RUHR UNIVERSITÄT BOCHUM RUB

Meson Molecules with Heavy Quarks

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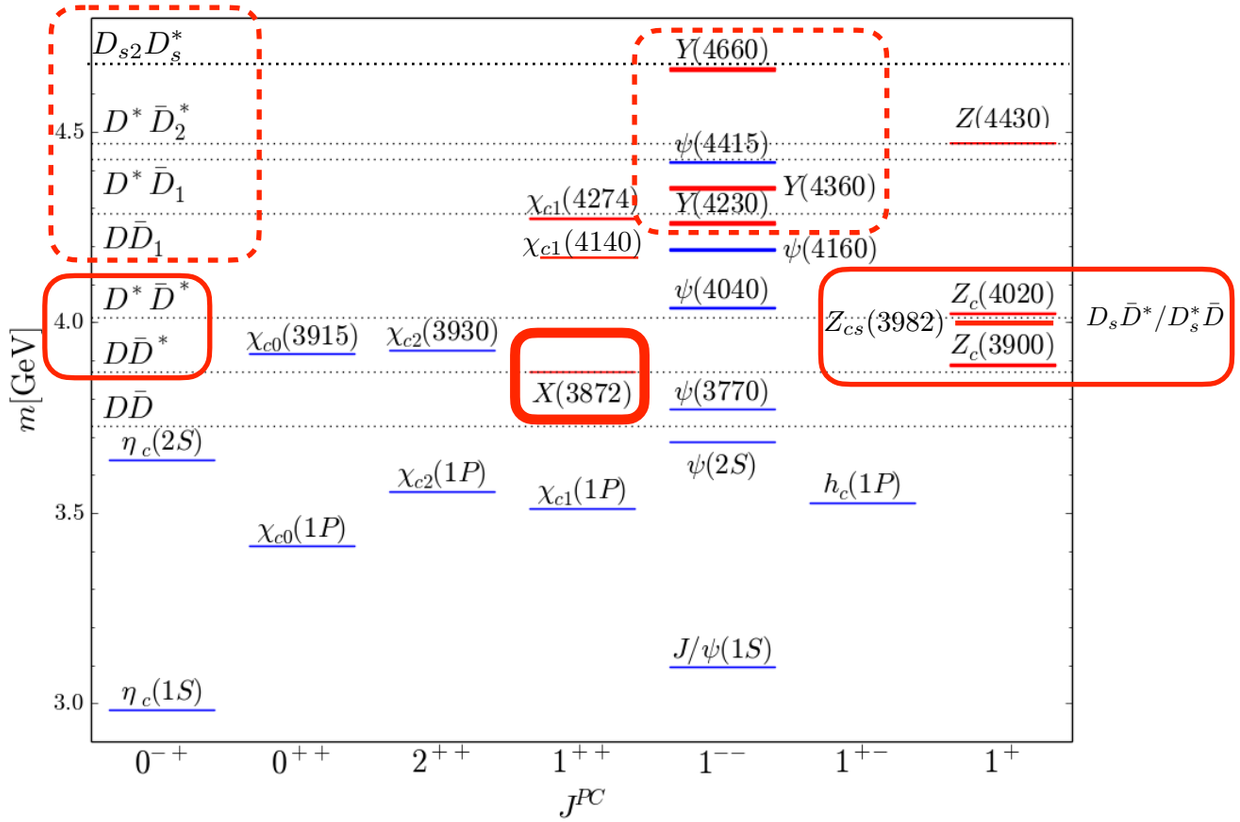
Meson 2023

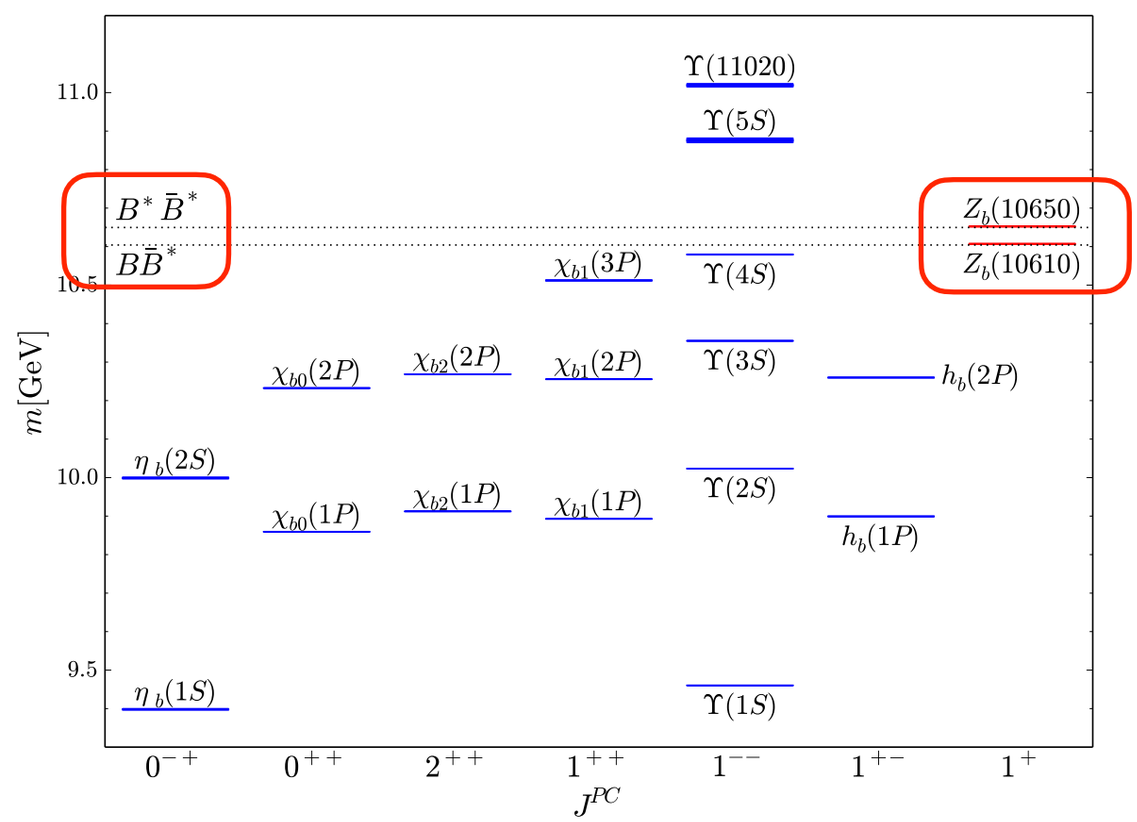
Based on *EPJA* 57, 101(2021); *PLB* 833(2022); *PRD* 105, 014024(2022); 2303.09441 [hep-ph]

in collaboration with

X. Dong, M. Du, E. Epelbaum, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, I. Matuschek, J. Nieves and Q. Wang

"Tetraquarks" in quarkonium spectrum





- Discovery of the X(3872) by Belle in 2003: New era in hadron spectroscopy XYZ exotics

  — Many are sitting near some two meson thresholds

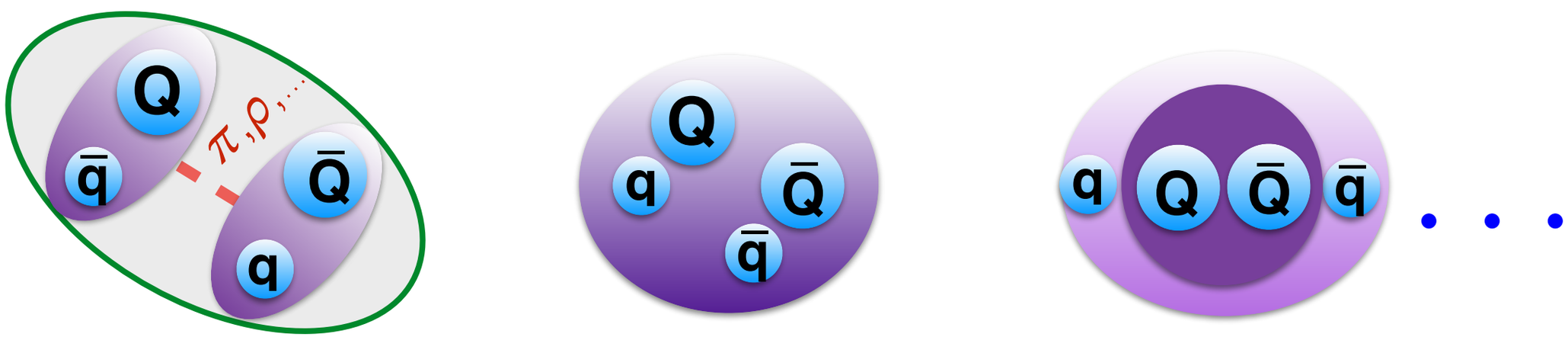
  — Some are charged $\Longrightarrow$ manifestly exotic: $Z_b(10610)/Z_b(10650)$, $Z_c(3900)/Z_c(4020)$, $Z_{cs}(3982)/Z_{cs}(4000)$

  — Abundance of Y-states ($J^{PC}=1^{--}$) near $D_J\, D_{(s)}^{(*)}$ (J=1,2) thresholds

talks by Frank Nerling and Leon von Detten on Thursday

Evidence for Exotic States near thresholds

- Heavy-light sector $D_{s0}(2317)$, $D_{s1}(2460)$, $X_{0/1}(2900)$,… $cqq\bar{q}$
- XYZ
  - X(3872), …
  - $Z_c(3900)$, $Z_c(4020)$, $Z_{cs}(3982)$ …
  - Y(4230), Y(4360), Y(4660), … $c\bar{c}q\bar{q}$
  - $Z_b(10610)$, $Z_b(10650)$ $b\bar{b}q\bar{q}$
  - X(6900) $cc\bar{c}\bar{c}$
- Pentaquarks $P_c(4312)$, $P_c(4440)$, $P_c(4457)$, $P_{cs}(4459)$ $c\bar{c}qqq$
- double c-quark Tcc $ccq\bar{q}$



molecule **tetraquark** **hadroquarkonium**

Size: $\sim 1/\sqrt{2\mu E_B} >> 1\text{fm}$ $\sim 1/\Lambda_{\text{QCD}} \sim 1\text{fm}$

Weinberg compositeness

Weinberg 1963-65

Physical coupling and ERE parameters via probability of a molecular component *X*

$$a = -2\,\frac{X}{1+X}\,\frac{1}{\gamma} + \mathcal{O}(1/\beta) \qquad r = -\frac{1-X}{X}\,\frac{1}{\gamma} + \mathcal{O}(1/\beta) \qquad g_R^2 = \frac{2\pi\gamma}{\mu^2}\,X + \mathcal{O}(1/\beta)$$

a < 0 — bound state

VB et al. PLB 2004

If $|a| \gg |r|,\ r \sim 1/\beta \quad \Longrightarrow \quad X \to 1$ ⟹ Molecule

If $|a| \ll |r|,\ r < 0 \quad \Longrightarrow \quad X \to 0$ ⟹ Compact state

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- Same information can be inferred from pole counting

Morgan 1992

— one near-thr. pole: $k_1 = -\frac{i}{a}\left[1 + \mathcal{O}\left(\frac{r}{a}\right)\right]$ $\quad |a| \gg |r|$ $\Longrightarrow$ Molecule

— two near-thr. poles $k_{1,2} = \pm i\sqrt{\frac{2}{a\,r}} + \frac{i}{r} + \mathcal{O}\left(\sqrt{\frac{a}{r^3}}\right)$ $\quad |a| \ll |r|$ $\Longrightarrow$ Compact state

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- Extensions mostly for resonances by Jido, Kamiya, Nieves, Oller, Oset, Sekihara,…

review
Kamiya and Hyodo 2017

- Recent generalisations to virtual states, coupled-channels, …

see also talk by Kinugawa on Thursday

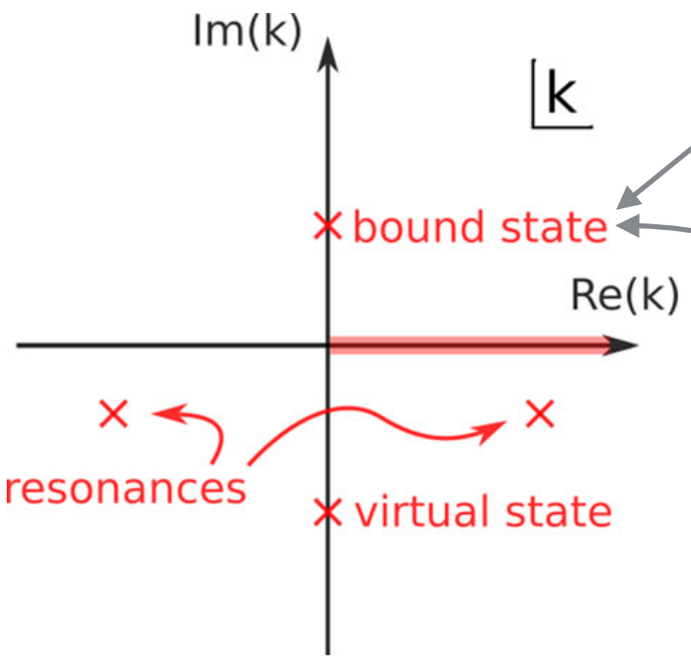
Matuschek et al. EPJA 57 (2021)
VB et al., PLB 833 (2022)

- Insights on range effects

Albaladejo, Nieves 2022, Li et al. 2022, Song et al 2022, Kinugawa, Hyodo 2022

Extensions beyond bound states

Matuschek, VB, Guo, Hanhart
EPJA 2021



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Compact bound st.

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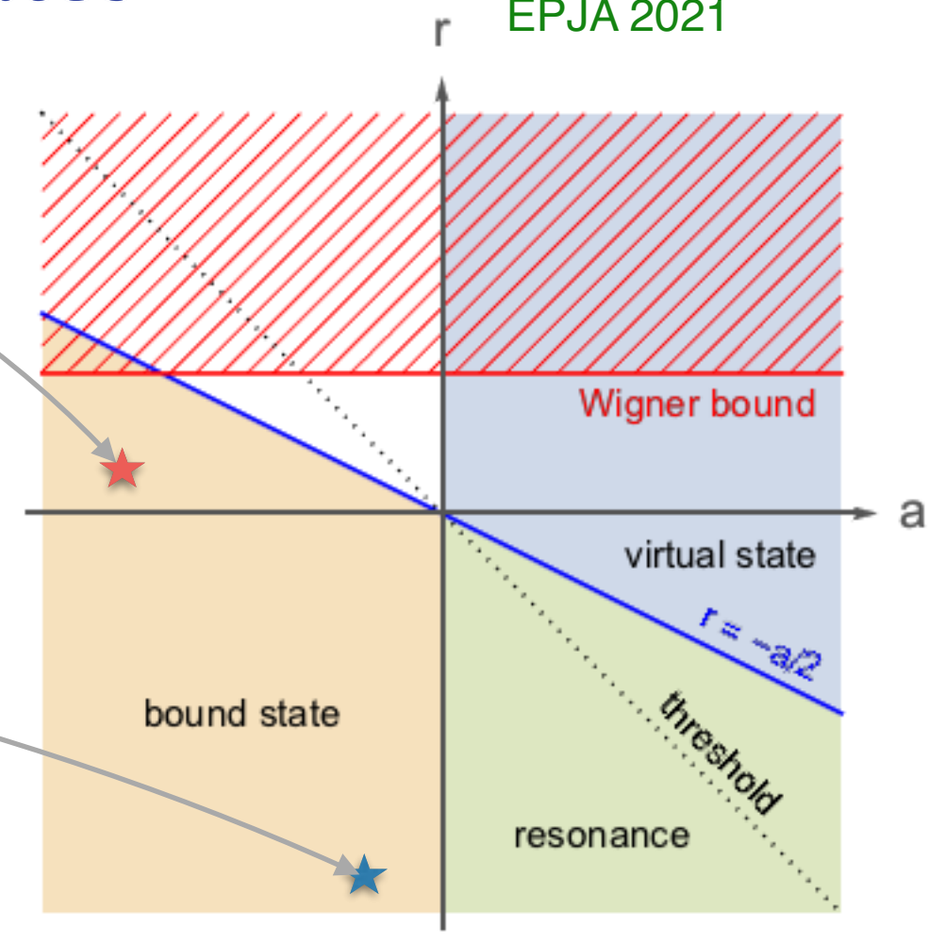

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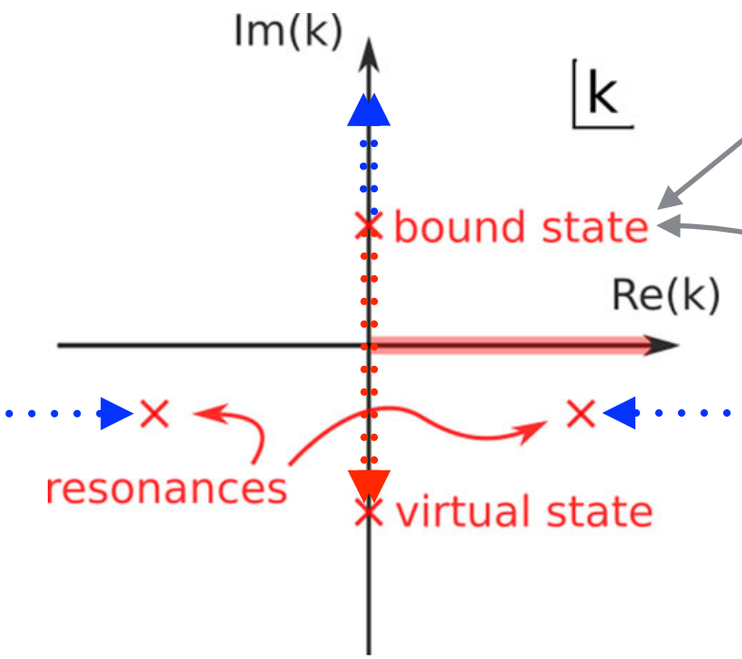

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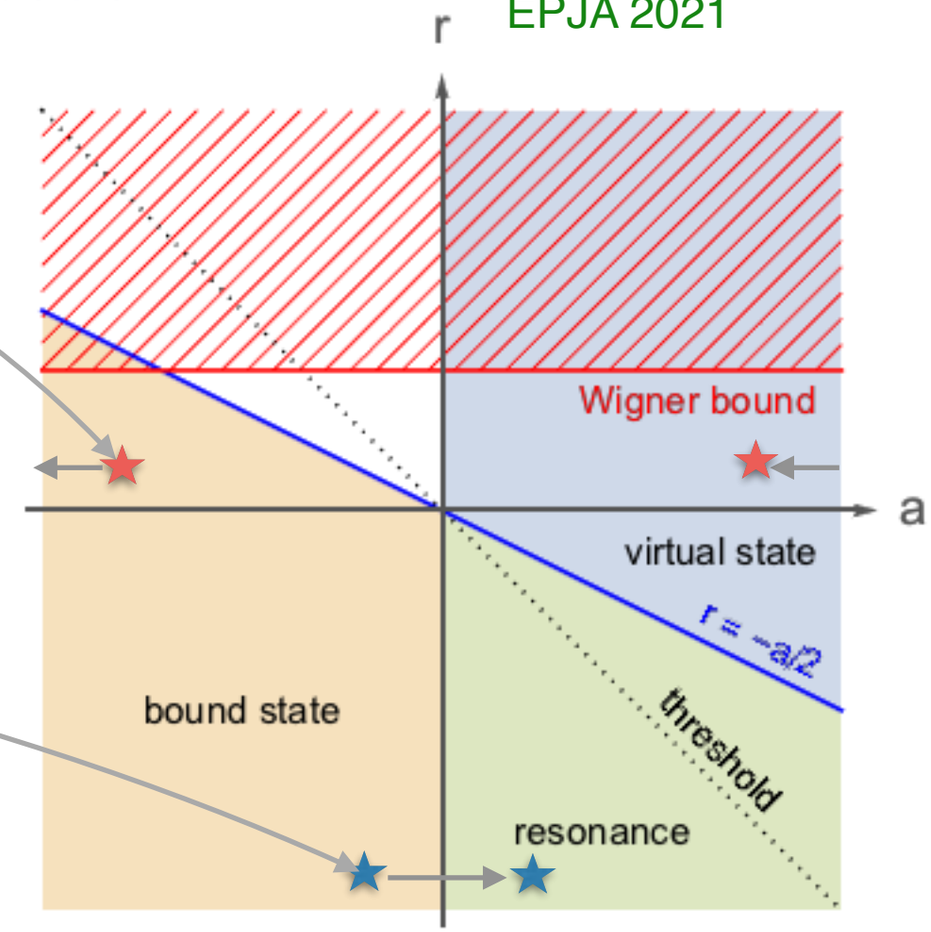

- Evolution of poles and analyticity → Extensions beyond bound states

Molecular pole: $k_1 = -\frac{i}{a}\left[1 + \mathcal{O}\left(\frac{r}{a}\right)\right]$ if sc. length changes sign → virtual state
$|a| \gg |r|$

Compact pole: $k_{1,2} = \pm i\sqrt{\frac{2}{a\,r}} + \frac{i}{r} + \mathcal{O}\left(\sqrt{\frac{a}{r^3}}\right)$ if sc. length changes sign → turns to a resonance
$|a| \ll |r|$

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Matuschek, VB, Guo, Hanhart
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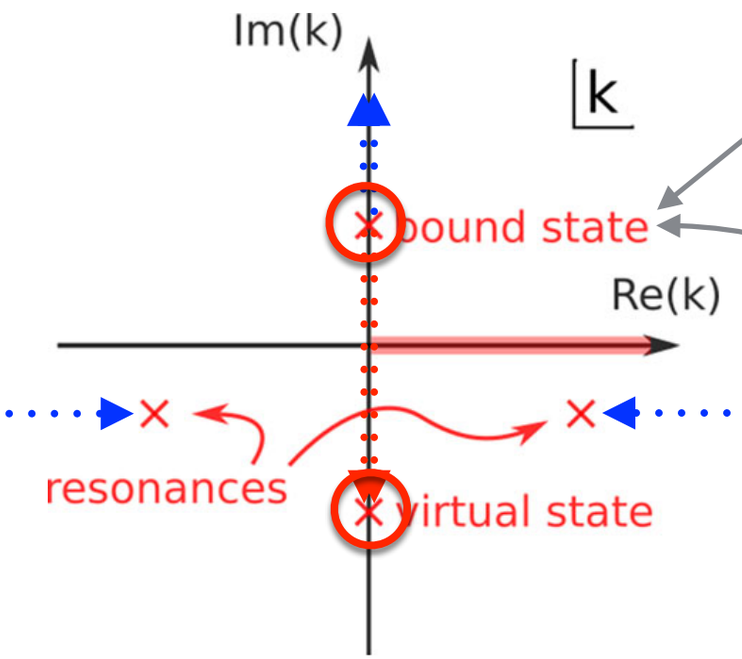

