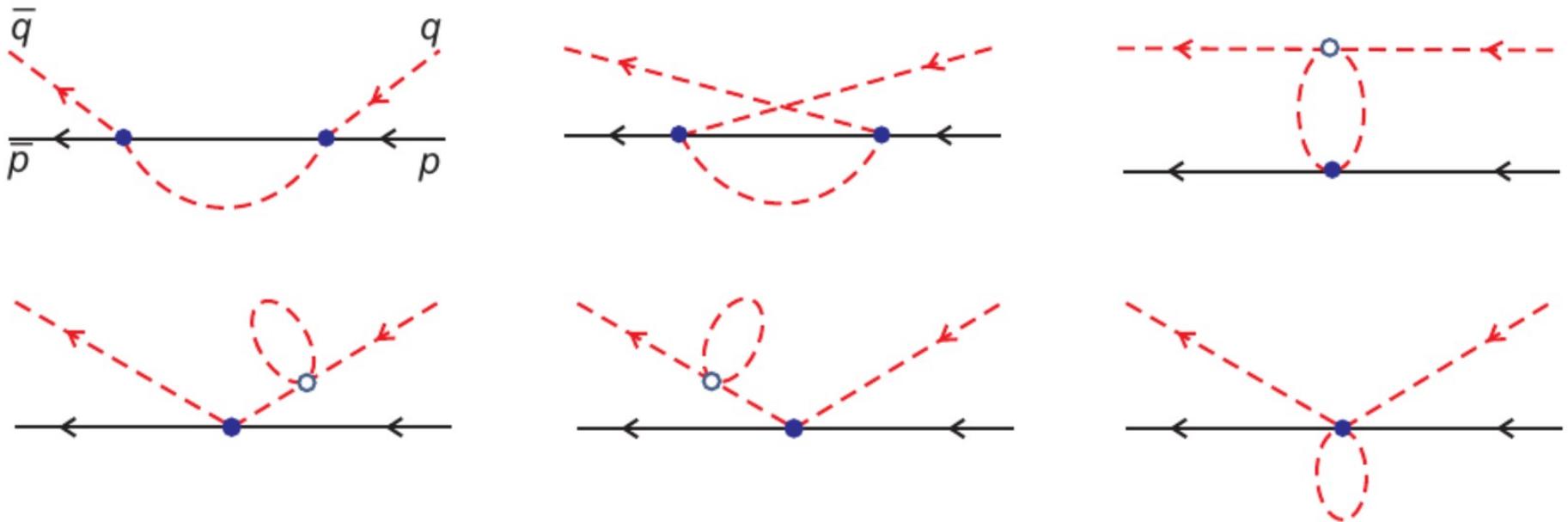
$$U(s) = U_{\text{close-by}}(s) + U_{\text{far-distant}}(s)$$

