Electroproduction of Hypernuclei

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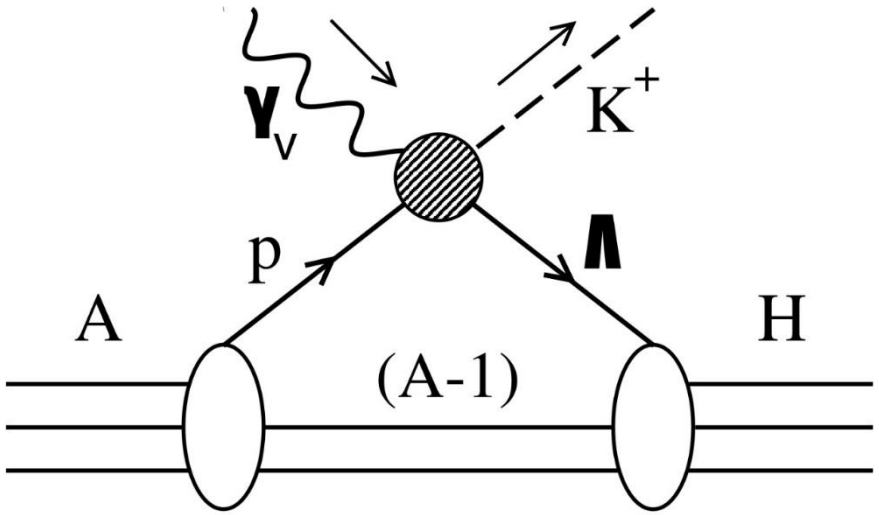
Motivation of studying electroproduction of hypernuclei in DWIA

$$e + A \rightarrow e' + H + K^+$$

- We obtain information on the spin-dependent part of the hyperon-nucleon interaction.
- The electro-magnetic part of the interaction is well known and can be treated perturbatively which simplifies description of the process.
- The impulse approximation with kaon distortion (DWIA) is well justified in considered kinematics. (P. Bydzovsky. D.Denisova, D.Skoupil, P.Vesely Phys. Rev. C 106, 044609(2022)). The IA formalism was developed and proved to work well. (F.Garibaldi et al, Phys Rev. C 99, 054309(2019))
- One can achieve a better experimental resolution than in hadron-induced reactions $\mathrm{A}(\pi^+, K^+)H$.
- Predictions of the cross section for planed JLab experiments (E12-15-008) are desired.

Impulse approximation

$$\gamma_\nu(P_\gamma) + A(P_A) \rightarrow H(P_H) + K^+(P_K)$$



Without Kaon distortion

- 3-momentum conservation in each vertex.
- The proton and Λ are bound particles but $\epsilon_p \approx \epsilon_\Lambda$ is assumed.
- The optimal factorization approximation (OFA) with the effective proton momentum in the nucleus $\vec{p}_{eff}$.
- Kaon momentum $|\overrightarrow{P_K}|$ is calculated from energy conservation in the many-body system – in general, the elementary amplitude is off-energy shell or two values of $|\overrightarrow{P_K}|$ are used.

Cross section

- The unpolarized triple differential cross section in electroproduction of hypernuclei in the laboratory frame

$$\frac{d^3\sigma}{dE'_e d\Omega'_e d\Omega_K} = \Gamma\left[\frac{d\sigma_T}{d\Omega_K} + \varepsilon_L \frac{d\sigma_L}{d\Omega_K} + \varepsilon \frac{d\sigma_{TT}}{d\Omega_K} + \sqrt{\varepsilon_L(\varepsilon+1)}\frac{d\sigma_{TL}}{d\Omega_K}\right]$$

- The transverse and longitudinal cross sections

$$\frac{d\sigma_T}{d\Omega_K} = \frac{\beta}{2(2J_A+1)}\sum_{Jm}\frac{1}{2J+1}\left(\left|A_{Jm}^{+1}\right|^2 + \left|A_{Jm}^{-1}\right|^2\right)$$

$$\frac{d\sigma_L}{d\Omega_K} = \frac{\beta}{2J_A+1}\sum_{Jm}\frac{1}{2J+1}\left|A_{Jm}^{0}\right|^2$$

- The reduced amplitudes are

$$A_{Jm}^{\lambda} = \frac{1}{[J]}\sum_{S\eta} F_{\lambda\eta}^{S} \sum_{LM} C_{LMS\eta}^{Jm} \sum_{\alpha'\alpha} R_{\alpha'\alpha}^{LM} H_{l'j'lj}^{LSJ} \left(\Psi_H \| [b_{\alpha'}^{+} \otimes a_{\alpha}]^J \| \Psi_A\right)$$

- with the single-particle states denoted as α=[nlj]
- $R_{a'a}^{LM}$-radial integrals

Elementary amplitude

- The invariant amplitude M

$$M \cdot \varepsilon = \overline{u_\Lambda}\gamma_5 \left(\sum_{j=1}^{6} M_j \cdot \varepsilon A_j \right) u_p = X_\Lambda^+ (\vec{J} \cdot \vec{\epsilon}) X_p$$

- The elementary amplitude $\vec{J}$ in the spherical coordinates

$$\vec{J} \cdot \vec{\epsilon} = \sum_{\lambda=\mp 1,0} (-1)^{-\lambda} J_\lambda^{(1)} \epsilon_{-\lambda}^{(1)}$$

- The spherical components of $J^{(1)}$ can be defined via 12 spherical amplitudes $F_{\lambda,\xi}^S$ with $S = 0, 1$ and $\lambda, \xi = \pm 1, 0$

$$J_\lambda^{(1)} = \sum_{\lambda,\xi,S} F_{\lambda,\xi}^S \sigma_\xi^S$$

For more information look forward to Dalibor Skoupil's presentation

OBDME

- The nuclear structure is included via the reduced one-body density matrix elements (OBDME) $(\Psi_H||[b^+_{\alpha'} \otimes a_\alpha]^J||\Psi_A)$ which are calculated

1) In shell-model calculations (D.J. Millener, Nucl. Phys. A 804, 84 (2008)).

