Systematics of strange hadron yields from heavy-ion collisions at few GeV

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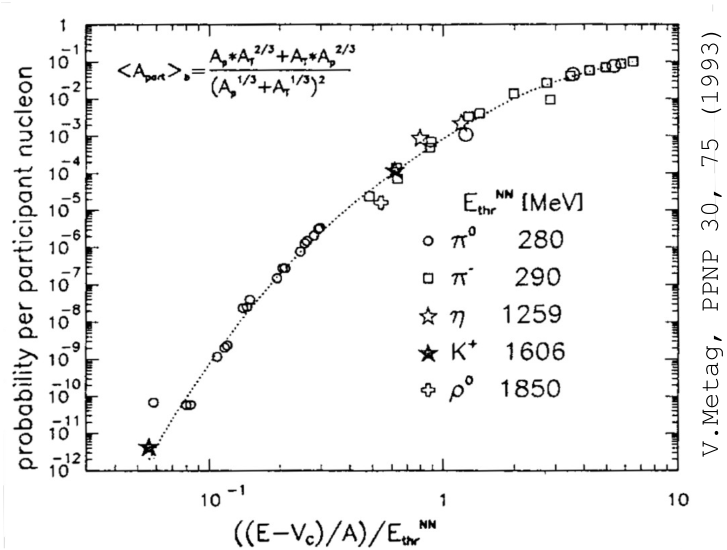
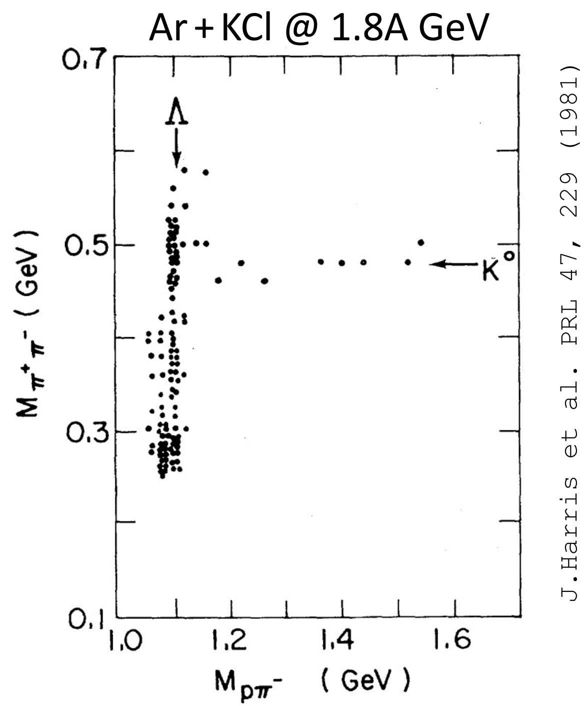
- A bit of history and motivation
- Glauber model extraction of $\langle A_{\mathrm{part}} \rangle$
- Global parametrization of yields
- Behaviour of α exponent in $P \sim \langle A_{\mathrm{part}} \rangle^{\alpha}$

- **Exploring the map of strangeness production in HI near threshold** throughout last 40 years

1981,82: First Bevalac results on K^+ and Λ

1993: V. Metag's systematic of meson production: 2 K^+ points





1990s
2000s Measurements of **KaoS**, **FOPI**, **HADES**, **STAR**
2010s

2023: ~100 published yields of strange hadrons ($K^{\pm 0}$ ϕ Λ) within $\sqrt{s_{NN}} \in [2-3]$ GeV

☞ Good to look at yields, assuming $P = f(\sqrt{s_{NN}}, \langle A_{part} \rangle)$, and to find the parametrizations

Motivation

Published $\langle A_{part} \rangle_b$: ambiguity of modelling

Ⓐ 56 values from **geometrical model**

(most of Fopi, some of KaoS, some of HADES)

Ⓑ 19 values from **optical Glauber model**

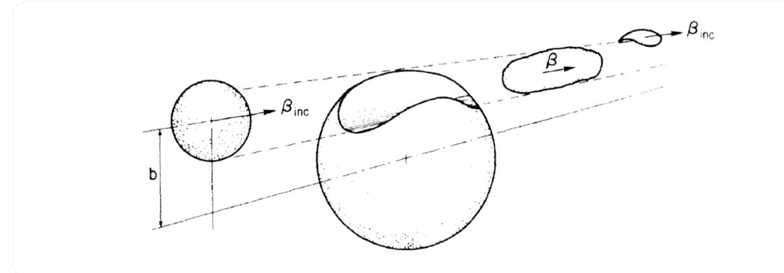
(some of KaoS)

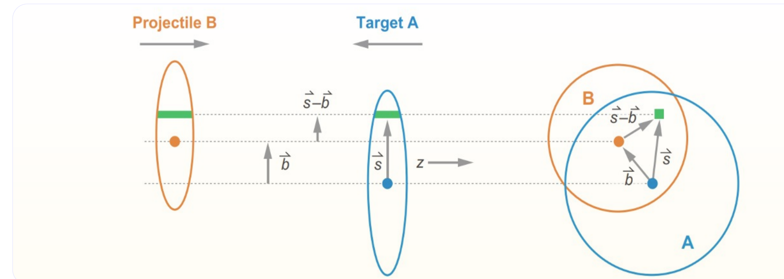
Ⓒ 22 values from **Glauber Monte Carlo**

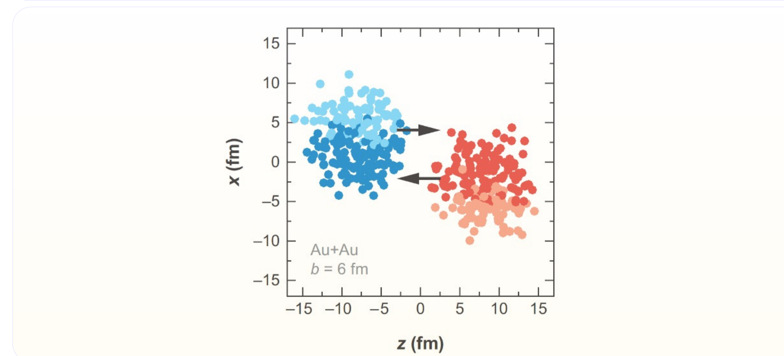
(most of Hades, STAR)

Ⓓ 6 unspecified

(Bevalac, some of Fopi)







J.Gosset et al., Phys. Rev. C 16, 629 (1977)

M. Miller et al. Ann.Rev.Nucl.Part.Sci. 57, 205 (2007)

Idea: upgrade all the data → to Glauber Monte Carlo.

For A_{part} obtained by non-GlauberMC, take stated centralities (MUL-based) and simulate Glauber MC.

(ZOO of) nuclear density profiles

- **2pF (two-parameter Fermi)**

$$\rho(r) = \rho_0 \frac{1}{1 + \exp\left(\frac{r-c}{z}\right)}$$

- **3pF (three-parameter Fermi)**

$$\rho(r) = \rho_0 \frac{1 + w\left(\frac{r}{c}\right)^2}{1 + \exp\left(\frac{r-c}{z}\right)}$$

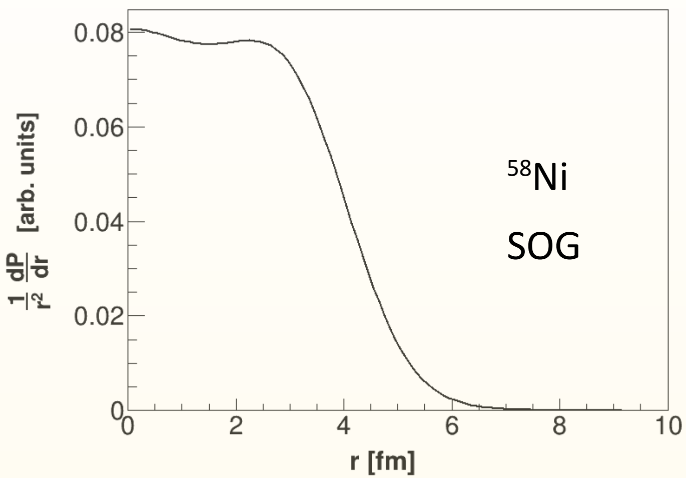
- **Deformed nucleus**

$$\rho(r) = \rho_0 \frac{1}{1 + \exp\frac{\left[r - R\left(1 + \beta_2 Y_{20} + \beta_4 Y_{40}\right)\right]}{a}}$$

