

# Correlation Function constraints on $S = -2$ meson-baryon interaction from UChPT

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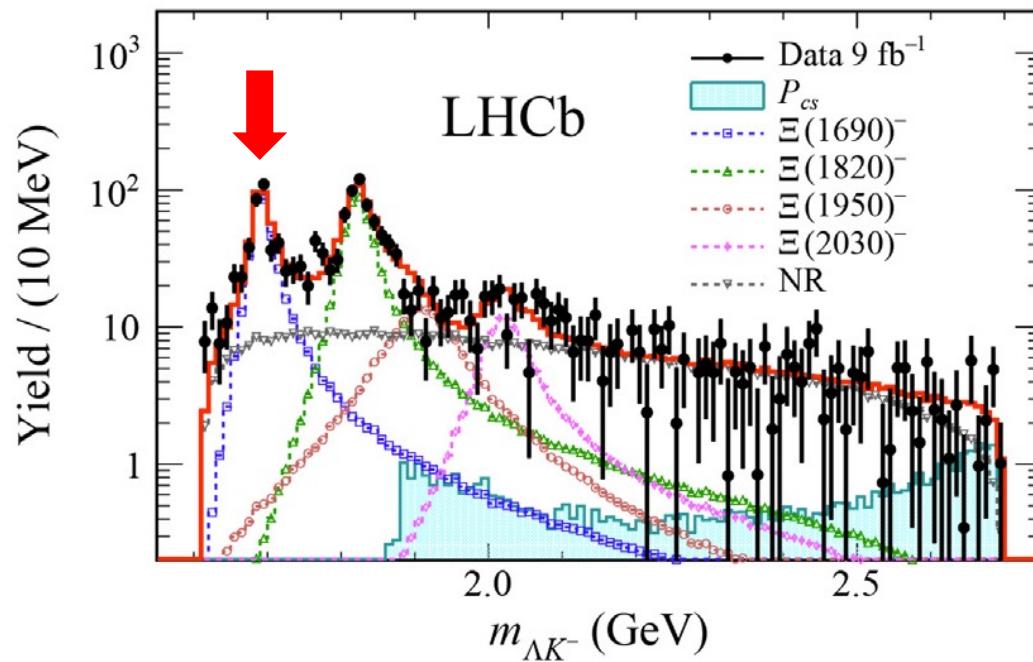
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## Introduction: Experimental Background

Observation of two  $K^-\Lambda$  structures in the  $\Xi_b^- \rightarrow K^- J/\psi \Lambda$



R. Aaij, et al., LHCb, Sci. Bull. 66 (2021) 1278

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$$M = 1692.0 \pm 1.3^{+1.2}_{-0.4} \text{ MeV},$$

$$\Gamma = 25.9 \pm 9.5^{+14.0}_{-13.5} \text{ MeV}.$$

PDG: \*\*\* status

$$J^P = \frac{1}{2}^-$$

Experimental evidence for the spin-parity  
from  $\Lambda_c^+ \rightarrow K^+ \pi^+ \Xi^-$  decay  
B. Aubert, et al., Phys. Rev. D 78 (2008) 034008

$$M = 1822.7 \pm 1.5^{+1.0}_{-0.6} \text{ MeV}$$

$$\Gamma = 36.0 \pm 4.4^{+7.8}_{-8.2} \text{ MeV}.$$

## Introduction: Experimental Background

Observation of three peaks in the  $\pi^+\Xi^-$  invariant mass distribution for  $\Xi_c^+ \rightarrow \pi^+\pi^+\Xi^-$

