Analysis of $\Xi(1620)$ resonance with chiral unitary approach

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T.Nisihibuchi and T.Hyodo, arXiv:2305.10753 [hep-ph]

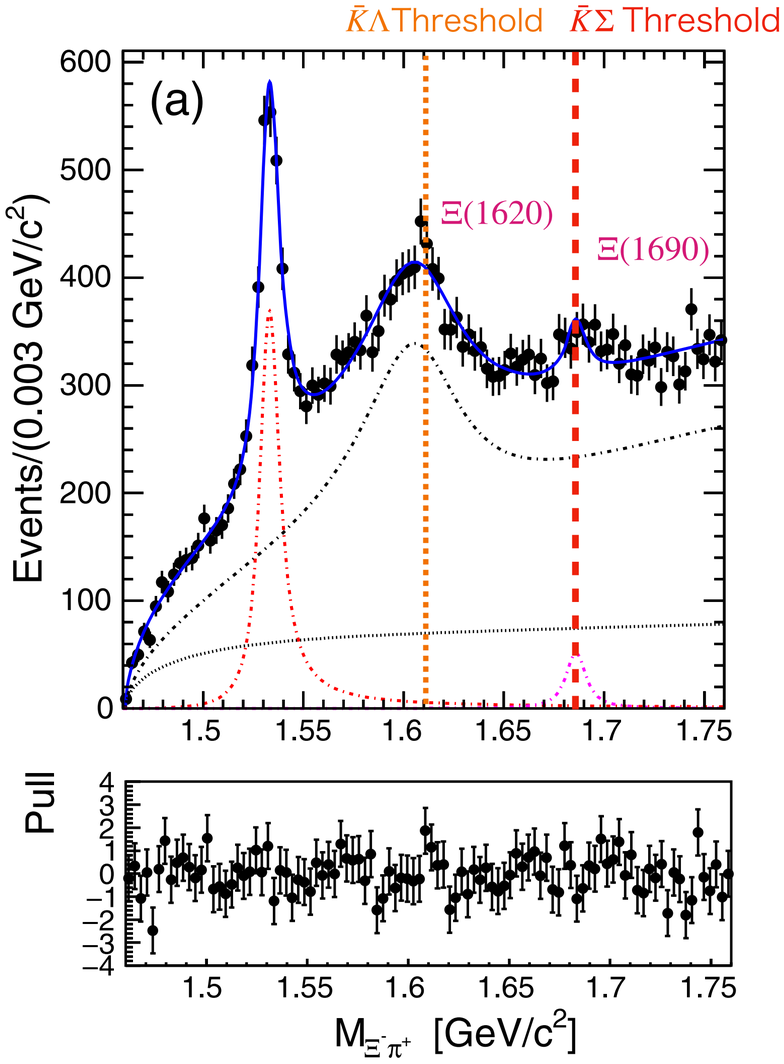
Result of Belle

Belle experiment of $\Xi_c \to \pi\pi\Xi$ (2019)[1]

- $\Xi(1620)$ and $\Xi(1690)$ peaks are observed in the invariant mass distribution of $\pi^+\Xi^-$.
- The mass M_R and width Γ_R of $\Xi(1620)$ are reported as follows.

$$M_R = 1610.4 \pm 6.0(\text{stat.})^{+6.1}_{-4.2}(\text{syst.})\ \text{MeV}$$
$$\Gamma_R = 59.9 \pm 4.8(\text{stat.})^{+2.8}_{-7.1}(\text{syst.})\ \text{MeV}$$

- Mass and width do not agree with values in previous theoretical studies.
- Peaks are close to thresholds of $\bar{K}\Lambda$ and $\bar{K}\Sigma$?



Invariant mass distribution of $\pi^+\Xi^-$ in the $\Xi_c \to \pi\pi\Xi$ decay [1].

[1]Belle collaboration, M. Sumihama et al., Phys. Rev. Lett. **122**, 072501 (2019).

Result of ALICE

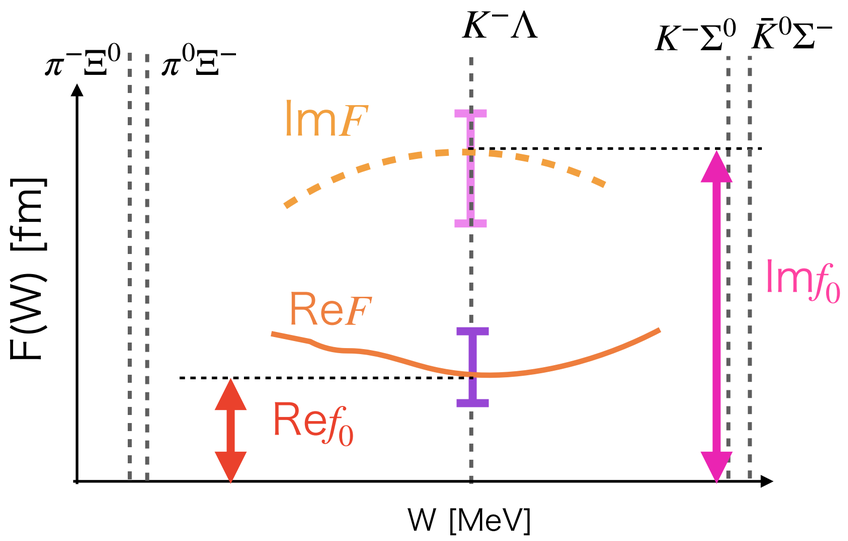
ALICE experiment(2021)[2]

The scattering length f_0 of $K^-\Lambda$ was determined with femtoscopy in Pb-Pb collisions as follows.

$$\mathrm{Re}f_0 = 0.27 \pm 0.12(\mathrm{stat.}) \pm 0.07(\mathrm{syst.})\ \mathrm{fm}$$
$$\mathrm{Im}f_0 = 0.40 \pm 0.11(\mathrm{stat.}) \pm 0.07(\mathrm{syst.})\ \mathrm{fm}$$

f_0 constrains real and imaginary parts of scattering amplitude at threshold.

Scattering length $f_0 \in \mathbb{C}$



Schematic figure of the scattering length

The error bar of Re f_0

The error bar of Im f_0

More quantitative comparison is possible than the spectrum fit.

[2]S. Acharya et al. (ALICE Collaboration)Phys. Rev. C **103**, 055201(2021).

Strategy of this study

Strategy of model construction

- **Model of previous study**[3] Set 1

Mass M_R and width Γ_R of $\Xi(1620)$

$M_R = 1607$ MeV, $\Gamma_R = 280$ MeV.

We construct theoretical models which reproduce new experimental results.

- **Model for Belle result (Model 1)**

Assume pole position from mass M_R and width Γ_R reported by Belle

- **Model for ALICE result (Model 2)**

Reproduce the $K^-\Lambda$ scattering length determined by ALICE

[3] A.Ramos, E.Oset and C.Bennhold, Phys. Rev. Lett. **89**.252001 (2002).

Formulation

Coupled-channel meson-baryon scattering amplitude $T_{ij}(W)$ at total energy W.