- **SOG (Sum of Gaussians)**

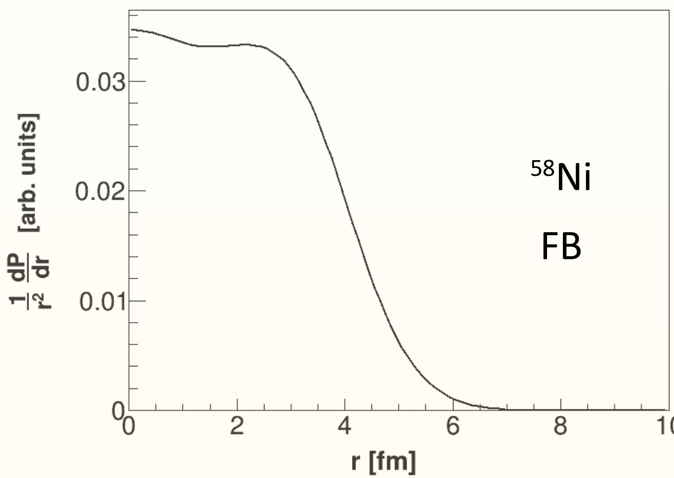
$$\rho(r) = \sum_i A_i \left[e^{-\left(\frac{r-R_i}{\gamma}\right)^2} + e^{-\left(\frac{r+R_i}{\gamma}\right)^2}\right]$$

$$A_i = \frac{Ze\,Q_i}{2\pi^{3/2}\gamma^3\left(1 + 2R_i^2/\gamma^2\right)}$$



- **FB (Fourier-Bessel)**

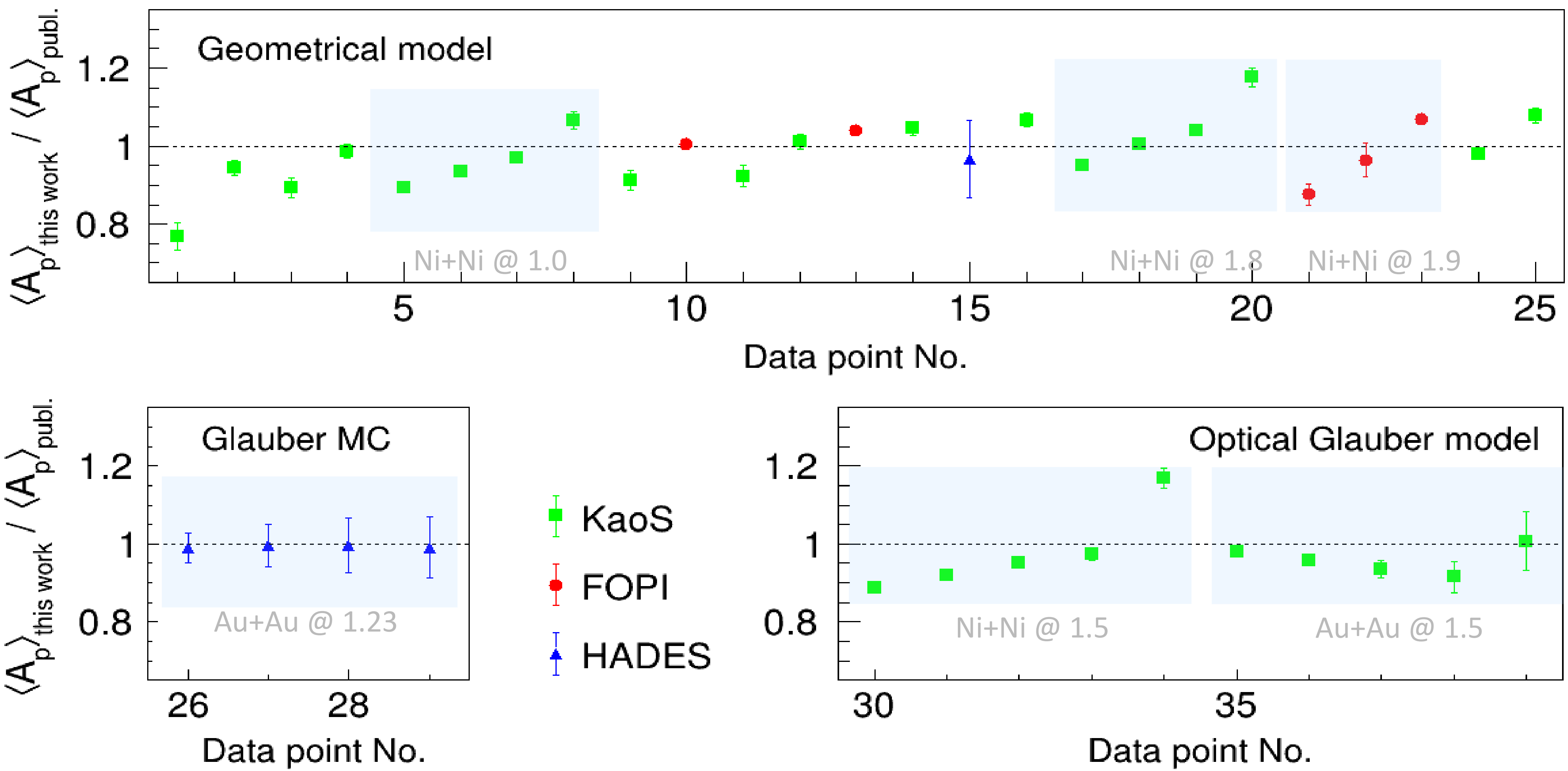
$$\rho(r) = \sum_\nu a_\nu\, j_0\left(\nu\pi\frac{r}{R}\right)$$



Refs: ➤ H. de Vries et al., Atom. Data Nucl. Data Tab. 36, 495 (1987) ➤ P. Möller *et al.*, Atom. Data Nucl. Data Tab. 59, 185 (1995)

How our analysis changed extracted ⟨Apart⟩ ?

Let's take K⁺ as an example:



was worth the effort ☺

Parameterizations of yields

Global parameterization $P = f[\sqrt{s_{NN}}, \langle A_{part} \rangle]$.

① Usual approach $P = N \cdot \langle A_{part} \rangle^{\alpha} \cdot \sqrt{s}^{\beta}$ (worse χ^2/ν)

② **Best-fit approach** $P = N \cdot \langle A_{part} \rangle^{\alpha} \cdot \exp\left[-\left(C \cdot \sqrt{s}\right)^{\beta}\right]$ C fitted for $K^{\pm}$ but adjusted for Λ, ϕ, K^0_S

Hadron	K^+	K^-	Λ	ϕ	K^0
No. points	40	25	12	9	11
χ^2/ν	3.6	2.2	1.4	0.2	2
N	$(3.0 \pm 1.0) \cdot 10^{-3}$	$(1.6 \pm 0.7) \cdot 10^{-4}$	$(5.1 \pm 1.0) \cdot 10^{-4}$	$(2.6 \pm 1.4) \cdot 10^{-5}$	$(4.5 \pm 0.9) \cdot 10^{-3}$
α	1.32±0.02	1.32±0.04	1.22±0.04	1.27±0.12	1.05±0.05
β	-6.2±0.5	-7.3±0.7	-67±6	-10.0±0.2	-5.7±0.1
C	0.32±0.01	0.32±0.01	0.41 (fixed)	0.35 (fixed)	0.32 (fixed)

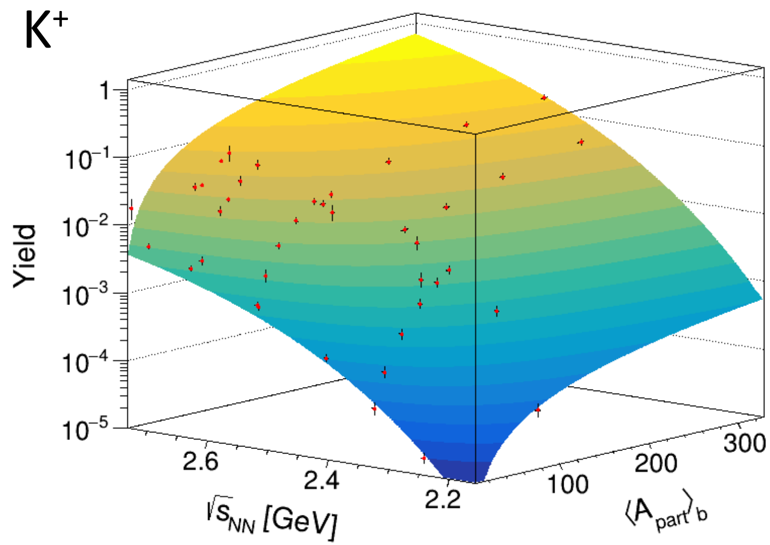
*(see **arXiv:2305.13760v1** for detailed information)*