$$M = 1610.4 \pm 6.0^{+6.1}_{-4.2} \text{ MeV},$$

$$\Gamma = 59.9 \pm 4.8^{+2.8}_{-7.1} \text{ MeV}.$$

M. Sumihama, et al., Phys. Rev. Lett. 122 (7) (2019) 072501.

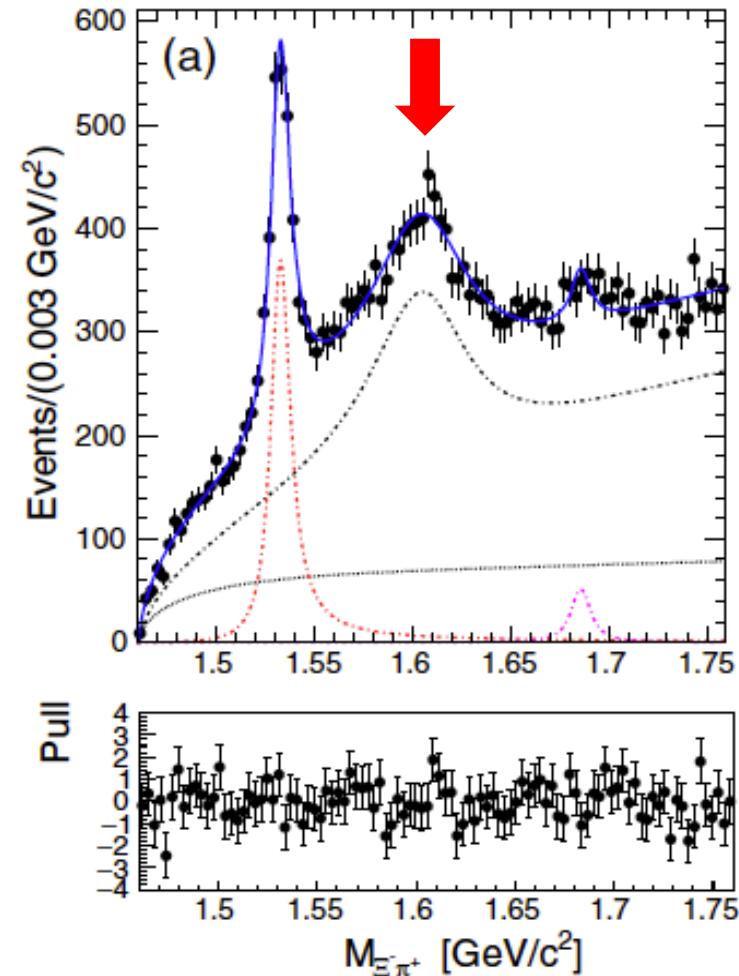
PDG: \* status

$$J^P = ?^?$$

Assumed to have spin-parity  $\frac{1}{2}^-$  as the S=-2 counterpart of the  $\Lambda(1405)$

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## *Introduction: Theoretical Framework and Historical Background*

Study of the **meson-baryon interaction** in the **S=-2, Q=0** sector.

6 channels involved in this sector:



**Interaction:** QCD is a gauge theory which **describes** the **strong interaction** governed by the effects of the color charge of its carriers: quarks and gluons.

Perturbative QCD is inappropriate to treat low energy hadron interactions.

**Chiral Perturbation Theory (ChPT)** is an effective theory with hadrons as degrees of freedom which respects the symmetries of QCD.

- limited to a moderate range of energies above threshold
- not applicable close to a resonance (singularity in the amplitude)

But it is not so straight forward ...

## *Introduction: Theoretical Framework and Historical Background*

The meson-baryon interaction within the previous energy range is dominated by the presence of the  $\Xi(1620)$  and  $\Xi(1690)$  resonances.

→ A nonperturbative resummation is needed!!!

In 2002, the problem was reformulated in terms of a Unitary extension of ChPT in coupled channels.

The pioneering work -- **A. Ramos, E. Oset, C. Bennhold, Phys. Rev. Lett. 89 (2002) 252001.**

- C. Garcia-Recio, M. F. M. Lutz, J. Nieves, Phys. Lett. B 582 (2004) 49–54.
- D. Gamermann, C. Garcia-Recio, J. Nieves, L. L. Salcedo, Phys. Rev. D 84 (2011) 056017.
- T. Sekihara, PTEP 2015 (9) (2015) 091.
- T. Nishibuchi, T. Hyodo, arXiv:2305.10753 [hep-ph].  
T. Nishibuchi talk,, Parallel sesión C Friday 23 June

→ In all these Works only a WT-like contact term was employed... some describe just one of the states properly, others can describe both states simultaneously yet only qualitatively

## *Motivation: Evolution of our chiral model*

In this sector, Born and NLO terms have been systematically ignored , assumed to play a very moderate role...  
How solid this assumption is?

- Evidences of their non-negligible function in  $\bar{K}N$  interaction:

J. A. Oller and U.-G. Meissner, Phys. Lett. B 500, 263 (2001)

Born contributions reach ~20% of the dominant WT contribution just 65 MeV above  $\bar{K}N$  threshold (S-wave)

→ Just the energy range between the lowest and the highest threshold in S=-2 sector is about 410 MeV wide

A. F., V. Magas and A. Ramos, Phys. Rev. C 99, no.3, 035211 (2019), Nucl. Phys. A 954, 58 (2016)

At slightly higher energies, the NLO and the Born terms are essential to reproduce the experimental total cross section from  $\bar{K}N \rightarrow \eta\Lambda, \eta\Sigma, K\Xi$  processes ( $\eta$  channel thresholds are around 200 MeV above  $\bar{K}N$  threshold)

The incorporation of the s- and u-channel diagrams as well as NLO may have additional implications...

- the inclusion of new pieces in the interaction kernel can affect the interplay among the channels of the basis

## *Motivation: Evolution of our chiral model*

Over the last years, Barcelona group has been working on the NLO contributions of the chiral Lagrangian

1. Paying attention on reactions particularly sensitive to NLO

e.g.  $\bar{K}N \rightarrow K\Xi$  (it does not proceed through WT at tree level)

A. F., V. Magas, A. Ramos, Phys. Rev. C 92, 015206 (2015)

2. Analysing the relative relevance between the Born terms and the NLO contributions in the previous type of reactions

Born and NLO terms play a similar role

A. Ramos, A. F., V. Magas, Nucl. Phys. A 954, 58 (2016)

3. Studying the constraining effect of isospin filtering reactions especially sensitive to NLO

More reliable values for the NLO LECs

A. F., V. Magas, A. Ramos, Phys. Rev. C 99 (2019) 035211

## *Formalism: Effective Chiral Lagrangian*

**Lagrangian:**

$$\mathcal{L}^{eff}(B, U) = \mathcal{L}_{MB}^{(1)}(B, U) + \mathcal{L}_{MB}^{(2)}(B, U)$$