$$\text{with } U_{\text{far-distant}}(s) = \sum_k c_k \xi^k(s)$$

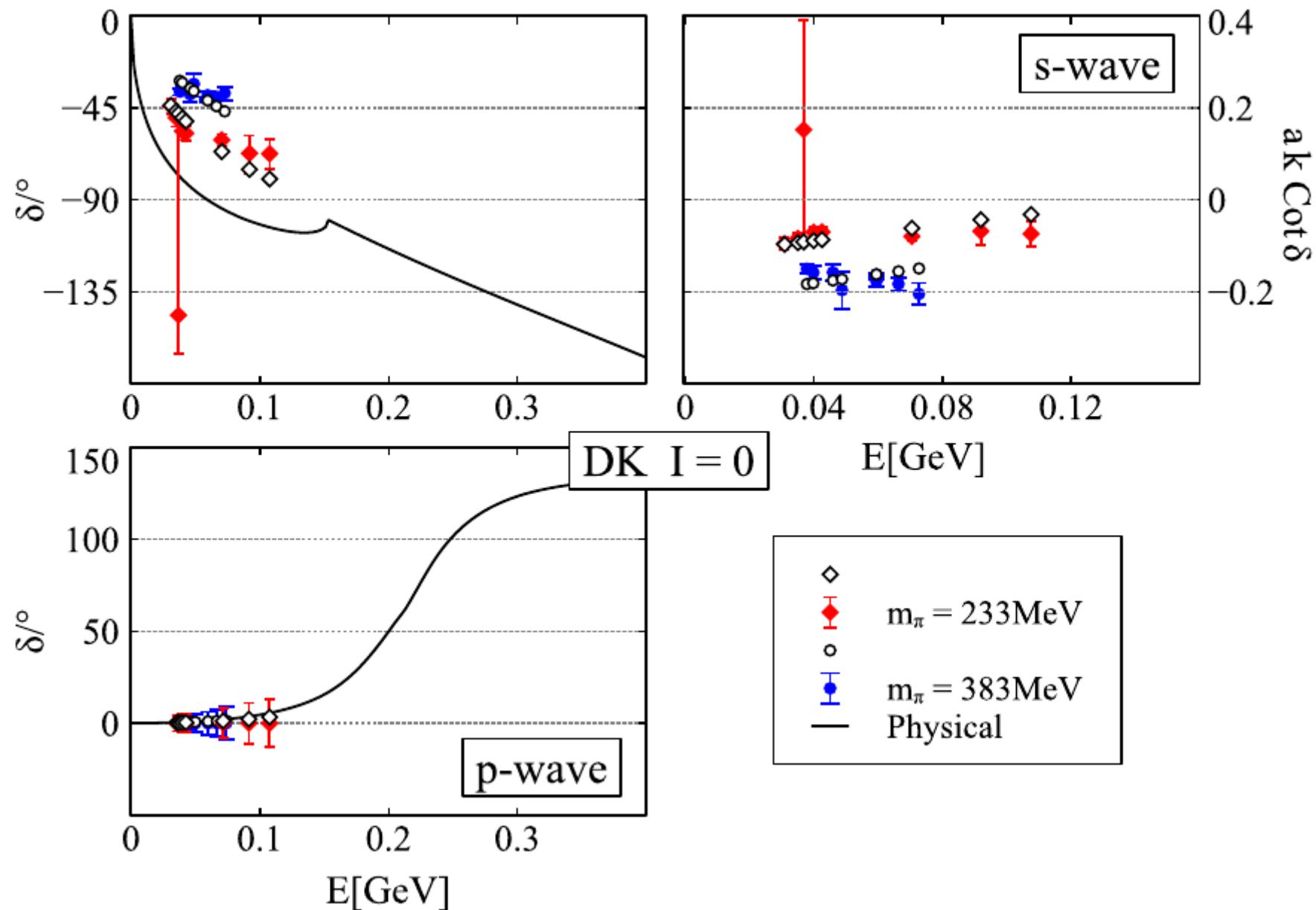
# Coupled-channel scattering with long range forces

$$T_{ab}^J(s) = U_{ab}^J(s) + \sum_{c,d} \int_{\mu_{thr}^2}^{\infty} \frac{d\bar{s}}{\pi} \frac{s - \mu_M^2}{\bar{s} - \mu_M^2} \frac{T_{ac}^J(\bar{s}) \rho_{cd}^J(\bar{s}) T_{db}^{J*}(\bar{s})}{\bar{s} - s - i\epsilon}$$