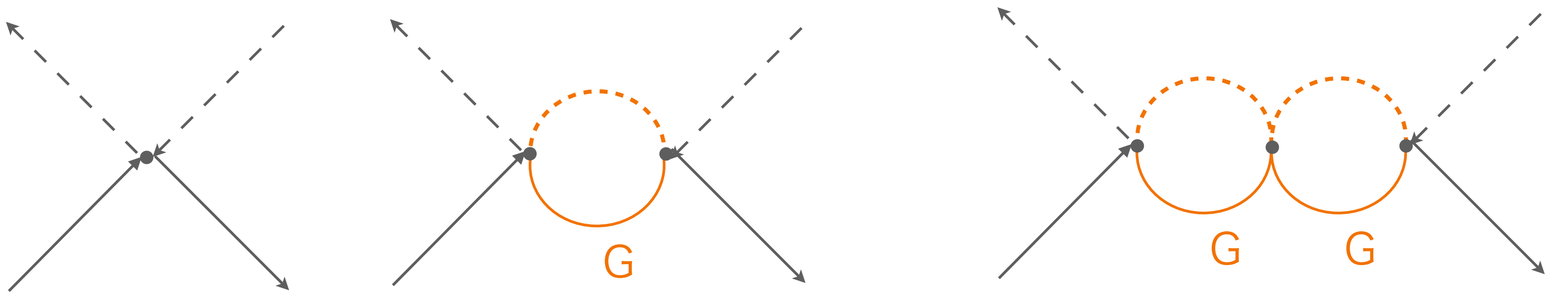
Scattering equation

$$T_{ij}(W) = V_{ij}(W) + V_{ik}(W) G_k(W) T_{kj}(W)$$

$V_{ij}(W)$···Interaction kernel

$G_i(W)$···Loop function

$$T_{ij}(W) = V_{ij}(W) + V_{ik}(W) G_k(W) V_{kj}(W) + V_{ik}(W) G_k(W) V_{kl}(W) G_l(W) V_{lj}(W) + \cdots$$



Meson-baryon multiple scattering

The solution of the equation is obtained as

$$T_{ij}(W) = [[V(W)]^{-1} - G(W)]^{-1}_{ij}$$

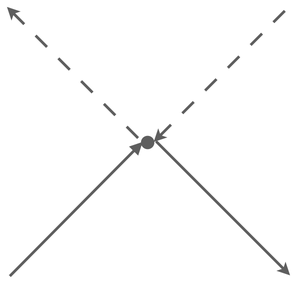
Formulation

$V_{ij}(W)$···Interaction kernel (Weinberg-Tomozawa term)
s-wave interaction satisfying chiral low energy theorem.

$$V_{ij}(W) = -\frac{C_{ij}}{4f_i f_j} N_i N_j (2W - M_i - M_j)$$

f_i : Meson decay constant, C_{ij} : Group theoretical coefficient,

M_i : Baryon Mass, N_i : Kinematical coefficient



$G_i(W, a_i)$···Loop function
(Divergence renormalized by dimensional regularization)

$$G_i(W) \to G_i(W, a_i)$$

W : Total energy, a_i : Subtraction constant

Model 1 construction

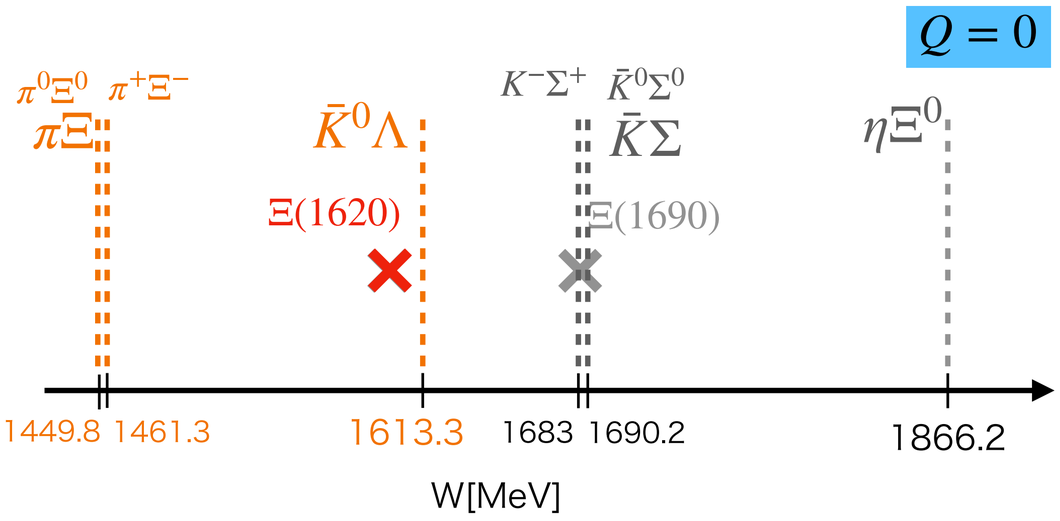
- Belle result : $M_R = 1610$ MeV, $\Gamma_R = 60$ MeV
- Based on the peak position, we define $z_{ex} = [1610 - 30i]$ MeV.
- z_{th} : Pole in theoretical model

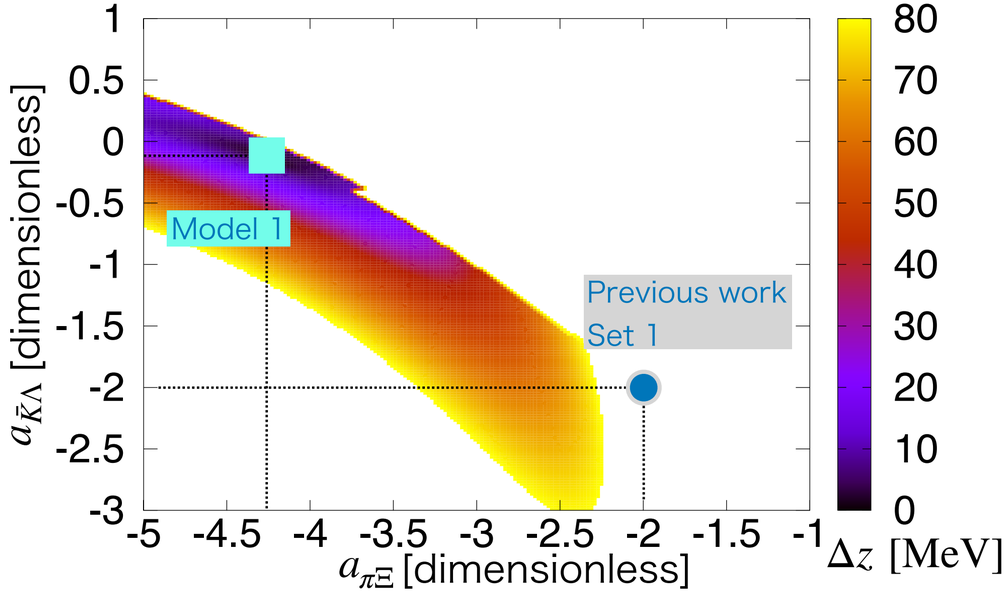
$$\Delta z = |z_{th} - z_{ex}|$$

- We minimize Δz by adjusting subtraction constants $a_{\pi\Xi}$ and $a_{\bar{K}\Lambda}$.

Named as Model 1

$\Delta z = 0.3$ MeV is achieved at $a_{\pi\Xi} = -4.26$ and $a_{\bar{K}\Lambda} = -0.12$[4].