→ derive an interaction kernel  $V_{ij}$

- **Leading order (LO)**

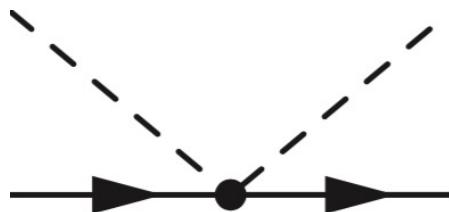
$$\mathcal{L}_{MB}^{(1)} = \langle \bar{B}(i\gamma_\mu D^\mu - M_0)B \rangle + \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\} \rangle + \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B] \rangle$$

**Lagrangian:**

$$\mathcal{L}^{eff}(B, U) = \mathcal{L}_{MB}^{(1)}(B, U) + \mathcal{L}_{MB}^{(2)}(B, U) \rightarrow \text{derive an interaction kernel } V_{ij}$$

$$\mathcal{L}_{MB}^{(1)} = \boxed{\langle \bar{B}(i\gamma_\mu D^\mu - M_0)B \rangle} + \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\} \rangle + \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B] \rangle$$

↓  
Weinberg-Tomozawa term (WT)



1. Dominant contribution.
2. Interaction mediated by the constant  $f$  of the leptonic decay

**Lagrangian:**

$$\mathcal{L}^{eff}(B, U) = \mathcal{L}_{MB}^{(1)}(B, U) + \mathcal{L}_{MB}^{(2)}(B, U) \quad \rightarrow \text{derive an interaction kernel } V_{ij}$$

- **Leading order (LO)**

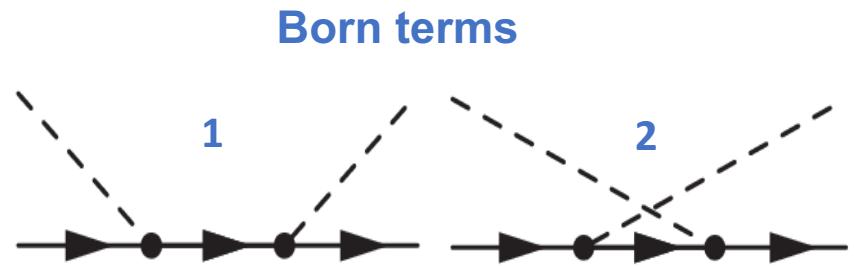
$$\mathcal{L}_{MB}^{(1)} = \langle \bar{B}(i\gamma_\mu D^\mu - M_0)B \rangle + \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\} \rangle + \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B] \rangle$$

1. Direct diagram (s-channel Born term)

$$V_{ij}^D = V_{ij}^D(D, F)$$

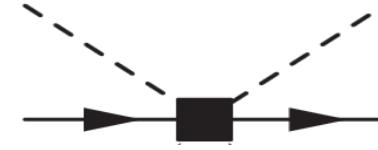
2. Cross diagram (u-channel Born term)

$$V_{ij}^C = V_{ij}^C(D, F)$$



## Formalism: Effective Chiral Lagrangian

- Next to leading order (NLO), just considering the **contact term**



$$\begin{aligned} \mathcal{L}_{\phi B}^{(2)} = & b_D \langle \bar{B} \{\chi_+, B\} \rangle + b_F \langle \bar{B} [\chi_+, B] \rangle + b_0 \langle \bar{B} B \rangle \langle \chi_+ \rangle + d_1 \langle \bar{B} \{u_\mu, [u^\mu, B]\} \rangle \\ & + d_2 \langle \bar{B} [u_\mu, [u^\mu, B]] \rangle + d_3 \langle \bar{B} u_\mu \rangle \langle u^\mu B \rangle + d_4 \langle \bar{B} B \rangle \langle u^\mu u_\mu \rangle \\ & - \frac{g_1}{8M_N^2} \langle \bar{B} \{u_\mu, [u_\nu, \{D^\mu, D^\nu\} B]\} \rangle - \frac{g_2}{8M_N^2} \langle \bar{B} [u_\mu, [u_\nu, \{D^\mu, D^\nu\} B]] \rangle \\ & - \frac{g_3}{8M_N^2} \langle \bar{B} u_\mu \rangle \langle [u_\nu, \{D^\mu, D^\nu\} B] \rangle - \frac{g_4}{8M_N^2} \langle \bar{B} \{D^\mu, D^\nu\} B \rangle \langle u_\mu u_\nu \rangle \\ & - \frac{h_1}{4} \langle \bar{B} [\gamma^\mu, \gamma^\nu] B u_\mu u_\nu \rangle - \frac{h_2}{4} \langle \bar{B} [\gamma^\mu, \gamma^\nu] u_\mu [u_\nu, B] \rangle - \frac{h_3}{4} \langle \bar{B} [\gamma^\mu, \gamma^\nu] u_\mu \{u_\nu, B\} \rangle \\ & - \frac{h_4}{4} \langle \bar{B} [\gamma^\mu, \gamma^\nu] u_\mu \rangle \langle u_\nu, B \rangle + h.c. \end{aligned}$$

terms NOT taken  
into account

- Contributions with  $g_3$  get cancelled
- **$b_0, b_D, b_F, d_1, d_2, d_3, d_4, g_1, g_2, g_4, h_1, h_2, h_3, h_4$**  are not well established, so they should be treated as parameters of the model!