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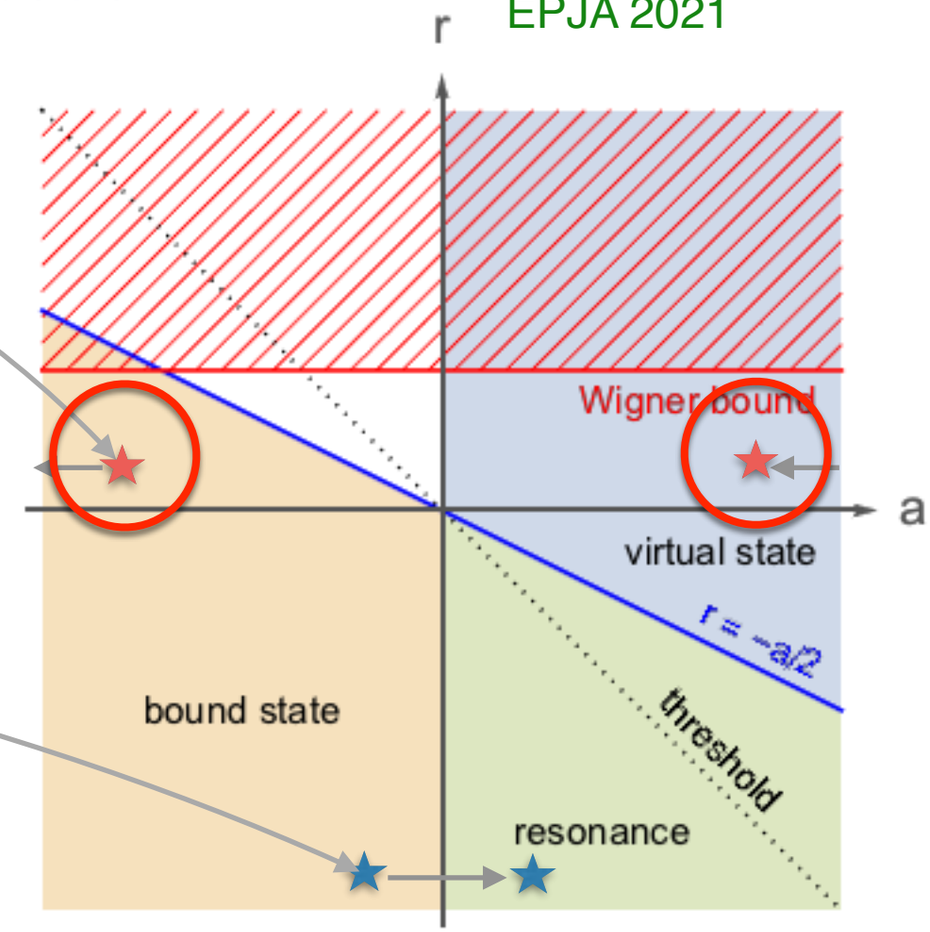

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Near thr. molecules

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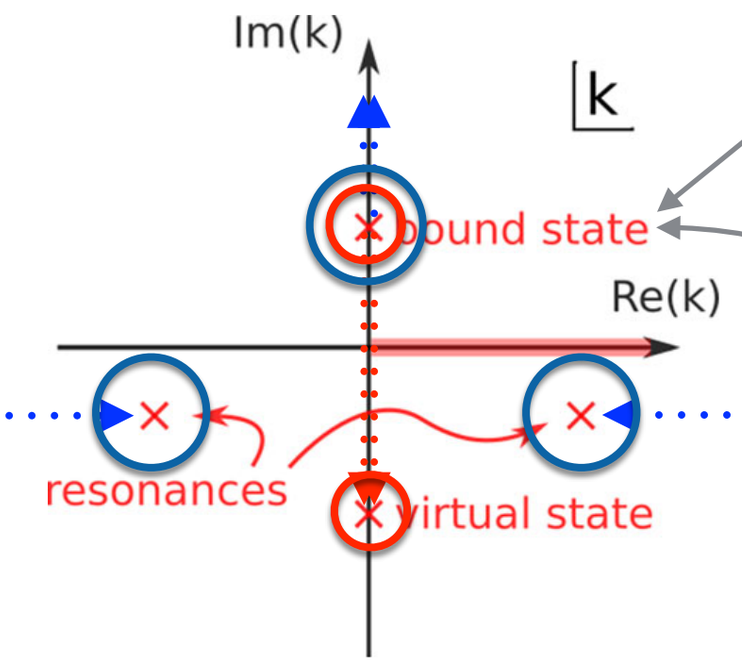

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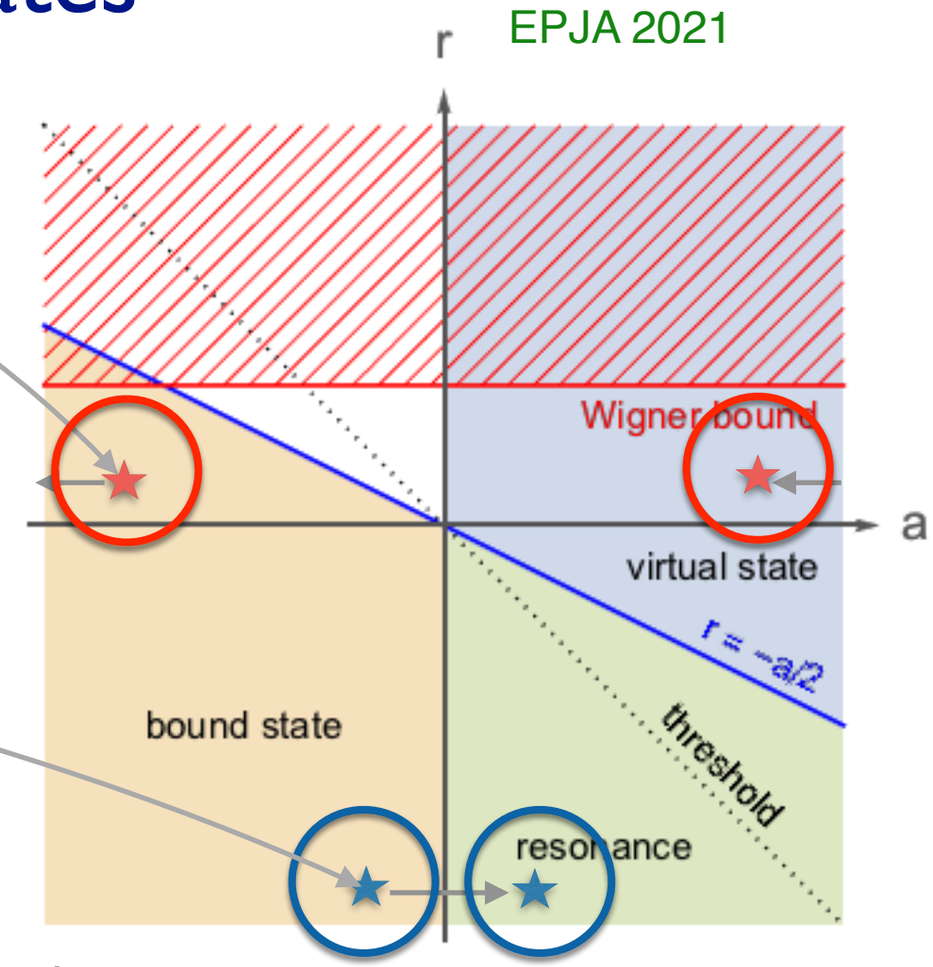

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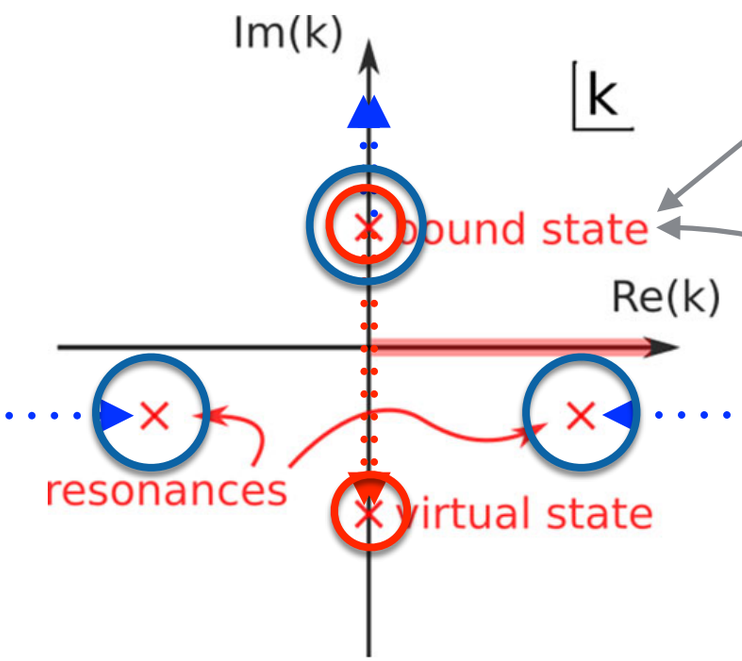
Near thr. compact states

Extensions beyond bound states

Matuschek, VB, Guo, Hanhart
EPJA 2021



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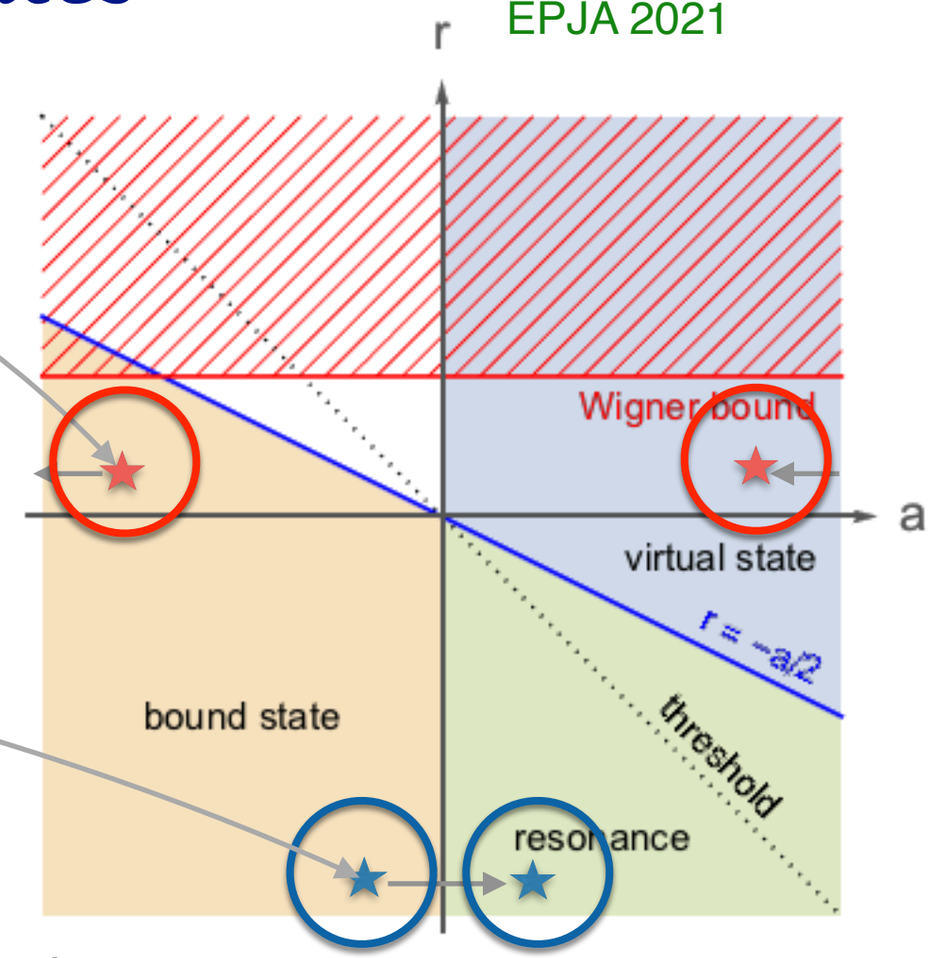

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Near thr. compact states

$$X_W = \sqrt{\frac{1}{1 + 2r/a}} \quad \Longrightarrow \quad \boxed{\bar{X} = \sqrt{\frac{1}{1 + |2r/a|}}}$$

both cases
subsumed here

- $\bar{X}$ allows one to test compositeness for bound/virtual states and resonances

Ex: proton-neutron bound and virtual states

Matuschek, VB, Guo, Hanhart
EPJA 2021

Deuteron

- $a = -5.41$ fm, $r = +1.75$ fm $\Longrightarrow$ large a: $|a| \gg |r|$

  $r \sim O(1/M_\pi)$

$\Longrightarrow$ Clear molecule

- But $X = \sqrt{\dfrac{1}{1+2r/a}} \simeq 1.7 \gg 1$

— X was derived in the zero-range approximation and has a pole when r/a is negative

- Meanwhile, $\bar{X} = \sqrt{\dfrac{1}{1+|2r/a|}} \approx 0.8$

1S0 pn virtual state

- $a = 23.74$ fm, $r = +2.75$ fm $\Longrightarrow$ large a: $|a| \gg |r|$

  Dumbrajs et al 1983

  $r \sim O(1/M_\pi)$

$\Longrightarrow$ Clear molecule

— both a and r changed the sign $\Longrightarrow$ no pole

- $X = \bar{X} \approx 0.9$

$\bar{X} \simeq 1$, as expected for a molecule up to the range corrections!

Identifying a molecule in observables

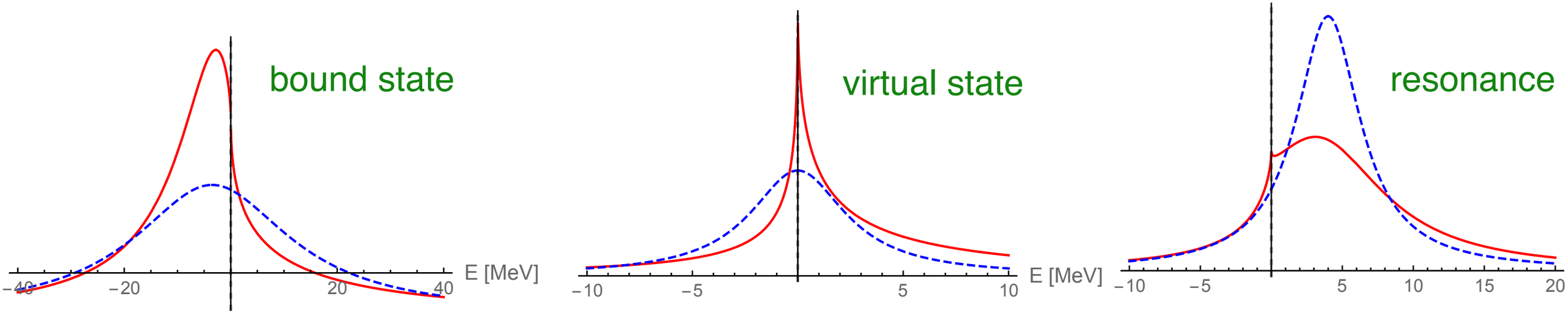
VB et al.(2004, 2005), Braaten et al.(2007), Hanhart et al.(2010), Oset et al.(2012), Oller et al.(2016), …

$$A_{\rm prod} = \frac{\rm const}{E + E_B + \frac{g_0^2 \mu}{2\pi}(ik+\gamma) + i\frac{\Gamma_0}{2}}$$

large g0 $\Longrightarrow$ molecule

g0 =0 $\Longrightarrow$ (Breit-Wigner) compact state

Typical shape of the production rate in the inelastic channels: Molecule vs Compact



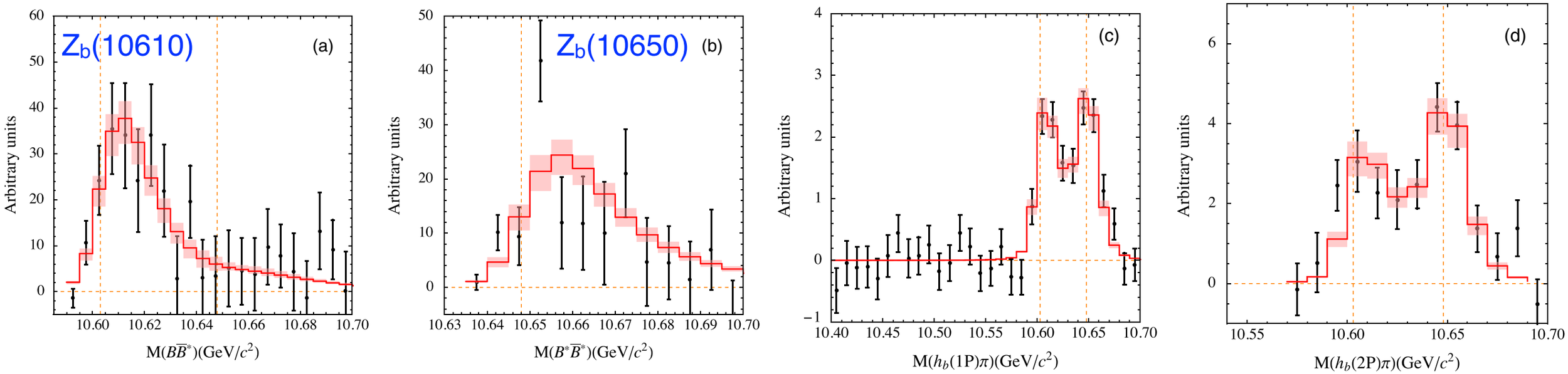
Molecular line shapes are strongly affected by threshold effects enhanced by nearby poles

Access to relevant info: poles and residues

Wang et al. PRD 98(2018)

VB et al. PRD 99(2019), 103(2021)

If there are coupled-channels a molecular pole can be anything: quasi-bound state, virtual states resonances



Fits to Elastic channels only: either bound or virtual states

Fits to all data (no pions): virtual states

Fits to all data (with pions): near-threshold resonances

— Access to relevant info: poles and residues

$\Longrightarrow$ Data in various channels are complementary and very important!

$\Longrightarrow$ An appropriate framework to analyse data: model independent; unitarity (2 and 3-body); analytic (cuts and poles) $\Longrightarrow$ EFT

χEFT approach at low energies

Weinberg (1992)

our works 2010-till now

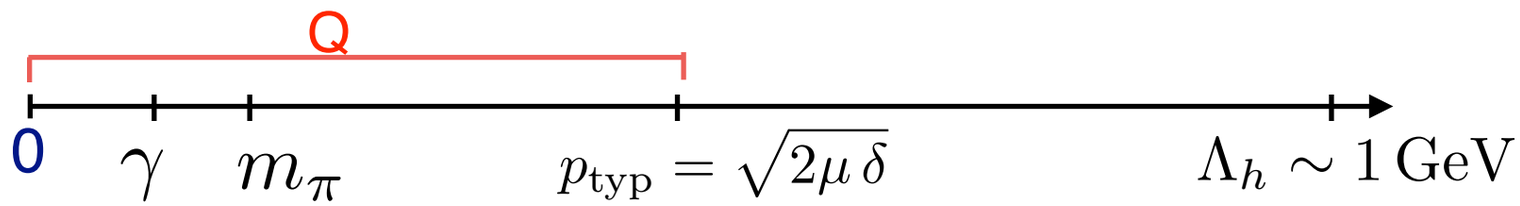
see also AlFiky et al 2006

Fleming et al 2007

- Elastic coupled-channel hadronic potential to a given order in $\chi = Q/\Lambda_h$

$$V^{\rm eff} = V_{\rm LO} + \chi V_{\rm NLO} + \chi^2 V_{\rm N^2LO} + \ldots$$

☛ typical scales



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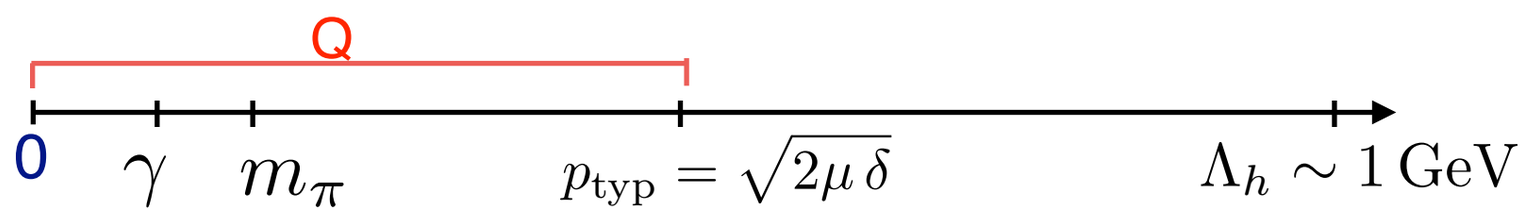

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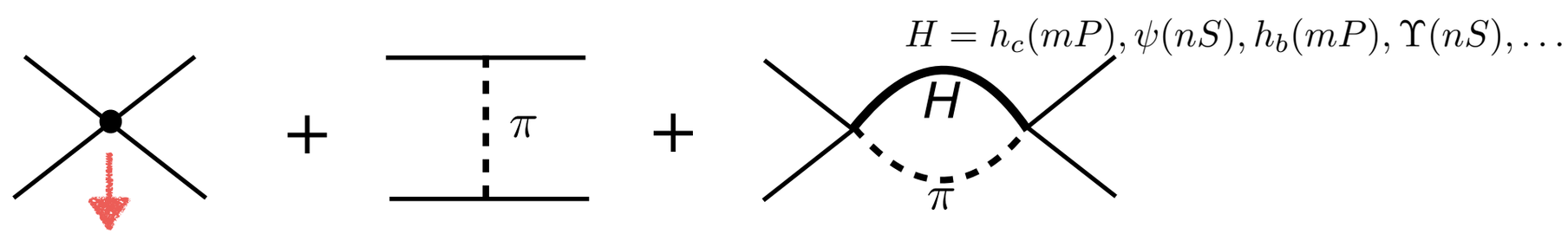

O(Q⁰) contact terms constrained by HQSS

Long range: OPE

Imaginary part from inelastic channels

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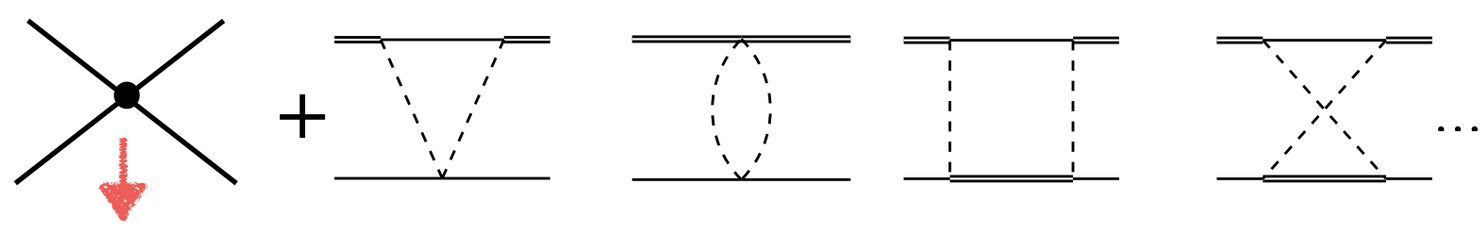

