Analysis of $\Xi(1620)$ resonance with chiral unitary approach

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Result of Belle

Belle experiment of $\Xi_c \to \pi\pi\Xi$ (2019)[1]
- $\Xi(1620)$ and $\Xi(1690)$ peaks are observed in the invariant mass distribution of $\pi^+\Xi^-$.  
- The mass $M_R$ and width $\Gamma_R$ of $\Xi(1620)$ are reported as follows.
  \[ M_R = 1610.4 \pm 6.0 \text{(stat.)}^{+6.1}_{-4.2} \text{(syst.)} \text{ MeV} \]
  \[ \Gamma_R = 59.9 \pm 4.8 \text{(stat.)}^{+2.8}_{-7.1} \text{(syst.)} \text{ MeV} \]
- Mass and width do not agree with values in previous theoretical studies.
- Peaks are close to thresholds of $\bar{K}\Lambda$ and $\bar{K}\Sigma$?

Result of ALICE

ALICE experiment(2021)[2]
The scattering length $f_0$ of $K^-\Lambda$ was determined with femtoscopy in Pb-Pb collisions as follows.

$$\text{Re}f_0 = 0.27 \pm 0.12(\text{stat.}) \pm 0.07(\text{syst.}) \text{ fm}$$

$$\text{Im}f_0 = 0.40 \pm 0.11(\text{stat.}) \pm 0.07(\text{syst.}) \text{ fm}$$

$f_0$ constrains real and imaginary parts of scattering amplitude at threshold.

More quantitative comparison is possible than the spectrum fit.

Strategy of this study

Strategy of model construction

- Model of previous study [3] Set 1
  Mass $M_R$ and width $\Gamma_R$ of $\Xi(1620)$
  $M_R = 1607$ MeV, $\Gamma_R = 280$ MeV.

We construct theoretical models which reproduce new experimental results.

- Model for Belle result (Model 1)
  Assume pole position from mass $M_R$ and width $\Gamma_R$ reported by Belle

- Model for ALICE result (Model 2)
  Reproduce the $K^-\Lambda$ scattering length determined by ALICE

Formulation

Coupled-channel meson-baryon scattering amplitude $T_{ij}(W)$ at total energy $W$.

Scattering equation

$$T_{ij}(W) = V_{ij}(W) + V_{ik}(W)G_k(W)T_{kj}(W)$$

$V_{ij}(W)$...Interaction kernel

$G_i(W)$...Loop function

$$T_{ij}(W) = V_{ij}(W) + V_{ik}(W)G_k(W)V_{kj}(W) + V_{ik}(W)G_k(W)V_{kl}(W)G_l(W)V_{lj}(W) + \cdots$$

The solution of the equation is obtained as

$$T_{ij}(W) = \left(\left[ [V(W)]^{-1} - G(W) \right]_{ij} \right)^{-1}$$
Formulation

$V_{ij}(W)$ \(\cdots\) Interaction kernel (Weinberg-Tomozawa term)

s-wave interaction satisfying chiral low energy theorem.

$$V_{ij}(W) = -\frac{C_{ij}}{4f_if_j}N_iN_j(2W - M_i - M_j)$$

$f_i$ : Meson decay constant, $C_{ij}$ : Group theoretical coefficient,

$M_i$ : Baryon Mass, $N_i$ : Kinematical coefficient

$G_i(W, a_i)$ \(\cdots\) Loop function

(Divergence renormalized by dimensional regularization)

$$G_i(W) \rightarrow G_i(W, a_i)$$

$W$ : Total energy, $a_i$ : Subtraction constant
Model 1 construction

- Belle result: $M_R = 1610$ MeV, $\Gamma_R = 60$ MeV
- Based on the peak position, we define $z_{\text{ex}} = [1610 - 30i]$ MeV.