## Motivation: Evolution of our chiral model

- Unitarized scattering amplitude from Chiral Lagrangian (**WT+Born+NLO**)

$$V_{ij} = V_{ij}^{WT} + V_{ij}^D + V_{ij}^C + V_{ij}^{NLO} \implies T = (1 - VG)^{-1}V \implies T_{ij}$$

Only S-wave contribution is taken into account

$$G_l = \frac{2M_l}{(4\pi)^2} \left\{ a_l(\mu) + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} + \frac{q_{cm}}{\sqrt{s}} \ln \left[ \frac{(s + 2\sqrt{s}q_{cm})^2 - (M_l^2 - m_l^2)^2}{(s - 2\sqrt{s}q_{cm})^2 - (M_l^2 - m_l^2)^2} \right] \right\}$$

$$a_{\pi^0 \Xi^0} = a_{\pi^+ \Xi^-} = a_{\pi \Xi}$$

$$a_{\bar{K}^0 \Lambda} = a_{\bar{K} \Lambda}$$

$$a_{\bar{K}^0 \Sigma^0} = a_{K^- \Sigma^+} = a_{\bar{K} \Sigma}$$

$$a_{\eta \Xi^0} = a_{\eta \Xi}$$

With isospin symmetry

subtraction constants for the dimensional regularization scale  $\mu = 630\text{MeV}$  in all the “I” channels.

## Formalism: Fitting procedure

- Decay constant  $f$

Model I:  $f = \alpha f_\pi$ ,  $\alpha = 1.197$  (fixed by BCN model)

Model II:  $f = \alpha f_\pi$ ,  $\alpha \in [1.190, 1.210]$  (constrained by the error band of BCN model)

- Axial vector couplings  $D, F$  (fixed by BCN model)
- 14 coefficients of the NLO lagrangian terms  $b_0, b_D, b_F, d_1, d_2, d_3, d_4$  (fixed by BCN model)  
LECs are assumed to be SU(3) symmetric
- 4 subtracting constants (isospín symmetry):  $a_{\pi\Xi}, a_{\bar{K}\Lambda}, a_{\bar{K}\Sigma}, a_{\eta\Xi}$   
Constrained to be within  $[-4, -1]$

## Fitting parameters and pole content

A. F., V. Valcarce, and V. K. Magas, *Phys.Lett.B* 841 (2023) 137927

Table 2: Values of the parameters for the different models described in the text.  
The subtraction constants are taken at a regularization scale  $\mu = 630$  MeV.

	<b>Model I</b>	<b>Model II</b>		
$a_{\pi\Xi}$	-2.7981	-2.7228		
$a_{K\Lambda}$	-1.0071	-1.0000		
$a_{K\Sigma}$	-3.0938	-2.9381		
$a_{\eta\Xi}$	-3.2665	-3.3984		
$f/f_\pi$	1.197 (fixed [20])	1.204		
<b>Model II</b>	<b><math>\Xi(1620)</math></b>		<b><math>\Xi(1690)</math></b>	
$M$ [MeV]	1608.51		1686.17	
$\Gamma$ [MeV]	170.00		29.72	
$\pi^+\Xi^-$	$g_i$	$ g_i $	$g_i$	$ g_i $
$\pi^0\Xi^0$	$1.73 + i0.85$	1.93	$0.51 + i0.25$	0.57
$\bar{K}^0\Lambda$	$-1.24 - i0.67$	1.41	$0.09 - i0.06$	0.11
$K^-\Sigma^+$	$-2.12 - i0.09$	2.12	$0.81 - i0.02$	0.81
$\bar{K}^0\Sigma^0$	$0.8 - i0.25$	0.84	$1.36 + i0.10$	1.36
$\eta\Xi^0$	$-0.36 + i0.31$	0.48	$-1.99 + i0.08$	1.99
	$-0.20 + i0.12$	0.24	$-1.04 + i0.06$	1.04

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<b>Model I</b>	<b><math>\Xi(1620)</math></b>	<b><math>\Xi(1690)</math></b>
$M$ [MeV]	1599.95	1683.04
$\Gamma$ [MeV]	158.88	11.51
	$g_i$	$ g_i $
$\pi^+\Xi^-$	$1.70 + i0.78$	1.87
$\pi^0\Xi^0$	$-1.22 - i0.62$	1.37
$\bar{K}^0\Lambda$	$-2.11 - i0.08$	2.11
$K^-\Sigma^+$	$0.81 - i0.22$	0.84
$\bar{K}^0\Sigma^0$	$-0.41 + i0.28$	0.50
$\eta\Xi^0$	$-0.23 + i0.13$	0.26
	$ g_i $	$ g_i $