O(Q²) contact terms constrained by HQSS

Simon Krug, Jabez Tom Chacko, VB, Hanhart (in preparation)

see also Meng, Wang, Zhu, … for loops in a different power counting

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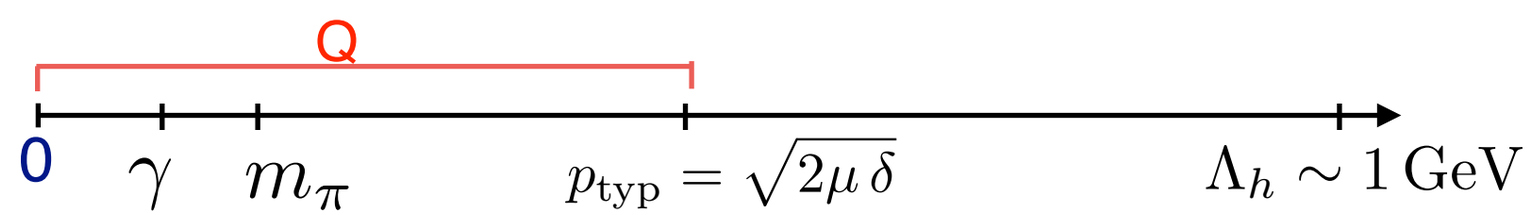

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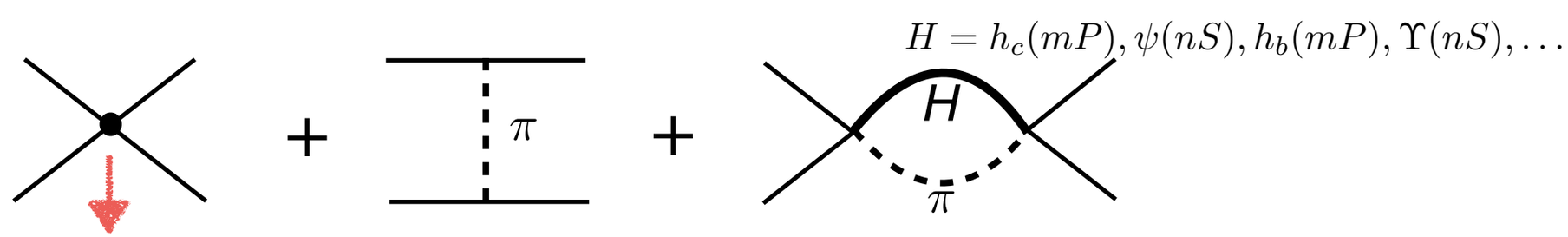

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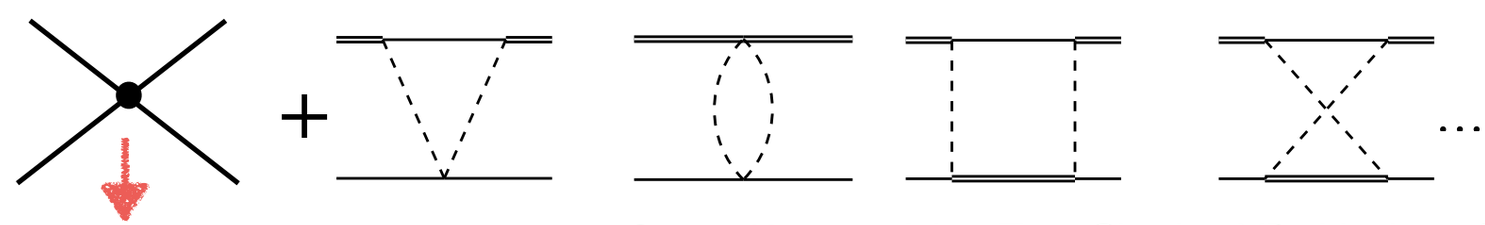

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- Amplitudes are solutions of the coupled-channel ($\alpha\beta$) integral equations

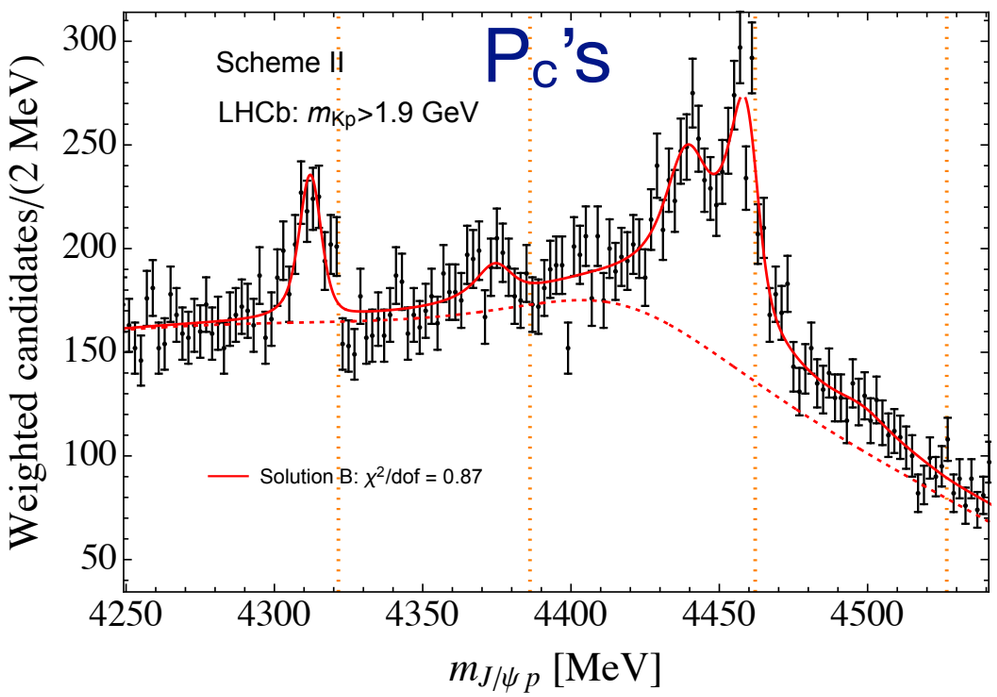
$$T_{\alpha\beta} = V^{\rm eff}_{\alpha\beta} - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\rm eff}_{\alpha\gamma} G_\gamma T_{\gamma\beta}$$

→ Consistent with Unitarity and analyticity

G - Green functions

Various applications

Du et al. PRL (2020), JHEP (2021)



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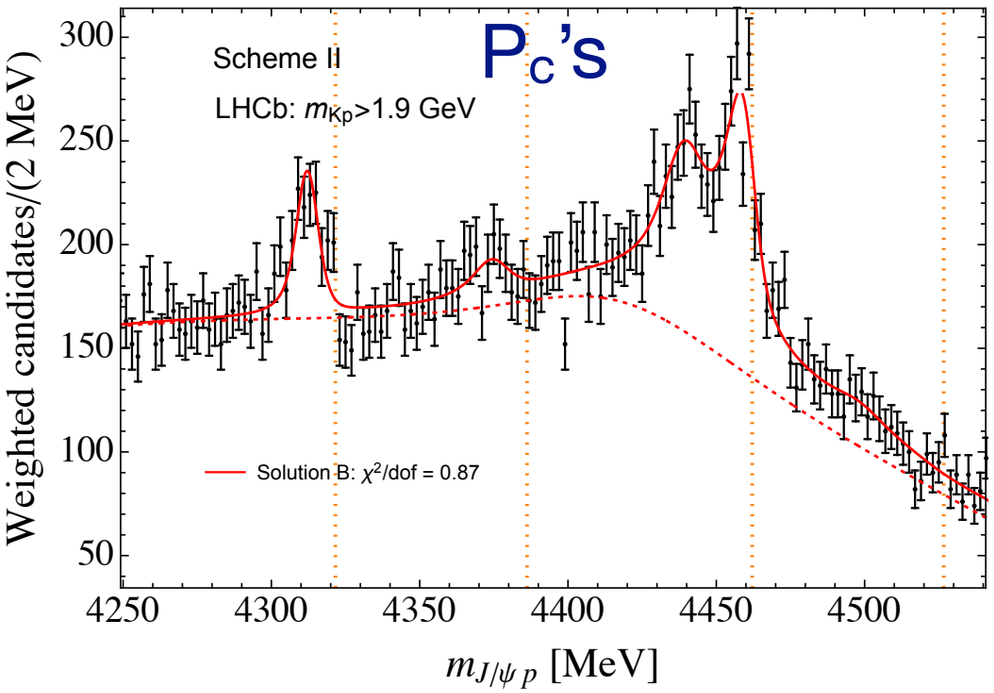
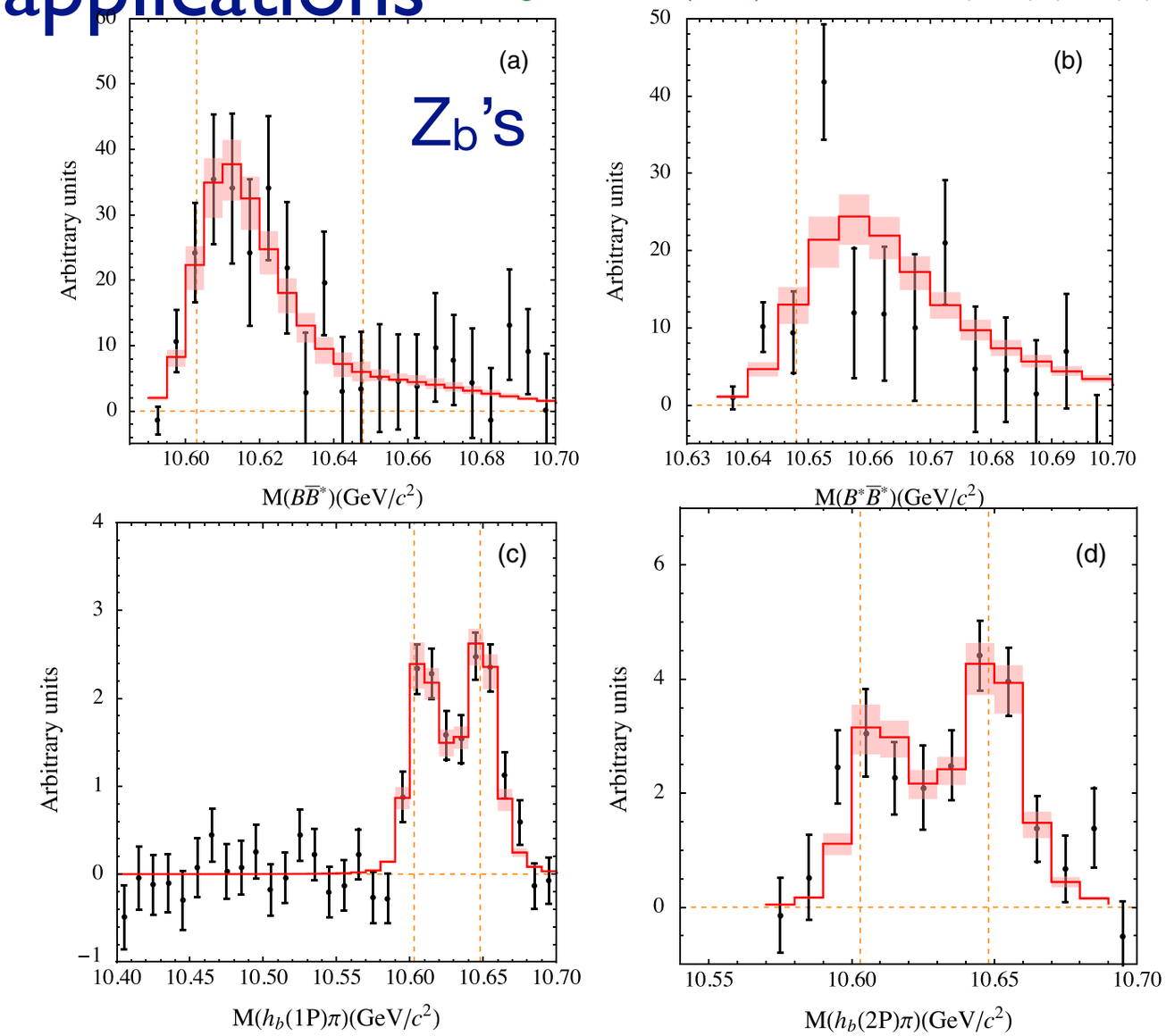


Wang et al PRD (2018). VB et al. PRD (2017), (2019), (2021)



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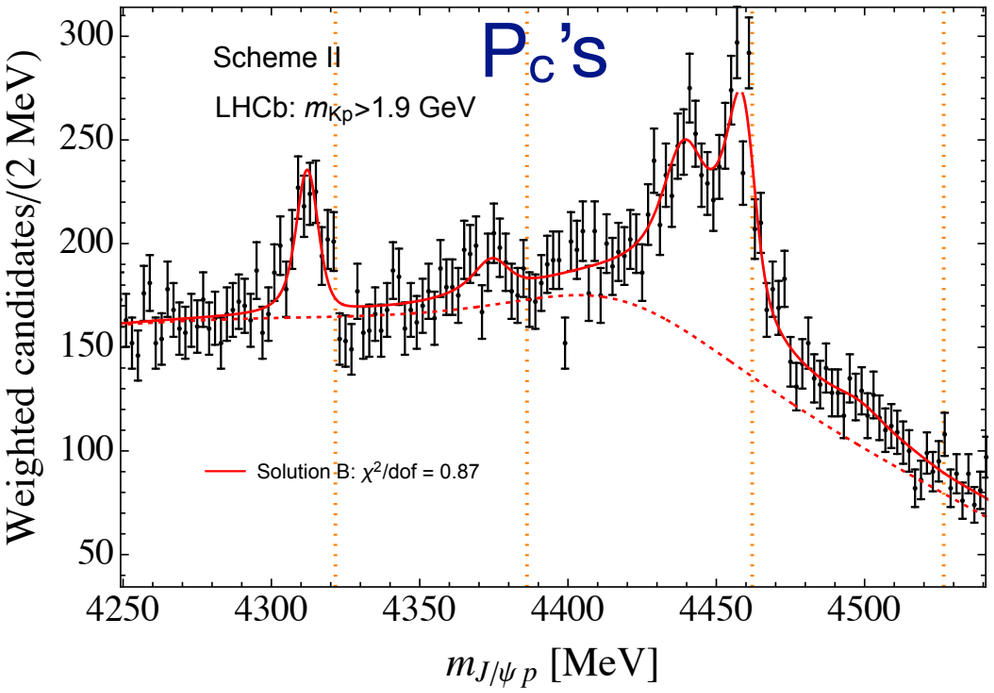

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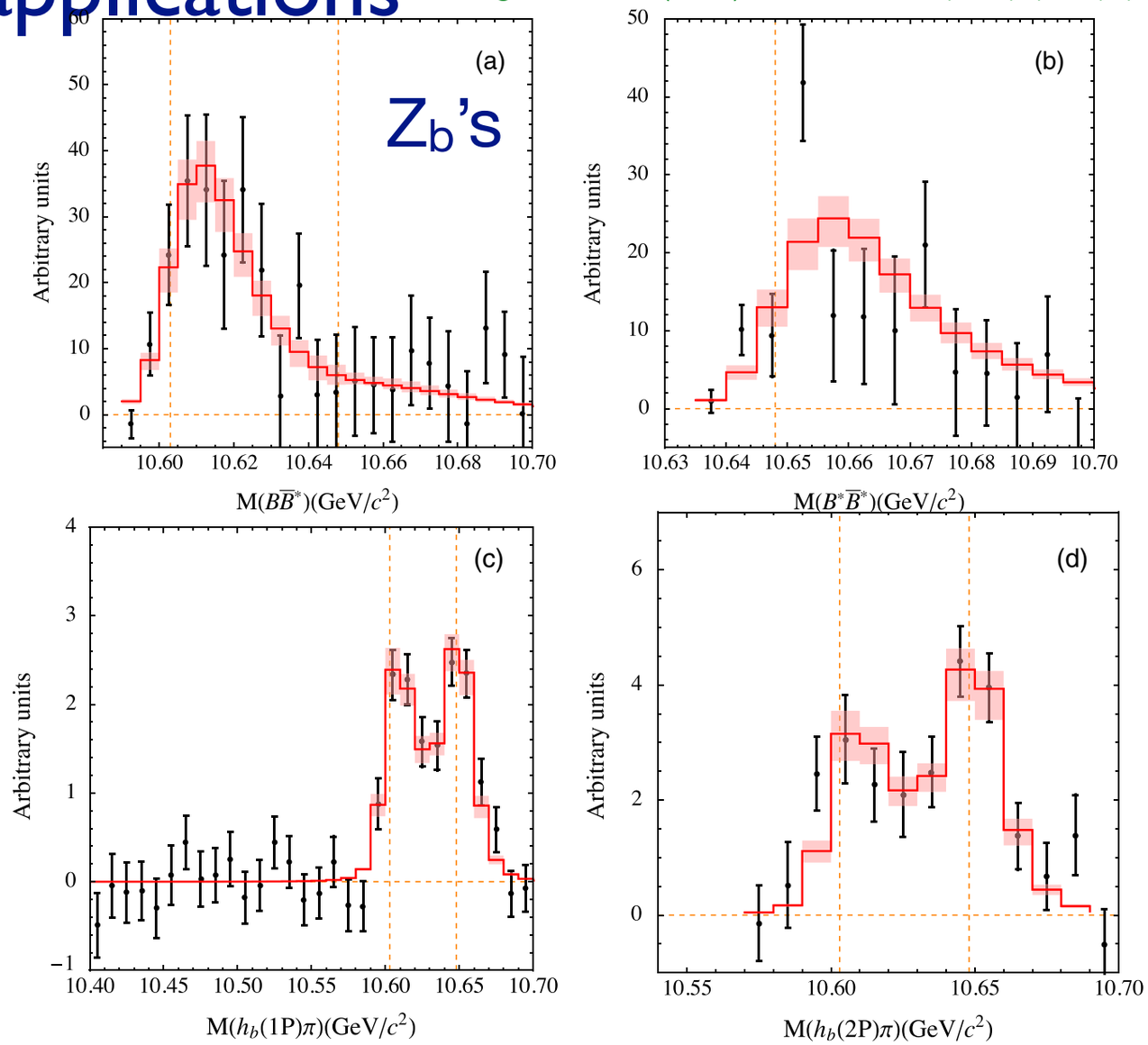
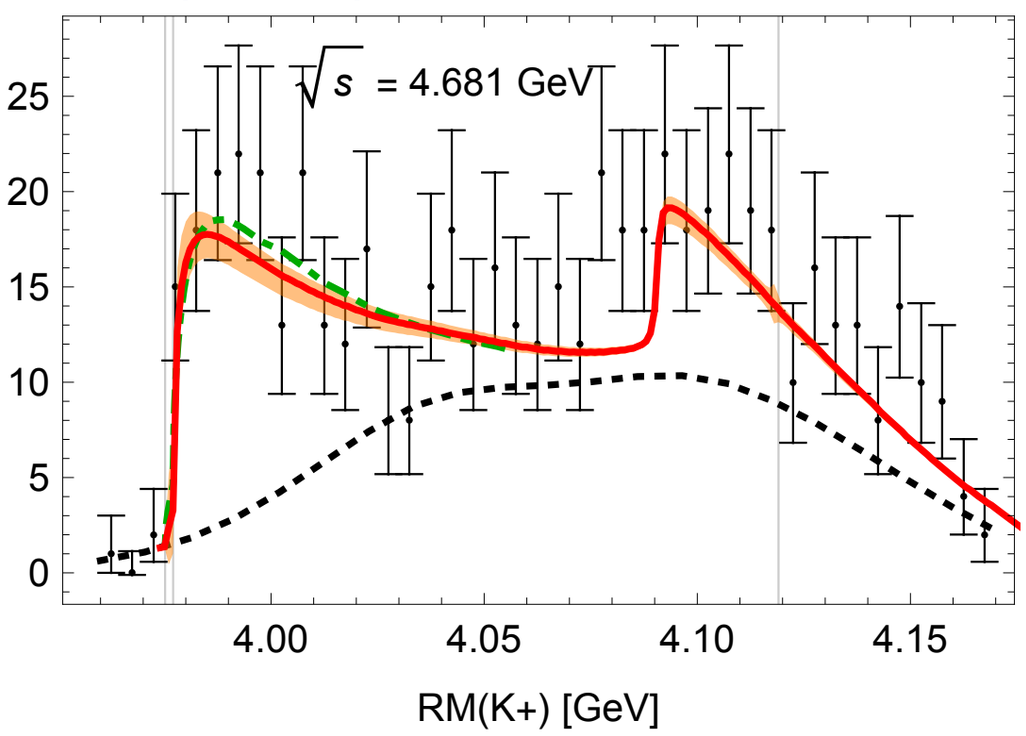


$Z_{cs}(3972)^+$ VB et al. PRD 105 (2022)



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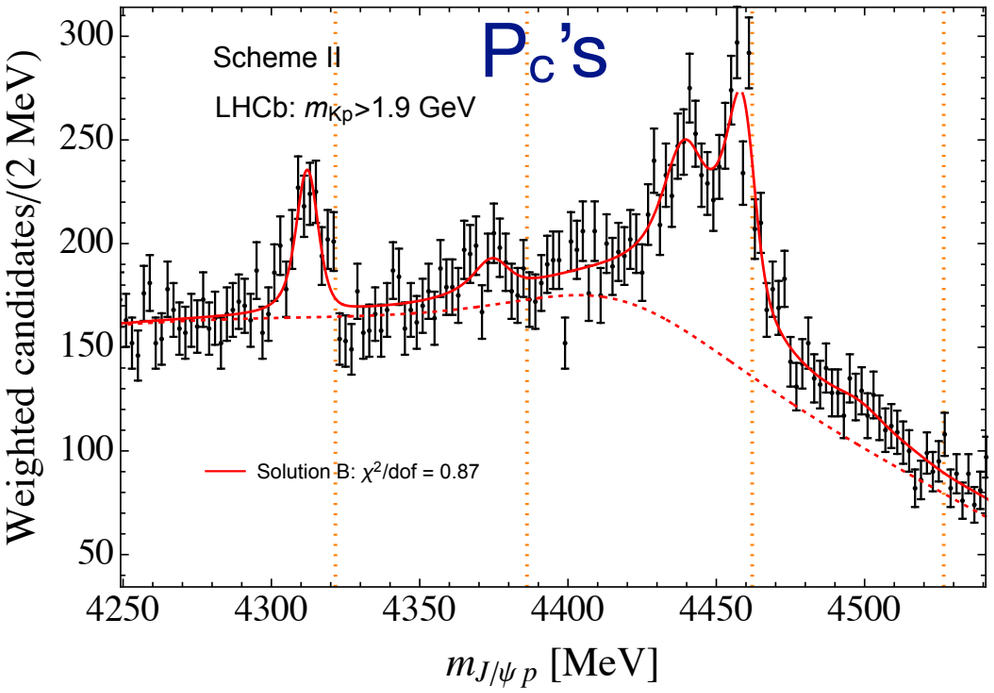