- $z_{\text{th}}$: Pole in theoretical model
  \[ \Delta z = |z_{\text{th}} - z_{\text{ex}}| \]
- We minimize $\Delta z$ by adjusting subtraction constants $a_{\pi\Xi}$ and $a_{\bar{K}\Lambda}$.

$\Delta z = 0.3$ MeV is achieved at $a_{\pi\Xi} = -4.26$ and $a_{\bar{K}\Lambda} = -0.12$[4].

Result of Model 1

- $\pi^+\Xi^-$-scattering amplitude of Model 1 (Thin lines)

- In comparison with previous study [3], distinct peak appears on real axis like Belle result.

- In comparison with Breit-Wigner distribution, the peak position is shifted and the shape is distorted by the threshold effect.

Model 2 construction

\[ f_0 \cdots K^- \Lambda \text{ scattering length} \]

ALICE experiment: \[ f_{\text{ALICE}} = 0.27 + 0.40i \text{ fm} \]

Previous work (Set 1): \[ f_0 = -0.07 + 0.21i \text{ fm} \]

Model 1: \[ f_0 = -0.75 + 0.93i \text{ fm} \]

They do not reproduce \( f_{\text{ALICE}} \).

\[ f_{\text{th}} : \text{scattering length in theoretical model} \]

\[ \Delta f = |f_{\text{th}} - f_{\text{ALICE}}| \]

We minimize \( \Delta f \) by adjusting subtraction constants \( a_{\pi \Xi} \) and \( a_{\bar{K} \Lambda} \).

\( \Delta f = 0.00 + 0.00i \text{ fm} \) is achieved at \( a_{\pi \Xi} = -2.90 \) and \( a_{\bar{K} \Lambda} = 0.36 \).
Result of Model 2

- We plot the scattering amplitude with [5] $a_{\pi\Xi} = -2.90$, $a_{\bar{K}\Lambda} = 0.36$ and $f_{th} = 0.27 + 0.40i$ fm.
- There are no peaks in the spectrum, but a cusp at the threshold.
- There are no poles on the physically relevant Riemann sheets.


The error bar of real part of $f_{\text{ALICE}}$  
The error bar of imaginary part of $f_{\text{ALICE}}$  

The scattering amplitudes of ALICE model
**Estimation of pole position**

If the absolute value of the scattering length $a_0 = -f_0$ is sufficiently large, the pole position $z$ can be estimated as follows.

$$z \sim \frac{-1}{2\mu_{K^0\Lambda}a_0^2} + M_\Lambda + m_{K^-}$$  \hspace{1cm} (a)

| Model name  | $a_0$ [fm]       | $|a_0|$ [fm] | Pole position by eq(a) [MeV] | Exact pole position [MeV] | Distance [MeV] |
|-------------|------------------|-------------|------------------------------|----------------------------|---------------|
| Model 1     | $0.80 - 0.92i$   | 1.21        | $1615 - 38i$ [bbtttt]        | $1609 - 30i$ [bbtttt]      | 9.43          |
| Model 2     | $-0.27 - 0.40i$  | 0.48        | $1701 + 228i$ [ttbtttt]      | $1655 + 53i$ [ttbtttt]     | 180.94        |

- Eq.(a) can predict the Riemann sheet that exact pole locates.
- The prediction for model with large $|a_0|$ gives accurate pole position.
- Pole position can be predicted with 10 MeV accuracy, for $|a_0| \sim 1$ fm.
Consistency of ALICE and Belle

Is there a model which satisfies both Belle and ALICE?

→ We consider the error of each experiment.

\[ M_R \approx 1610.4^{+6.1}_{-7.3} \text{ MeV}, \Gamma_R \approx 59.9^{+5.6}_{-8.5} \text{ MeV} \]

\[ \text{Re}f_0 \approx 0.27 \pm 0.14 \text{ fm}, \text{Im}f_0 \approx 0.40 \pm 0.13 \text{ fm} \]

• Figure shows

  ○ Set 1, 2, …, 5 in previous study[5],
  ○ the region which satisfies assumption of pole at \( M_R - i\Gamma_R/2 \),
  ○ the region which reproduce \( K^-\Lambda \) scattering length.
Consistency of ALICE and Belle

Is there a model which satisfies both Belle and ALICE?

