Second-order pion-nucleus potential for scattering and photoproduction

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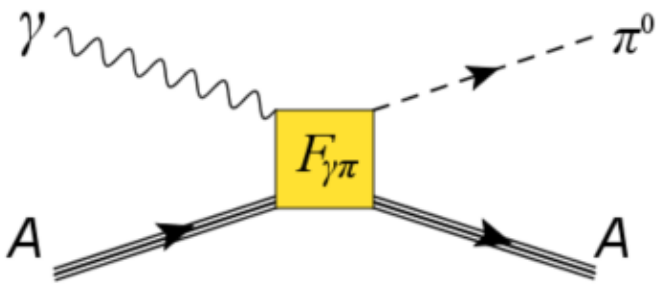
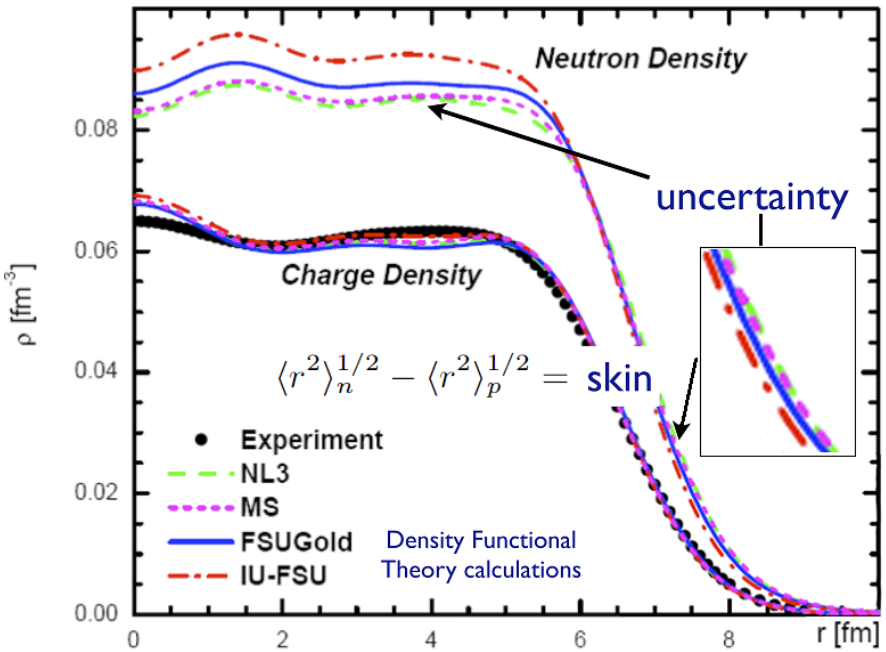
MESON 2023

Outlook

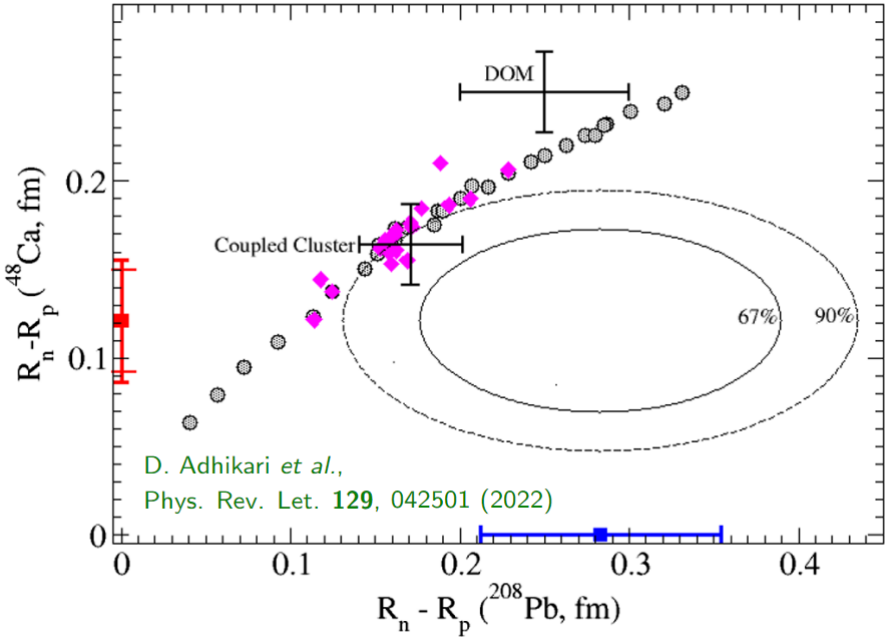


- Motivation: neutron skin
- Minimal model for π^0 photoproduction on nuclei
- Similarity to pion production in neutrino scattering
- Common approach for scattering and photoproduction
- Scattering: new potential, fits and prediction
- Application of our model to photoproduction