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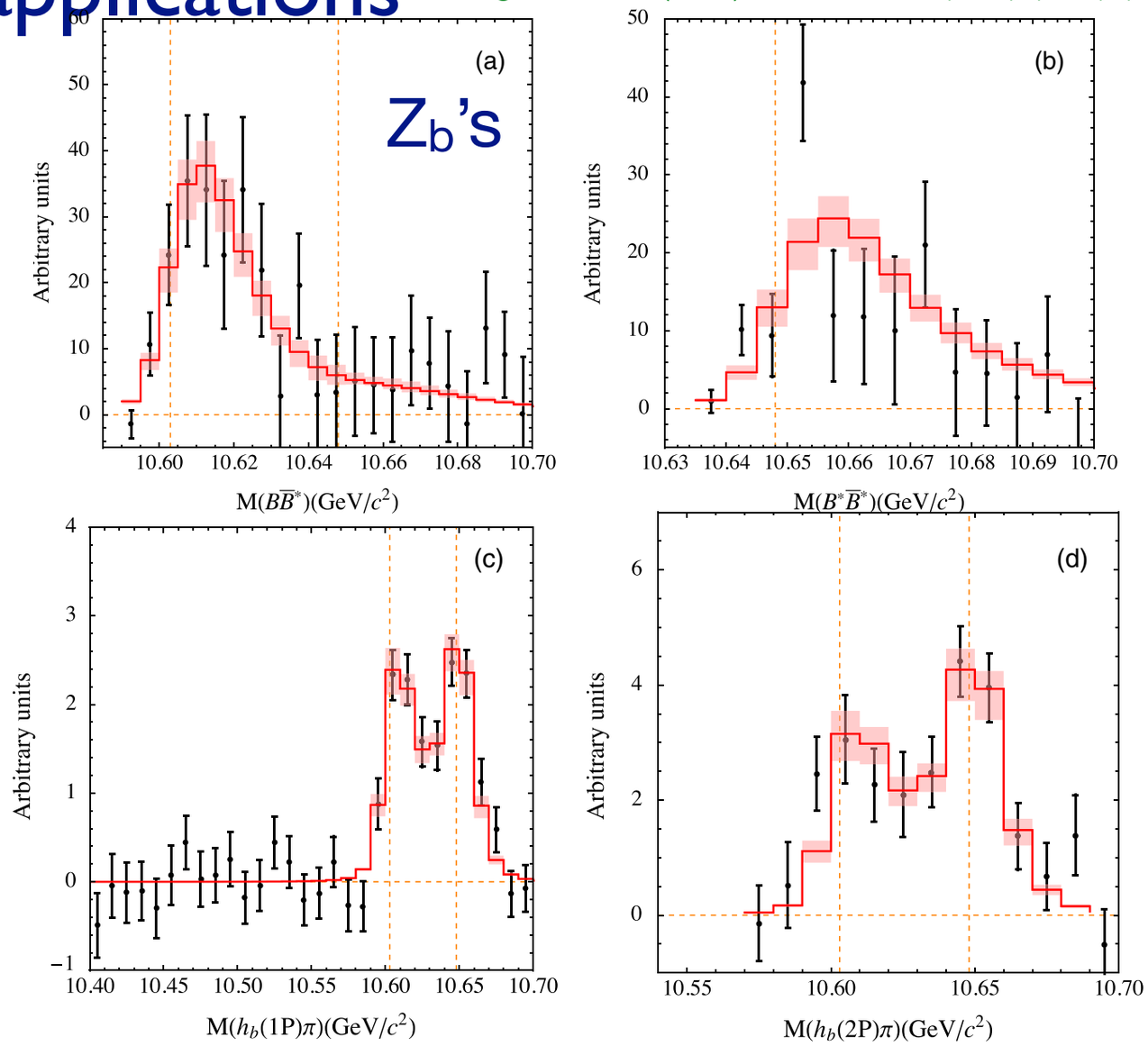
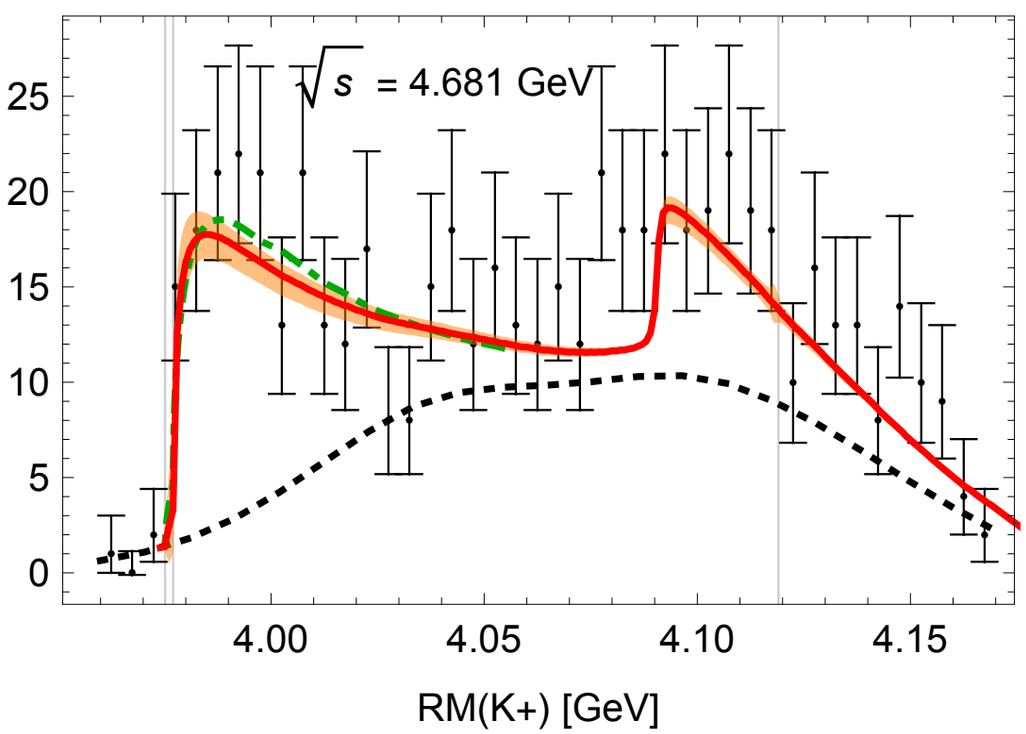
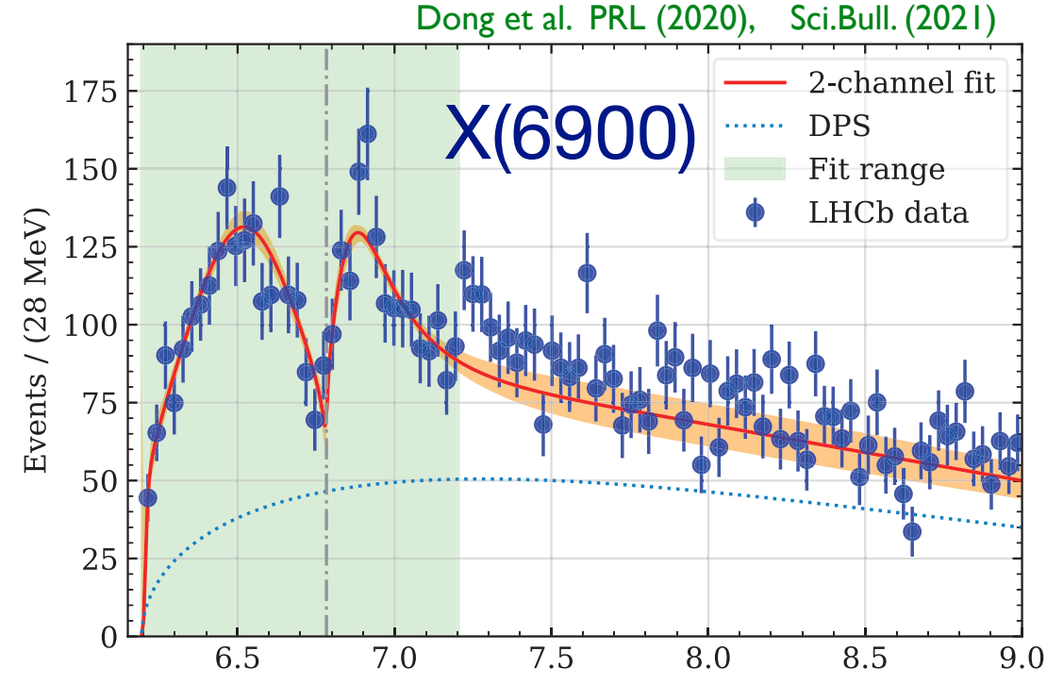


$Z_{cs}(3972)^+$ VB et al. PRD 105 (2022)



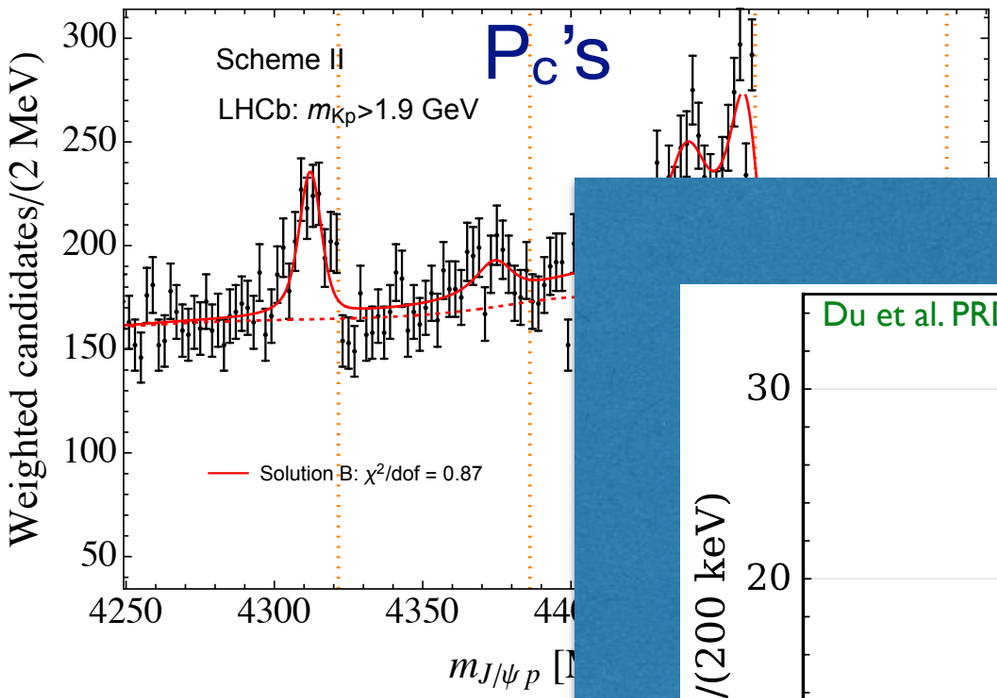
Dong et al. PRL (2020), Sci.Bull. (2021)

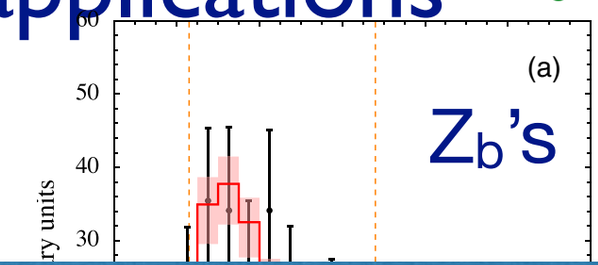


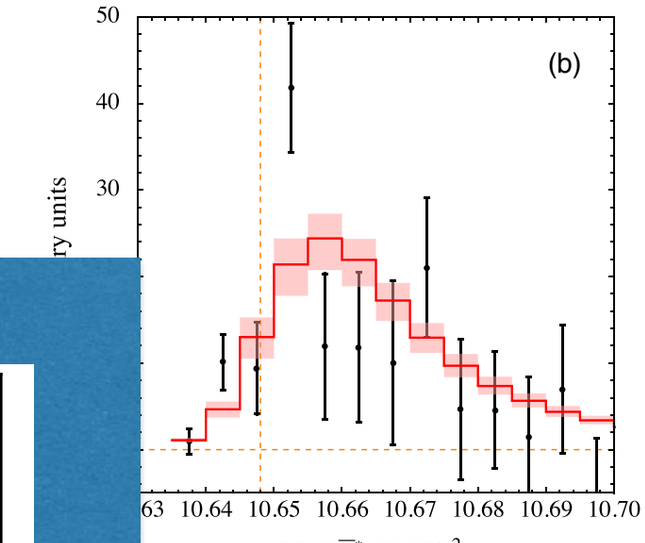
Various applications

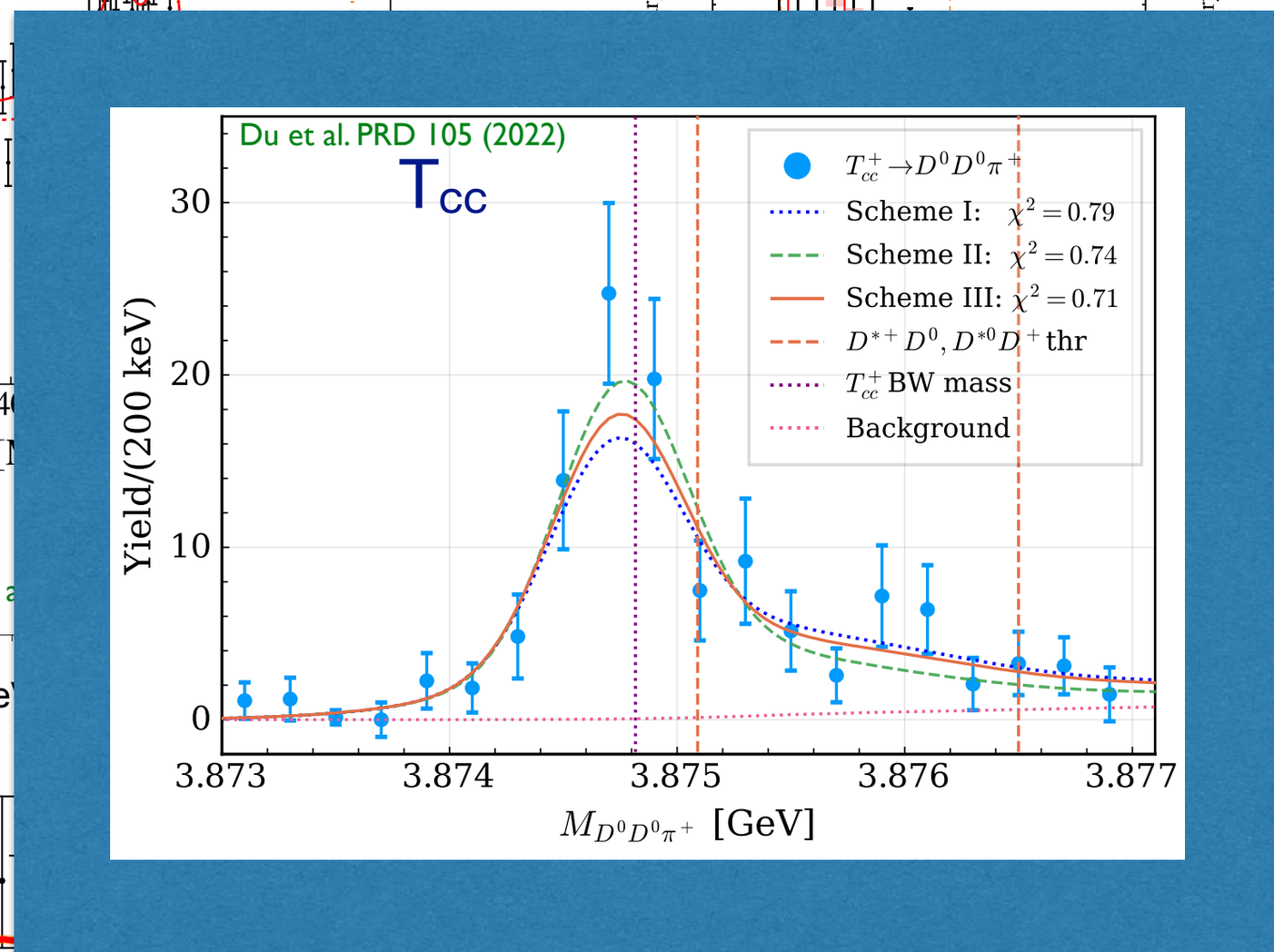
Wang et al PRD (2018). VB et al. PRD (2017), (2019), (2021)

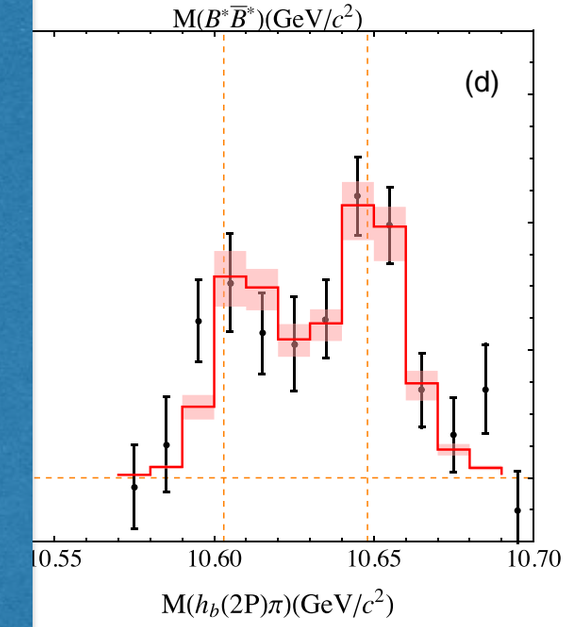
Du et al. PRL (2020), JHEP (2021)



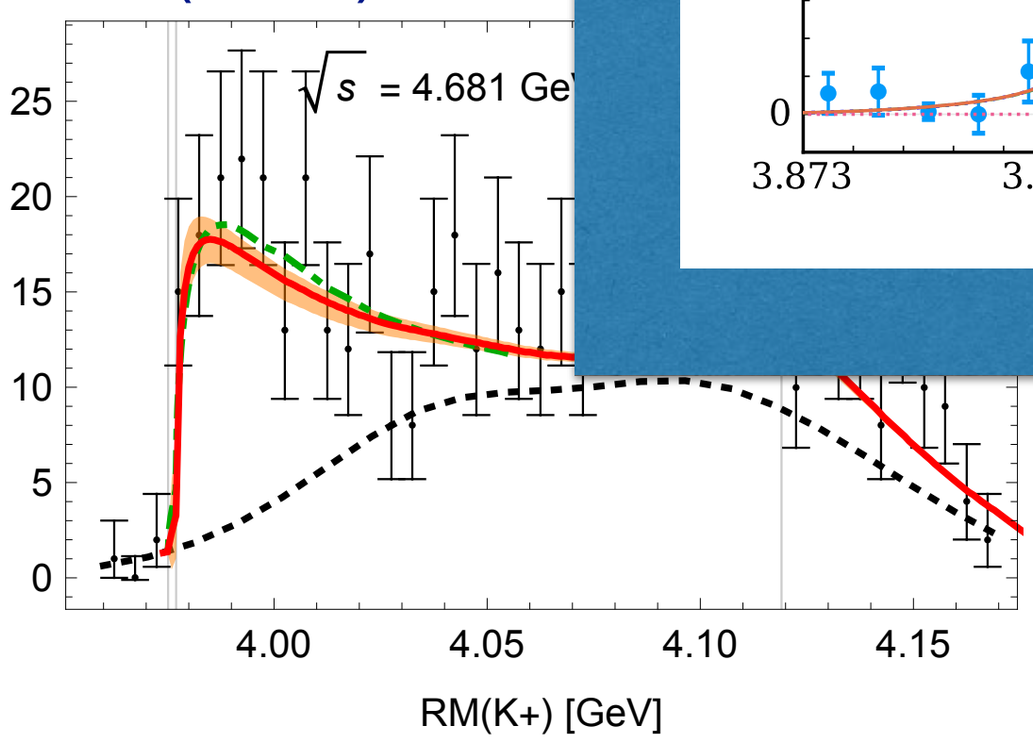








$Z_{cs}(3972)^+$ VB et a



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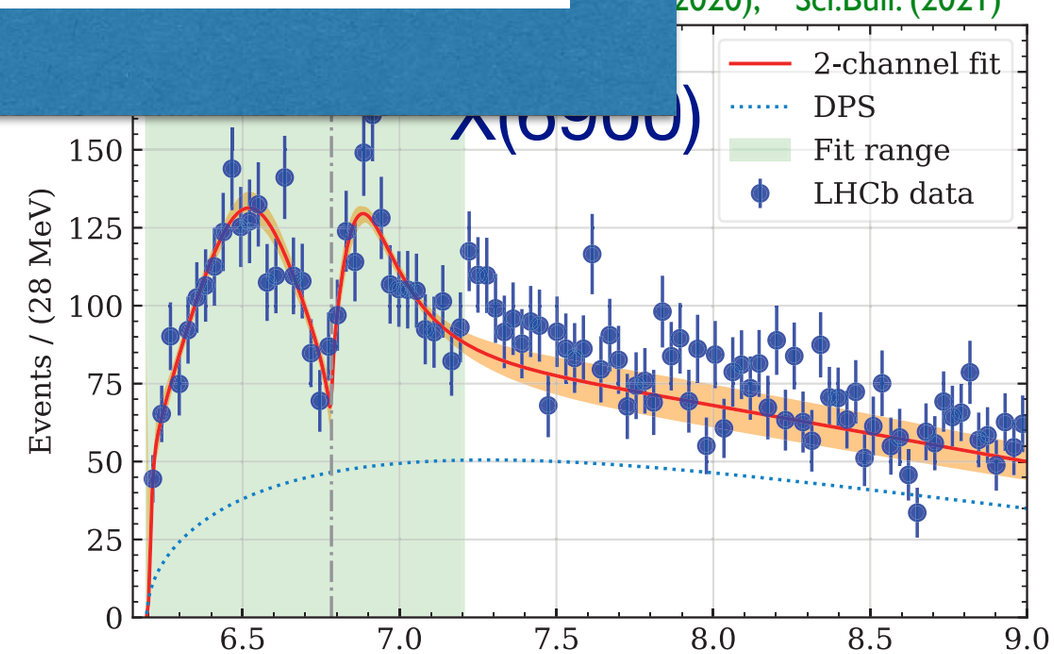