Δz density plot at $a_{\bar{K}\Lambda}$-$a_{\pi\Xi}$ plane

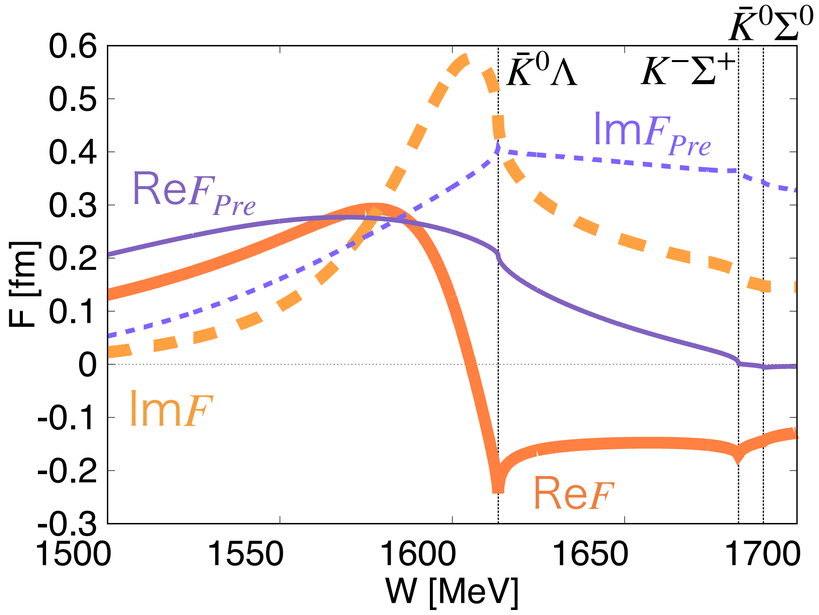
[4]T.Nisihibuchi and T.Hyodo, EPJ Web of Conferences **271**, 10002 (2022); arXiv:2305.10753 [hep-ph]

Result of Model 1

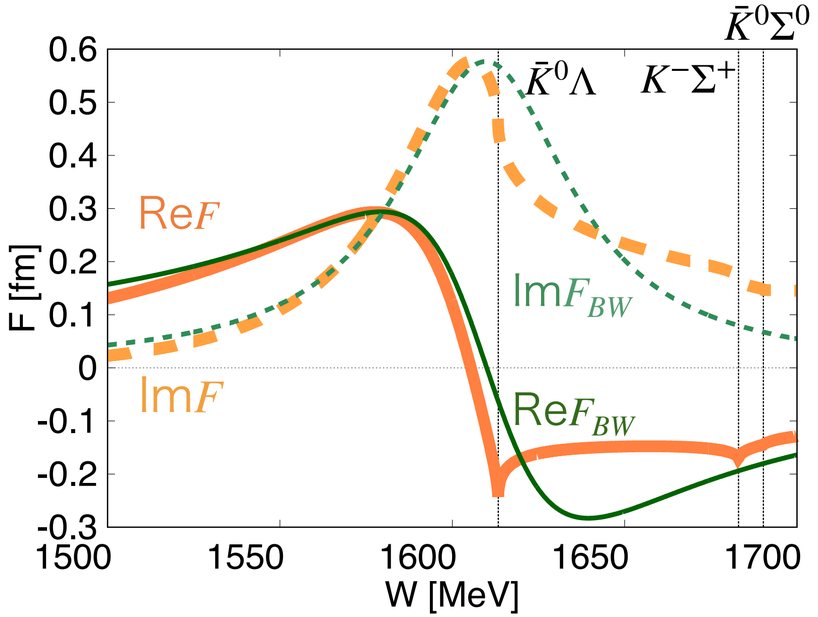
- $\pi^+\Xi^-$scattering amplitude of Model 1(Thin lines)

  - In comparison with previous study[3], distinct peak appears on real axis like Belle result.

  - In comparison with Breit-Wigner distribution, the peak position is shifted and the shape is distorted by the threshold effect.



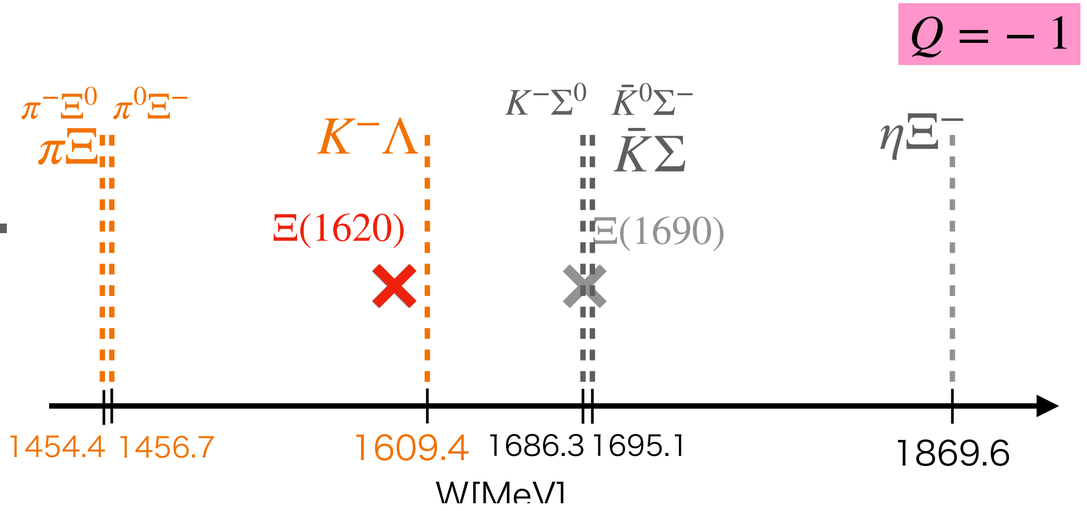
▲comparison with previous study



▲comparison with Breit-Wigner distribution

[3] A.Ramos, E.Oset and C.Bennhold Phys. Rev. Lett. **89**.252001 (2002).

Model 2 construction



- $f_0 \cdots K^-\Lambda$ scattering length

ALICE experiment: $f_{\mathrm{ALICE}} = 0.27 + 0.40i$ fm

Previous work(Set 1): $f_0 = -0.07 + 0.21i$ fm

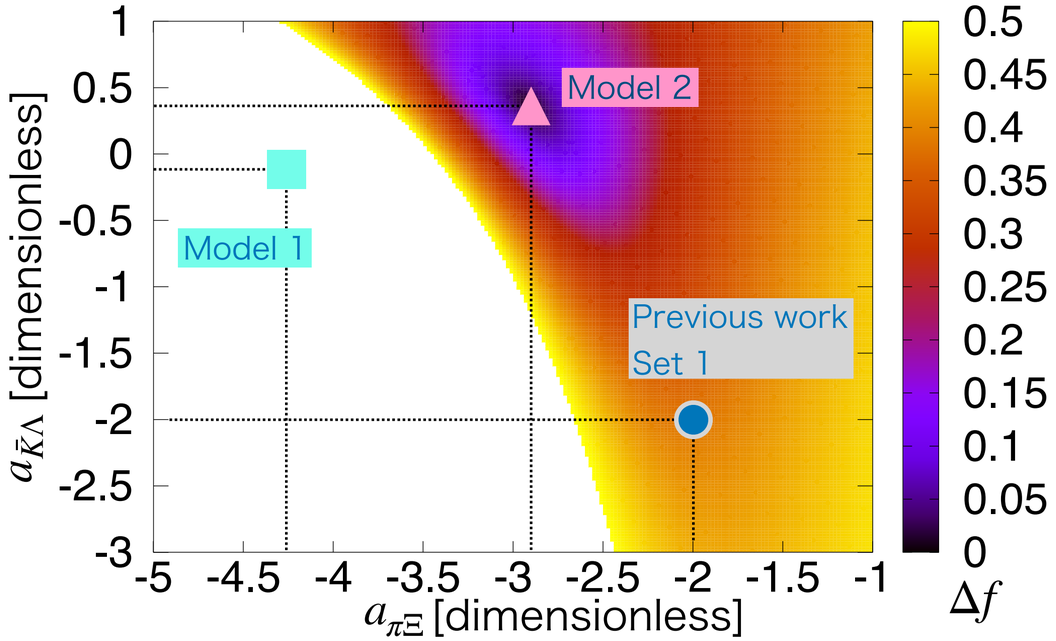
Model 1: $f_0 = -0.75 + 0.93i$ fm

→ They do not reproduce f_{ALICE}.