2) In TDΛ assuming N-Λ particle-hole excitations (arXiv:2306.01308).

3) In EMPMΛ assuming also nucleus-core excitations (arXiv:2306.01308).

Fermi motion effect

- In OFA the elementary amplitude is calculated at $\vec{p}_{eff}$.
- Results with three values of $\vec{p}_{eff}$ (Fermi motion effects) are compared:

1) frozen p: $\vec{p}_{eff} = 0 \Rightarrow \vec{p}_{\Lambda} = \vec{\Delta}$

2) frozen Λ: $\vec{p}_{eff} = -\vec{\Delta} \Rightarrow \vec{p}_{\Lambda} = 0$

Both cases with on-shell elementary amplitude ⇒ 2 values of $|\vec{P}_K|$

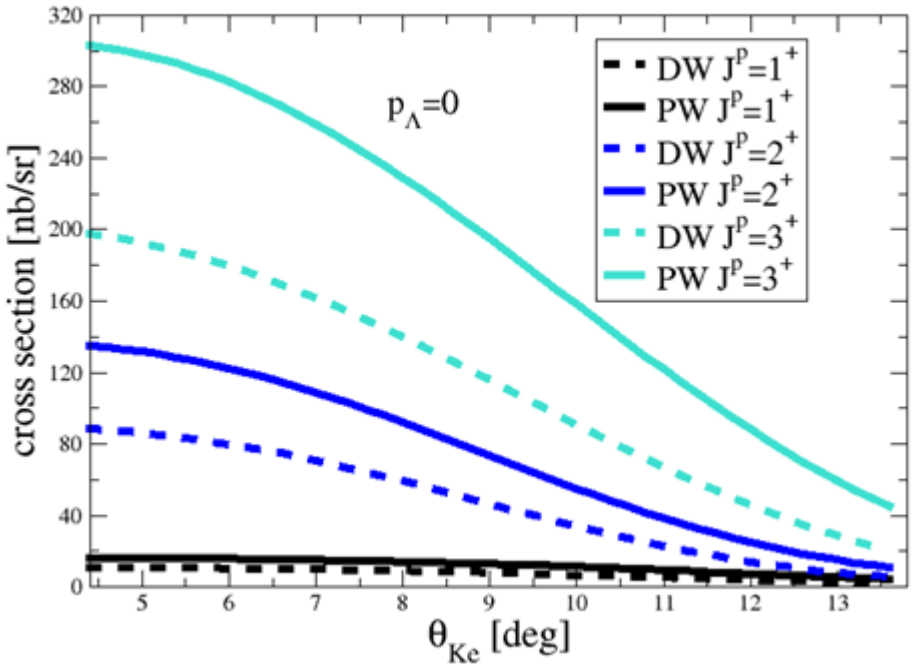
3) optimum: $\vec{p}_{eff}=\vec{p}_{opt}$ satisfies energy conservation both in the many-body and 2-body systems =>elementary amplitude is on-shell ⇒ 1 values of $|\vec{P}_K|$

$$\sqrt{m_{\Lambda}^2 + \left(\vec{\Delta} + \vec{p}_{opt}\right)^2} - \sqrt{m_p^2 + \vec{p}_{opt}^{\,2}} = \sqrt{M_{\mathrm{H}}^2 + \vec{\Delta}^{\,2}} - M_A$$