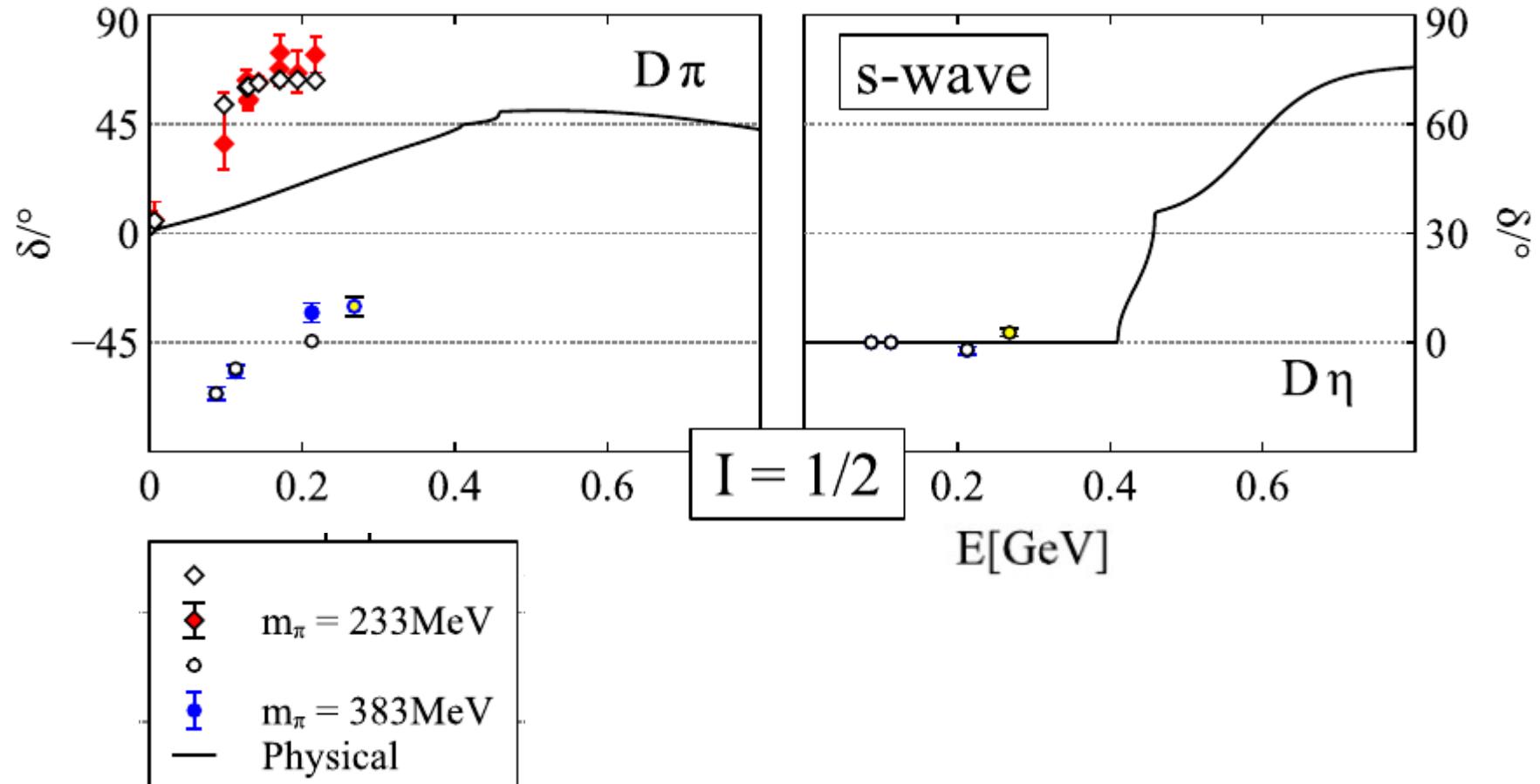
- ✓ Compute  $T_{ab}^J(s)$  from the Chiral Lagrangian (GPA)
- ✓  $T_{ab}^J(s)$  is computed in terms of non-linear integral equations
  - use perturbation theory for  $U_{ab}^J(s)$  followed by a conformal expansion
  - truncate  $U_{ab}^J(s)$  at the one-loop level and fit LEC to Lattice QCD data



# Lattice ensembles from HSC



# Lattice ensembles from HSC



- at unphysical quark masses (published HSC ensemble)
- black line: prediction of phase shifts at physical quark masses
- red points were predicted (by now results from HSC )

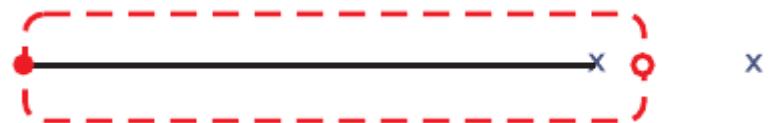
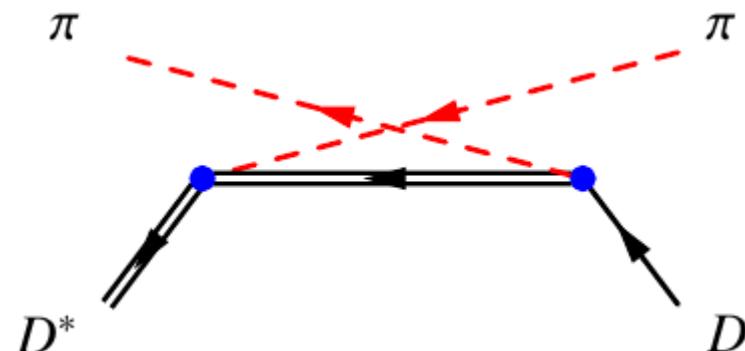
# Anomalous thresholds and coupled-channel unitarity