- f_{th} :scattering length in theoretical model

$$\Delta f = |f_{\mathrm{th}} - f_{\mathrm{ALICE}}|$$

- We minimize Δf by adjusting subtraction constants $a_{\pi\Xi}$ and $a_{\bar{K}\Lambda}$.



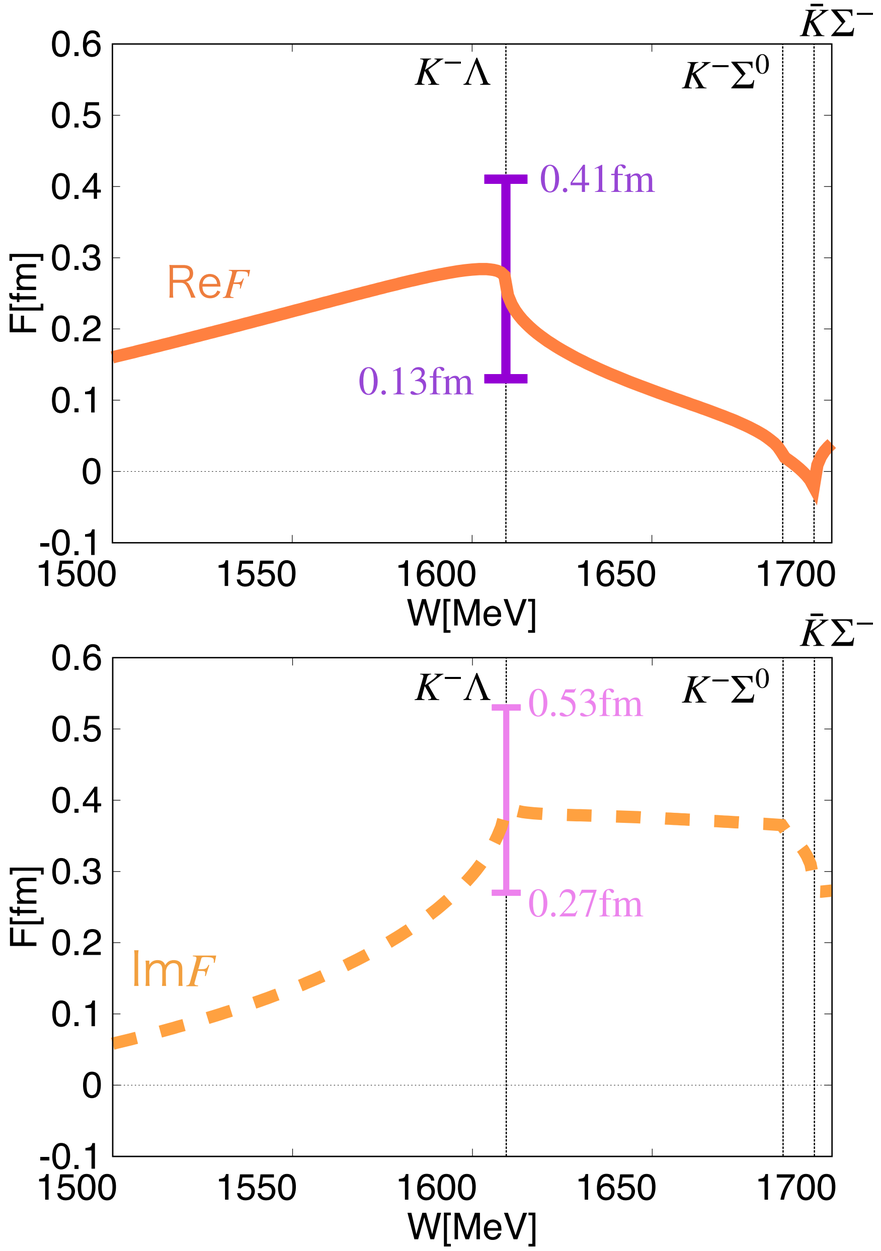
$\Delta f = 0.00 + 0.00i$ fm is achieved at $a_{\pi\Xi} = -2.90$ and $a_{\bar{K}\Lambda} = 0.36$

Result of Model 2

- We plot the scattering amplitude with[5] $a_{\pi\Xi} = -2.90$, $a_{\bar{K}\Lambda} = 0.36$ and $f_{\rm th} = 0.27 + 0.40i$ fm.
- There are no peaks in the spectrum, but a cusp at the threshold.
- There are no poles on the physically relevant Riemann sheets.

The error bar of real part of $f_{\rm ALICE}$

The error bar of imaginary part of $f_{\rm ALICE}$



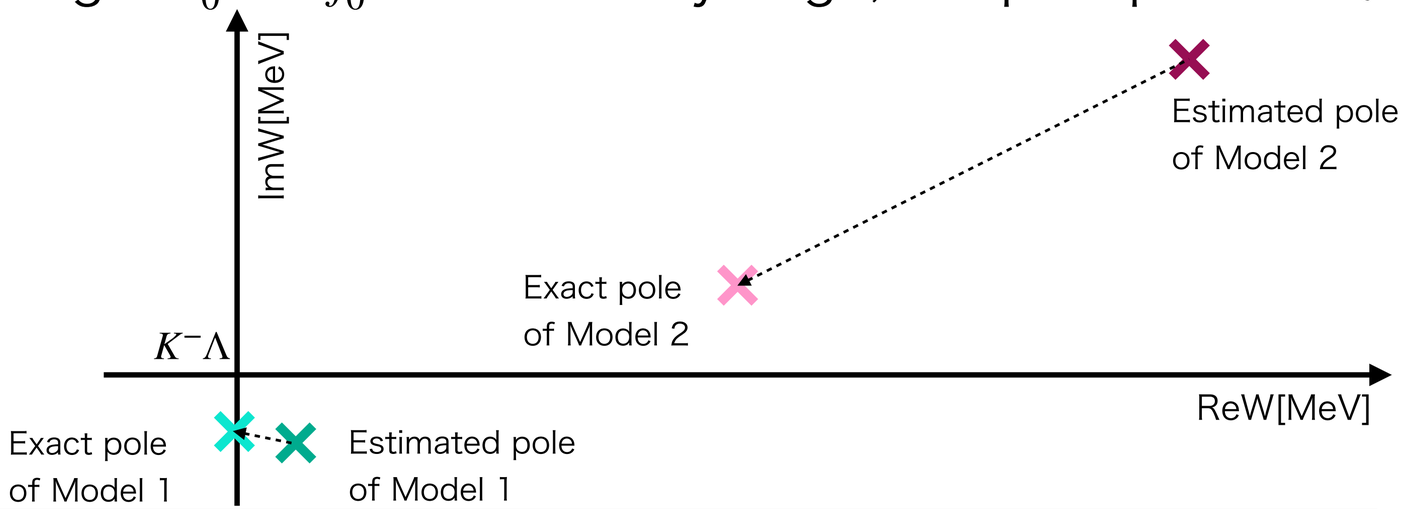
▲The scattering amplitudes of ALICE model

[5]T.Nisihibuchi and T.Hyodo, arXiv:2305.10753 [hep-ph]

Estimation of pole position

If the absolute value of the scattering length $a_0 = -f_0$ is sufficiently large, the pole position z can be estimated as follows.

$$z \sim \frac{-1}{2\mu_{K^-\Lambda} a_0^2} + M_\Lambda + m_{K^-} \qquad \text{(a)}$$



Model name	a_0 [fm]	$\lvert a_0 \rvert$ [fm]	Pole position by eq(a) [MeV]	Exact pole position [MeV]	Distance[MeV]
Model 1	$0.80 - 0.92i$	1.21	$1615 - 38i$ [bbtttt]	$1609 - 30i$ [bbtttt]	9.43
Model 2	$-0.27 - 0.40i$	0.48	$1701 + 228i$ [ttbttt]	$1655 + 53i$ [ttbttt]	180.94

- Eq.(a) can predict the Riemann sheet that exact pole locates.
- The prediction for model with large $|a_0|$ gives accurate pole position.
- Pole position can be predicted with 10 MeV accuracy, for $|a_0| \sim 1$ fm.