$\vec{\Delta} = \vec{P}_{\gamma} - \vec{P}_K$

Comparison of the PWIA and DWIA

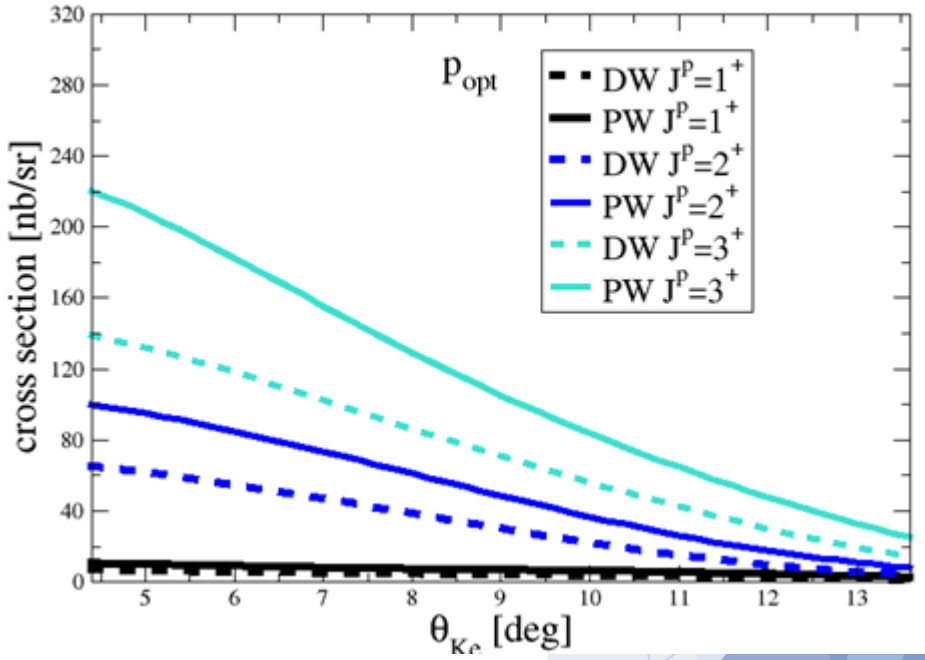
p_{eff} = 0.233GeV/c



$E_H \approx$11 MeV

θ_{Ke} = 6°

p_{eff} = 0.099GeV/c



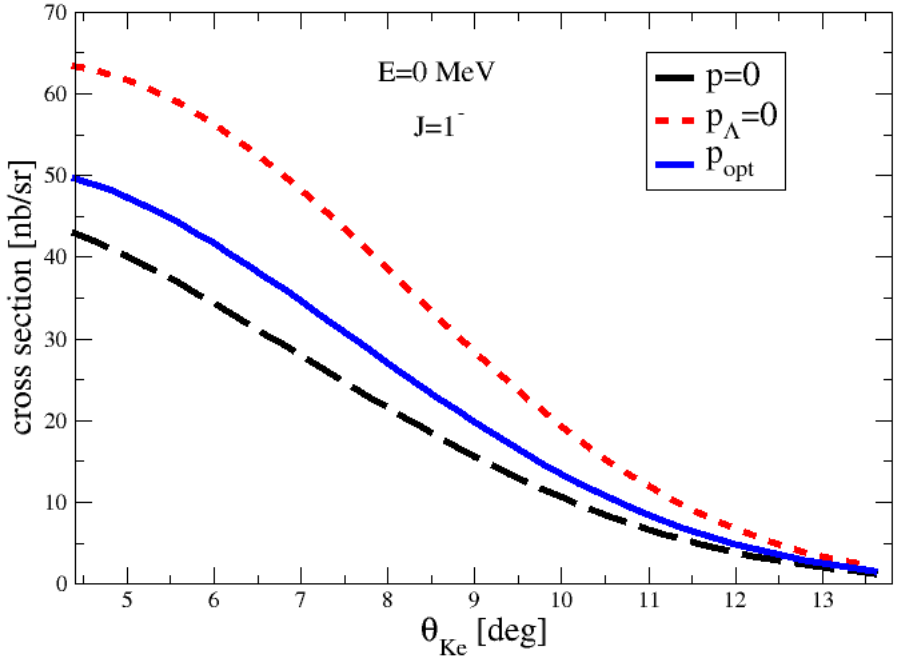
Reaction $^{12}C(e,e'K^+)^{12}_{\Lambda}B$ E_i =3.77 GeV $E_f{}'$ =1.56 GeV θ_e = 6° $\Phi_K = 180°$

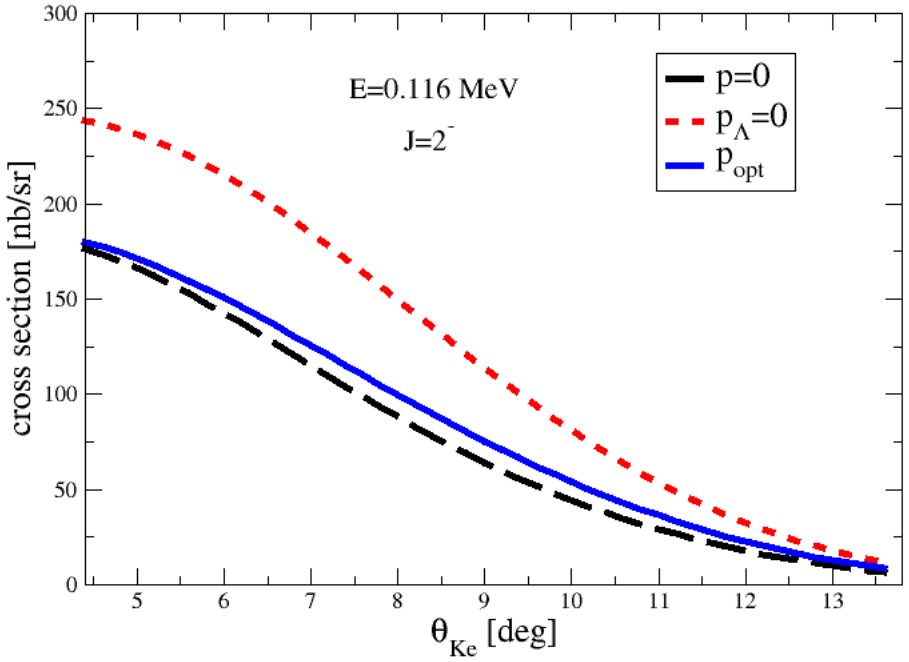
Calculations are with elementary amplitude BS3. (Rev. C 97, 025202(2018).)

The nuclear structure (OBDME) is from shell-model calculations by John Millener. (Nucl. Phys. A 804, 84 (2008))