T_{cc} : an excellent case for χEFT

Mikhasenko talk on Friday

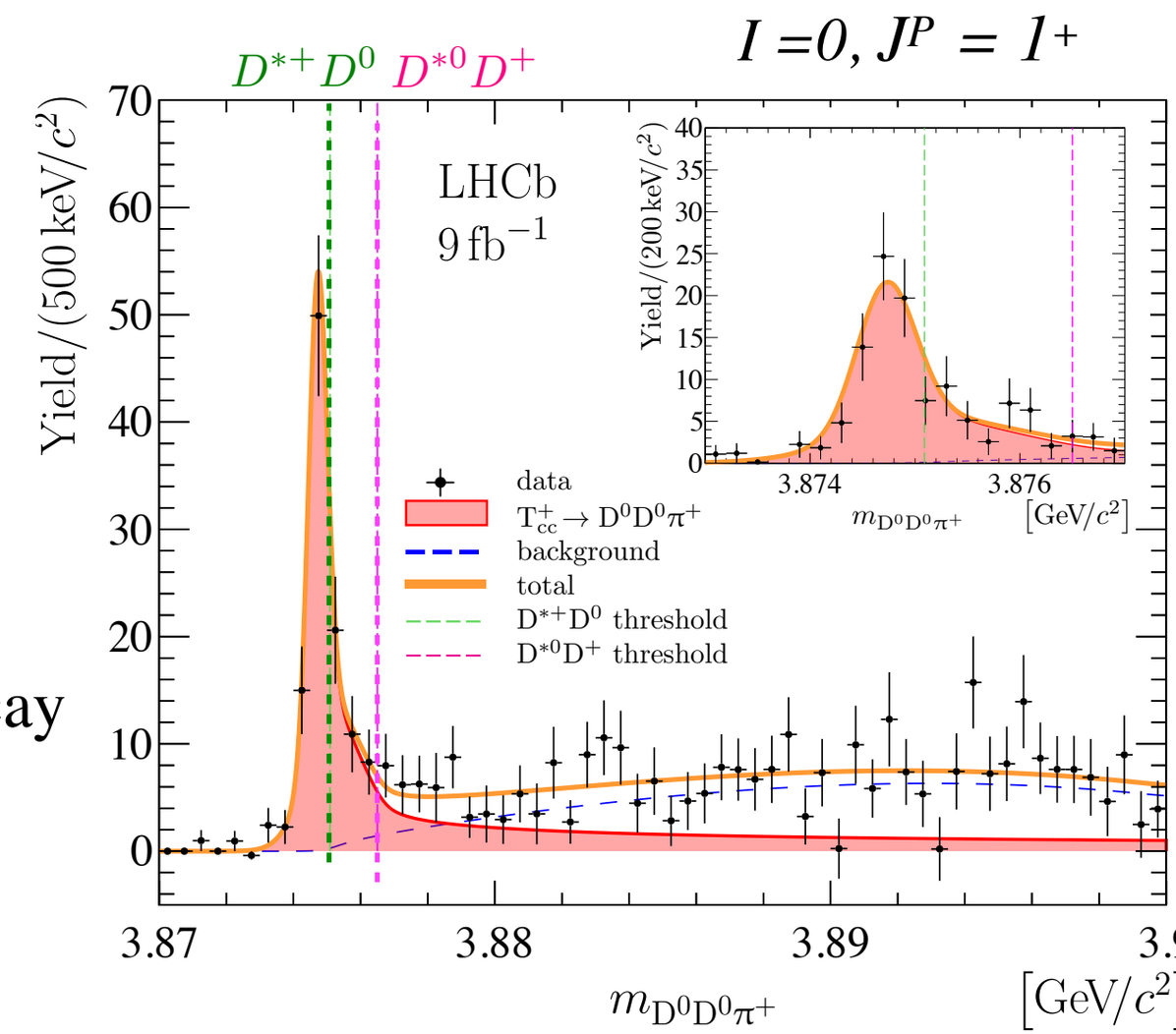
— Expansion in χEFT : $\chi = \frac{\sqrt{2\mu\Delta_M}}{\Lambda_\chi} < 0.1$

$$\Delta_M = m(D^+D^{*0}) - m(D^0D^{*+})$$

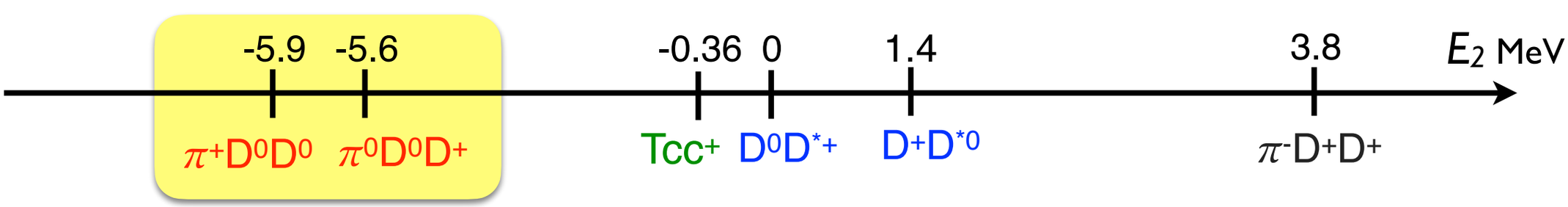
— No admixture of inelastic channels

— Width: almost entirely from the only strong decay

$$T_{cc}^+ \to D^0D^{*+} \to D^0D^0\pi^+ / D^0D^+\pi^0$$



Aaij et al [LHCb] Nature Physics (2022)
Nature Comm.(2022)



We study: Tcc properties including pions and various cuts (3-body, left-hand cuts)

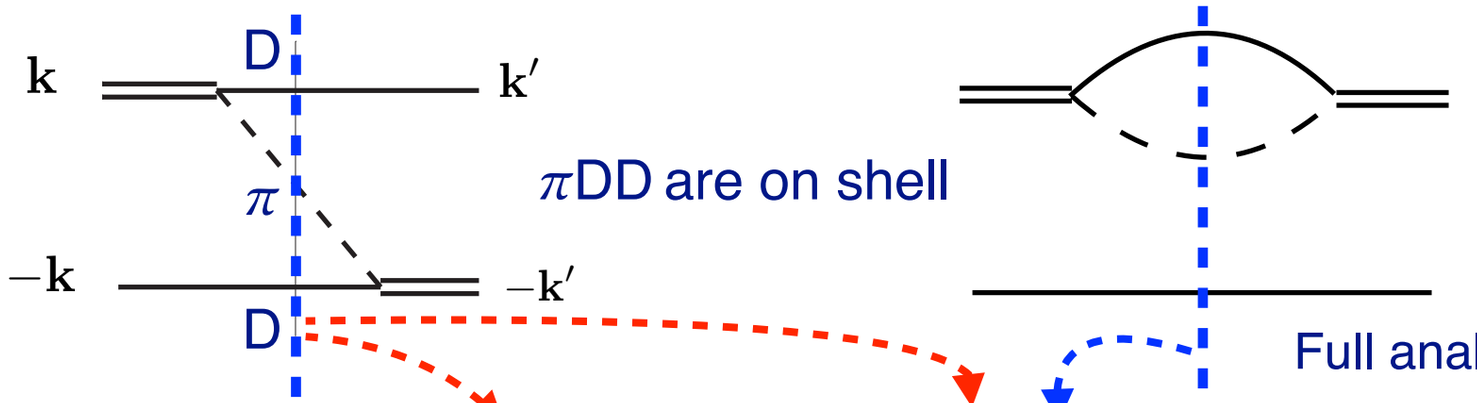
$\Rightarrow$ Proper analytic structure of the scattering amplitude

Tcc in χEFT at leading order

Du et al. PRD 105, 014024 (2022)

- 3-body cut stems from one-pion exchange (OPE) and self energies in the Green funct.

$$G_{3\text{body}} \to \delta(E_D(k) + E_D(k') + E_\pi(\mathbf{k} - \mathbf{k}') - E)$$



πDD are on shell

$$T_{\alpha\beta} = V^{\text{eff}}_{\alpha\beta} - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\text{eff}}_{\alpha\gamma} G_\gamma T_{\gamma\beta}$$

Full analogy to the X(3872) from Faddeev-type 3-body Eqs.

VB et al. PRD84 2011

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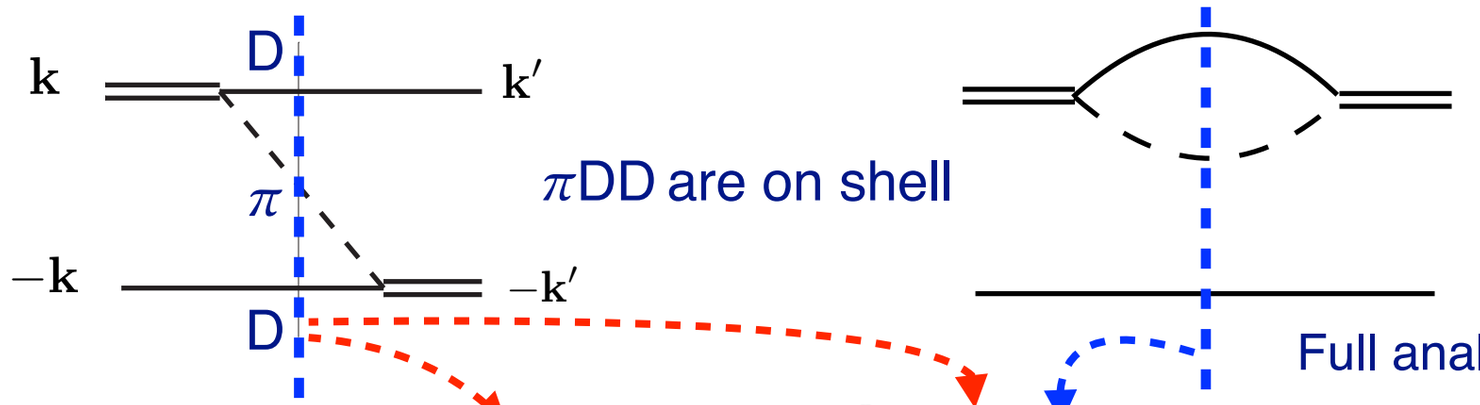

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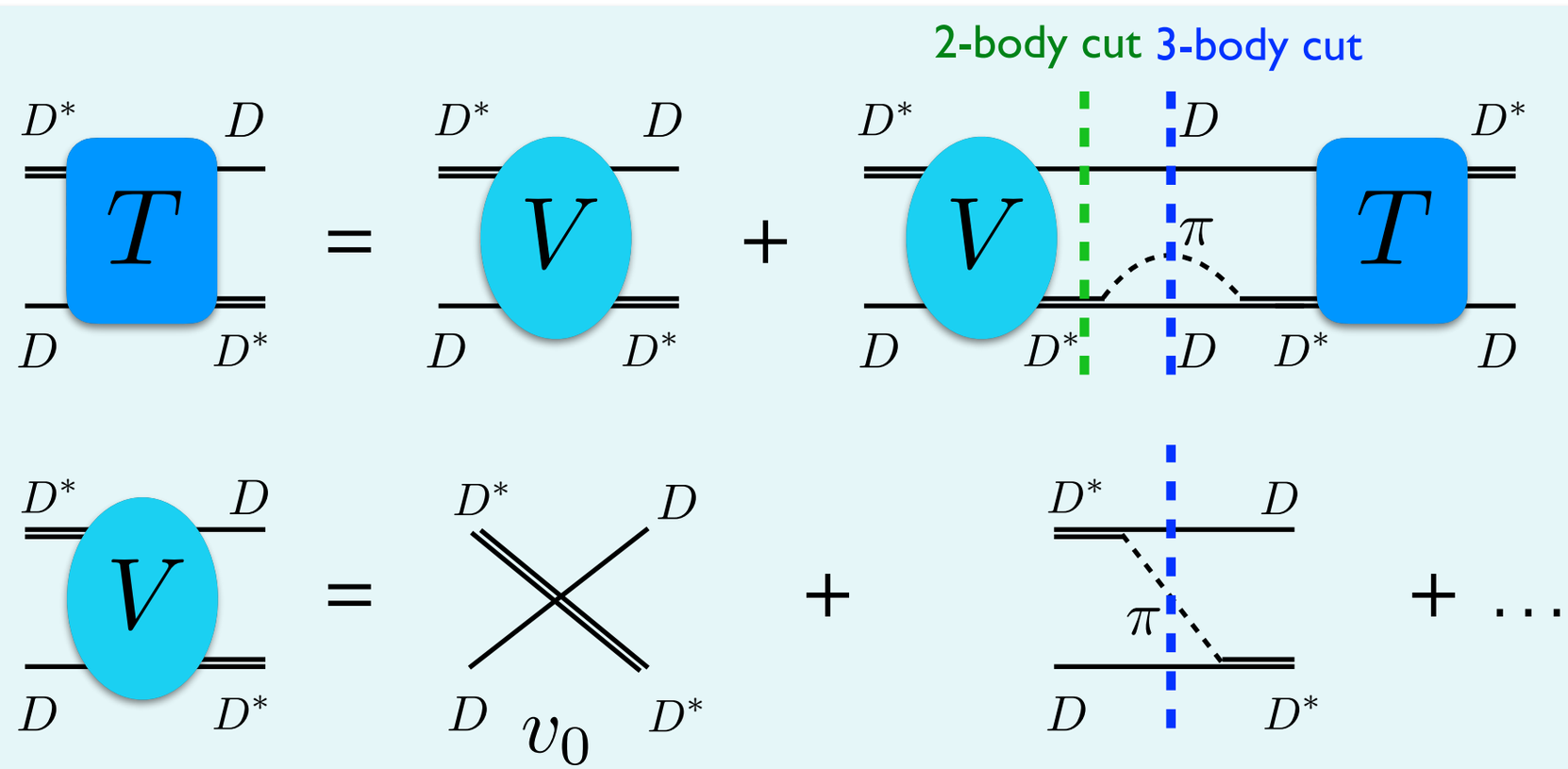
VB et al. PRD84 2011

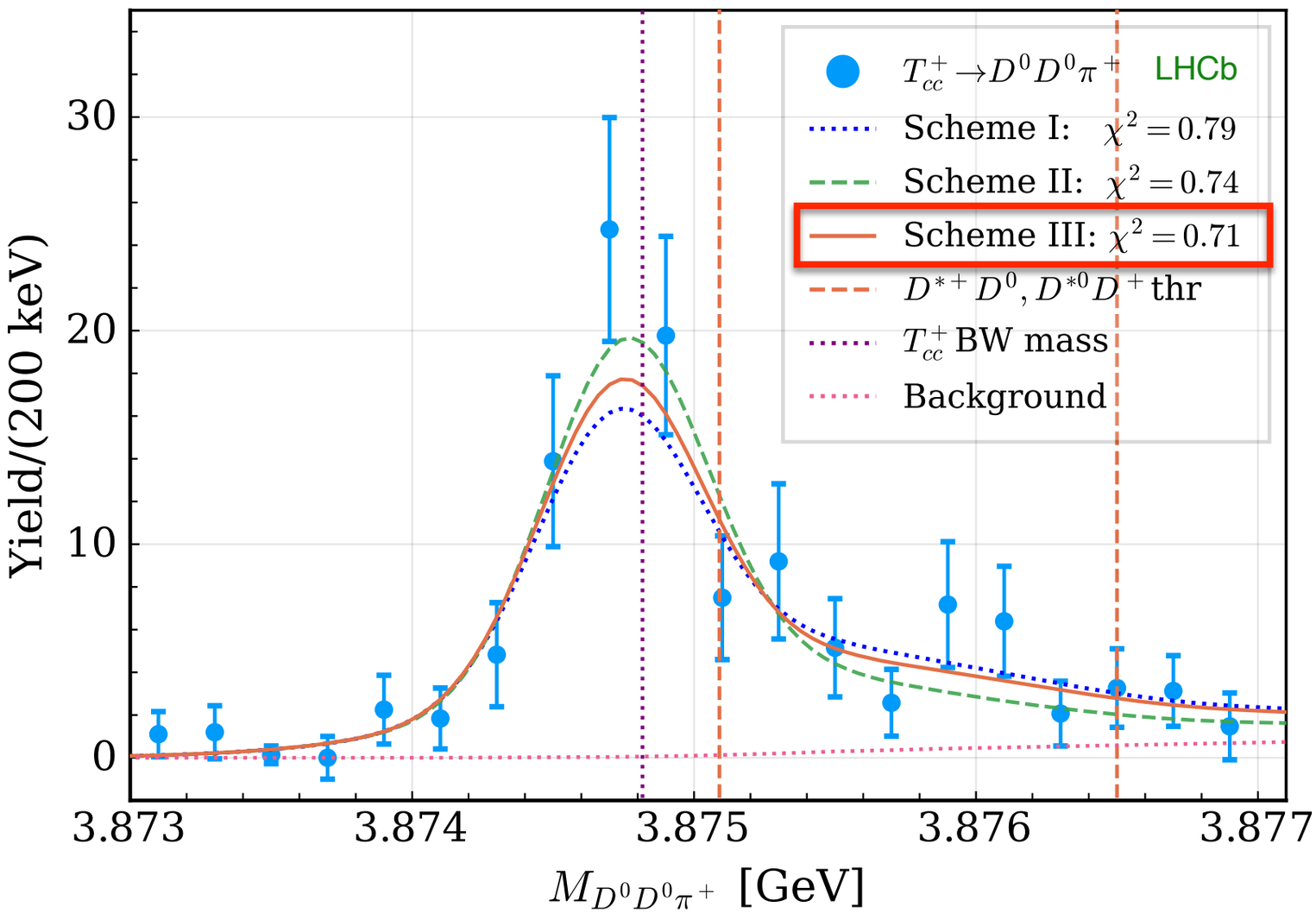
- Coupled-channel D^0D^{*+}/D^+D^{*0} scattering amplitude
- Point-like production source
- Only 2 params to be fitted

  v0 and overall Norm



χEFT fits to the $D^0D^0\pi^+$ mass spectrum

with resolution



The pole

III
full 3-body unitarity:
OPE + dynamical D^* width
$-356^{+39}_{-38} - i(28 \pm 1)$
0.71

Re part of the T_{cc}^+ pole: Inconclusive about the role of 3-body effects with current exp. precision

Can be reanalysed if more precise data emerge

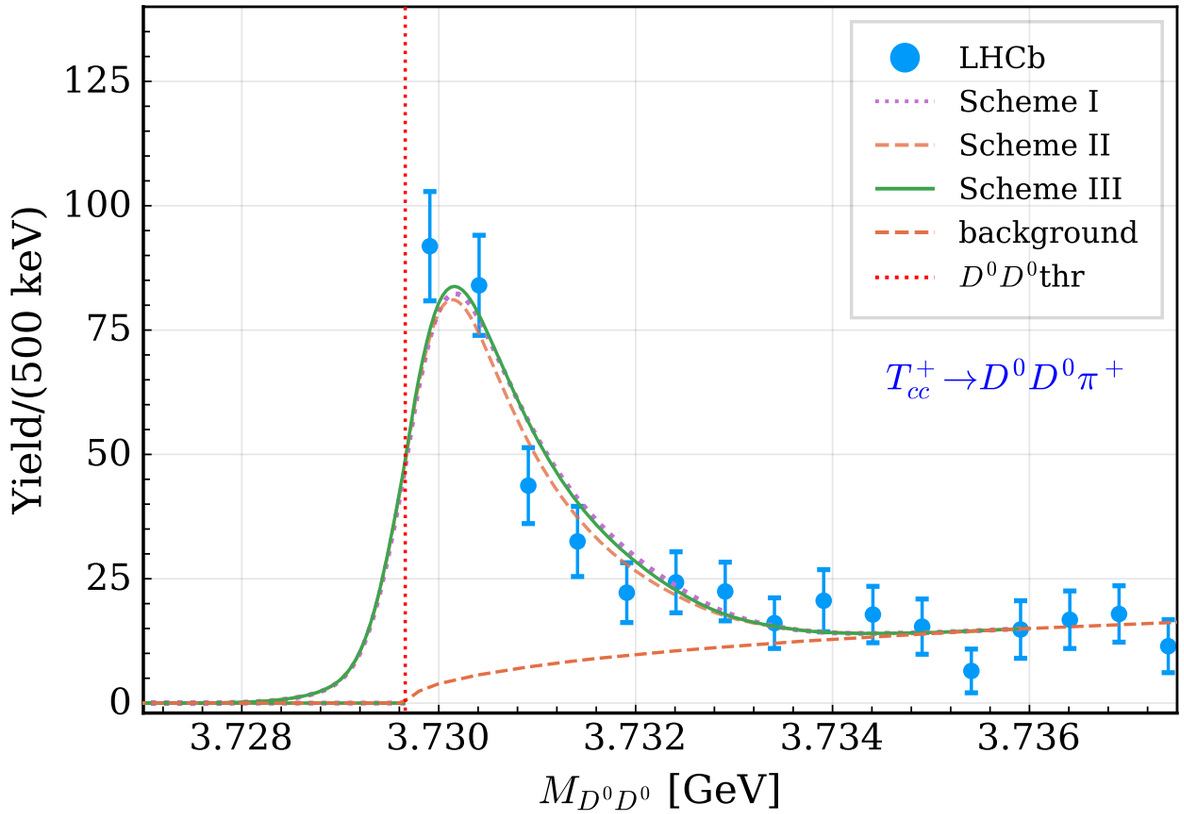
Im part of the T_{cc}^+ pole: Controlled by 3-body effects

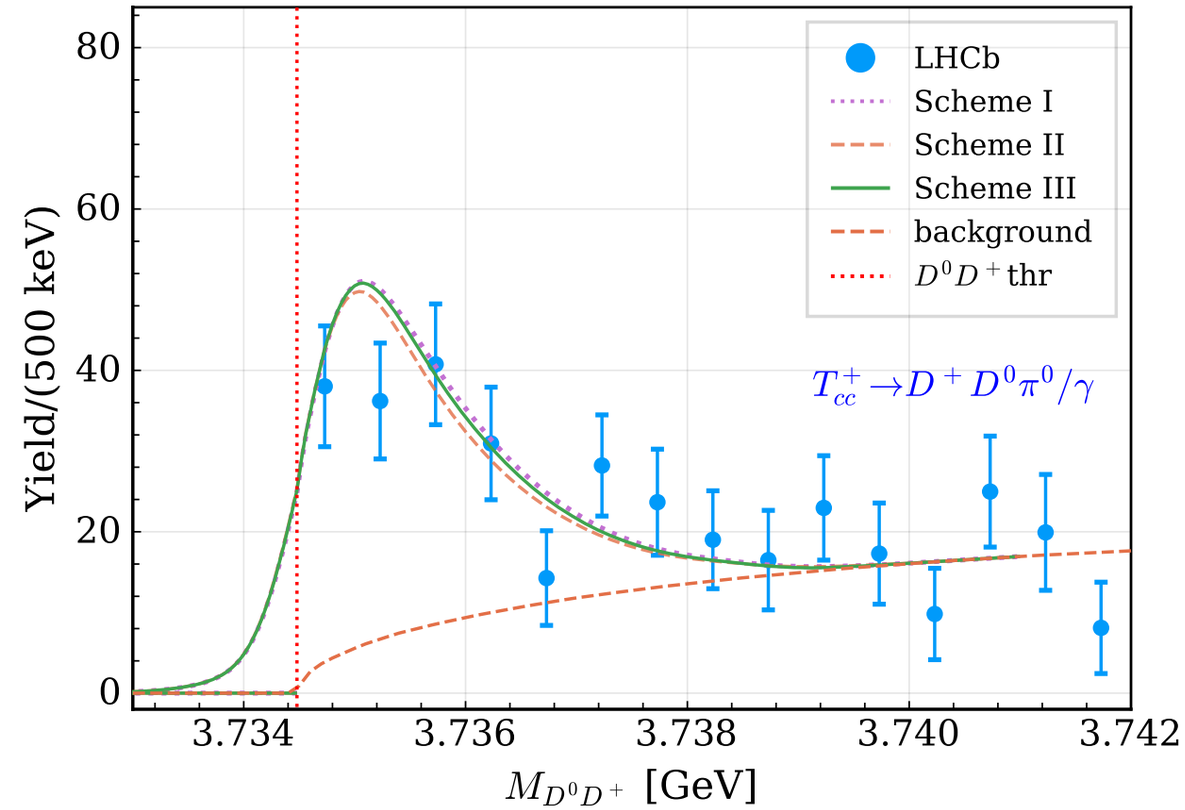
$$\Gamma_{T_{cc}}^{3-\text{body}} = 56 \pm 2\,\text{keV}$$

Various predictions

D^0D^0 and D^0D^+ spectra

with resolution





Heavy quark spin partners

see also Albaladejo PLB 829 (2022) in contact EFT

$$V^{I=0}(D^*D^* \to D^*D^*, 1^+) = V^{I=0}(D^*D \to D^*D, 1^+)$$

$$\delta_{cc}^{*+} = m_{T_{cc}^{*+}} - m_c^* - m_0^* = -503(40)\ \text{keV}$$

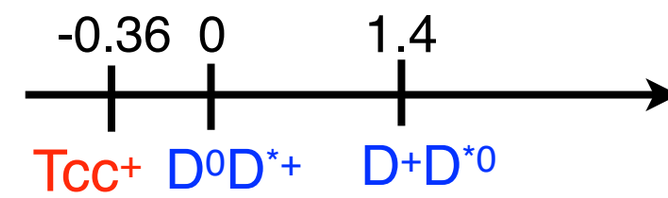
⇒ (quasi)bound D*D* state ~ 0.5 MeV below the threshold

Low-energy parameters

Du et al. PRD 105, 014024 (2022)

Scattering amplitude in the 1st (close to the pole) channel :

$$T_{D^{*+}D^0\to D^{*+}D^0}(k) = -\frac{2\pi}{\mu_{c0}}\left(\frac{1}{a_0}+\frac{1}{2}r_0k^2-ik+\mathcal{O}(k^4)\right)^{-1}$$



$$r_0' = r_0 - \Delta r$$

Eff. range in the 1st channel

$$\Delta r = -\sqrt{\frac{\mu_2}{2\mu_1^2\delta_2}} \simeq -3.8 \text{ fm}$$

Negative "correction" from 2nd $D^{*0}D^+$ channel caused by isospin breaking δ_2

$$\delta_2 = m_{\text{thr2}} - m_{\text{thr1}}$$

VB et al., PLB 833 (2022)

a_0 [fm]	r_0 [fm]	r_0' [fm]	X_A
$\left(-6.72^{+0.36}_{-0.45}\pm0.27\right)-i\left(0.10^{+0.03}_{-0.03}\pm0.03\right)$	$-2.40\pm0.01\pm0.85$	$1.38\pm0.01\pm0.85$	$0.84\pm0.01\pm0.06$

$$r_0' \ll |a_0|$$

- r_0' positive and is of natural size
- Contrib. to r_0' from OPE is ~ 0.4 fm

Tcc+ is consistent with a pure isoscalar molecule!