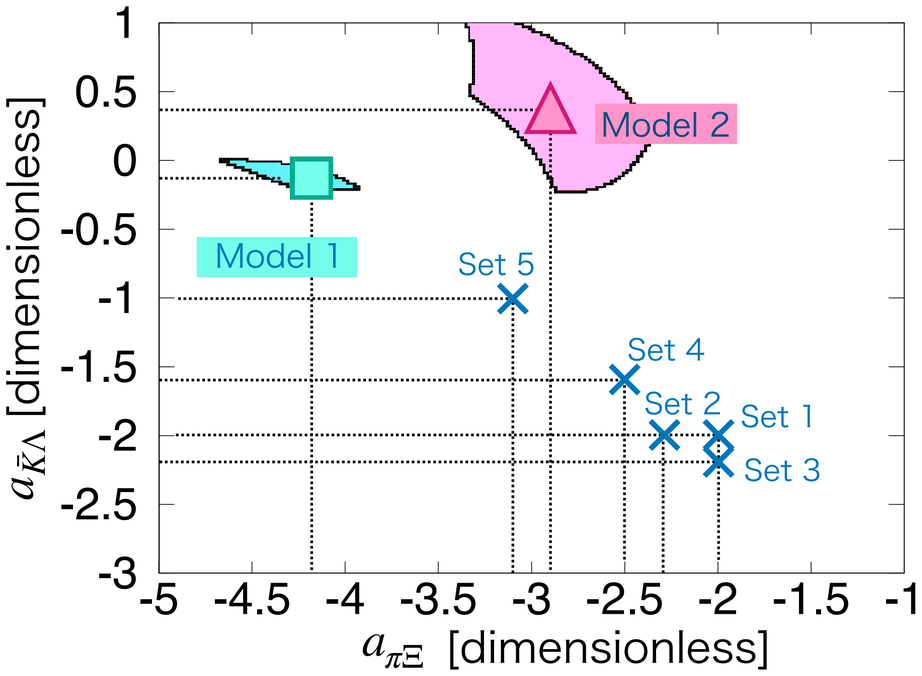
Consistency of ALICE and Belle

Is there a model which satisfies both Belle and ALICE?

→We consider the error of each experiment.

$$\mathrm{M_R} \simeq 1610.4^{+6.1}_{-7.3}\ \mathrm{MeV}, \Gamma_R \simeq 59.9^{+5.6}_{-8.5}\ \mathrm{MeV}$$

$$\mathrm{Re}f_0 \simeq 0.27 \pm 0.14\ \mathrm{fm}, \mathrm{Im}f_0 \simeq 0.40 \pm 0.13\ \mathrm{fm}$$



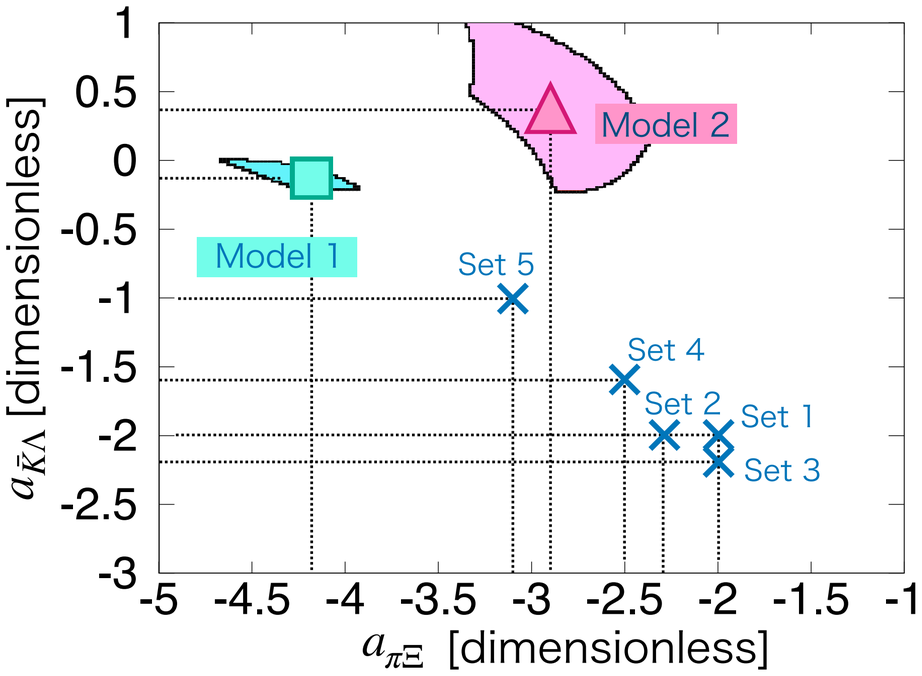
- Figure shows
  - Set 1,2,⋯,5 in previous study[5],
  - the region which satisfies assumption of pole at $\mathrm{M_R} - i\Gamma_R/2$,
  - the region which reproduce $K^-\Lambda$ scattering length.

Consistency of ALICE and Belle

Is there a model which satisfies both Belle and ALICE?

- There is no overlap region which satisfies both ALICE scattering length and the assumption of subthreshold pole.
- The assumption of Model 1 may not be adequate.

  Belle used Breit-Wigner fit to the spectrum, and they did not determine the pole position.
- To compare with Belle data, it is better to use the $\pi\Xi$ spectrum directly.



[5]T.Nisihibuchi and T.Hyodo, arXiv:2305.10753 [hep-ph]

Conclusion

- We construct the models to reproduce the Belle data of the $\pi\Xi$ spectrum (Model 1) and the $K^-\Lambda$ scattering length by ALICE (Model 2).
- In Model 1, we find that the near-threshold resonance peak is distorted by the threshold effect.
- In Model 2, the scattering amplitude shows the cusp at $K^-\Lambda$ threshold. There are no pole of $\Xi(1620)$ in physically relevant Riemann sheets.
- There is no parameter region which satisfies both ALICE scattering length and the assumption of QB state (subthreshold pole) near the $K^-\Lambda$ threshold.
- Future plan: study of $\Xi(1690)$, calculation of $\Xi_c \to \pi\pi\Xi$ decay.