Fermi motion effect in DWIA

Experimental value for the ground-state doublet at θ_{Ke} = 6°: 253.4±38 nb/sr





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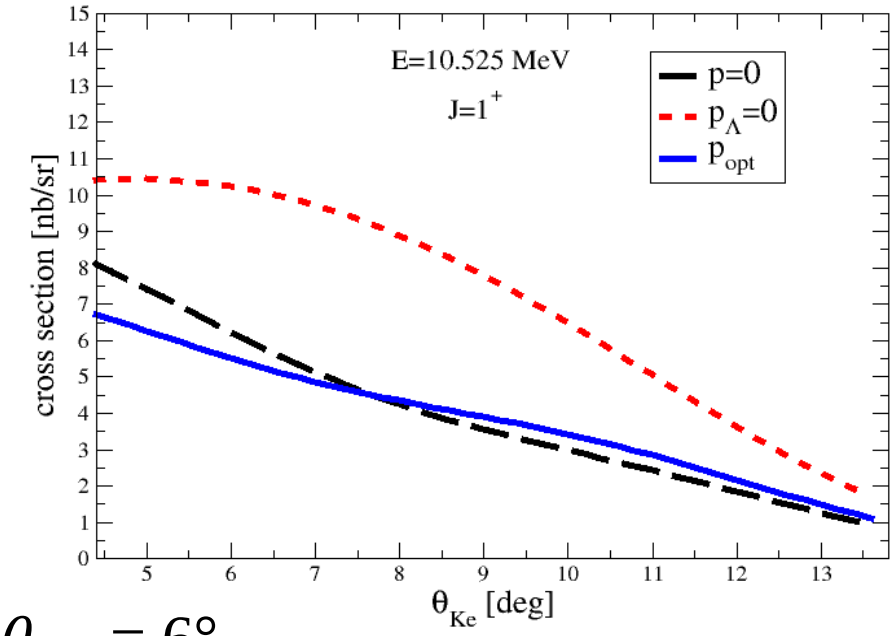
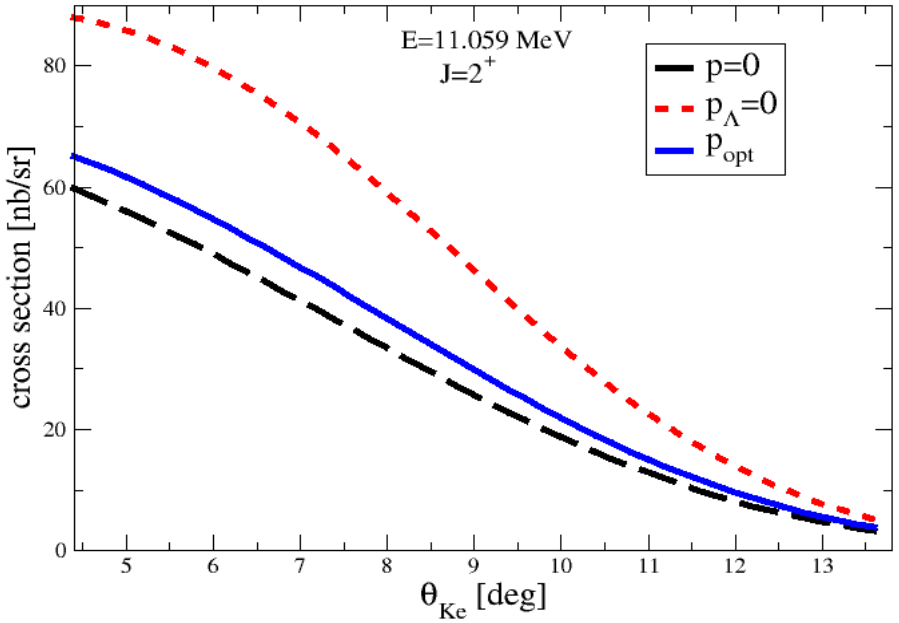


The $d\sigma_L$ depend on the A^0_{Jm} with the largest value for m = 0. The F^1_{00} dominates => the behavior of $d\sigma_L$ is affected by the selection rule for C^{J0}_{L010}.