- There is no overlap region which satisfies both ALICE scattering length and the assumption of subthreshold pole.
- The assumption of Model 1 may not be adequate.
  
  Belle used Breit-Wigner fit to the spectrum, and they did not determine the pole position.
  
  To compare with Belle data, it is better to use the $\pi\Xi$ spectrum directly.

Conclusion

○ We construct the models to reproduce the Belle data of the $\pi\Xi$ spectrum (Model 1) and the $K^-\Lambda$ scattering length by ALICE (Model 2).

○ In Model 1, we find that the near-threshold resonance peak is distorted by the threshold effect.

○ In Model 2, the scattering amplitude shows the cusp at $K^-\Lambda$ threshold. There are no pole of $\Xi(1620)$ in physically relevant Riemann sheets.

○ There is no parameter region which satisfies both ALICE scattering length and the assumption of QB state (subthreshold pole) near the $K^-\Lambda$ threshold.

○ Future plan: study of $\Xi(1690)$, calculation of $\Xi_c \rightarrow \pi\pi\Xi$ decay.

Back up New studies for $\Xi$ excited states

LHCb Collaboration(2021)[6]

- $\Xi^-(1690)$ and $\Xi^-(1820)$ are observed in $\Xi_b^- \rightarrow J/\psi \Lambda K^-$ decay.
- Mass $M_R$ and width $\Gamma_R$ of $\Xi^-(1690)$ are reported as follows.
  \[
  M_R = 1692.0 \pm 1.3\,(\text{stat.})^{+1.2}_{-0.4}\,(\text{syst.}) \text{ MeV} \\
  \Gamma_R = 25.9 \pm 9.5\,(\text{stat.})^{+14.0}_{-13.5}\,(\text{syst.}) \text{ MeV}
  \]

New theoretical analysis of $\Xi(1620)$ and $\Xi(1690)$ (2023)[7]
The study based on chiral unitary approach which is added the Born and NLO terms.

$\Xi(1620)$  $M_R = 1599.95 \text{ MeV}, \Gamma_R = 158.88 \text{ MeV}.$ \quad $M_R = 1608.51 \text{ MeV}, \Gamma_R = 170.00 \text{ MeV}.$
$\Xi(1690)$  $M_R = 1683.04 \text{ MeV}, \Gamma_R = 11.51 \text{ MeV}.$ \quad $M_R = 1686.17 \text{ MeV}, \Gamma_R = 29.72 \text{ MeV}.$


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**Back up Definition of scattering length**

- In this study, we define the scattering length \( f_0 \) as follows.
  (It is the value of scattering amplitude at threshold energy.)

\[
f(k) = \frac{1}{\frac{1}{f_0} + \frac{d_0}{2}k^2 + \cdots - ik}
\]

- \( f(k) \) : Scattering amplitude
- \( k \): Complex momentum
- \( r_0 \): effective range

- But in general, scattering length \( a_0 \) is defined as follow.
  (It is reverse sign of \( f_0 \).)

\[
f(k) = \frac{1}{-\frac{1}{a_0} + \frac{r_0}{2}k^2 + \cdots - ik}
\]
Back up The roles of subtraction constants

- By changing subtraction constants, the effects from outside of model space can be absorbed.

When the subtraction constants closer to natural value, outside effect become smaller.

\[\pi, K^\Lambda, K^\Sigma, \eta, \Xi(1620), \Xi(1690)\]

1456.3 1613.3 1686.1 1866.2

W[MeV]

Effects from other channels (\(\Xi_{usss}, K^*\Lambda, K^*\Sigma, \pi K\Lambda, \cdots\)).