[5]T.Nisihibuchi and T.Hyodo, arXiv:2305.10753 [hep-ph]

Back up New studies for Ξ excited states

LHCb Collaboration(2021)[6]

- $\Xi^-(1690)$ and $\Xi^-(1820)$ are observed in $\Xi_b^- \to J/\psi \Lambda K^-$ decay.
- Mass M_R and width Γ_R of $\Xi^-(1690)$ are reported as follows.

$$M_R = 1692.0 \pm 1.3(\text{stat.})^{+1.2}_{-0.4}(\text{syst.})\ \text{MeV}$$
$$\Gamma_R = 25.9 \pm 9.5(\text{stat.})^{+14.0}_{-13.5}(\text{syst.})\ \text{MeV}$$

New theoretical analysis of $\Xi(1620)$ and $\Xi(1690)$ (2023)[7]

The study based on chiral unitary approach which is added the Born and NLO terms.

$\Xi(1620)$	$M_R = 1599.95$ MeV, $\Gamma_R = 158.88$ MeV.	$M_R = 1608.51$ MeV, $\Gamma_R = 170.00$ MeV.
$\Xi(1690)$	$M_R = 1683.04$ MeV, $\Gamma_R = 11.51$ MeV.	$M_R = 1686.17$ MeV, $\Gamma_R = 29.72$ MeV.

[6]R. Aaij, et al., Sci. Bull. 66 (2021) 1278–1287. [7]Feijoo, A. and Valcarce, V. and Magas, V. K., arXiv:2303.01323 [hep-ph].

Back up Definition of scattering length

- In this study, we define the scattering length f_0 as follows.

  (It is the value of scattering amplitude at threshold energy.)

$$f(k) = \frac{1}{\frac{1}{f_0} + \frac{d_0}{2}k^2 + \cdots - ik}$$

$f(k)$: Scattering amplitude

k: Complex momentum

r_0: effective range

- But in general, scattering length a_0 is defined as follow.

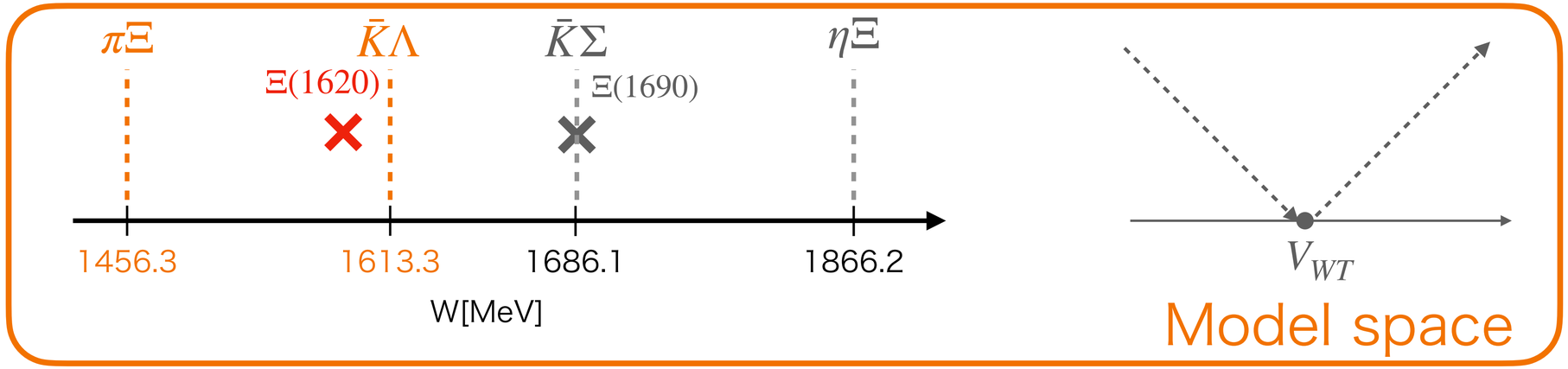
  (It is reverse sign of f_0.)

$$f(k) = \frac{1}{-\frac{1}{a_0} + \frac{r_0}{2}k^2 + \cdots - ik}$$

Back up The roles of subtraction constants

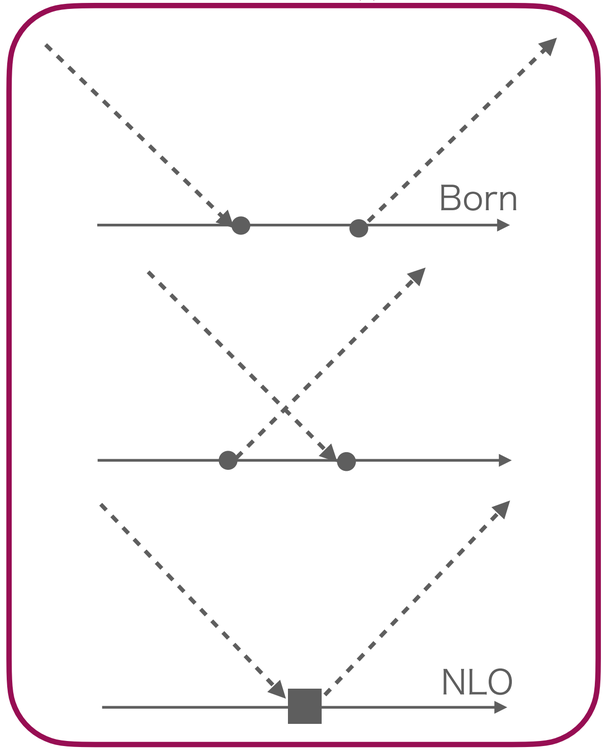
- By changing subtraction constants, the effects from outside of model space can be absorbed.

  ➡ When the subtraction constants closer to natural value, outside effect become smaller.



Effects from other channels(Ξ^*_{uss},$\bar{K}^*\Lambda$,$\bar{K}^*\Sigma$,$\pi\bar{K}\Lambda$,$\cdots$).

Effects from except V_{WT} .



[8] T.Hyodo, D.Jido and A.Hosaka Phys. Rev. C 78.025203 (2008)

Back up Detail of loop function

Loop function $G_i(W)$

$$G_i(W) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_i^2 + i0^+}\frac{1}{(P-q)^2 - M_i^2 + i0^+}$$

Loop function $G_i(W, a_i)$ (Removed divergence by dimensional regularization)

$$G_i(W, a_i) = \frac{1}{16\pi^2}\left[a_i(\mu_{reg}) + \ln\frac{mM}{\mu_{reg}^2} + \frac{M^2 - m^2}{2W^2}\ln\frac{M^2}{m^2} + \frac{\lambda^{1/2}}{2W^2}\left\{\ln(W^2 - m^2 + M^2 + \lambda^{1/2})\right.\right.$$

$$\left.\left. + \ln(W^2 + m^2 - M^2 + \lambda^{1/2}) - \ln(-W^2 + m^2 - M^2 + \lambda^{1/2}) - \ln(-W^2 - m^2 + M^2 + \lambda^{1/2})\right\}\right]$$

$$\lambda^{1/2} = \sqrt{W^4 + m_k^4 + M_k^4 - 2W^2m_k^2 - 2m_k^2M_k^2 - 2M_k^2W^2}$$

Back up Lednický and Lyuboshitz model

When Coulomb interaction is not at work, the correlation function can be described analytically with the Lednický and Lyuboshitz model.

$$f^s(k^*) = \left(\frac{1}{f_0^s} + \frac{1}{2} d_0^s k^{*2} - ik^* \right)^{-1}$$

$f_0^s(k)$: complex s-wave scattering length

d_0^s : Effective range

$$C(k^*)_{\text{Lednický}} = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^s(k^*)}{R_{\text{inv}}} \right|^2 \left(1 - \frac{d_0^s}{2\sqrt{\pi} R_{\text{inv}}} \right) + \frac{2\text{Re}\, f^s(k^*)}{\sqrt{\pi} R_{\text{inv}}} F_1(2k^* R_{\text{inv}}) + \frac{\text{Im}\, f^s(k^*)}{R_{\text{inv}}} F_2(2k^* R_{\text{inv}}) \right]$$