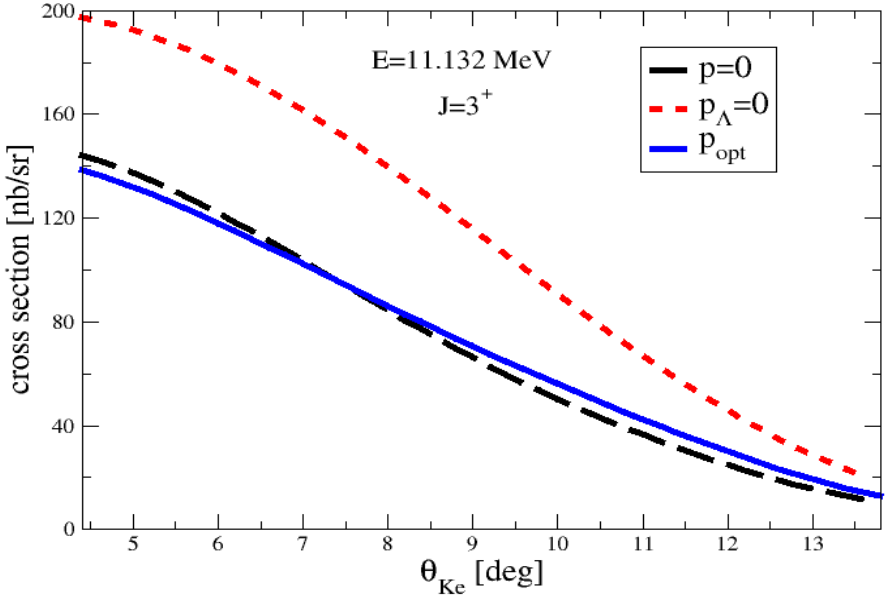
θ_{Ke} = 6°,

p_{eff} = 0.0 GeV/c, p_Λ = 0.301 GeV/c

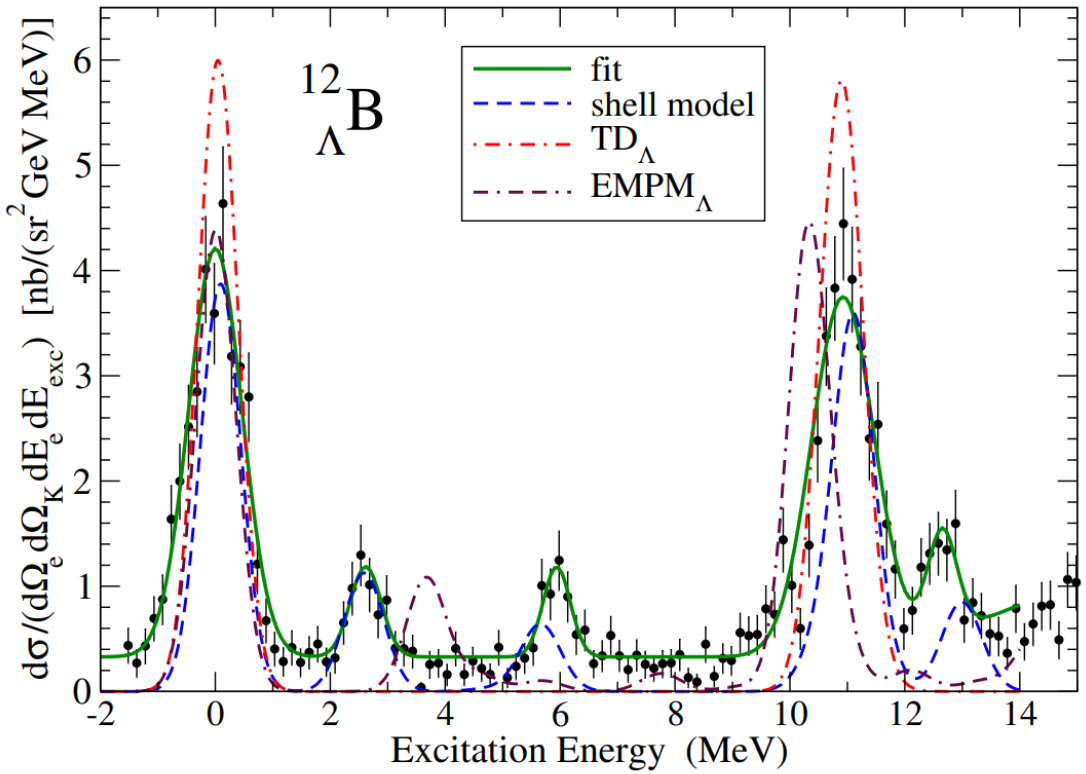
p_{eff} = 0.233GeV/c, p_Λ = 0.0 GeV/c

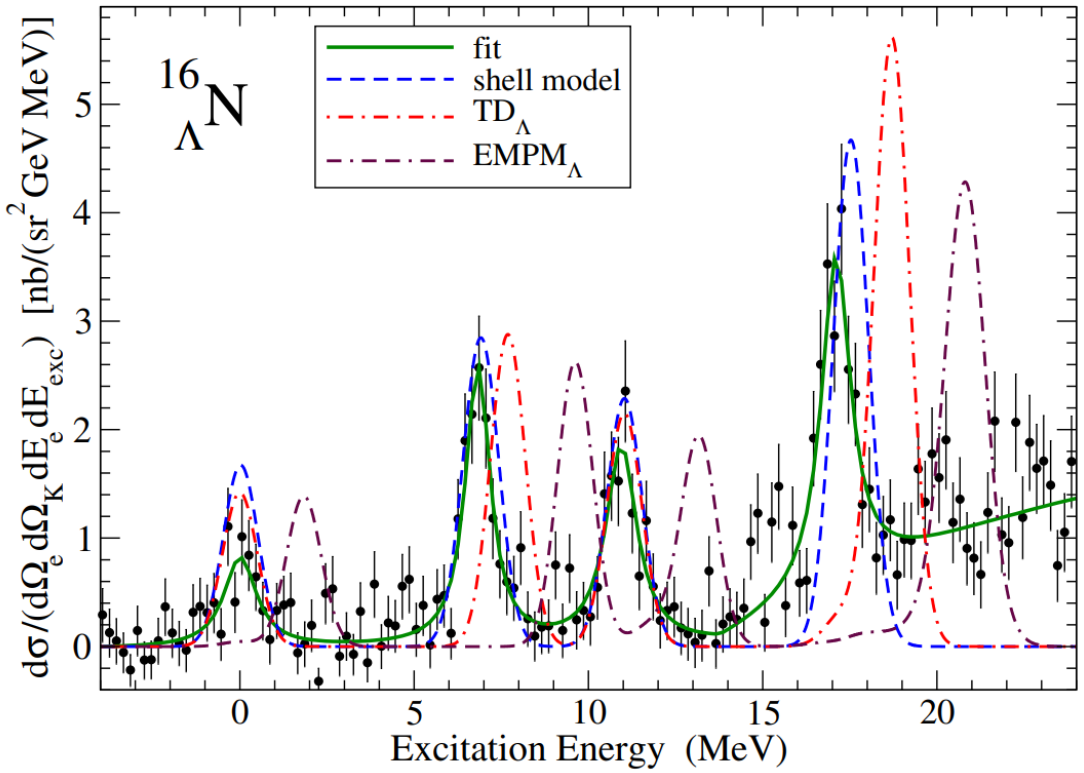
p_{eff} = 0.099 GeV/c, p_Λ = 0.170 GeV/c

More results can be found in Phys. Rev. C 106, 044609(2022)