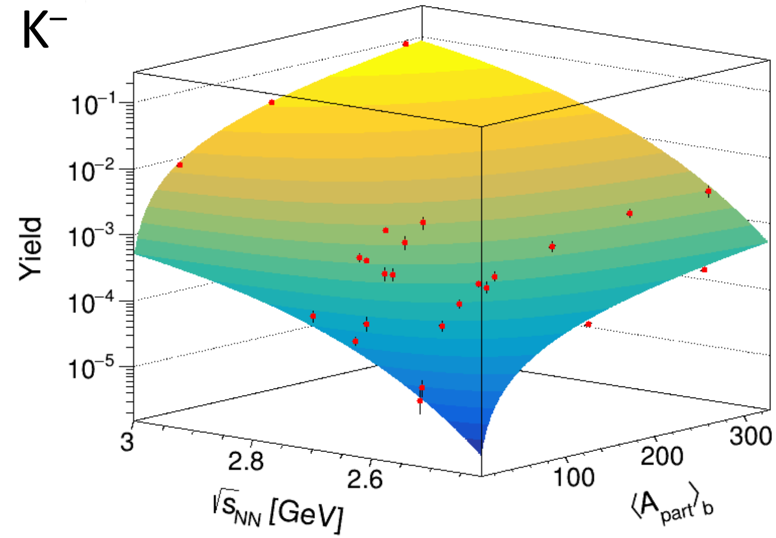
Good or rather good χ^2/ν.

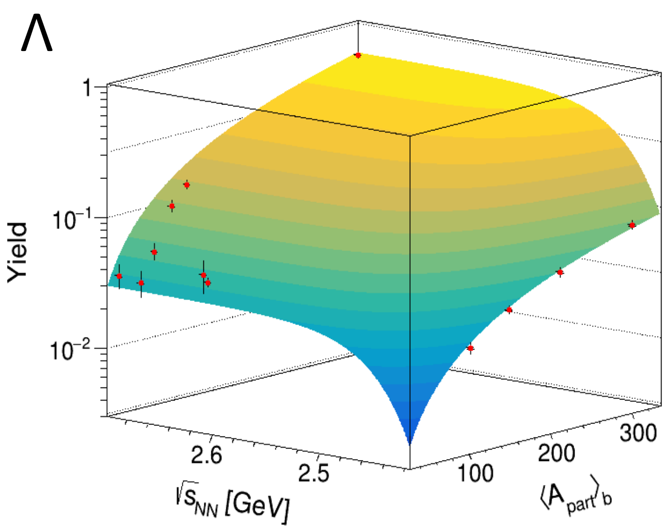
Usable for yield prediction (see *arXiv* for cov. matrices). E.g. for K^+, $P[\sqrt{s_{NN}} = 2.55\ \text{GeV}, \langle A_{part} \rangle = 100] = 0.0453\,(18)$

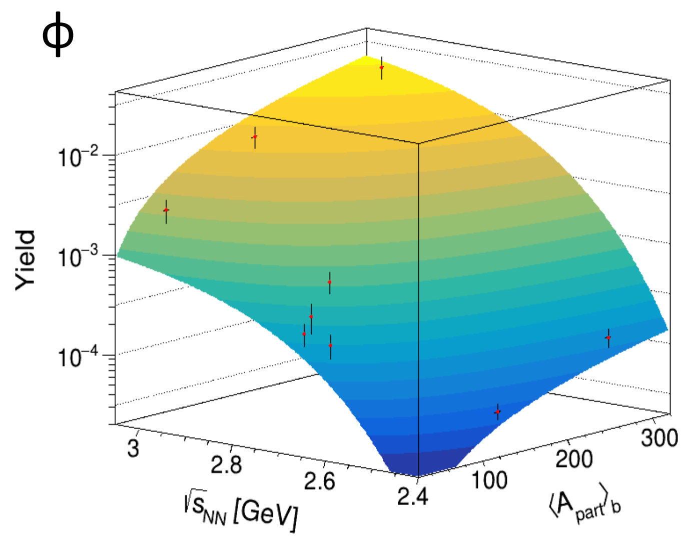
α parameters very close together, although $\alpha(K^0_s)$ away from the others.

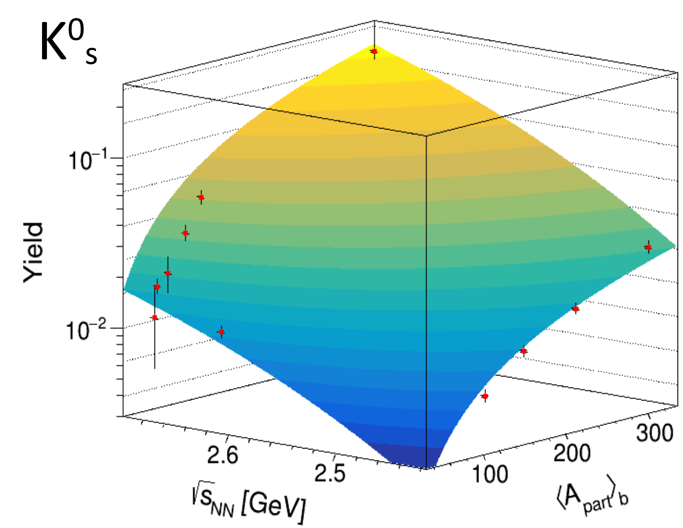
How do these functions look?



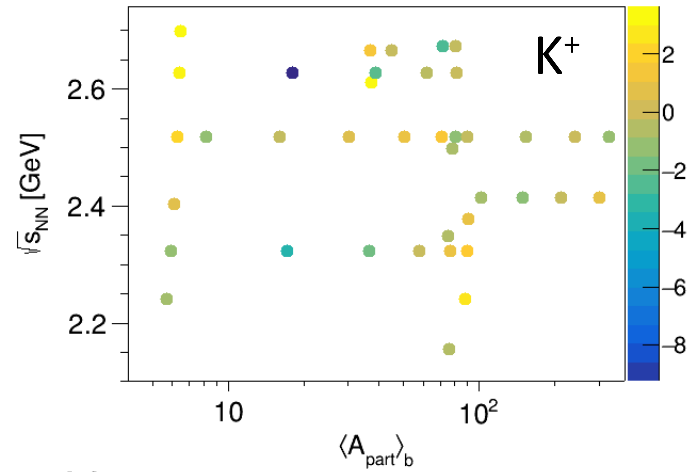


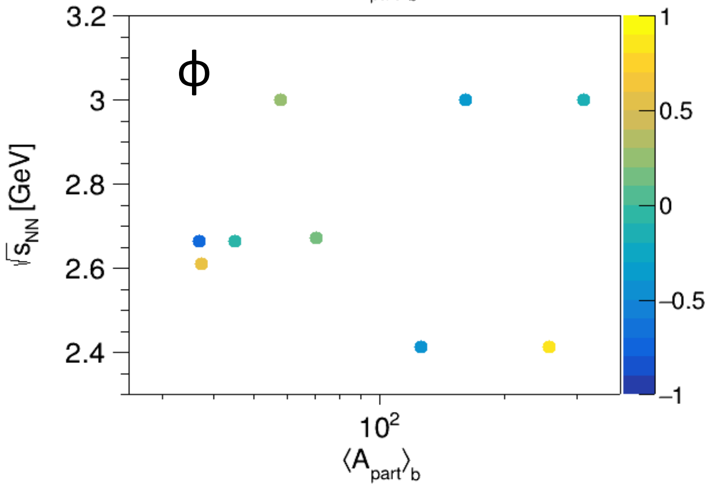




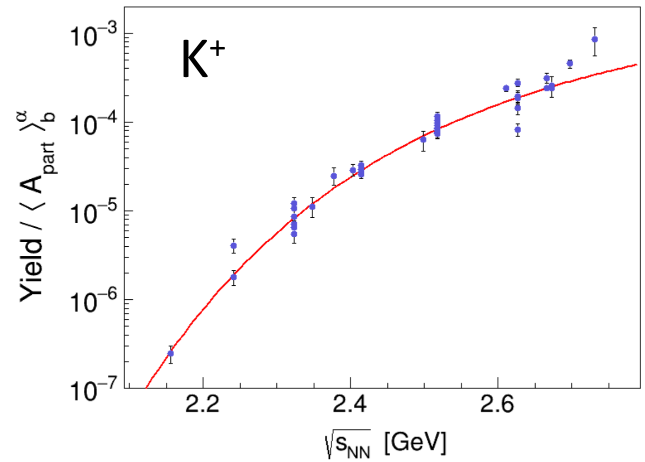


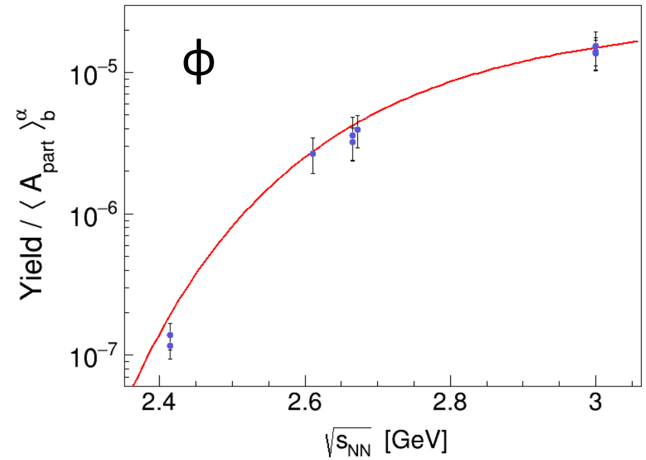
- Standard deviations between exp. data points and fit prediction:





- Projection onto $\sqrt{s_{NN}}$ by dividing Yield per $\langle A_{part} \rangle^{\alpha}$





Nearly all the points remain within $\pm 3\sigma$ range

α exponent dependency on beam energy

Let's examine α in $P \sim \langle A_{part} \rangle^{\alpha}$.