F_1, F_2 : Analytic functions

ρ_S :Weight factor

(the normalized emission probability for a state of total spin S)

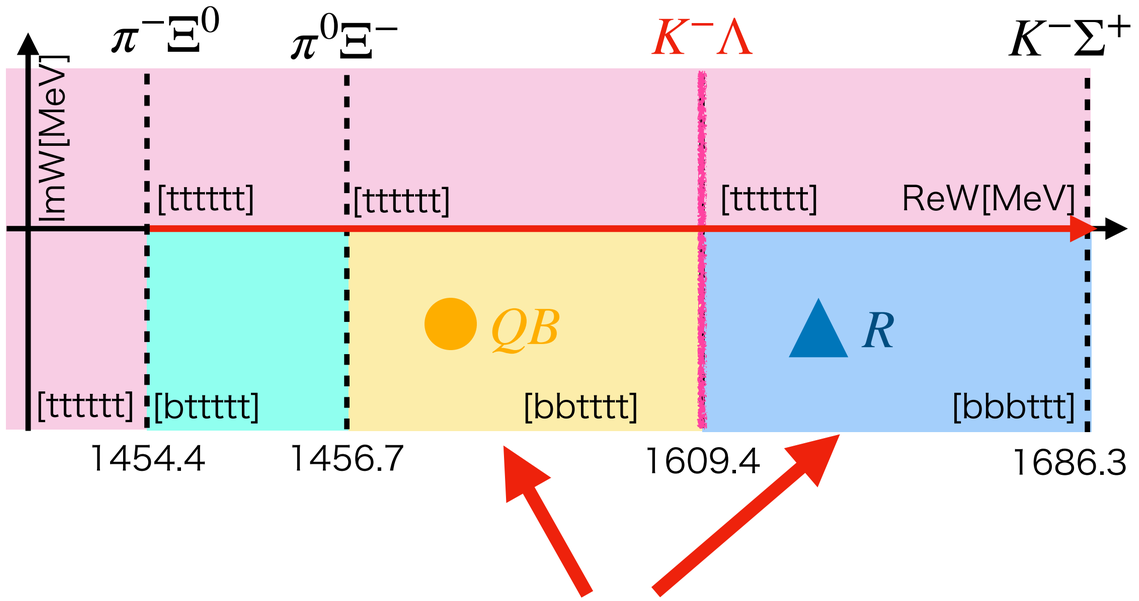
$$\rho_S = \frac{(2S+1)}{[(2j_1+1)(2j_2+1)]}$$

Back up Riemann sheets

Physically relevant Riemann sheet

- On the complex energy plane, Riemann sheets are separated at each threshold energy.(1st sheet : [t] / 2nd sheet : [b])

- If a pole exists at the [bbtttt] sheet or the [bbbttt] one, scattering amplitude on real axis is directly affected by the pole.(physically relevant)



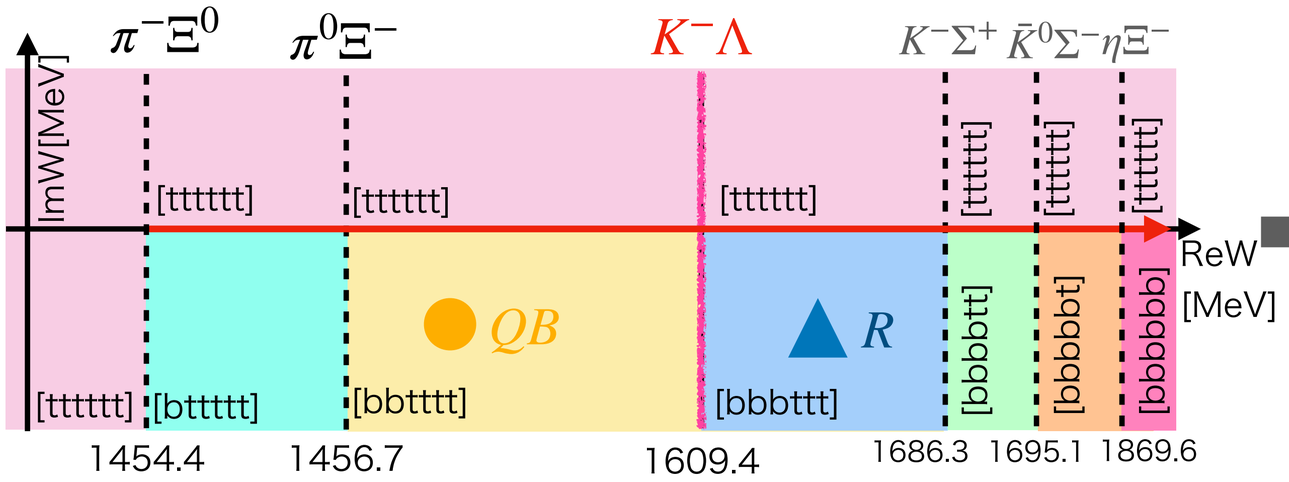
Physically relevant sheet

[bbtttt] : $1456.7 \leq \mathrm{Re}W \leq 1609.7,\ \mathrm{Im}W < 0$

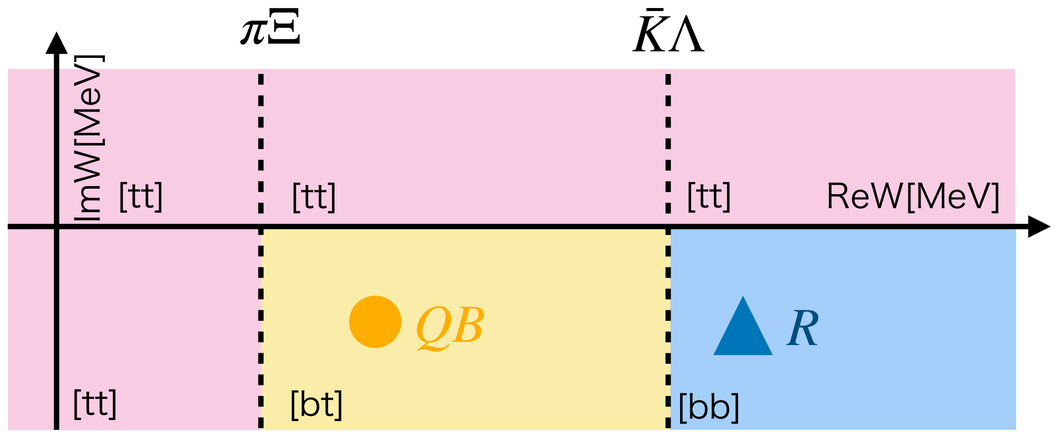
[bbbttt] : $1609.7 \leq \mathrm{Re}W \leq 1686.3,\ \mathrm{Im}W < 0$

Back up Riemann sheets

Considering pole trajectory on Riemann sheets



6 channels(difficult)