Nuclear structure effects

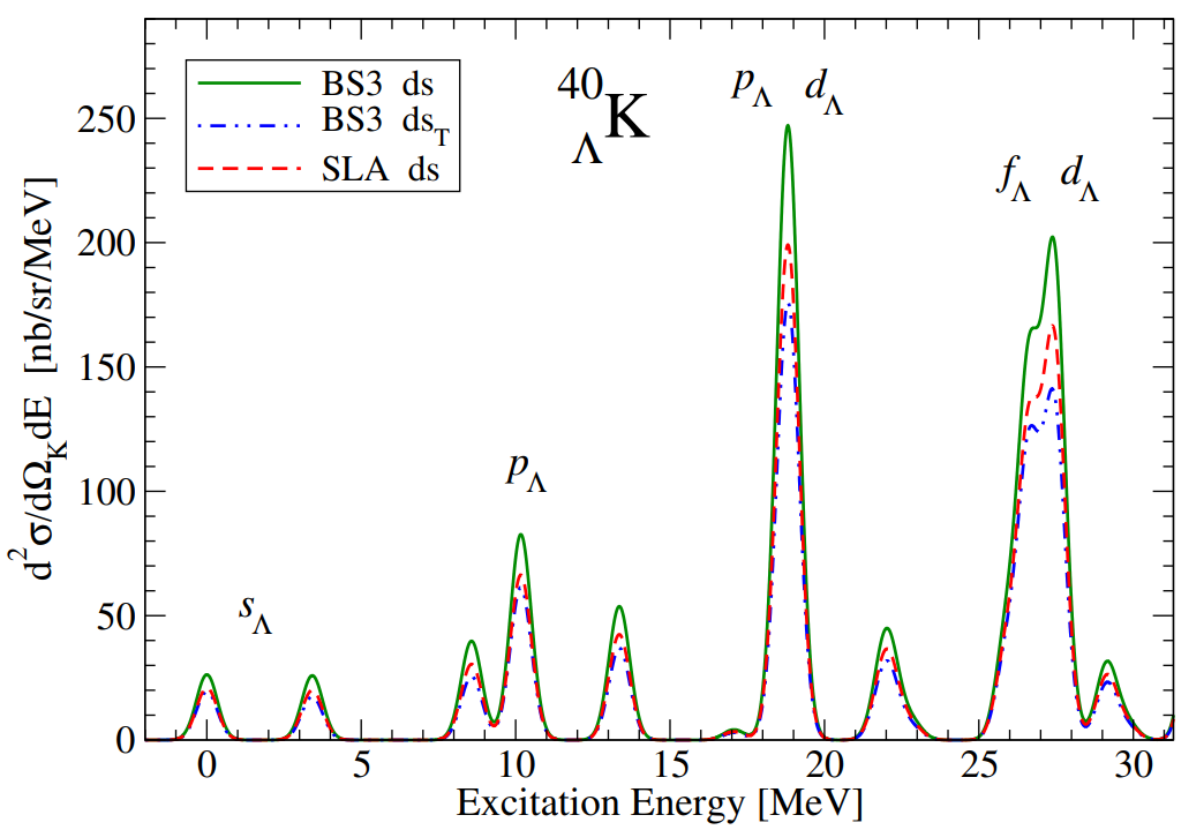


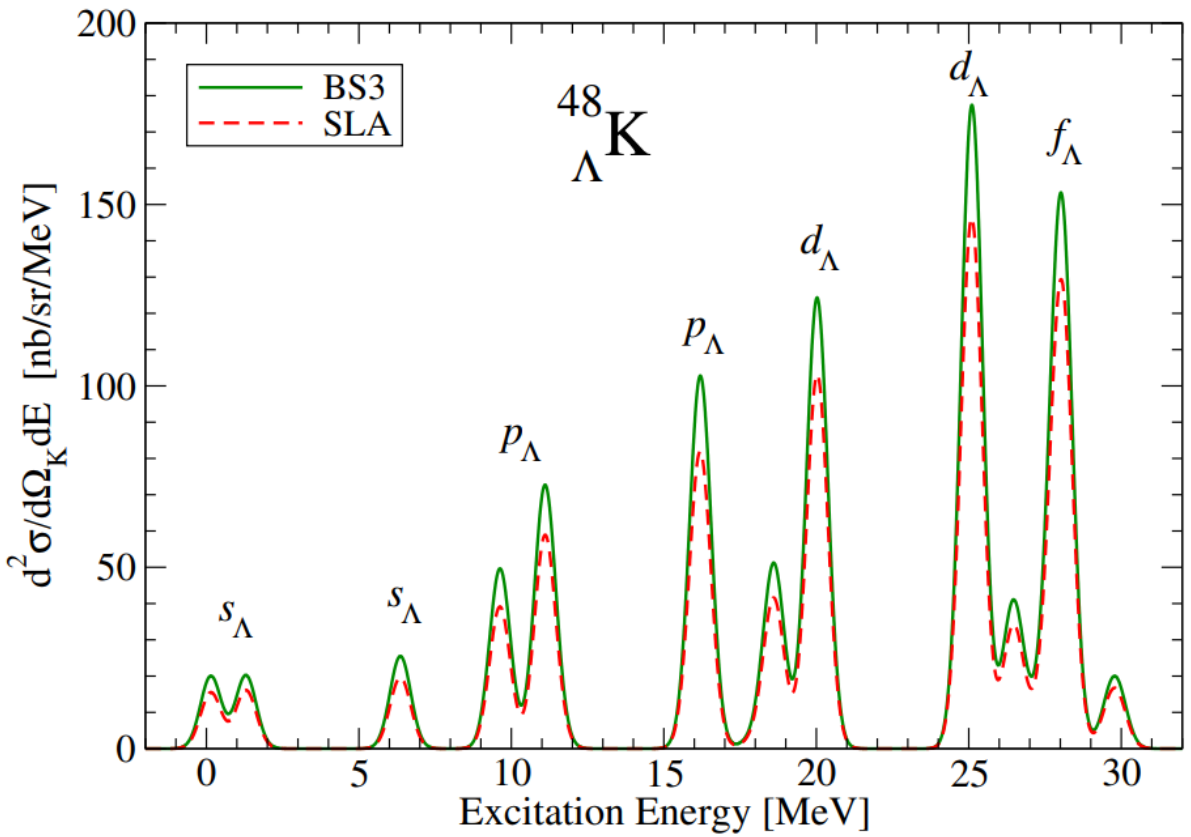


NΛ G-matrix Nijmegen-F interaction was used with the $k_F = 1.1 fm^{-1}$ in TDΛ and EMPMΛ.

Data is from arXiv:2306.01308

Prediction calculations





- Kinematics of the planned experiment E12-15-008 in Jlab
- TDΛ formalism with G-matrix Nijmegen-F interaction was used with the $k_F = 1.25 fm^{-1}$

Summary and outlook.