T_{cc} on lattice

- HAL QCD Collaboration at m_π = 146 MeV: 2302.04505 [hep-lat]

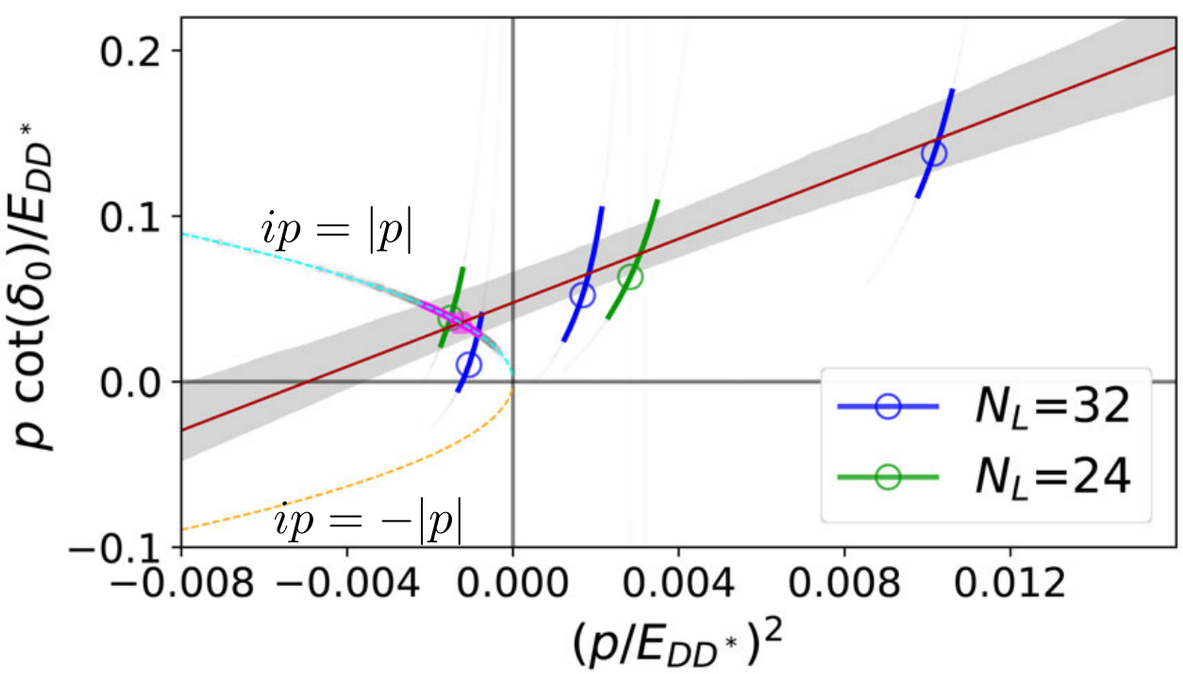
  — calculate the DD* scattering potential
  — use it to calculate the phase shifts above the two-body threshold ⇒ virtual state

- DD* phase shifts δ(E) are extracted using the Lüscher method

  — m_π = 391 MeV, one volume L=16 Cheung et al. (Hadron Spectrum collaboration), JHEP 11, 033 (2017)

  — m_π = 350 MeV, one volume L=16 Chen et al. , PLB 833, 137391 (2022).

  — m_π = 280 MeV, two volumes L = 24 and 32 Padmanath and Prelovsek, PRL 129, 032002 (2022)



— phase shifts parameterised using the ERE:

$$p\cot\delta = \frac{1}{a} + \frac{1}{2}rp^2 + \mathcal{O}(p^4)$$

— Intersection with

$ip = -|p|$ gives the bound state pole

$ip = |p|$ gives the virtual state pole

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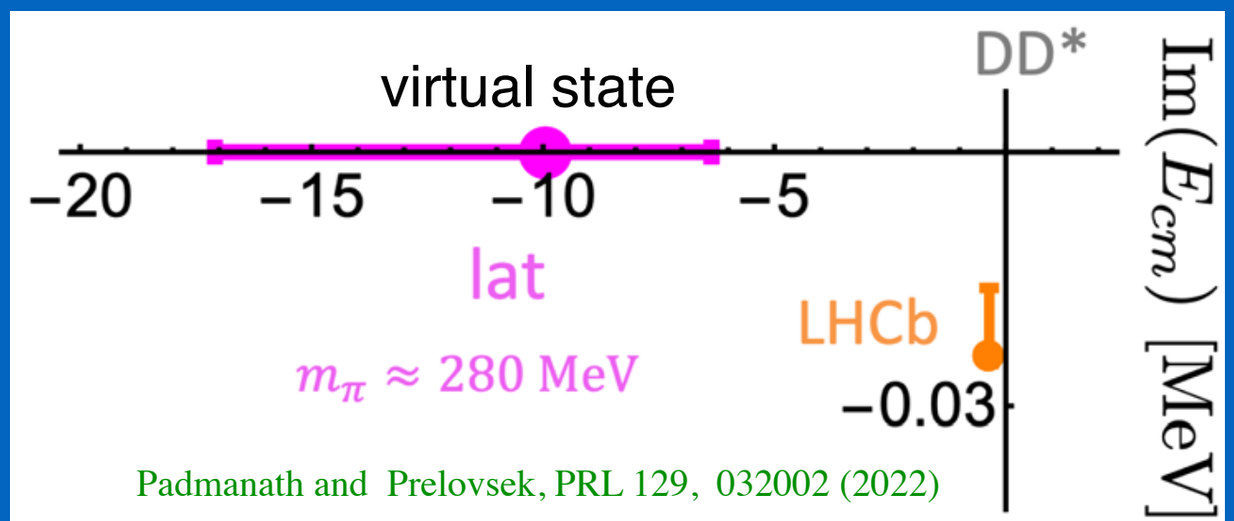

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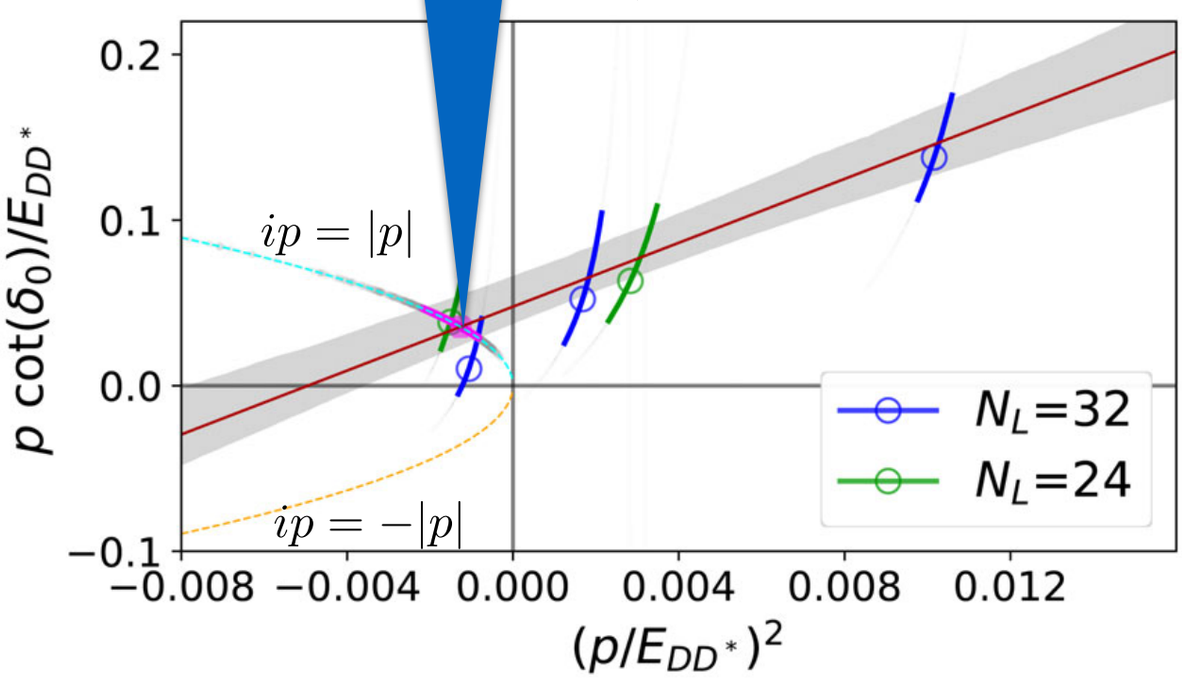

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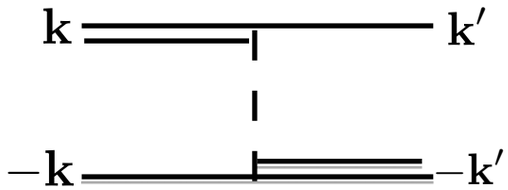
ERE applicability range

- Convergence Radius of the ERE is set by the nearest possible singularity

— 3-body threshold is far away: $E_2 = m_\pi - \Delta M = 158\,\mathrm{MeV}$ for masses from Padmanath, Prelovsek (2022)

$\Rightarrow$ static OPE is justified



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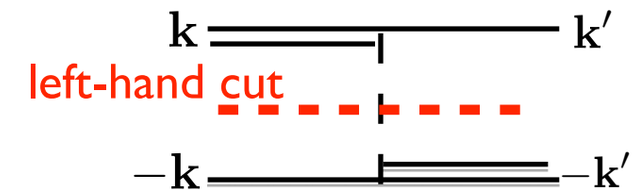
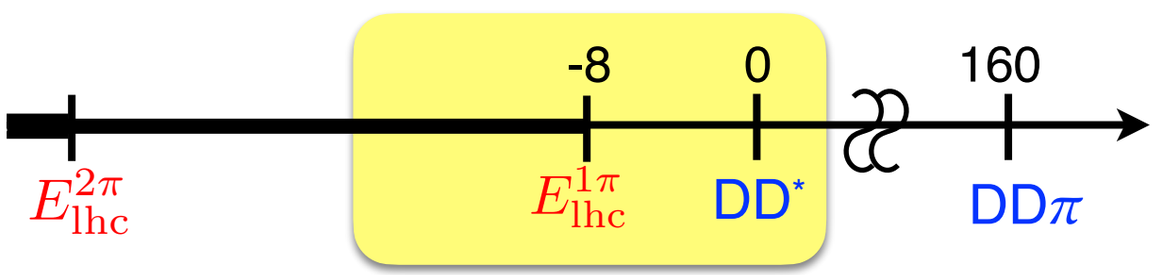


— Left-hand cuts (lhc) from crossed channels in partial-wave scattering amplitude

$\Rightarrow$ left-hand cut branch point is at $(p_{\rm lhc}^{1\pi})^2 = \dfrac{\Delta M^2 - m_\pi^2}{4}$

Numerically $E_{\rm lhc}^{1\pi} = \dfrac{(p_{\rm lhc}^{1\pi})^2}{2\mu} = -8\,\mathrm{MeV}$ $\Rightarrow$ $E_{\rm lhc}^{1\pi}$ sets the range of convergence of the ERE!



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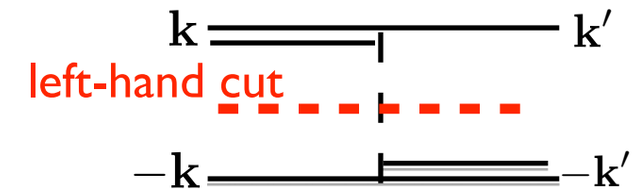

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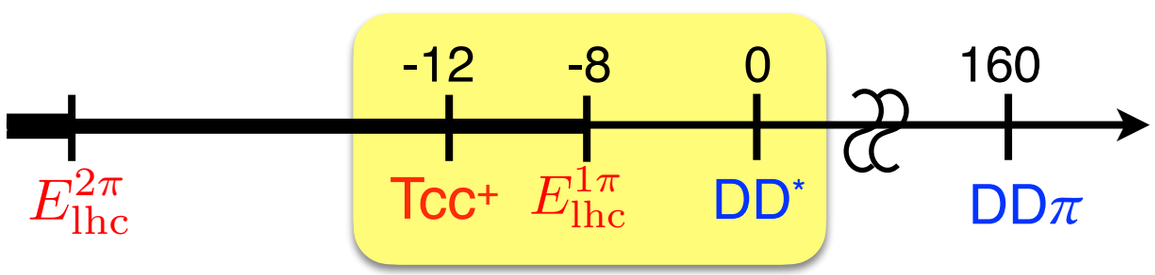


$\Rightarrow$ ERE is not applicable

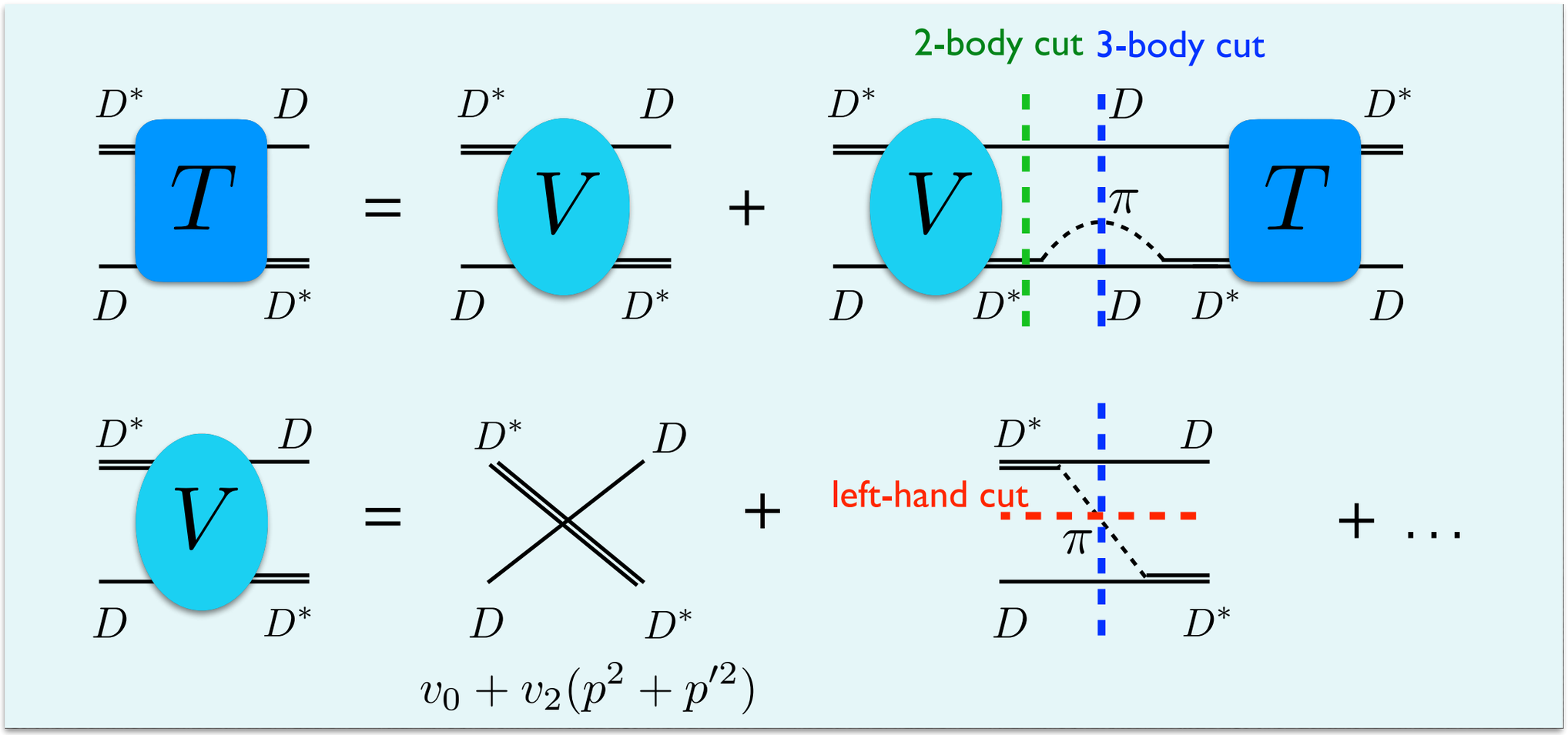
To extract Tcc pole accurately $\Rightarrow$ Calculate $p\,\cot\delta$ including the scale $E_{\rm lhc}^{1\pi}$ explicitly!

Analysis of lattice data including the left-hand cut

Du, Filin, VB, Epelbaum, Dong, Guo, Hanhart, Nefediev, Nieves and Wang 2303.09441 [hep-ph]

submitted to PRL

- $p \cot \delta$ from scattering T matrix including all relevant cuts



- Chiral extrapolation of D*Dπ coupling is included along the lines of

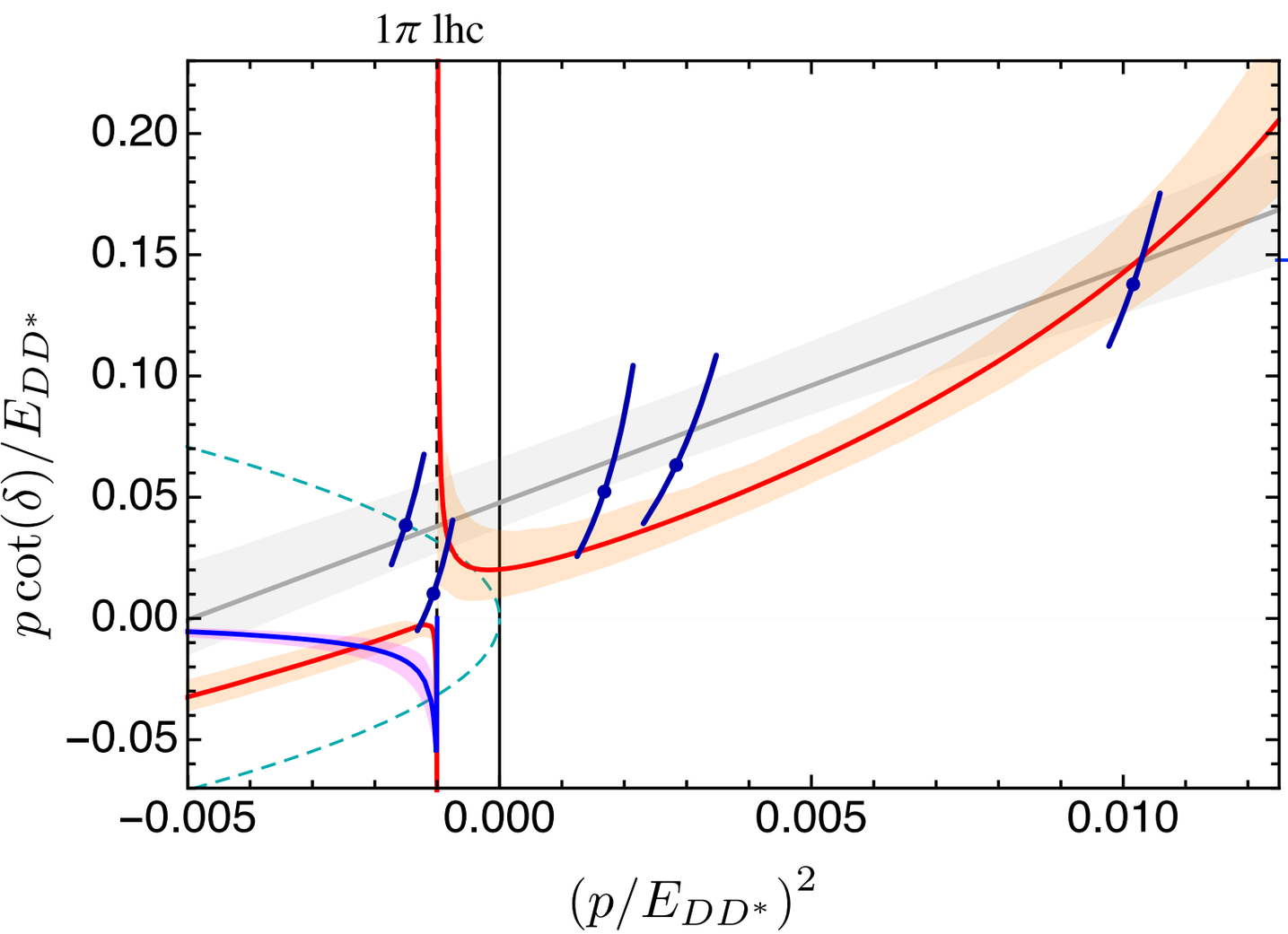
  Becirevic and Sanfilippo, PLB 721, 94 (2013) VB et al PLB 726, 537 (2013)

- Next-to-leading lhc is from 2π cuts — much further away

- Similar in spirit to the analysis of NN scattering at unphysical m_π

  VB, Epelbaum, Filin, Gegelia PRC92 014001(2015), PRC94 014001(2016)

Tcc from fits to phase shifts including 1π lhc



- Our LO χEFT analysis with 1π lhc:

—Fit is significantly different from the ERE

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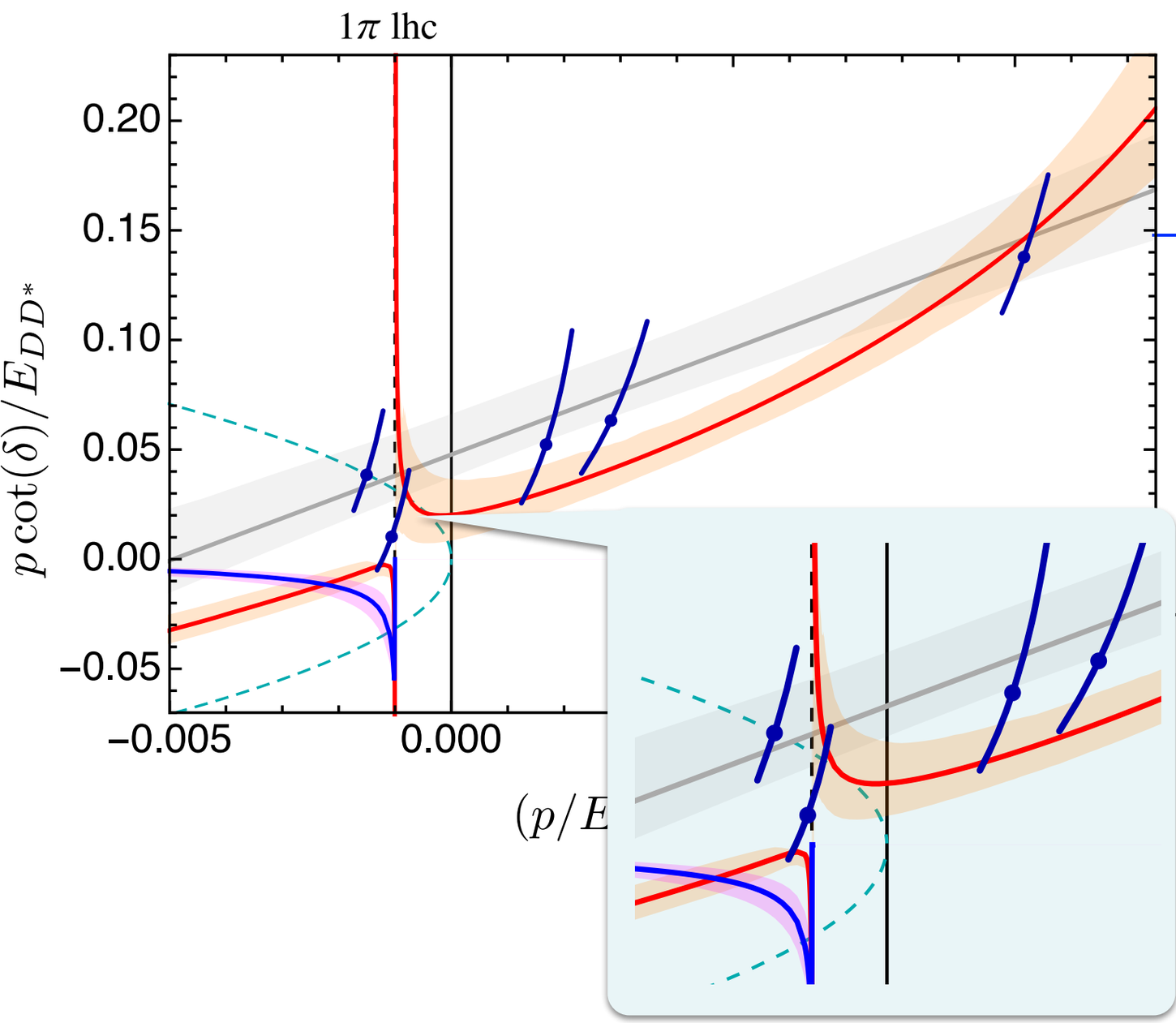

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T_{CC} poles:

— 2 virtual states or a resonance very near thr.

Zeros:

— $p\cot\delta$ has a pole near lhc $\Rightarrow T$ has a zero

$$-\frac{2\pi}{\mu}T^{-1}(E) = p\cot\delta - ip$$

Repulsive OPE + attractive short range!

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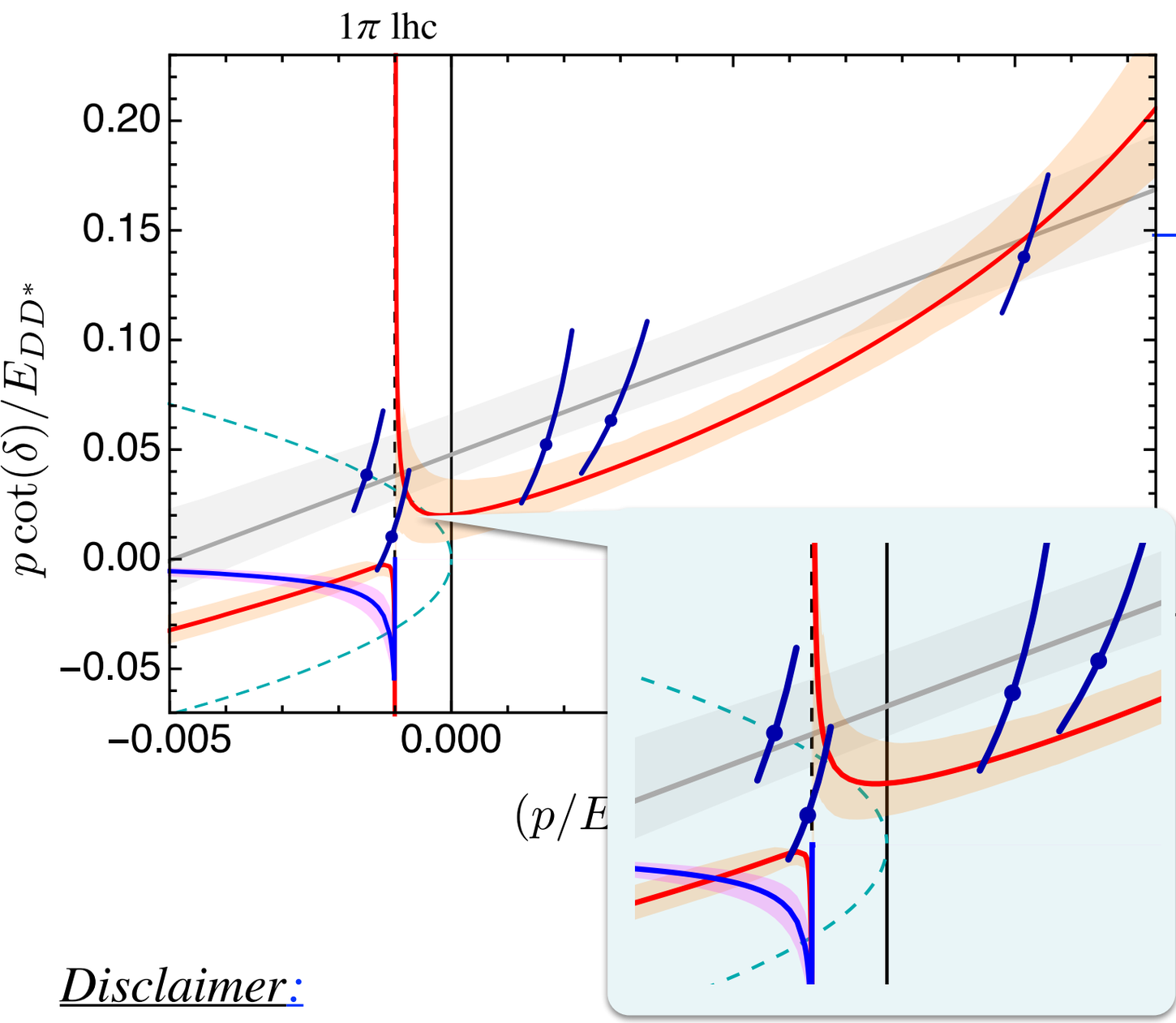

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Disclaimer:

- phase shifts used here for granted $\Rightarrow$ to be revisited
  - — lhc requires a modification of the Lüscher method — Raposo and Hansen, PoS LATTICE2022, 051 (2023)
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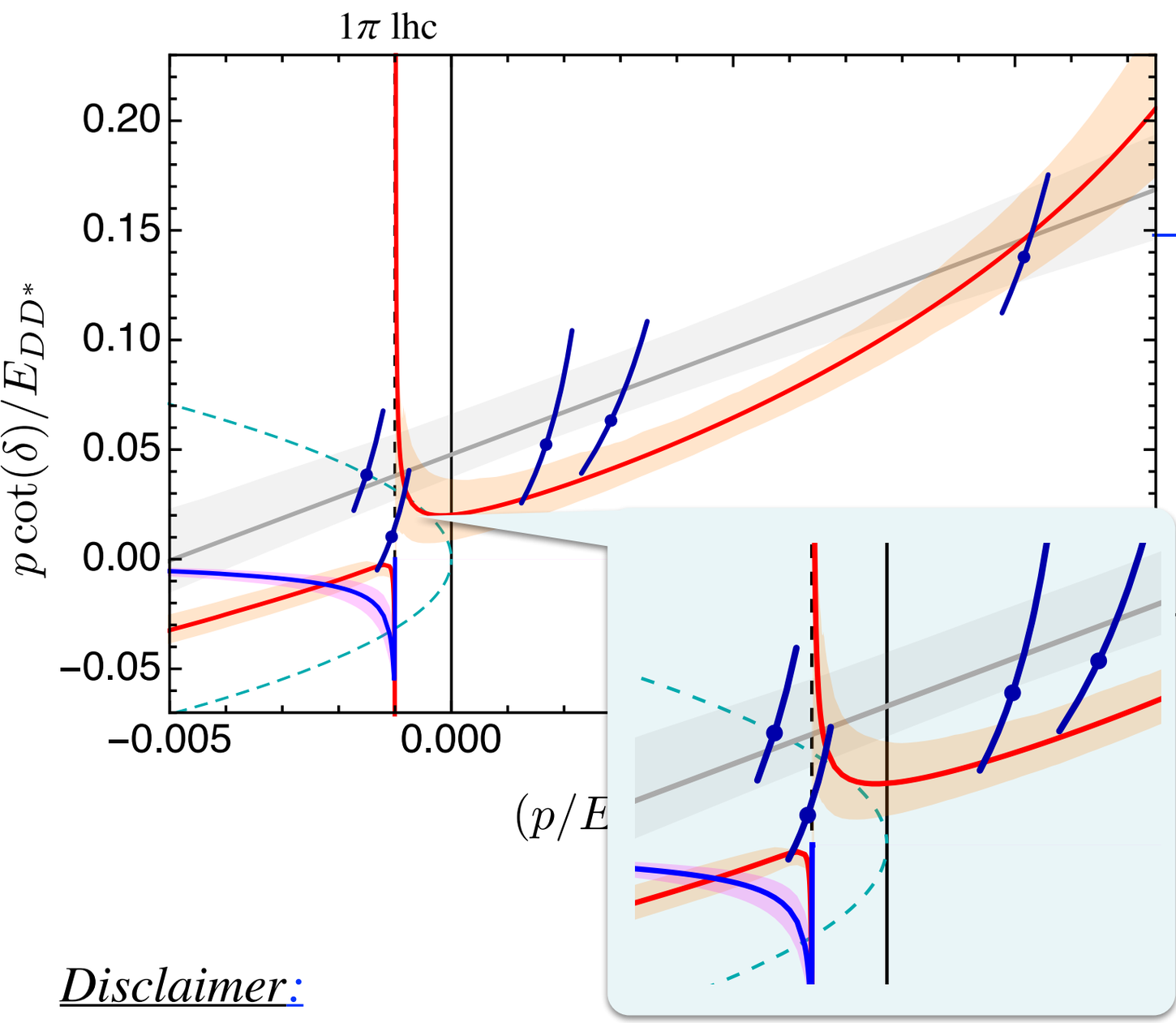

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But the method should be used in the future analyses of DD*, $\overline{D}$D*, BB*, …

Summary and conclusions

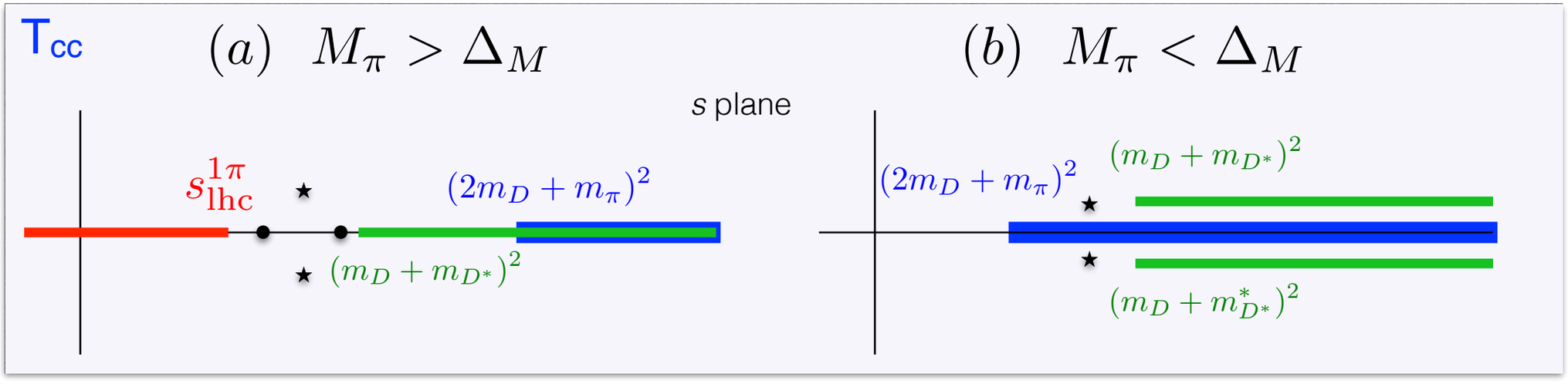
- Hadronic Molecules — specific subclass of exotic states:

—generated dynamically near hadronic thresholds

—do exist in nuclear and hadron physics

- Coupled-channel chiral EFT approach: right tool for a systematic analysis

χEFT: correct analytic structure of the scattering amplitude including relevant cuts



— Reliable extraction of the T_{cc} pole

— HQ spin partner (I=0, J^P= 1^+) should exist near the D*D* threshold

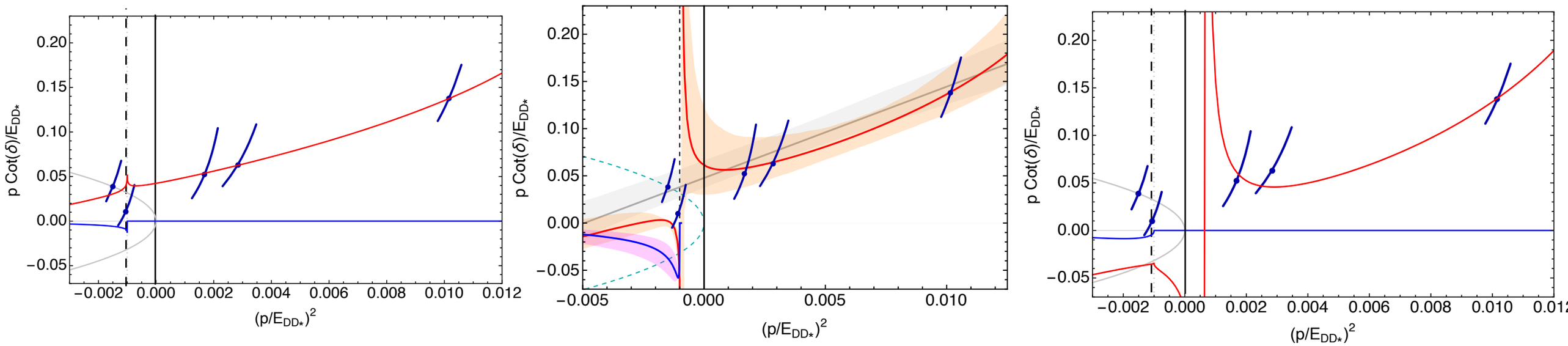
- Compositeness of near threshold states: Weinberg's criterion and extensions

Thanks for your attention!

Dependence on the pion coupling

- Importance of lhc is controlled by its position and strength (discontinuity)

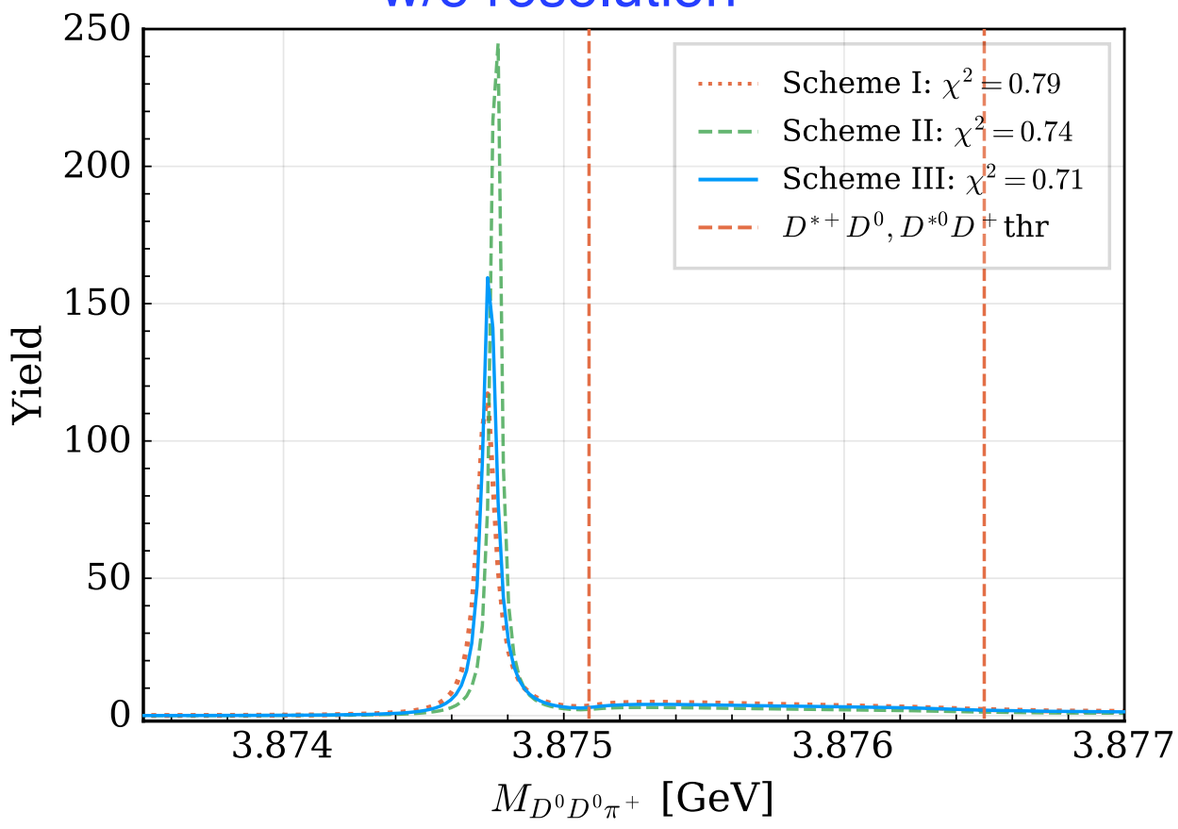
$\frac{1}{10} V^{\rm OPE}_{{\rm DD}^*\to{\rm DD}^*}(m_\pi = 280{\rm MeV})$ $\quad$ $V^{\rm OPE}_{{\rm DD}^*\to{\rm DD}^*}(m_\pi = 280{\rm MeV})$ $\quad$ $10\ V^{\rm OPE}_{{\rm DD}^*\to{\rm DD}^*}(m_\pi = 280{\rm MeV})$



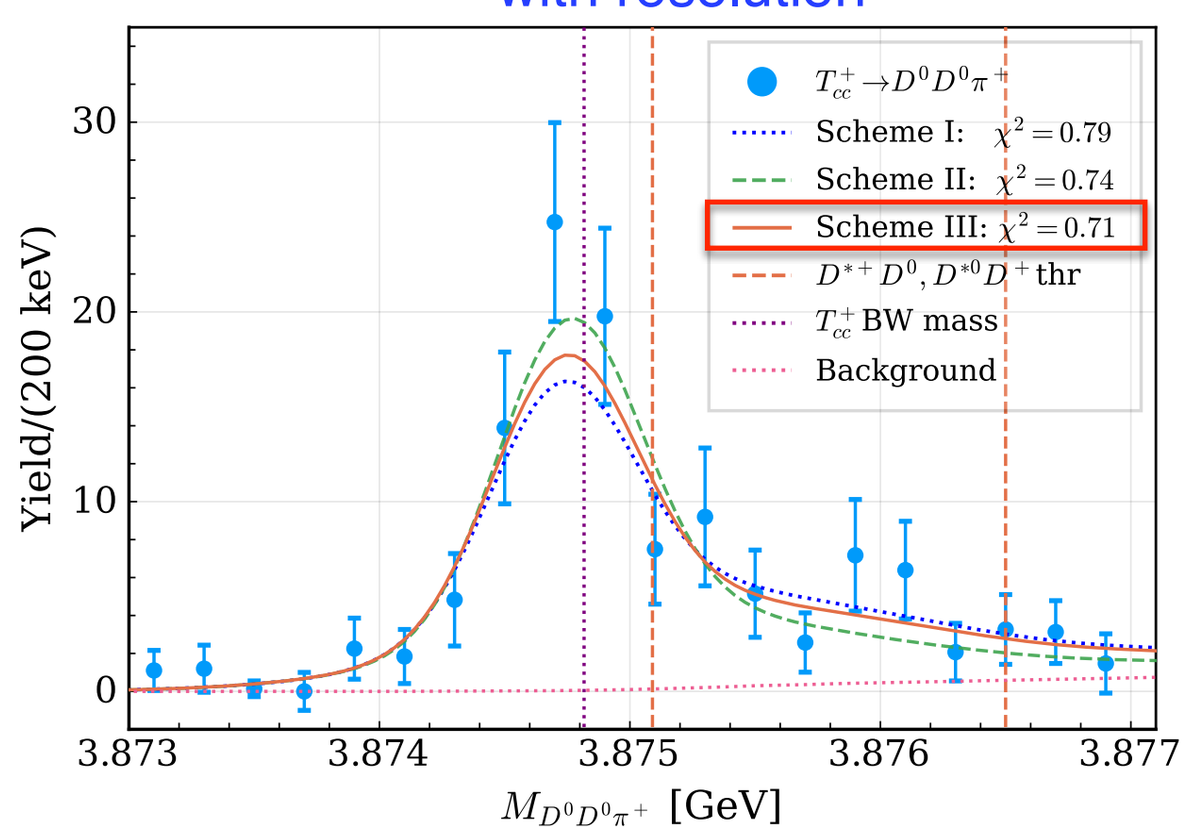
- The smaller the coupling the closer the fit is to the ERE

Fits to the $D^0D^0\pi^+$ mass spectrum

w/o resolution



with resolution



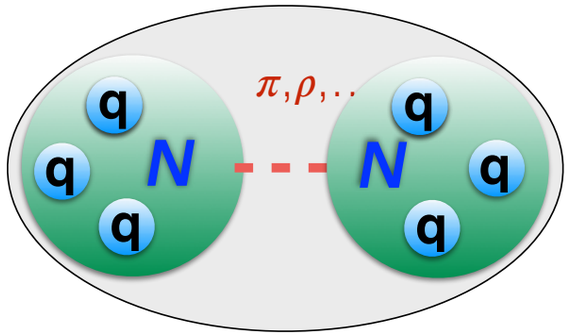
Scheme	I	II	III
Description	2-body unitarity: No OPE, static D^* width	Incomplete 3-body unitarity: No OPE, dynamical D^* width	full 3-body unitarity: OPE + dynamical D^* width
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$
χ^2	0.79	0.74	0.71

Real part of the pole: all Fits are consistent within 1σ —more precise data are needed

Width of Tcc+ : Accuracy requires 3-body effects

$\Gamma_{T_{cc}}^{3-\text{body}} = 56 \pm 2\,\text{keV}$ $\xrightarrow[\text{OPE}]{\text{remove}}$ 36 keV $\xrightarrow[\text{dynam.width}]{\text{remove}}$ 74 keV

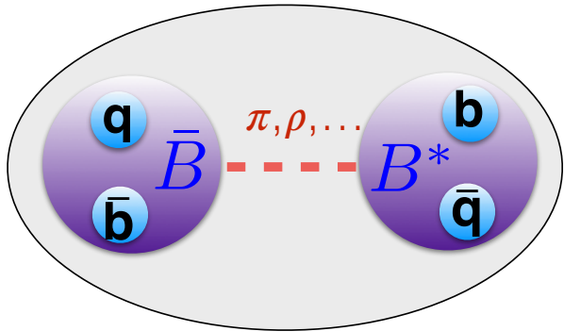
EFT's for doubly-heavy near-threshold states



Voloshin, Okun (1976)

Törnqvist (1991)

Ericson, Karl (1993)



$D, D^*, B, B^*, \ldots$

$\Lambda_c, \Sigma_c, \ldots$

— Contact EFT AlFiky (2006), Voloshin, Nieves, Albaladejo…

no pions
no coupled channels

Limited applicability range in energy

— X-EFT Fleming et al (2007), Mehen, Hammer, Braaten…

perturbative pions
no coupled channels

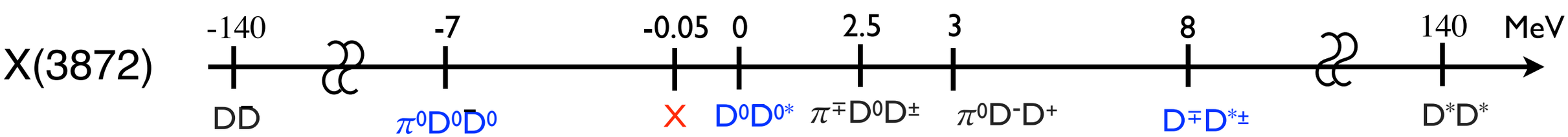
Applicable if the system is close to the unitary limit → universality

— χEFT

our works (2011– till now)

Fully non-perturbative pions together with coupled-channels

- Larger applicability range allows for direct fits to experimental line shapes
- Analytic structure of the amplitude is more complete



What do we know about Tcc+?