The parametrization assumes that:

$$\alpha = \text{const}\,(\sqrt{s})$$

But is it true? We can check it
by selecting experiments where yields
are available for a range of centralities.

Hypothesis ①

$$\alpha = \text{Linear function}\,(\sqrt{s})$$

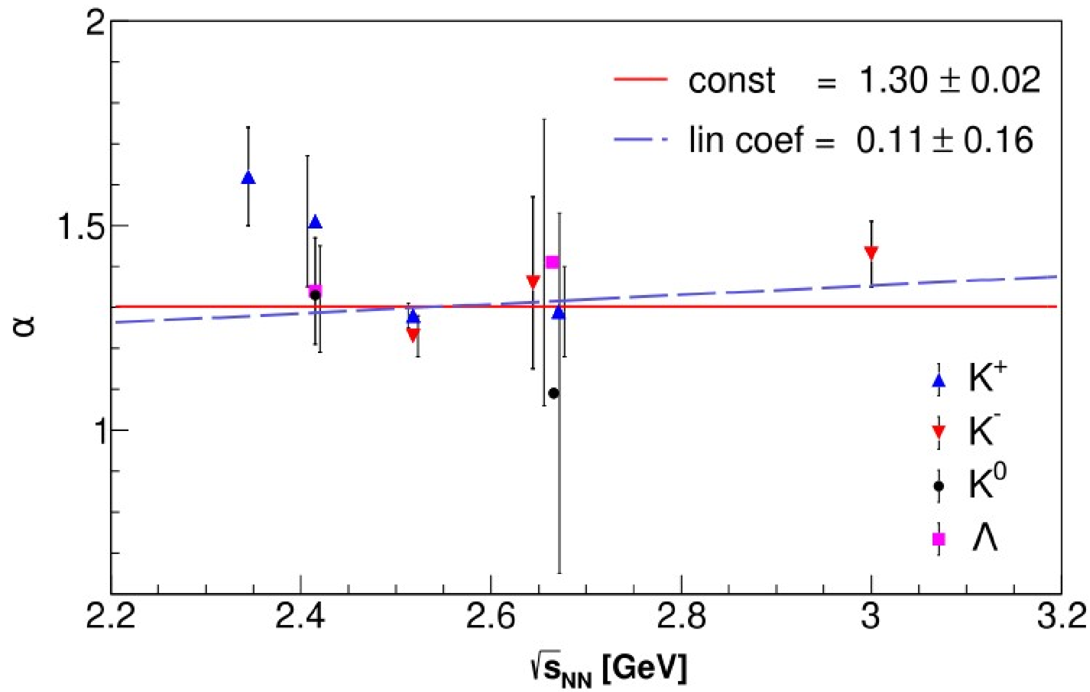
$\Rightarrow$ Linear coefficient: 0.11 ± 0.16

$\Rightarrow$ agrees with 0.

Hypothesis ②

$$\alpha = \text{const}\,(\sqrt{s})$$

$\Rightarrow$ Constant $= 1.30 \pm 0.02$ ($\chi^2/\nu = 1.4$)



$P \sim \langle A_{part} \rangle^{\alpha}$ dependency :

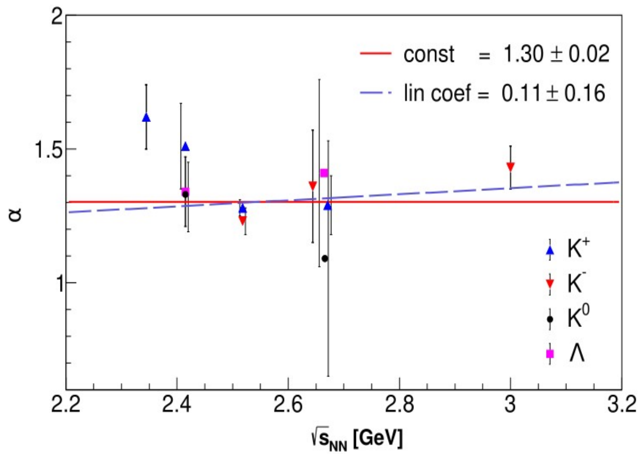
- **common** for K^+, K^-, K^0_s and Λ
- **independent of $\sqrt{s}$**

rises with $\langle A_{part} \rangle$ stronger than linearly .

Global α exponent: relation to other expts

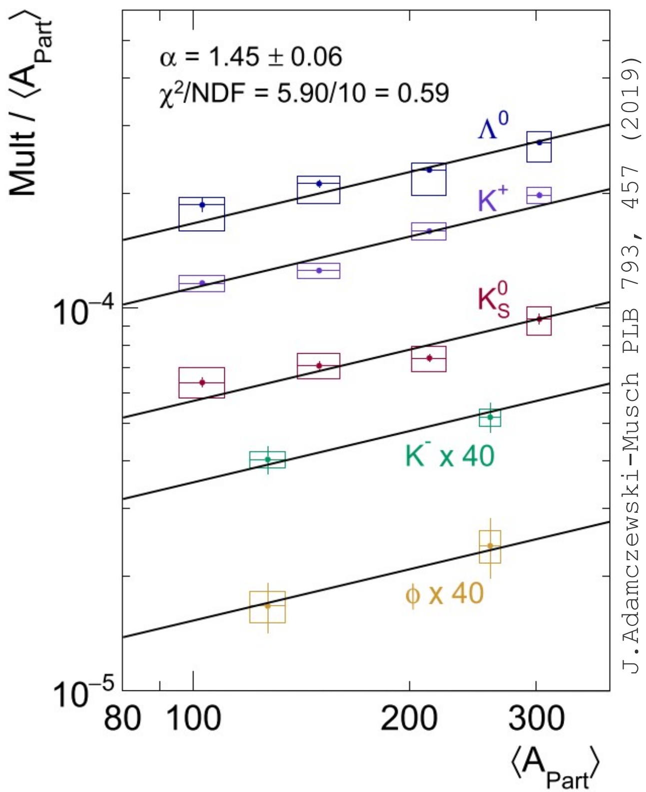
- **Inclusive $\sqrt{s_{NN}}$ = [2.3 – 3] GeV**
  Fit to all hadrons:

  $\alpha = 1.30 \pm 0.02$



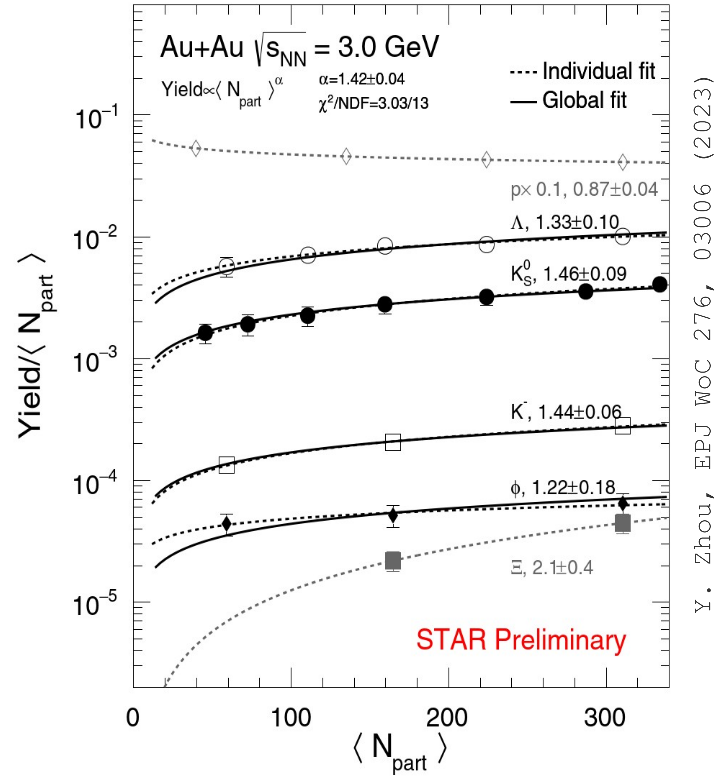
- **Au+Au $\sqrt{s}$ = 2.4 GeV [HADES]**
  Fit to 5 hadrons:

  $\alpha = 1.45 \pm 0.06$



- **Au+Au $\sqrt{s}$ = 3.0 GeV [STAR]**
  Fit to four hadrons:

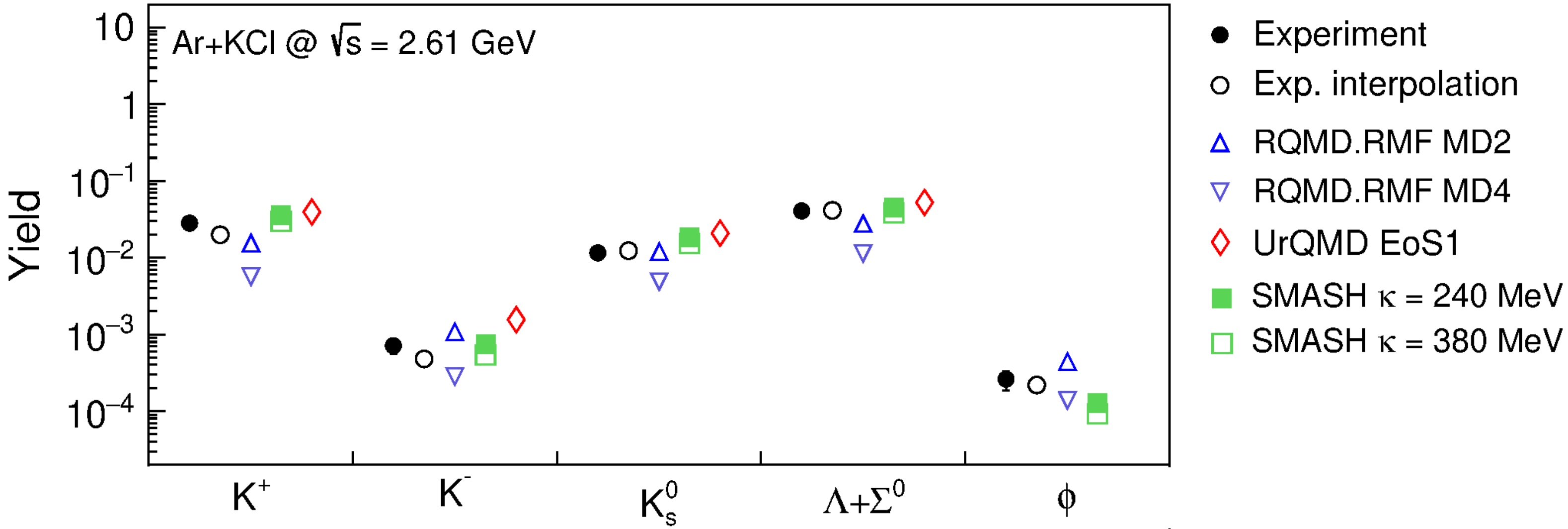
  $\alpha = 1.42 \pm 0.04$



$\langle A_{part} \rangle^{\alpha}$ dependency *common for "bulk strangeness"* is a good hypothesis also in 2 experiments, although some 2.5 .. 3 σ tension between results.

Predictive power of global parametrization

Benchmark point: Ar+KCl @ √s = 2.61 GeV (HADES) : all the yields are available at the same centrality.
Let's compare deviations from exp. data of: parametrization and transport models (public versions).

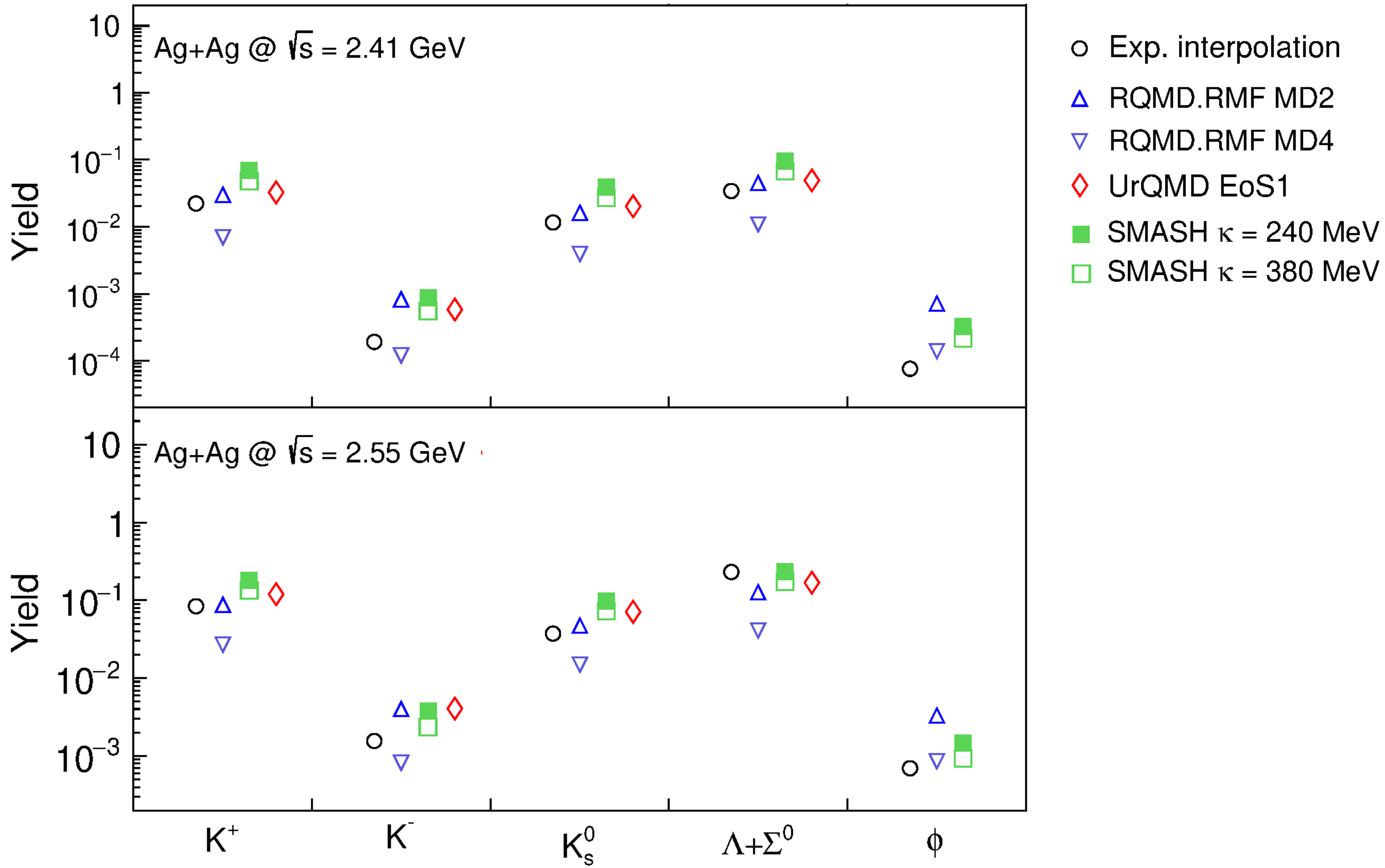


Hadron	K^+	K^-	Λ	ϕ	K^0	Σ dev
Phenom. parametrization	3.3	1.5	0	0.5	0.7	6.0
RQMD.RMF MD2	5.2	2.4	4	2.5	0.5	14.6
RQMD.RMF MD4	9.3	2.9	9	1.7	6.6	29.4
SMASH κ = 240	3.3	0.2	1.2	1.8	7	13.5
SMASH κ = 380	0.8	1.1	0.8	2.3	4	9.0
UrQMD Hard EoS	4.6	5.6	3.1	3.6	8.5	26.7

Phenomenological parametrization currently offers better overall estimation of yields than all the benchmarked transport codes (public versions) ☺

How parametrization predicts unpublished yields

Predictions of strange hadron yields from HADES on Ag+Ag @ √s = 2.41 and 2.55 GeV.