R. Aaij, et al., *Sci. Bull.* 66 (2021) 1278–1287

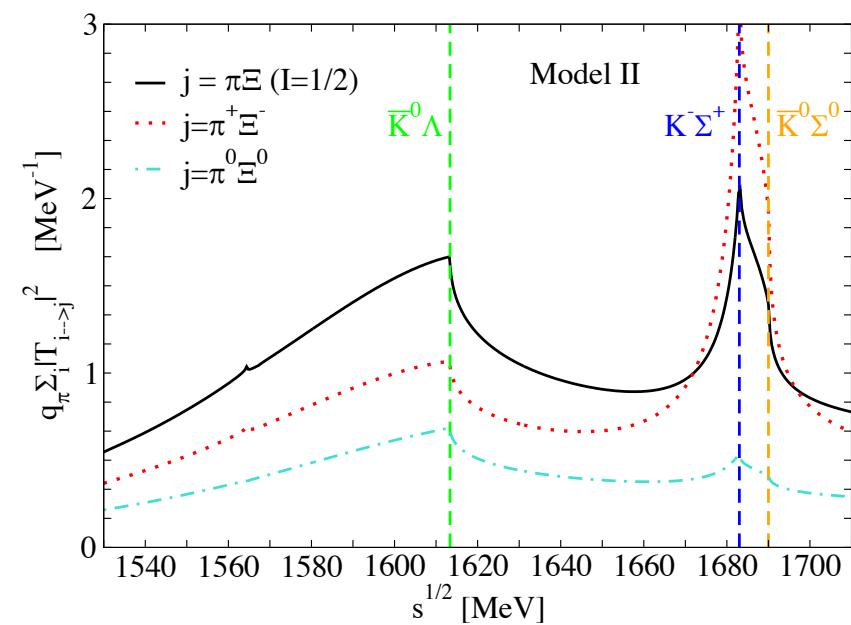
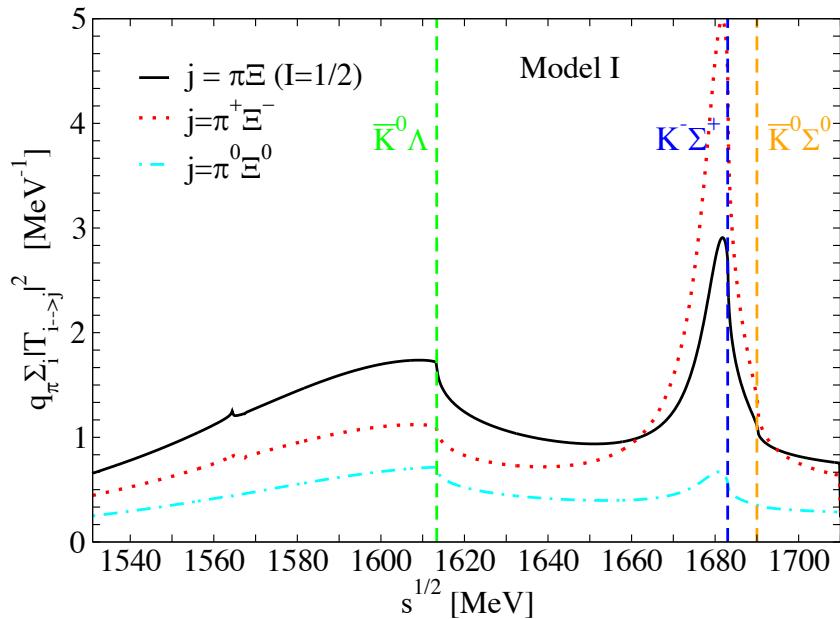
$M = 1610.4 \pm 6.0^{+5.9}_{-3.5}$  MeV,  $\Gamma = 59.9 \pm 4.8^{+2.8}_{-3.0}$  MeV.

M. Sumihama, et al., *Phys. Rev. Lett.* 122 (7) (2019) 072501

$M = 1692.0 \pm 1.3^{+1.2}_{-0.4}$  MeV,  $\Gamma = 25.9 \pm 9.5^{+14.0}_{-13.5}$  MeV.