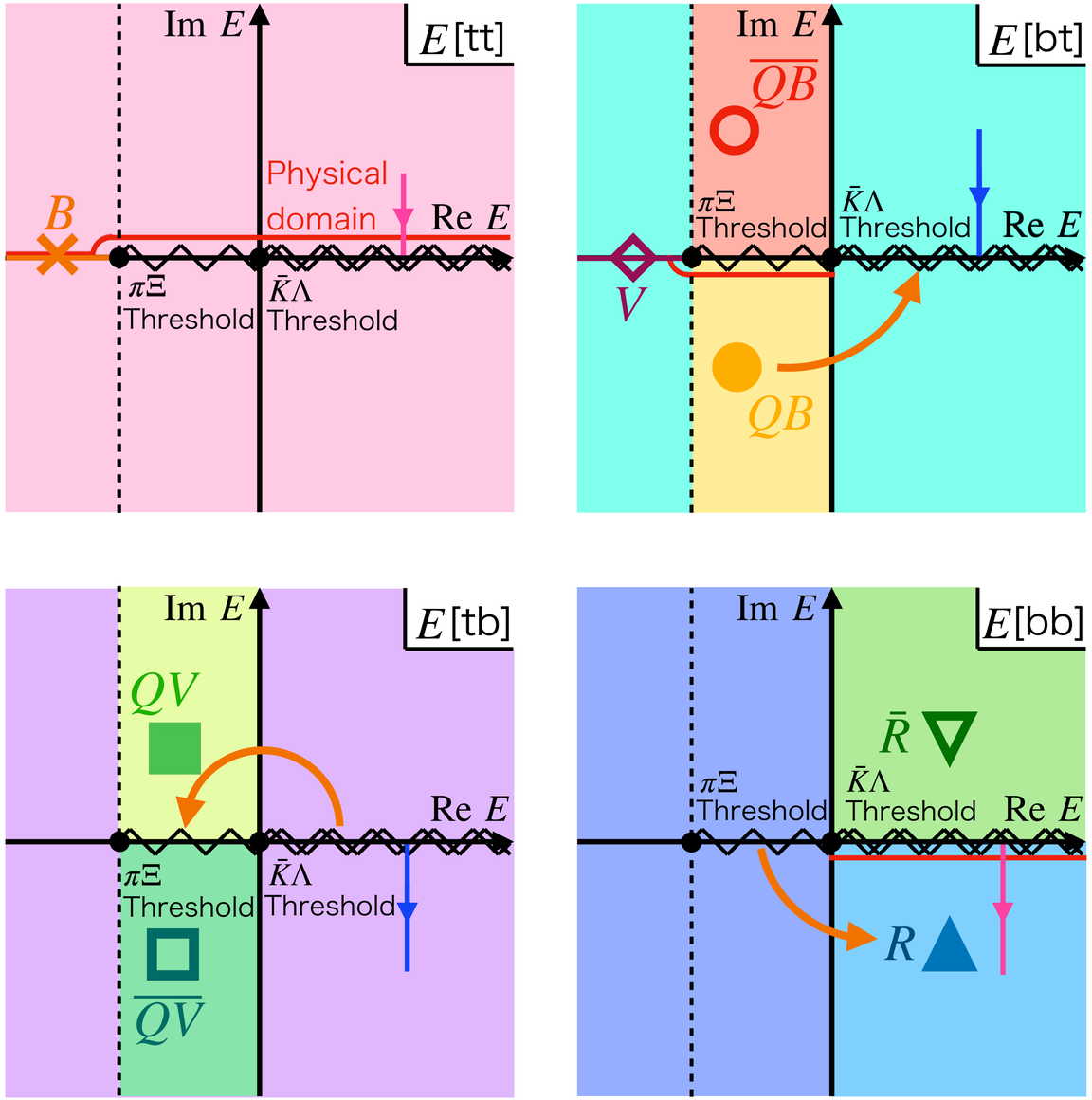


Only 2 channels(easy)

- Treating iso partner as same, so we consider the case with 2 channels.
- In this case, $K^-\Sigma^+$, $\bar{K}^0\Sigma^-$ and $\eta\Xi^-$ channels are negligible.

Back up Riemann sheets

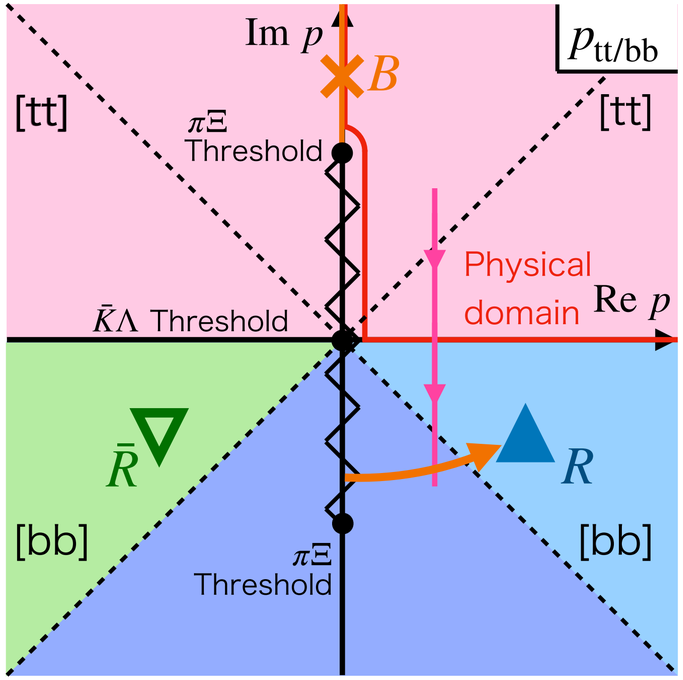
- Pole trajectory in complex energy plane
- The case of 2ch, then we have 4 riemann sheets.
- From QB in [bt] sheet to R in [bb], the pole moves through the upper half plane of the [tb] sheet.



Riemann sheets of complex E plane([tt],[tb],[bt],[bb])

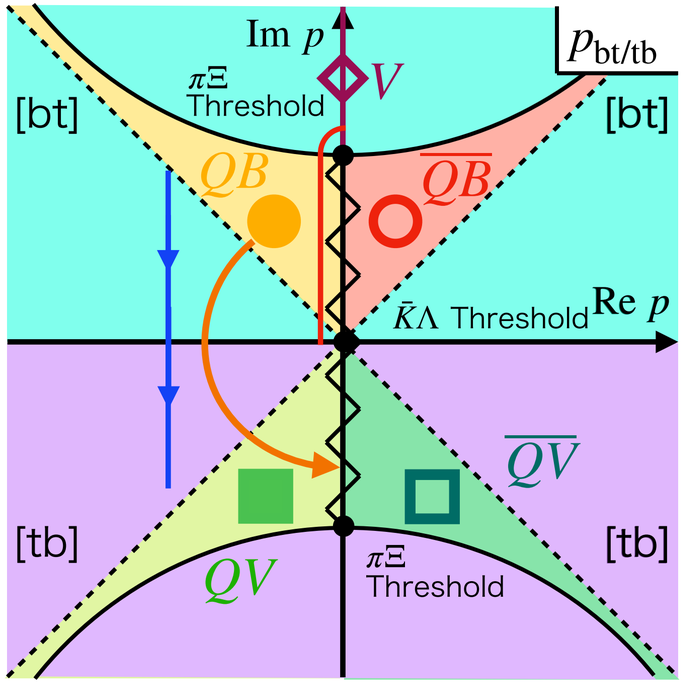
Back up Riemann sheets

- Pole trajectory in complex momentum plane
- From QB in [bt] sheet to R in [bb], the pole must go across the cut.
- The pole cannot go across the cut in upper half plane(forbidden in [tt])
- The pole in model 2 in [tb] represents the state between QB and R(QV).



(a)Complex p plane

[tt]-[bb] sheet



(b)Complex p plane

[bt]-[tb] sheet