Feel free to include our parametrization into predictions or comparisons to exp data ☺

Summary

- $\sim$**100** published yields of strange hadrons (**$K^{\pm 0}$ ϕ Λ**) within **$\sqrt{s_{NN}} \in [2-3]$ GeV**

- Calculations of $\langle A_{part} \rangle$ using **TGlauberMC** for all the data points
  - Improvement of $\langle A_{part} \rangle$ estimation methods (changes up to 20% wrt to published values)
- **Yield parametrization as $f(\sqrt{s_{NN}}, \langle A_{part} \rangle)$** . See *arXiv:2305.13760v1* for details.
  - reasonable χ^2/ν , nearly all data points remain within 3σ
- Tracing **α exponent** of $P \sim \langle A_{part} \rangle^{\alpha}$
  - α seems not to depend on hadron specie and not change with $\sqrt{s_{NN}}$ (within 2 – 3 GeV)
  - Overall **$\alpha = 1.30 \pm 0.02$** (common scaling)
- Benchmark: **Ar+KCl @ $\sqrt{s_{NN}}$ = 2.61 GeV**
  - Parametrization seems **better** than the public versions of RQMD.RMF, SMASH, UrQMD.
- **Predictons** for strangeness yields for **Ar+Ag @ $\sqrt{s_{NN}}$ = 2.41 and 2.55 GeV**

Thank You!

Backup slides

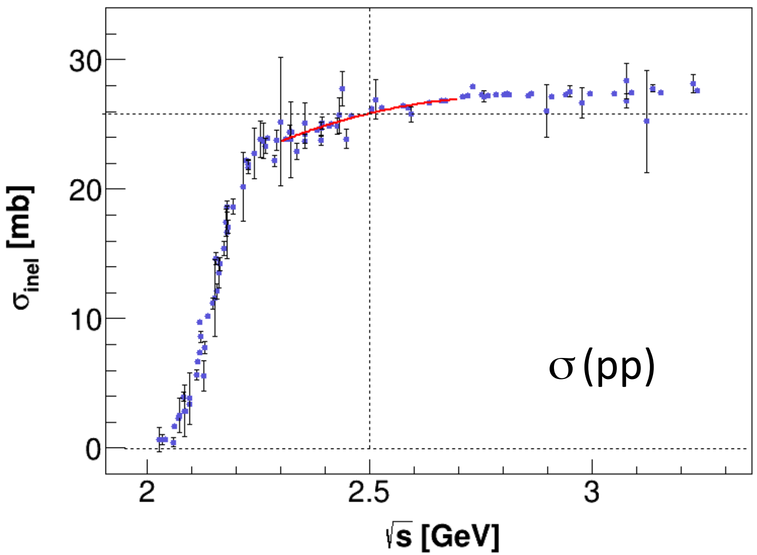
Inelastic NN cross sections

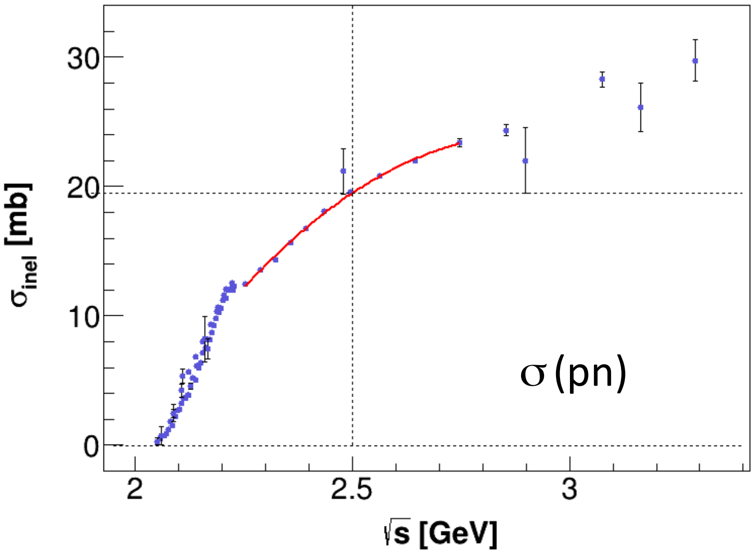
① σ(pp) is different from σ(pn) and σ(np)
② Assumption: isospin symmetry [$\sigma_{nn} = \sigma_{pp}$]

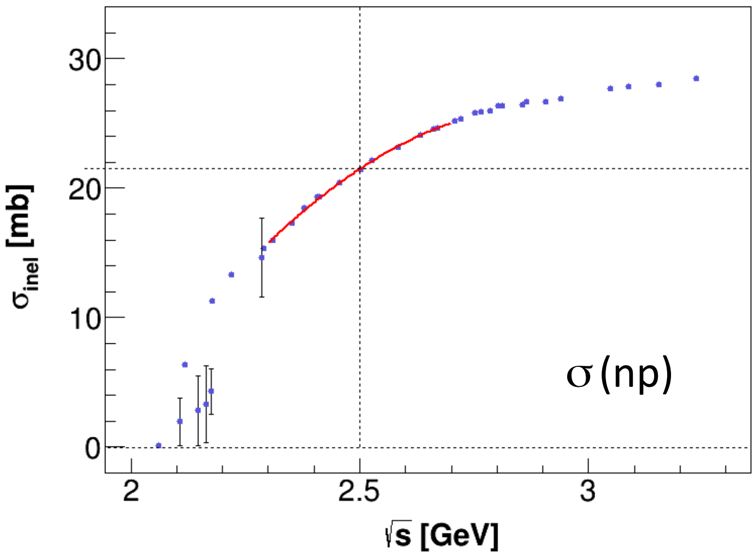
$$\sigma_{NN} = \frac{Z_p Z_t \sigma_{pp} + N_p N_t \sigma_{nn} + (Z_p N_t + N_p Z_t)\sigma_{np}}{A_p A_t}$$

③ Experimentally, σ(pn) is not the same as σ(np)
④ σ(np) at low √s and σ(pn) at higher √s are rare

[③,④] contribute to systematic errors







Refs. :
- PDG
- B. Kardan's Ms. C.

Glauber model extraction of $\langle A_{part} \rangle$

Tool: TGlauberMC (freely available code from PHOBOS)

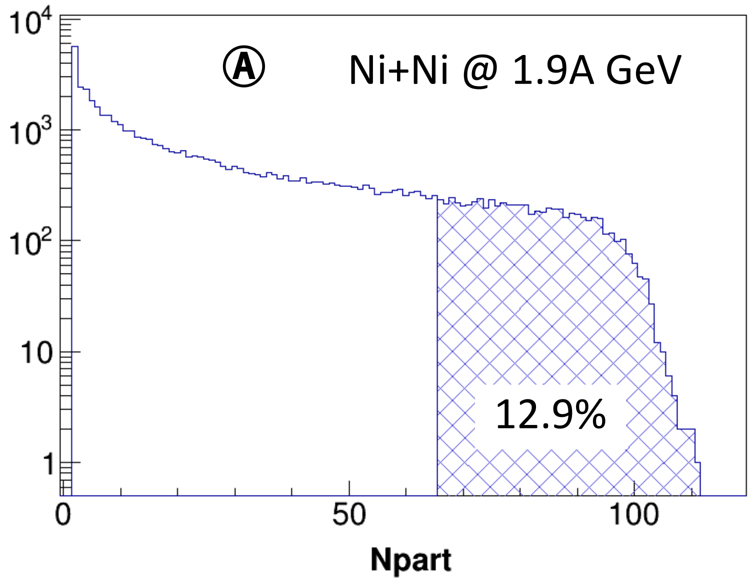
Method:
① For every data point find centrality [%]
② Simulate via TGlauberMC
③ Cut *Npart* at given centrality
④ Find $\langle A_{part} \rangle$ for accepted sample

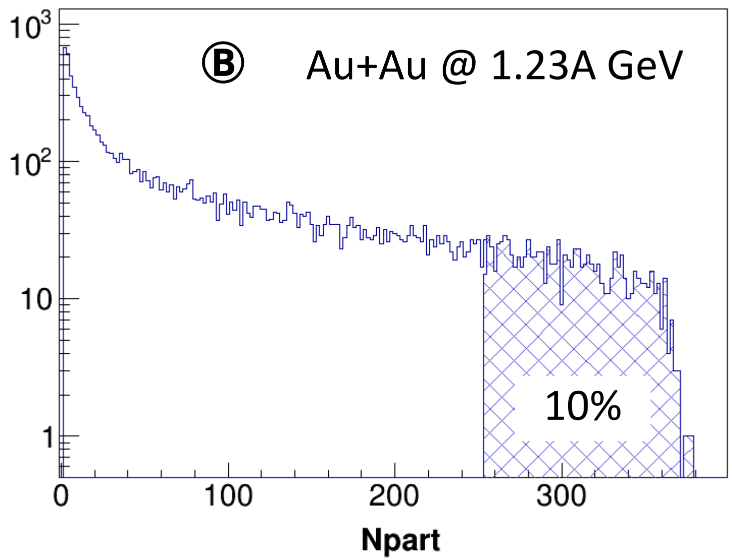
...but: iterate [②, ④] over σ_{NN} and shape variants