Results:  $\pi\Xi$  invariant mass

A. F., V. Valcarce, and V. K. Magas, *Phys.Lett.B* 841 (2023) 137927



Results:  $\Xi(1690)$  branching ratios

$$\frac{\Gamma_{\Xi(1690)}^{\pi\Xi}}{\Gamma_{\Xi(1690)}^{\bar{K}\Sigma}} = \frac{\Gamma_{\Xi(1690)}^{\pi^+\Xi^-} + \Gamma_{\Xi(1690)}^{\pi^0\Xi^0}}{\Gamma_{\Xi(1690)}^{K^-\Sigma^+} + \Gamma_{\Xi(1690)}^{\bar{K}^0\Sigma^0}} = 0.25$$

$$\frac{\Gamma_{\Xi(1690)}^{\bar{K}\Sigma}}{\Gamma_{\Xi(1690)}^{\bar{K}\Lambda}} = \frac{\Gamma_{\Xi(1690)}^{K^-\Sigma^+} + \Gamma_{\Xi(1690)}^{\bar{K}^0\Sigma^0}}{\Gamma_{\Xi(1690)}^{\bar{K}\Lambda}} = 3.22$$

A. F., V. Valcarce, and V. K. Magas, Phys.Lett.B 841 (2023) 137927

### $\Xi(1690)$ BRANCHING RATIOS

#### $\Gamma(\Lambda\bar{K})/\Gamma_{\text{total}}$

VALUE	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
seen	104	BIAGI	87	SPEC	$\Xi^-$ Be 116 GeV

#### $\Gamma_1/\Gamma$

$$\Gamma_R^i \sim p_i |g_{R,i}|^2 M_i/M_R$$

#### $\Gamma(\Sigma\bar{K})/\Gamma(\Lambda\bar{K})$

VALUE	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
$0.75 \pm 0.39$	75	ABE	02c	BELL	$e^+ e^- \approx \gamma(4S)$
$2.7 \pm 0.9$		DIONISI	78	HBC	$K^- p$ 4.2 GeV/c
$3.1 \pm 1.4$		DIONISI	78	HBC	$K^- p$ 4.2 GeV/c

#### $\Gamma_2/\Gamma_1$

R. L. Workman, et al., Review of Particle Physics, PTEP 2022 (2022) 083C01.

#### $\Gamma(\Xi\pi)/\Gamma(\Sigma\bar{K})$

VALUE	DOCUMENT ID	TECN	CHG	COMMENT
<0.09	DIONISI	78	HBC	$K^- p$ 4.2 GeV/c

#### $\Gamma_3/\Gamma_2$

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## Extension to the S=-2, Q=-1 sector

- The interaction kernel for the S=-2, Q=-1 sector can be built from the S=-2, Q=0 employing isospin symmetry arguments:  
 $V_{ij}(S = -2, Q = O) \rightarrow V_{ij}(I = 1/2), V_{ij}(I = 3/2) \rightarrow V_{ij}(S = -2, Q = -1)$
- Once again, the Bethe-Salpeter equation is solved in the same footing to get the T-matrix:

$$V_{ij} = V_{ij}^{WT} + V_{ij}^D + V_{ij}^C + V_{ij}^{NLO} \implies T = (1 - VG)^{-1}V \implies T_{ij}$$