## ✓ Consider p-wave scattering of $\pi D$ with $I = 1/2$

- couples to a p-wave  $\pi D^*$  channel

## ✓ Anomalous threshold occurs at physical masses

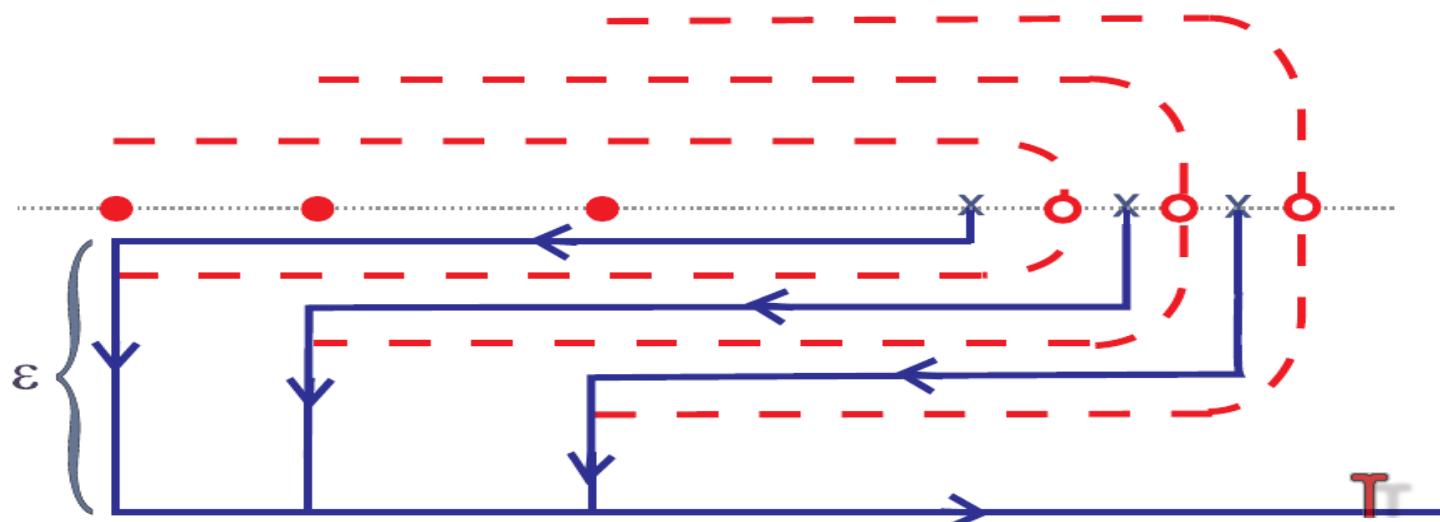
- assume first  $M_{D^*} < M_D + m_\pi$  (can be tuned on Lattice QCD ensembles)
- for  $m_\pi = 150$  MeV we find a normal system
- for  $m_\pi = 145$  MeV we find an anomalous reaction  $\pi D \rightarrow \pi D^*$