Examples:

Ⓐ TGlauberMC simulation of Ni+Ni @ 1.9A GeV
(input: σ_{NN} = 26.4 mb) ,
⊕ selection of 12.9% central events
→ $\langle A_{part} \rangle$ = 80.0

Ⓑ TGlauberMC simulation of Au+Au @ 1.23A GeV
(input: σ_{NN} = 23.7 mb) ,
⊕ selection of 10% central events
→ $\langle A_{part} \rangle$ = 300.8

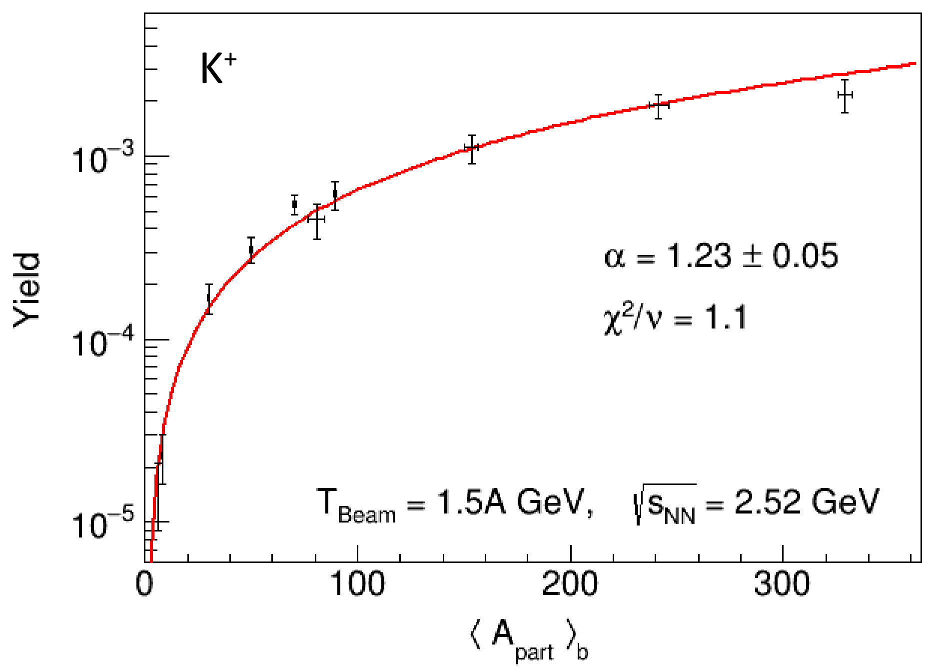


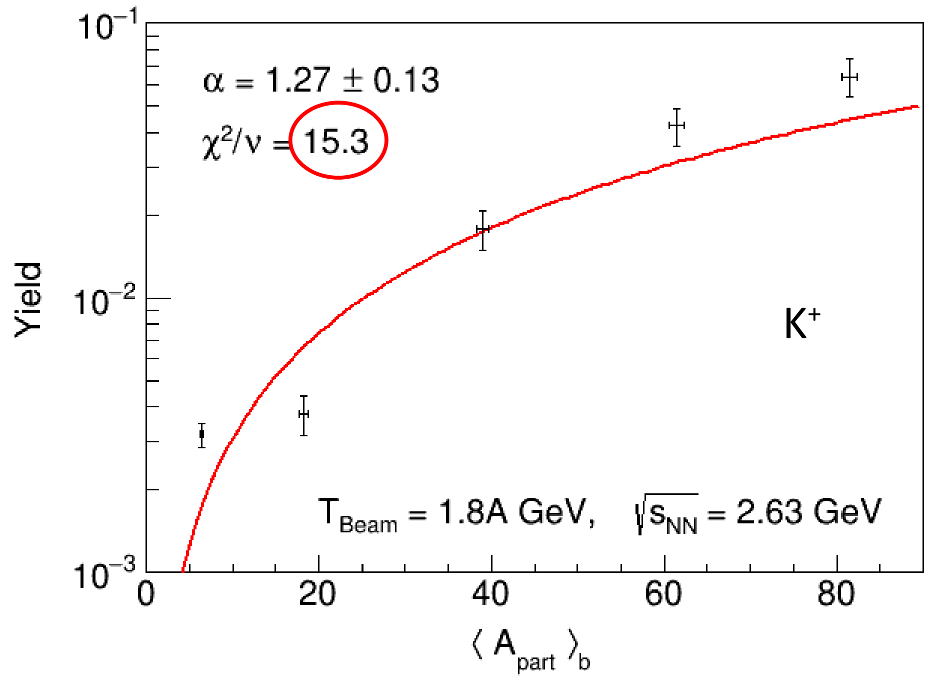


α exponent dependency on beam energy

Data is often available for similar but not the same beam energies (e.g. T_{Beam} = 1.756 vs 1.8 A GeV).

- 7 single-energy cases : enough points at the same beam energy, so the fit is stable.
- 5 adjacent-energy cases : points were fitted using the best-fit function (2)
- 1 hopeless case : fit of K^+ data at T_{Beam} = 1.8A GeV gives very bad χ^2/ν $\rightarrow$ unstable





➤ For ”adjacent-energy cases” the fit stability of α was traced, if 1 point was removed from highest or lowest energy. It contributed to systematic errors. Currently, $\Delta\alpha = \sqrt{(\Delta\alpha_{stat})^2 + (\Delta\alpha_{syst})^2}$

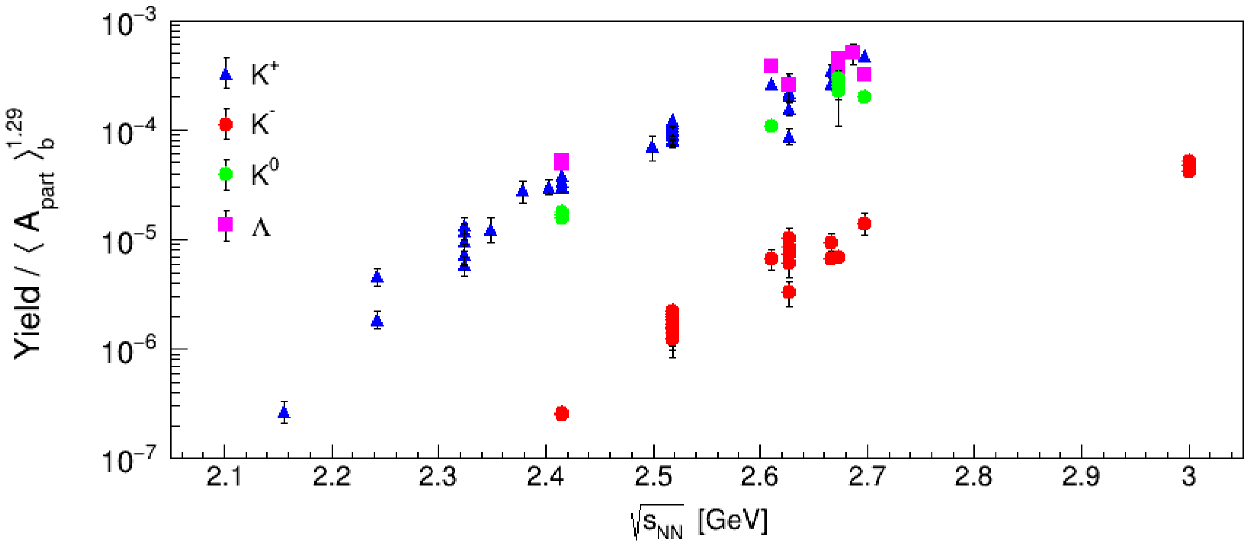
Common scaling of yields with $\sqrt{s}$?