**$\Lambda K^-$  CF data available → unprecedented opportunity to fit a model for the first time in this sector**

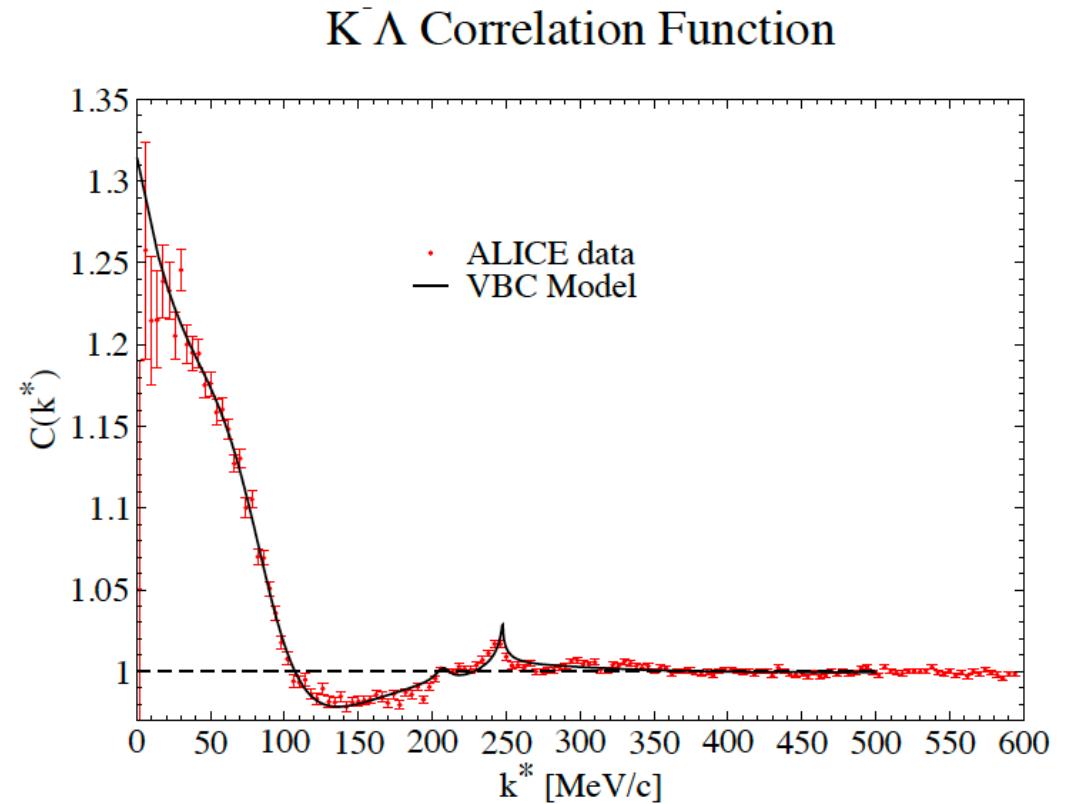
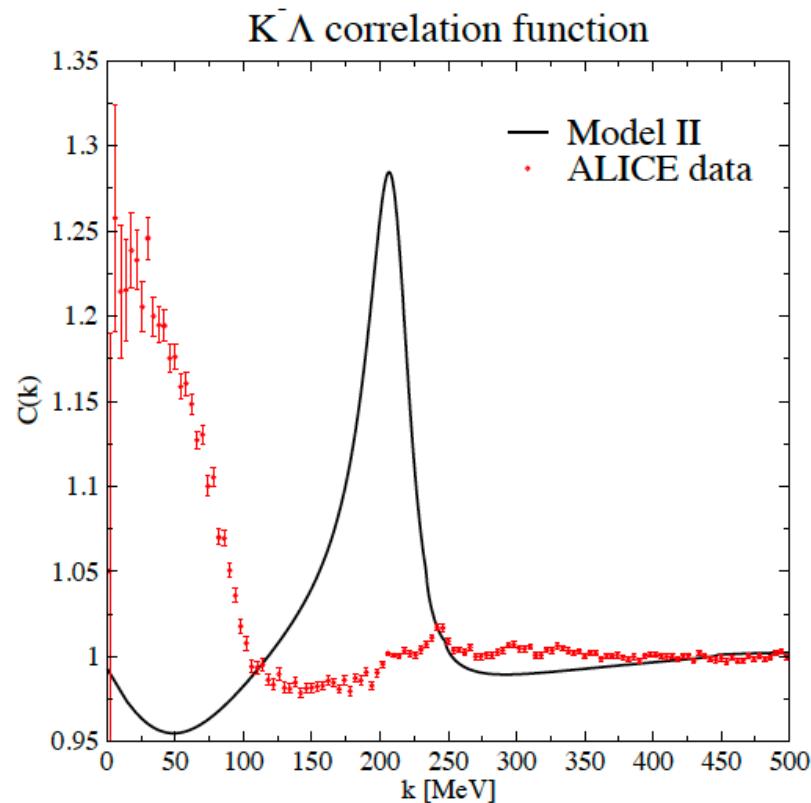
ALICE Collaboration,  
e-Print: 2305.19093 [nucl-ex]

**Valentina's talk a few minutes ago!!!**

$$C(k) \simeq 1 + \int_0^\infty 4\pi r^2 dr S_{12}(r) \left[ \sum_\beta \omega_\beta |\Psi_{\alpha\beta}(r)|^2 - |j_0(kr)|^2 \right]$$

$$\Psi_{\alpha\beta}(r) = \delta_{\alpha\beta} j_l(k_\alpha r) + \frac{1}{\pi} \int j_l(qr) dq q^2 \frac{1}{E - E_1^\beta(q) - E_2^\beta(q) + i\epsilon} T_{\alpha\beta;l}(q, k_\alpha; E)$$