Effects from except \(V_{WT}\).
Loop function $G_i(W)$

$$G_i(W) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m^2 + i0^+} \frac{1}{(P - q)^2 - M_i^2 + i0^+}$$

Loop function $G_i(W, a_i)$ (Removed divergence by dimensional regularization)

$$G_i(W, a_i) = \frac{1}{16\pi^2} \left[ a_i(\mu_{reg}) + \ln \frac{mM}{\mu_{reg}^2} + \frac{M^2 - m^2}{2W^2} \ln \frac{M^2}{m^2} + \frac{\lambda^{1/2}}{2W^2} \left\{ \ln(W^2 - m^2 + M^2 + \lambda^{1/2}) ight. \\
+ \ln(W^2 + m^2 - M^2 + \lambda^{1/2}) - \ln(-W^2 + m^2 - M^2 + \lambda^{1/2}) - \ln(-W^2 - m^2 + M^2 + \lambda^{1/2}) \right\} \right]$$

$$\lambda^{1/2} = \sqrt{W^4 + m_k^4 + M_k^4 - 2W^2m_k^2 - 2m_k^2M_k^2 - 2M_k^2W^2}$$
Back up Lednický and Lyuboshitz model

When Coulomb interaction is not at work, the correlation function can be described analytically with the Lednický and Lyuboshitz model.

\[ f^s(k^*) = \left( \frac{1}{f_0^s} + \frac{1}{2} d_0^s k^* - i k^* \right)^{-1} \]

\[ f_0^s(k) : \text{complex s-wave scattering length} \]

\[ d_0^s : \text{Effective range} \]

\[ C(k^*)_{\text{Lednický}} = 1 + \sum_s \rho_s \left[ \frac{1}{2} \left| \frac{f^s(k^*)}{R_{\text{inv}}} \right|^2 \left( 1 - \frac{d_0^s}{2\sqrt{\pi} R_{\text{inv}}} \right) \right] + \frac{2 \text{Re} f^s(k^*)}{\sqrt{\pi} R_{\text{inv}}} F_1(2k^* R_{\text{inv}}) + \frac{\text{Im} f^s(k^*)}{R_{\text{inv}}} F_2(2k^* R_{\text{inv}}) \]

\[ \rho_s = \frac{(2S + 1)}{[(2j_1 + 1)(2j_2 + 1)]} \]
**Back up Riemann sheets**

**Physically relevant Riemann sheet**

- On the complex energy plane, Riemann sheets are separated at each threshold energy. (1st sheet: [t] / 2nd sheet: [b])

- If a pole exists at the [bbtttt] sheet or the [bbtttttt] one, scattering amplitude on real axis is directly affected by the pole. (physically relevant)

![Graph showing physically relevant Riemann sheets](image)

**Physically relevant sheet**

\[
[bbbtttt] : 1456.7 \leq \text{Re}W \leq 1609.7, \text{Im}W < 0
\]

\[
[bbtttt] : 1609.7 \leq \text{Re}W \leq 1686.3, \text{Im}W < 0
\]
Considering pole trajectory on Riemann sheets

- Treating iso partner as same, so we consider the case with 2 channels.
- In this case, $K^-\Sigma^+$, $\bar{K}^0\Sigma^-$ and $\eta\Xi^-$ channels are negligible.
Back up Riemann sheets

- Pole trajectory in complex energy plane
- The case of 2ch, then we have 4 riemann sheets.
- From QB in [bt] sheet to R in [bb], the pole moves through the upper half plane of the [tb] sheet.

Riemann sheets of complex $E$ plane([tt],[tb],[bt],[bb])
Back up Riemann sheets

- Pole trajectory in complex momentum plane
- From QB in [bt] sheet to R in [bb], the pole must go across the cut.
- The pole cannot go across the cut in upper half plane (forbidden in [tt]).
- The pole in model 2 in [tb] represents the state between QB and R(QV).