Aaij et al [LHCb] Nature Physics (2022)

- LHCb reports a clear peak in $D^0 D^0 \pi^+$ spectrum right below the $D^{*+} D^0$ threshold

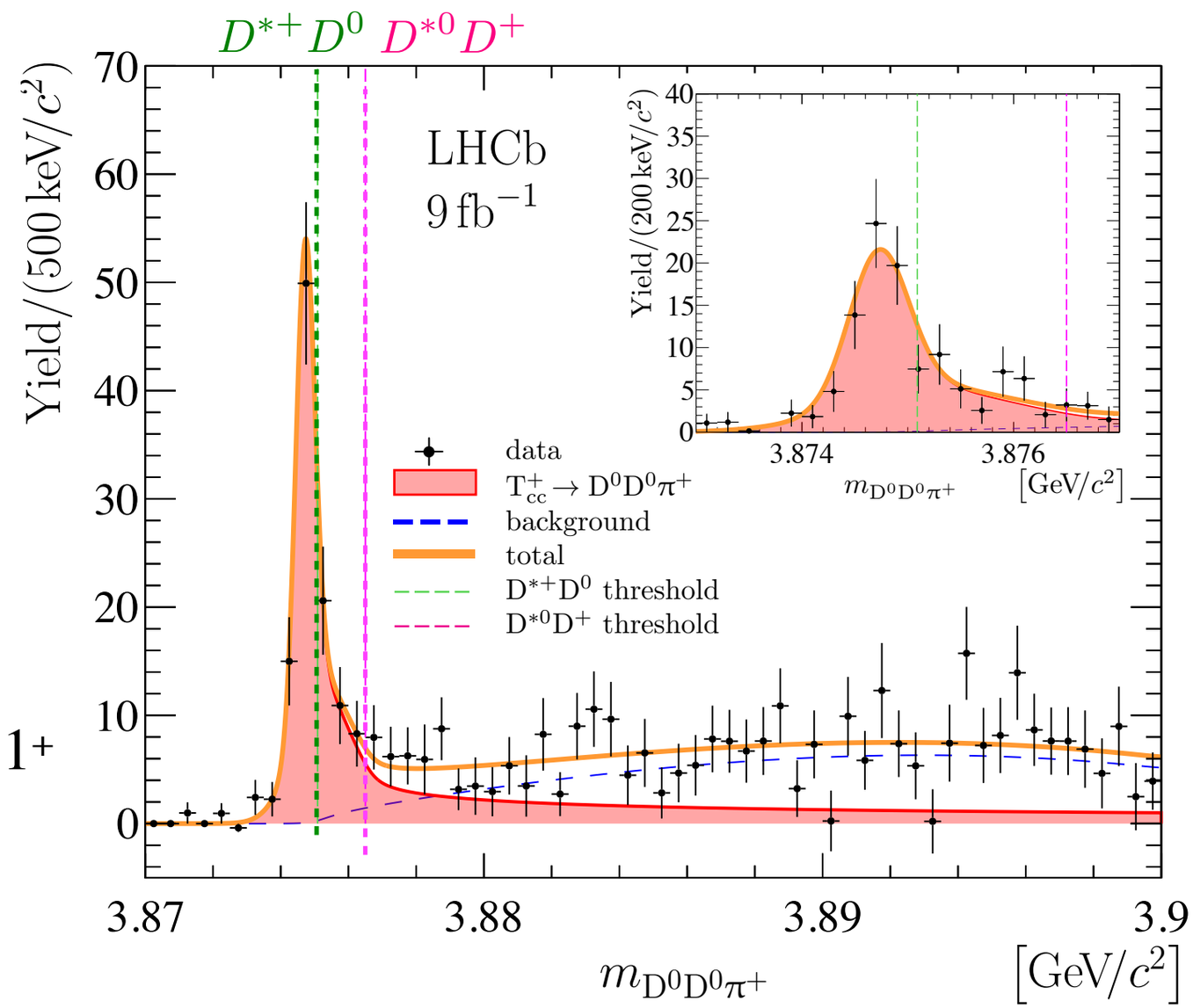
- Unitarized Breit-Wigner fit

$$\delta m_{\rm pole} = -360 \pm 40^{+4}_{-0} \text{ keV}$$

$$\Gamma_{\rm pole} = 48 \pm 2^{+0}_{-14} \text{ keV}$$

$$\Gamma_{\rm pole} \ll \Gamma_{D^{*+}}$$

LHCb: Nature Comm.(2022)



- Proceed via D^* $DD^* \to D[D\pi]$ 90%

- No signal in $D^{*+}D^+$— isoscalar; $J^P = 1^+$

- Several studies on lattice at different m_π. Conclusion: Tcc is probably a virtual state

Padmanath and Prelovsek, PRL 129, 032002 (2022), Chen et al. , PLB 833, 137391 (2022), Lyu et al [HAL QCD]: 2302.04505 [hep-lat]

- Plenty of theoretical studies; in particular, the Tcc width is addressed in

Meng et al (2021), Fleming et al (2021), Ling et al (2022), Feijoo et al. (2021), Yan et al. (2022), Albaladejo (2022), Dai et al. (2023),…

One pion exchange and 3-body cut

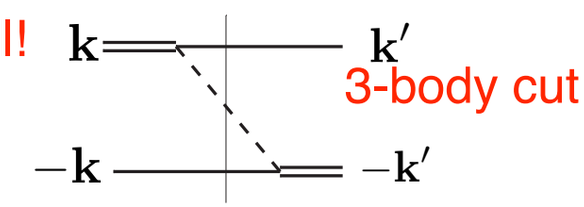
- OPE potential:

$$V_{\mathrm{DD}^*\to\mathrm{DD}^*}(\mathbf{k},\mathbf{k}') \propto \frac{g_c^2}{(4\pi f_\pi)^2}\,\tau_1\cdot\tau_2\,\frac{(\epsilon_1\cdot\vec{q})\,(\epsilon_2'^{\,*}\cdot\vec{q})}{2E_\pi(\mathbf{k}-\mathbf{k}')}\left(\frac{1}{D_{DD\pi}(\mathbf{k},\mathbf{k}')}+\frac{1}{D_{D^*D^*\pi}(\mathbf{k},\mathbf{k}')}\right)$$

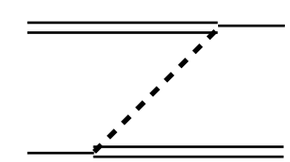
☛ TOPT propagators with NR heavy mesons and relativistic pions $E_\pi=\sqrt{m_\pi^2+(\mathbf{k}-\mathbf{k}')^2}$

$$D_{DD\pi}(\mathbf{k},\mathbf{k}') = m+m+\frac{k^2}{2m}+\frac{k'^2}{2m}+E_\pi(\mathbf{k}-\mathbf{k}')-E \quad\Rightarrow$$

goes on shell!



$$D_{D^*D^*\pi}(\mathbf{k},\mathbf{k}') = m_*+m_*+\frac{k^2}{2m_*}+\frac{k'^2}{2m_*}+E_\pi(\mathbf{k}-\mathbf{k}')-E \quad\Rightarrow$$

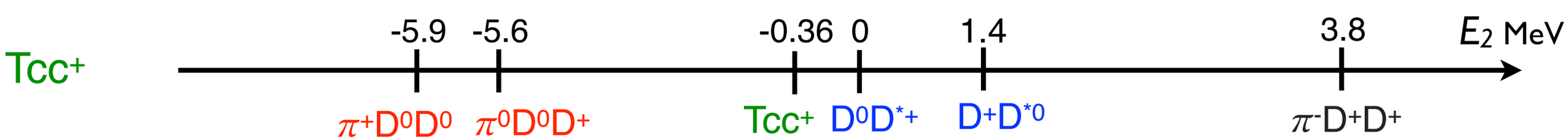


3-body cut condition

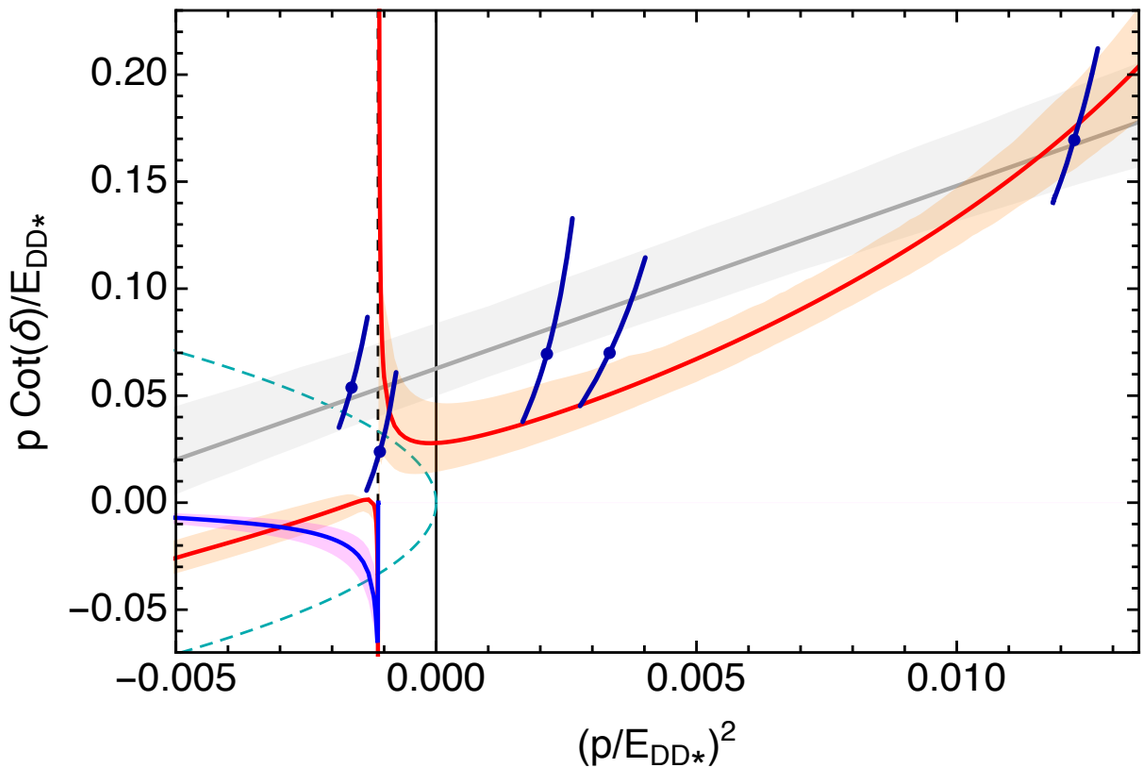
☛ For each $E>E_{\mathrm{thr}}\equiv 2m+m_\pi$, there are real values of k and k′ such that $D_{DD\pi}(k,k',E)=0$

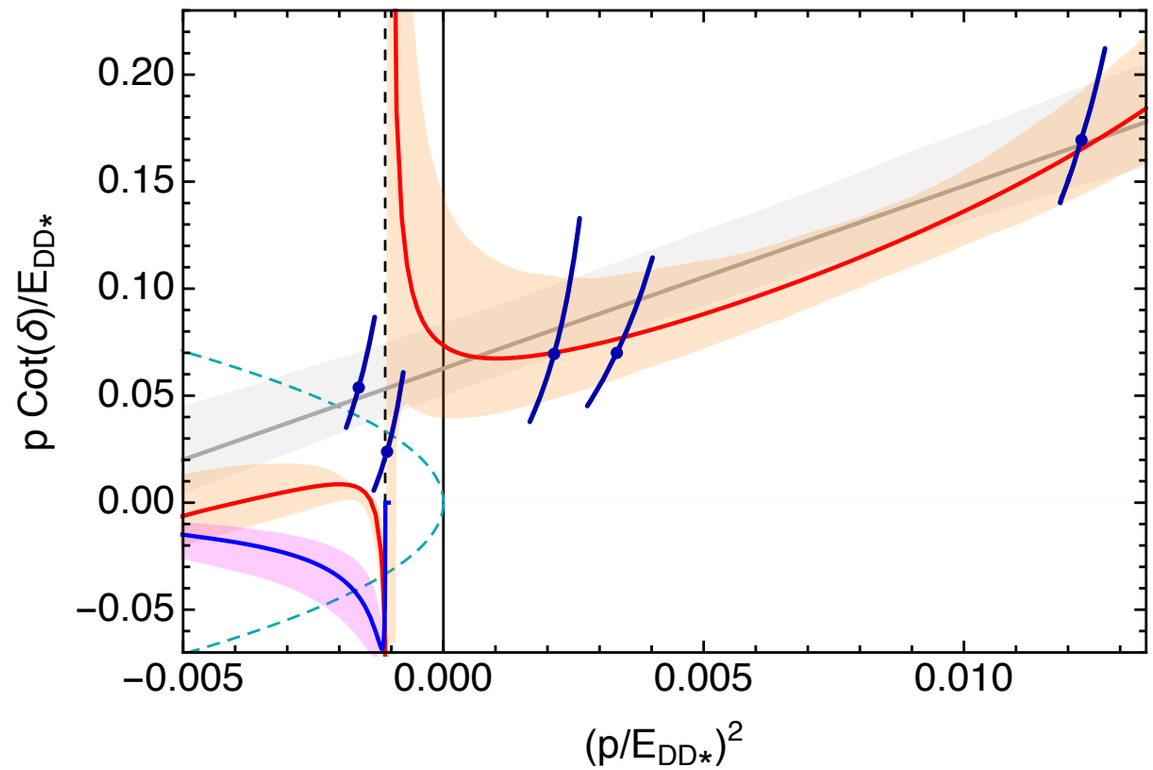
☛ If we count energy relative to 2-body DD* threshold $E=m+m_*+E_2$

3-body branch point reads (k=k′=0) $E_2=m_\pi-\Delta M=m_\pi+m-m_*$



Fits to data for smaller c-quark masses





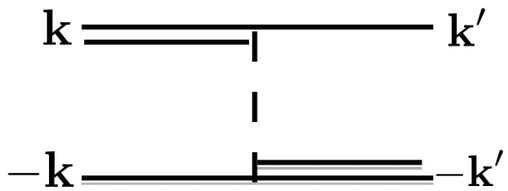
Conclusions are the same as for the other fit

ERE applicability range

- Convergence Radius of the ERE is set by the nearest singularity irrespective of its origin

— For masses from Padmanath and Prelovsek (2022) 3-body cut starts at $E_2 = m_\pi - \Delta M = 158\,\mathrm{MeV}$

$\Rightarrow$ 3-body effects are small, static OPE is justified



— Partial-wave scattering amplitude may also have left hand cuts (lhc)

$$\int dz \frac{1}{(\mathbf{k}-\mathbf{k}')^2 + m_\pi^2 - \Delta M^2} = \frac{1}{2kk'} \log \frac{(k+k')^2 + m_\pi^2 - \Delta M^2}{(k-k')^2 + m_\pi^2 - \Delta M^2} \xrightarrow[k=k'=p]{\text{on shell}} \frac{1}{2p^2} \log \frac{4p^2 + m_\pi^2 - \Delta M^2}{m_\pi^2 - \Delta M^2}$$

$\Rightarrow$ left-hand cut branch point is at $(p_{\rm lhc}^{1\pi})^2 = \dfrac{\Delta M^2 - m_\pi^2}{4}$

Numerically $(p_{\rm lhc}^{1\pi})^2 = -(126\,\mathrm{MeV})^2 \quad \Rightarrow E_{\rm lhc}^{1\pi} = \dfrac{(p_{\rm lhc}^{1\pi})^2}{2\mu} = -8\,\mathrm{MeV}$

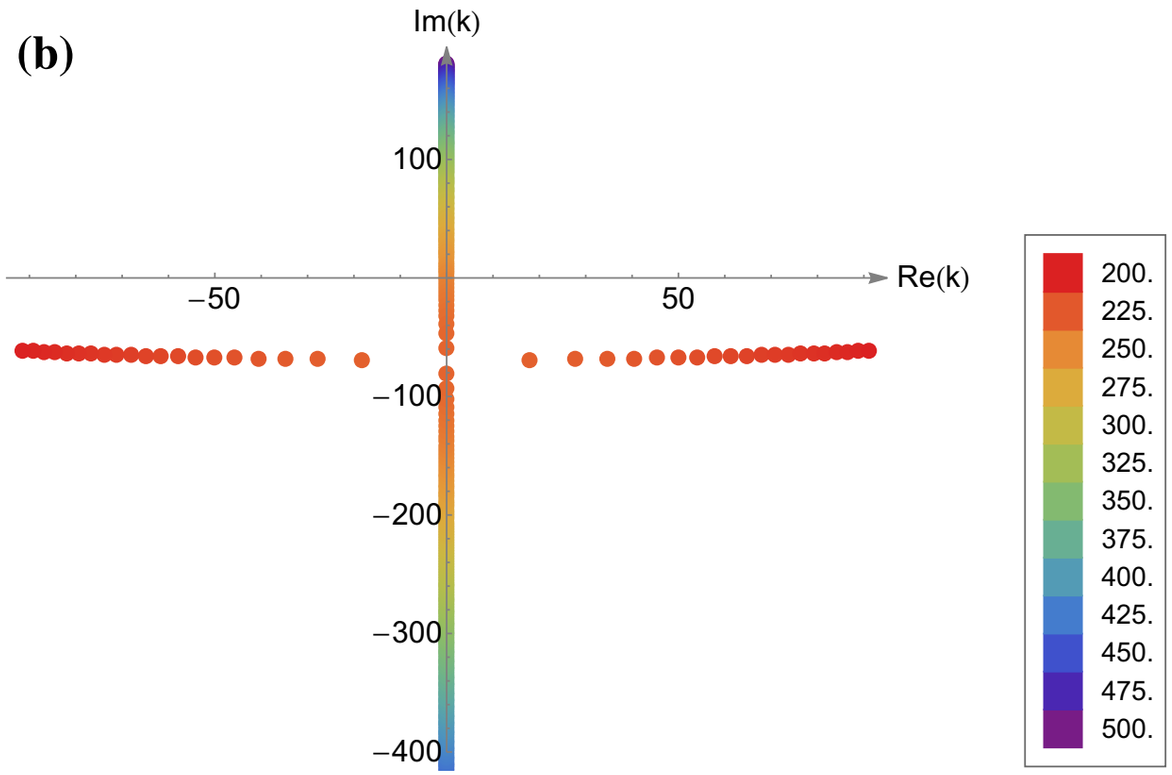
$\Rightarrow$ $E_{\rm lhc}^{1\pi}$ sets the range of convergence of the ERE: $E \ll |E_{\rm lhc}^{1\pi}|$

If $|E_{\rm lhc}^{1\pi}| \leq |E_{\rm Tcc}| \quad \Rightarrow$ ERE is not applicable

To extract Tcc pole accurately $\Rightarrow$ Calculate $p \cot\delta$ including the scale $E_{\rm lhc}^{1\pi}$ explicitly!

D_{s0}*(2317) pole trajectory vs kaon mass

Matuschek, VB, Guo, Hanhart
EPJA 2021



Use ChiPT approach for ΦD scattering

$$\mathcal{L}^{(1)} = \mathcal{D}^\mu D^\dagger \mathcal{D}_\mu D - m_D^2 D^\dagger D$$

$$\mathcal{L}^{(2)} = D\left(-h_0 \langle \chi_+ \rangle - h_1 \chi_+ - h_2 \langle u_\mu u^\mu \rangle - h_3 u_\mu u^\mu\right) \bar{D} + \mathcal{D}_\mu D \left(h_4 \langle u^\mu u^\nu \rangle - h_5 \{u^\mu u^\nu\}\right) \mathcal{D}_\nu \bar{D}$$

$$U = \exp\left(\frac{\sqrt{2} i \Phi}{F_\pi}\right), \quad u^2 = U$$

$$\mathcal{D}_\mu = \partial_\mu + \Gamma_\mu ,$$

$$\Gamma_\mu = \frac{1}{2}\left(u^\dagger \partial_\mu u + u\, \partial_\mu u^\dagger\right)$$

$$\Phi = \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix}$$

Fix all LECs from lattice studies: scattering lengths of I=3/2 Dπ, $D_s\pi$, D_sK, I=0,1 DbarK for the pion masses from 301, 362, 511, 617 MeV

Liu, Orginos, Guo, Hanhart, Meißner 2013

Predict the pole of D_{s0}*(2317) and its kaon mass dependence