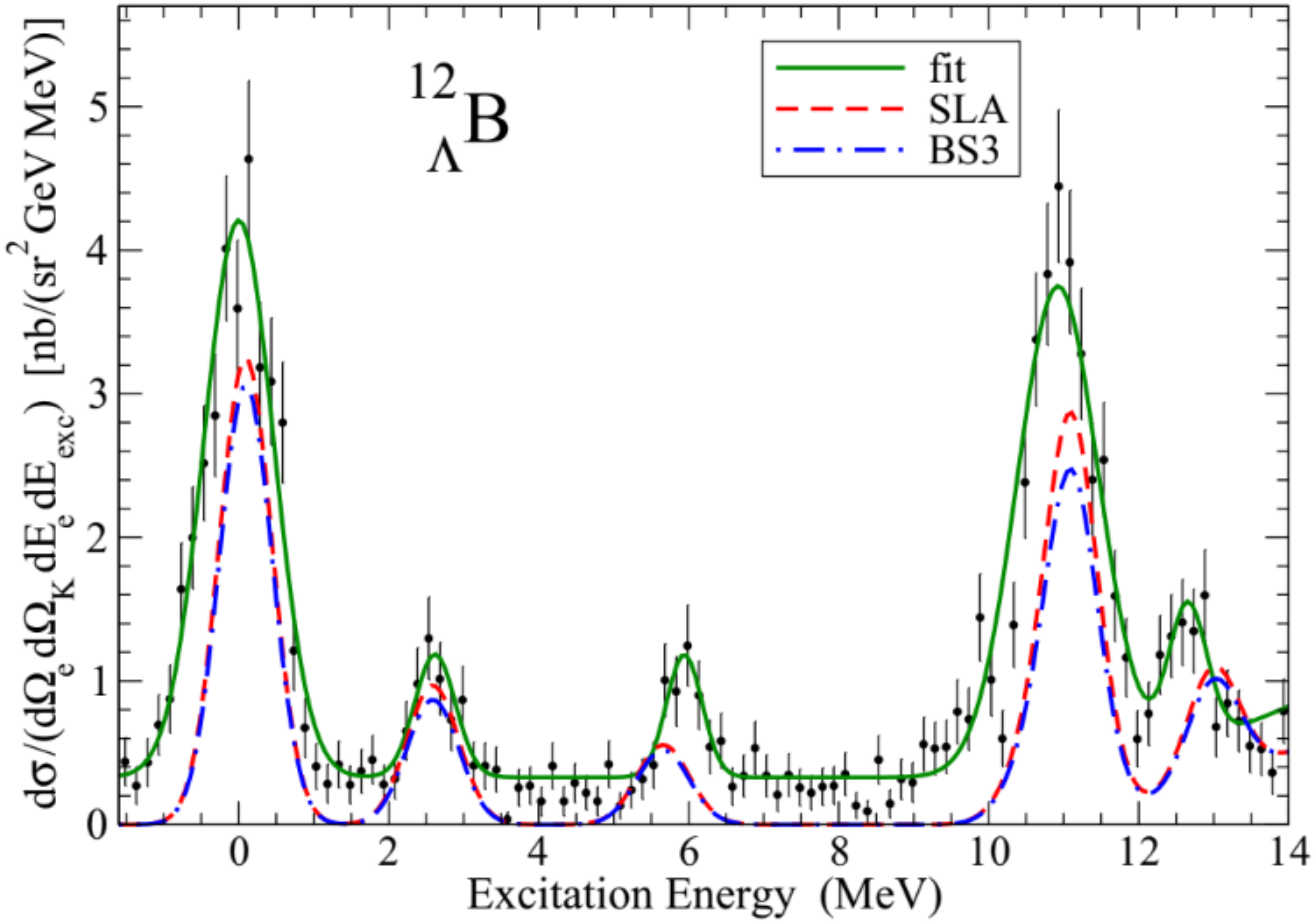
A general two-component form of the elementary amplitude was derived and used to show a dependence on the proton Fermi motion in the target nucleus.

Fermi-motion effect is more important, especially in the longitudinal cross sections, for smaller angles and energies above 2 GeV.

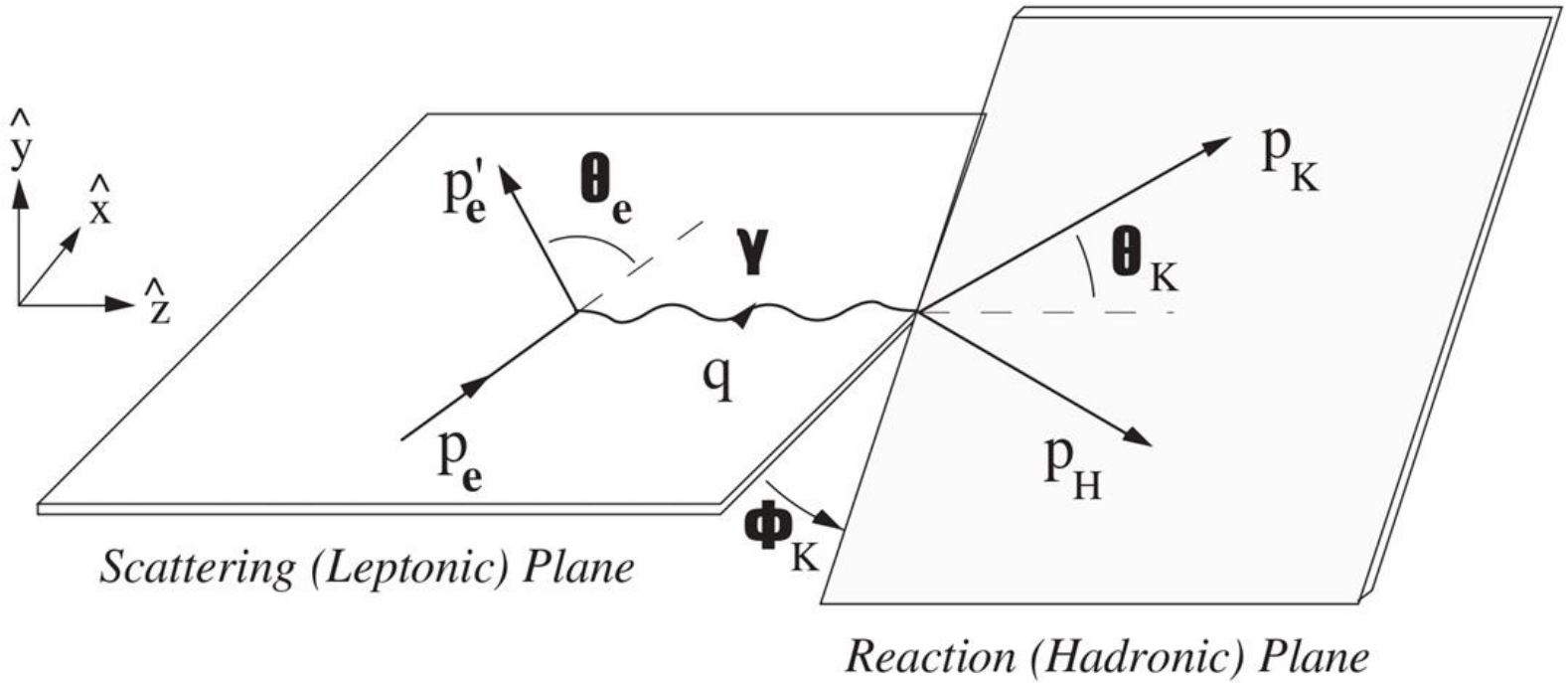
Comparison of PWIA and DWIA cross sections calculated in the optimum on-shell approximation show importance of the kaon distortion (about 30 %). Therefore, the DWIA and the optimum on-shell approximation are preferable for the further calculations as the results are close to the experimental cross section.