$\Lambda\bar{\Lambda}$  Correlation function: fit and values of the parameters



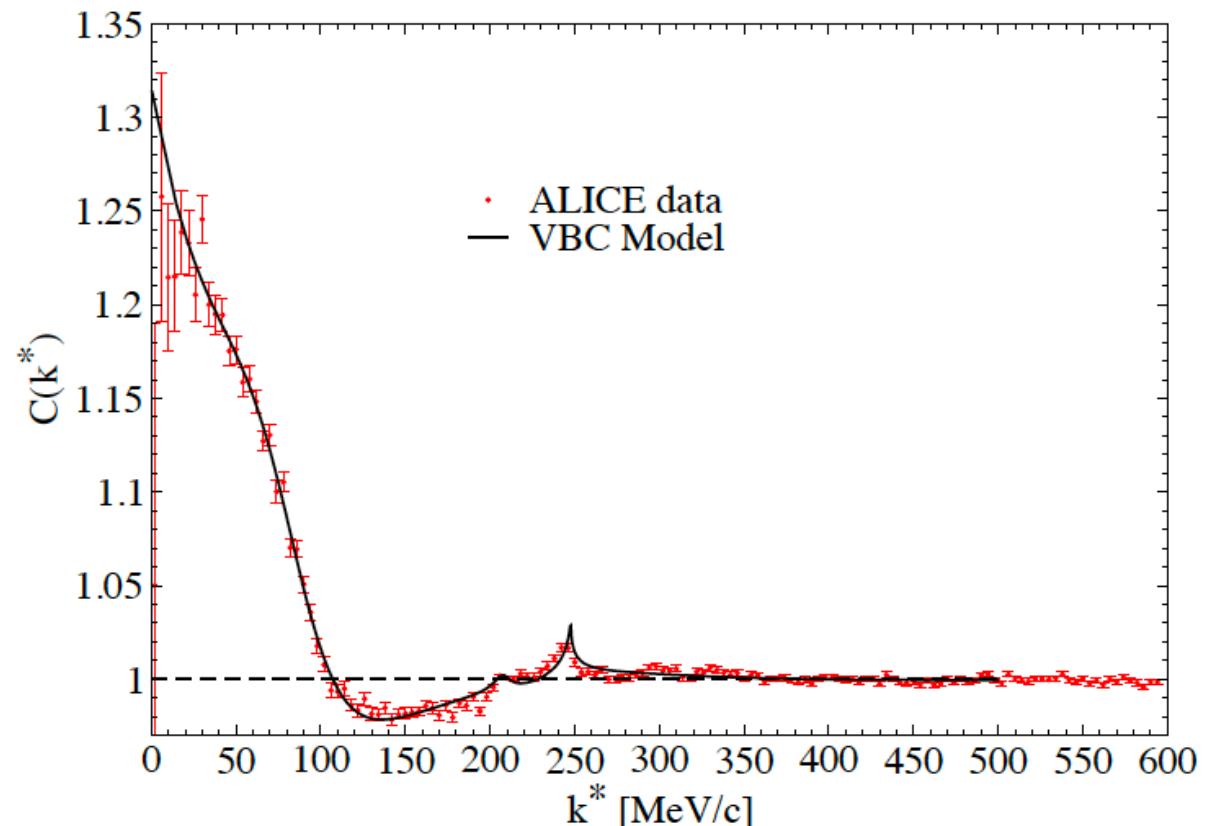
## $\Lambda K^-$ Correlation function: fit and values of the parameters

	WT	full
$a_{\pi\Xi}$	$-4.000 \pm 2.515$	$-3.981 \pm 2.375$
$a_{K\Lambda}$	$-0.002 \pm 2.993$	$-2.232 \pm 0.313$
$a_{K\Sigma}$	$-2.655 \pm 0.031$	$-1.705 \pm 0.055$
$a_{\eta\Xi}$	$-1.548 \pm 0.068$	$-3.998 \pm 1.221$
$b_0$ [GeV $^{-1}$ ]		$-1.067 \pm 0.010$
$b_D$ [GeV $^{-1}$ ]		$0.049 \pm 0.011$
$b_F$ [GeV $^{-1}$ ]		$0.265 \pm 0.007$
$d_1$ [GeV $^{-1}$ ]		$-0.202 \pm 0.027$
$d_2$ [GeV $^{-1}$ ]		$-0.260 \pm 0.021$
$d_3$ [GeV $^{-1}$ ]		$-0.748 \pm 0.380$
$d_4$ [GeV $^{-1}$ ]		$-0.406 \pm 0.010$
$\chi^2_{\text{dof}}$	2.92	0.95
$f_0$ [fm]	$-0.050 + i 0.268$	$0.202 + i 0.633$

$$f = f_\pi$$

*Work in progress...*

## $K^- \Lambda$ Correlation Function



Full model (WT+Born+NLO)

$$M = 1615.46 \text{ MeV} \quad M = 1687.69 \text{ MeV}$$

$$\Gamma = 20.92 \text{ MeV} \quad \Gamma = 17.16 \text{ MeV}$$

$$(- - + +) \quad (+ + - +)$$

	$ g_i $	$ g_i $
$\pi\Xi(1456)$	0.631	0.581
$\bar{K}\Lambda(1611)$	0.919	0.576
$\bar{K}\Sigma(1689)$	2.15	1.54
$\eta\Xi(1866)$	2.75	0.727

Change of paradigm!!!

work in progress...

$$M = 1610.4 \pm 6.0^{+5.9}_{-3.5} \text{ MeV}, \Gamma = 59.9 \pm 4.8^{+2.8}_{-3.0} \text{ MeV}.$$

$$M = 1692.0 \pm 1.3^{+1.2}_{-0.4} \text{ MeV}, \Gamma = 25.9 \pm 9.5^{+14.0}_{-13.5} \text{ MeV}.$$

## CONCLUSIONS

We have studied the meson-baryon interaction in the S=-2 sector with in the UChPT scheme taking into account higher order contributions for the first time.

- The Born terms and the NLO contributions have shown to play a key role in order to reasonably reproduce the  $\Xi(1620)$  and  $\Xi(1690)$  states
- The interpretation of such states as meson-baryon molecules provides a plausible explanation of the branching ratios

We have performed, **for the first time**, a fit to CF data and proved that Femtoscopy measurements are a very powerful tool to constrain the theoretical models and to unravel the new physics masked in the new parametrizations, specially in those cases where we cannot have access to scattering data.

- In particular, the experimental  $\Lambda K^-$  Correlation Function was crucial in order to change the paradigm in the molecular interpretation of the  $\Xi(1620)$  state.

# Thank you for your attention!