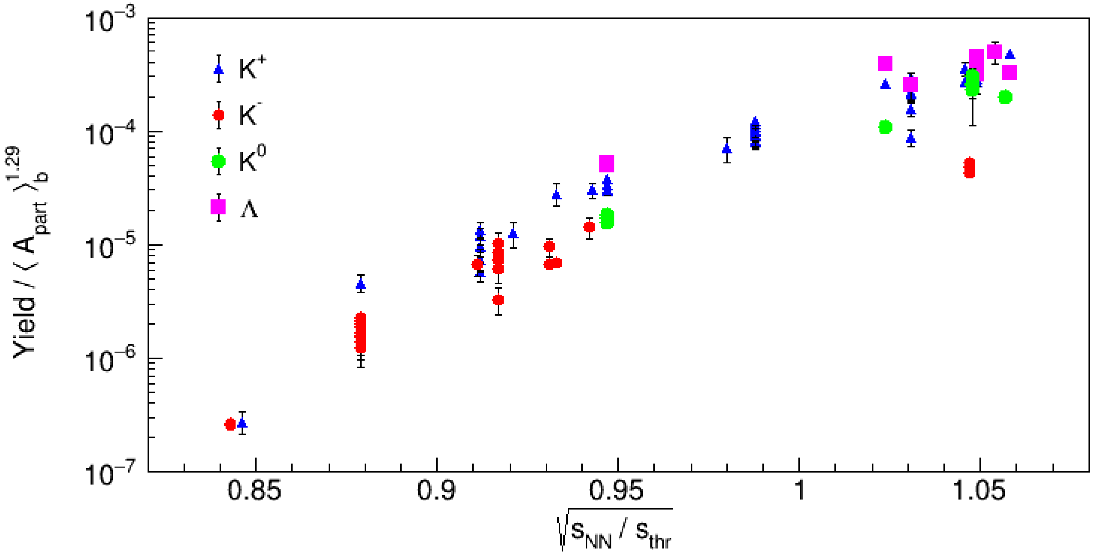
Let's take $\alpha = const = 1.29 \pm 0.03$

and try $\dfrac{P}{\langle A_{part} \rangle^{1.29}}$ vs $\sqrt{s}$.

$\Rightarrow$ per hadron, a narrow band.

At given $\sqrt{s}$, most data mutually agree (but not everywhere).





Per hadron, let's divide $\sqrt{s}$ by $\sqrt{s_{\text{threshold, NN}}}$:

For $K^{+,0}$ and Λ , $\sqrt{s_{\text{threshold, NN}}} = 2.55$ GeV
For K^- , $\sqrt{s_{\text{threshold, NN}}} = 2.87$ GeV .

$\Rightarrow$ scaled yields are within corridor, but :
at given $\sqrt{s}$, 1 order of magnitude - wide

K^0 vs K^+ : isospin + asymmetric EoS
K^- : absorption, feeding from ϕ decays, ...

$\Rightarrow$ K^- at $\sqrt{s_{NN}} = 3$ GeV : deviation from others?

Yields and Apart data for strangeness

- Data on yields and $\langle A_{part} \rangle$ for K^+, K^-, K^0, Λ, ϕ and even Ξ^- :) @ T_{Beam} = [0.6 .. 10.7] A GeV.
  Here: K^+ data [link to table]

K+	Tb/A	√s	√s_th	As published					Overlap	TGlauberMC		
System	[GeV]	[GeV]	[GeV]	<Apart>_b	type	P	dP	Cen [%]	<Ap>b	<Ap>b	Δ(<Ap>b)	Ref.
Al+Al	1,91	2,666	2,549	42	geom	0,035	0,0049	0 .. 8.6	34.8	36,78	1,13	P. Gasik et al. (FO
Ni+Ni	1,06	2,348	2,549	75	geom	0,0033	0,000825	0 .. 12.9	73.5	75,31	0,75	D. Best et al. (FOF
Ni+Ni	1,45	2,499	2,549	75	geom	0,0195	0,005005	0 .. 12.9	76.6	78,05	0,7	D. Best et al. (FOF
Ni+Ni	1,93	2,673	2,549	75	geom	0,0825						
Ni+Ni	1,91	2,666	2,549	46,5 ± 2	geom	0,03598						
Au+Au	1,23	2,415	2,549	303 ± 11,0	glauMC	0,0598						
Au+Au	1,23	2,415	2,549	213,1 ± 11,1	glauMC	0,0339						
Au+Au	1,23	2,415	2,549	149,8 ± 9,7	glauMC	0,0188						
Au+Au	1,23	2,415	2,549	103,1 ± 6,8	glauMC	0,012						
Ar+KCl	1,756	2,611	2,549	38,5 ± 3,9	geom	0,028						
Au+Au	1,5	2,518	2,549	16	glauOpt	0,00328						
Au+Au	1,5	2,518	2,549	88,2	glauOpt	0,024						
Au+Au	1,5	2,518	2,549	164,8	glauOpt	0,0606						
Au+Au	1,5	2,518	2,549	252	glauOpt	0,116						
Au+Au	1,5	2,518	2,549	336,2	glauOpt	0,158						
Ni+Ni	1,5	2,518	2,549	7	glauOpt	0,00119						
Ni+Ni	1,5	2,518	2,549	31	glauOpt	0,00815						
Ni+Ni	1,5	2,518	2,549	52,8	glauOpt	0,0168						
Ni+Ni	1,5	2,518	2,549	77	glauOpt	0,028						
Ni+Ni	1,5	2,518	2,549	101	glauOpt	0,0311						
C+C	1	2,324	2,549	6	geom	0,00008						
C+C	1,8	2,627	2,549	6	geom	0,00318	0,00032	0 ..100	4.8	6,4	0,11	F. Laue et al. (Kao
Ni+Ni	1	2,324	2,549	16,2	geom	0,00023	0,000045	41.0 .. 59.6	13.7	17,25	0,37	R. Barth et al. (Ka
Ni+Ni	1	2,324	2,549	37,6	geom	0,00074	0,000135	17.9 .. 41.0	32.4	36,45	0,46	R. Barth et al. (Ka
Ni+Ni	1	2,324	2,549	61,6	geom	0,00179	0,00033	11.4 .. 17.9	54.3	57,62	0,82	R. Barth et al. (Ka
Ni+Ni	1	2,324	2,549	85,7	geom	0,00322	0,00056	0 .. 11.4	74.3	76,49	0,81	R. Barth et al. (Ka
Ni+Ni	1,8	2,627	2,549	15,4	geom	0,00375	0,00061	41.0 .. 59.6	14.5	18,11	0,36	R. Barth et al. (Ka
Ni+Ni	1,8	2,627	2,549	37,3	geom	0,0178	0,0028	17.9 .. 41.0	34.5	38,77	0,45	R. Barth et al. (Ka
Ni+Ni	1,8	2,627	2,549	61,3	geom	0,0423	0,0066	11.4 .. 17.9	58.1	61,64	0,54	R. Barth et al. (Ka
Ni+Ni	1,8	2,627	2,549	85,7	geom	0,0638	0,0099	0 .. 11.4	79.5	81,52	0,59	R. Barth et al. (Ka
C+C	0,8	2,242	2,549	6	geom	0,0000175	0,0000032	0 .. 100	3.9	5,67	0,11	A. Foerster et al. (
C+C	1,5	2,518	2,549	6	geom	0,0013	0.00016	0 .. 100	4.73	6,27	0,11	A. Foerster et al. (
Au+Au	0,8	2,242	2,549	98,5	geom	0,00147	0,00027	0 .. 100	80.2	88	2,5	A. Foerster et al. (
Au+Au	1	2,324	2,549	98,5	geom	0,0045	0.0007	0 .. 100	84.7	89,8	2,6	A. Foerster et al. (
C+C	2	2,698	2,549	6	geom	5,30E-03	0.00058	0 .. 100	4.83	6,47	0,1	A. Foerster et al. (Ka
Au+Au	0,6	2,156	2,549	98,5	geom	7,30E-05	0.000016	0 .. 100	65.1	75,7	3,6	A. Foerster et al. (Ka
C+C	1,2	2,403	2,549	6	geom	3,10E-04	0.000046	0 .. 100	4.58	6,07	0,12	A. Foerster et al. (Ka
Au+Au	1,135	2,378	2,549	98,5	geom	9,40E-03	0.0021	0 .. 100	86.0	90,9	2,6	A. Foerster et al. (Ka
Ni+Ni	1,93	2,673	2,549	72.7	geom	0,066	0.003	0 .. 21.4	68.5	71,29	0,69	M. Menzel et al. (K
Ne+NaF	2,1	2,732	2,549	unknown	geom	0.0171	0.0060	0 .. 100		9,78	0,22	S. Schnetzer et al.
Au+Au	10,7	4,859	2,549	354	geom	24.2	0.9	0 .. 5		341,5	3,8	L. Ahle et al. (E-80
Au+Au	10,7	4,859	2,549	312	geom	19.7	0.6	5 .. 12		278,2	3,9	L. Ahle et al. (E-80
Au+Au	10,7	4,859	2,549	248	geom	13.3	0.4	12 .. 23		205,7	4,2	L. Ahle et al. (E-802
Au+Au	10,7	4,859	2,549	164	geom	8.0	0.3	23 .. 39		128,1	4,6	L. Ahle et al. (E-802