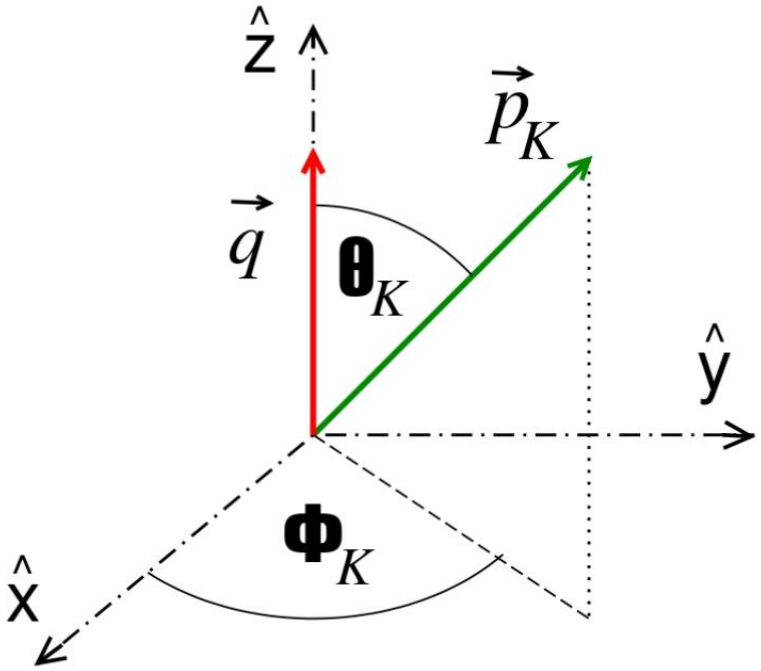
We showed that the TDΛ and EMPMΛ approach with the Nijmegen YN and NNLOsat interactions, used to calculate the OBDME and the single-particle wave functions, are appropriate in description of experimental data for the p-shell hypernuclei giving similar results as the shell-model calculations. We can therefore use this formalism to predict the excitation spectra of medium-mass hypernuclei which will be measured in a planned experiment in JLab. We have predicted the spectra for ${}^{40}_{\Lambda}K$ and ${}^{48}_{\Lambda}K$ which will be measured in the planned experiment in JLab.

Thank you for attention


$^{12}_{\Lambda}B$
fit
SLA
BS3
$d\sigma/(d\Omega_e d\Omega_K dE_e dE_{exc})$ [nb/(sr^2 GeV MeV)]
Excitation Energy (MeV)
0
1
2
3
4
5
0
2
4
6
8
10
12
14

Kinematics


ŷ
x̂
ẑ
p'e
θe
γ
pK
θK
q
pe
pH
ΦK
Scattering (Leptonic) Plane
Reaction (Hadronic) Plane


ẑ
$\vec{p}_K$
$\vec{q}$
θ_K
ŷ
Φ_K
x̂

Determination of P_K

- $\vec{P}_K$ is calculated from energy conservation in the many-body system (Lab)

$$\mathrm{E}_\gamma + M_A = \sqrt{m_K^2 + \vec{P}_K^2} + \sqrt{M_\mathrm{H}^2 + (\vec{P}_\gamma - \vec{P}_K)^2}$$

- Then, the energy conservation in the elementary system (on-shell amplitude)

$$\mathrm{E}_\gamma + \sqrt{{m_p}^2 + \vec{p}_p^2} = \sqrt{m_K^2 + \vec{P}_K^2} + \sqrt{m_\Lambda^2 + (\vec{P}_\gamma - \vec{P}_K + \vec{p}_p)^2}$$

is not satisfied for arbitrary proton momentum.

- Assuming, that $\epsilon_\mathrm{p} - \epsilon_\Lambda \approx 0$ and a given momentum transfer $\vec{\Delta} = \vec{P}_\gamma - \vec{P}_K$ the equations can be solved simultaneously for optimum proton momentum $\vec{p}_{opt}$

$$\mathrm{E}_\gamma - \sqrt{m_K^2 + \vec{P}_K^2} = \sqrt{m_\Lambda^2 + \left(\vec{\Delta} + \vec{p}_p\right)^2} - \sqrt{m_p^2 + \vec{p}_p^2} = \sqrt{M_\mathrm{H}^2 + \vec{\Delta}^{\,2}} - M_A$$

Radial integral

- The kaon distortion is included in the radial integral:

$$R_{a'a}^{LM} = \int_0^\infty d\xi\ \xi^2 R_{a'}^{\Lambda}(\xi)^*\ F_{LM}\ (\Delta B\xi)\ R_a^{p}(\xi)$$

- With $F_{LM}\ (\Delta B\xi)$ determined from

$$e^{(iB\vec{\Delta}\vec{\xi})} X_K^* \left(\vec{p_K}, B\vec{\xi}\right) = \sum_{LM} F_{LM}\ (\Delta B\xi)\ Y_{LM}(\hat{\xi})$$

where $B = \frac{A-1}{A-1+\frac{m_\Lambda}{m_N}}$

- Where X_K^* is the kaon distortion calculated in eikonal approximation assuming the first-order optical potential.

$$X_K^* = e^{-a\ \sigma_{tot}^{KN}\ \frac{1-i\alpha}{2} \int_0^\infty dt\ \rho(B\vec{\xi}+\vec{p_K}t)}$$