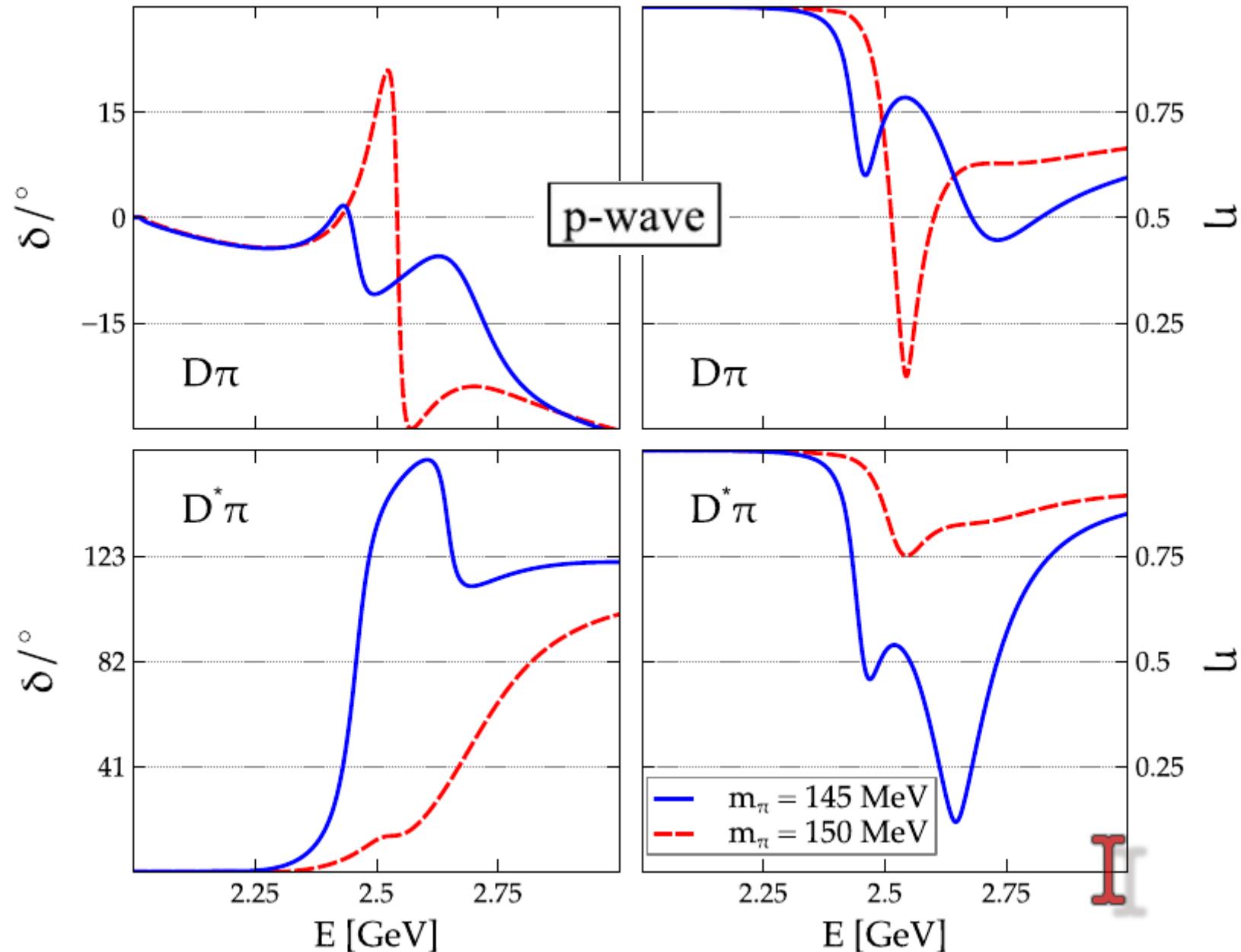
# Anomalous thresholds and coupled-channel unitarity

$$T_{ab}^J(s) = U_{ab}^J(s) + \sum_{c,d} \int_{\mu_{thr}^2}^{\infty} \frac{d\bar{s}}{\pi} \frac{s - \mu_M^2}{\bar{s} - \mu_M^2} T_{ac}^J(\bar{s}) \rho_{cd}^J(\bar{s}) T_{db}^{J*}(\bar{s})$$

- ✓ How to solve this equation with anomalous left-hand cut lines?
- ✓ Analytic continuation as implied by deformed s-channel cut lines
  - suggested already by Mandestam et al in the 1960s
  - a first implementation in hadron physics only 2018



# P-wave state from anomalous threshold effects



# Summary & Outlook

## ✓ Chiral extrapolation of hadron masses

- resummed  $\chi$ PT : use physical masses in the loops
  - chiral expansion with up, down and strange quarks works
- here we considered open-charm meson masses and scattering processes at  $N^3LO$ 
  - fits to Lattice QCD masses of ground states with  $J^P = 0^-, 1^-$  and Lattice QCD data from HSC on scattering
- predict a large number of low-energy constants for the chiral Lagrangian of QCD
  - quantitative results for s-wave and p-wave  $\pi D, \eta D$  and  $\pi D^*, \eta D^*$  phase shifts ...
  - predict that QCD forms exotic flavour sextet states with  $J^P = 0^+$  and  $J^P = 1^+$
  - p-wave resonance from unitarized anomalous threshold effects

## ✓ QCD spectroscopy with coupled-channel dynamics

- current QCD lattice data provide low-energy constants for hadron-hadron scattering
- use as input in systematic coupled-channel computations (GPA)
- analyze and predict the quark-mass dependence of hadron resonances in QCD

thanks to: Yonggoo Heo, Xiao-Yu Guo and Csaba Korpa