π^0 photoproduction – a tool for studying neutron distributions



CREX and PREX-II data from JLab



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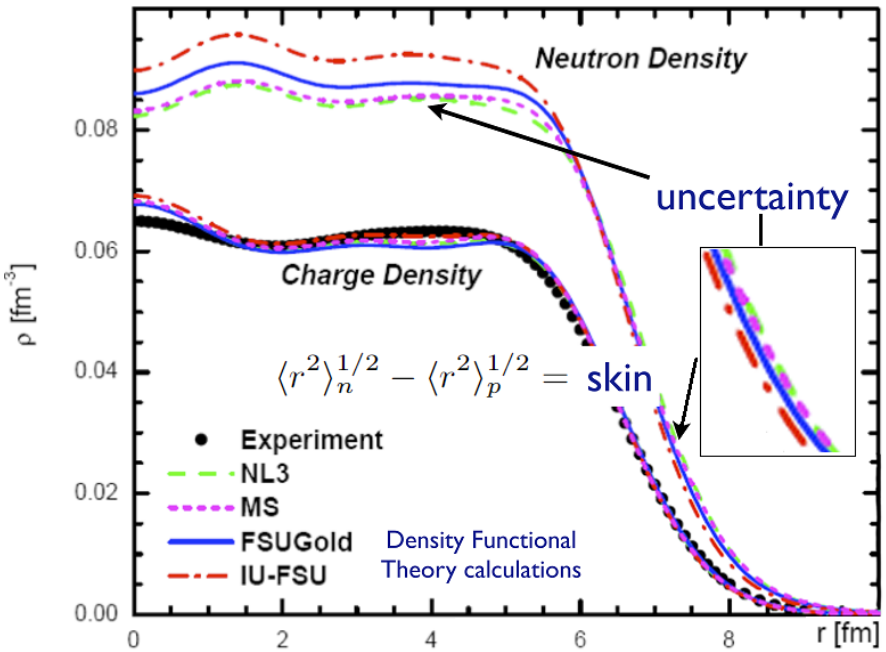

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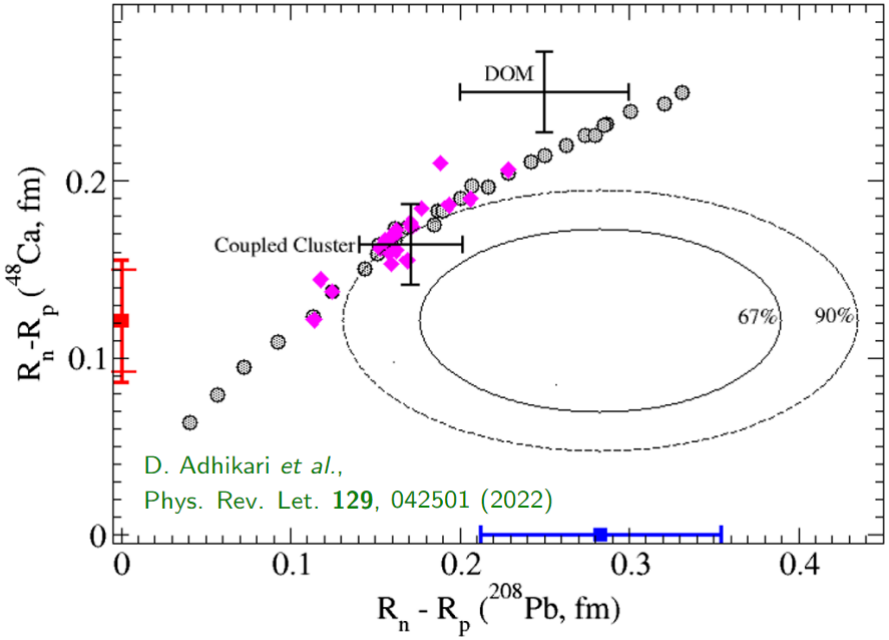
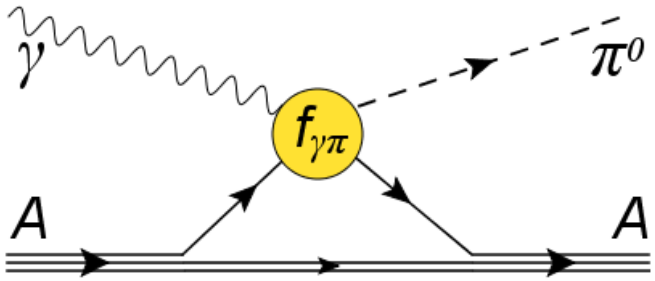
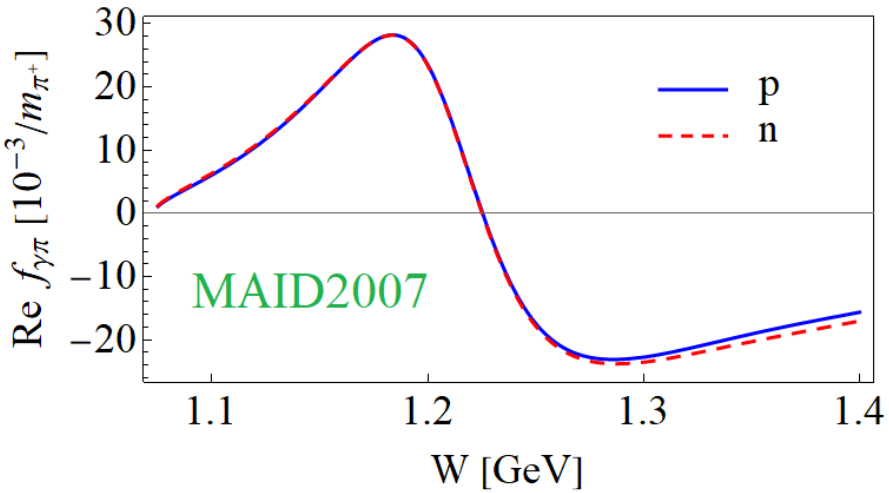






π^0 photoproduction provides nucleon FF:

$$V_{\gamma\pi} \propto f_{\gamma\pi} F_N(q)$$

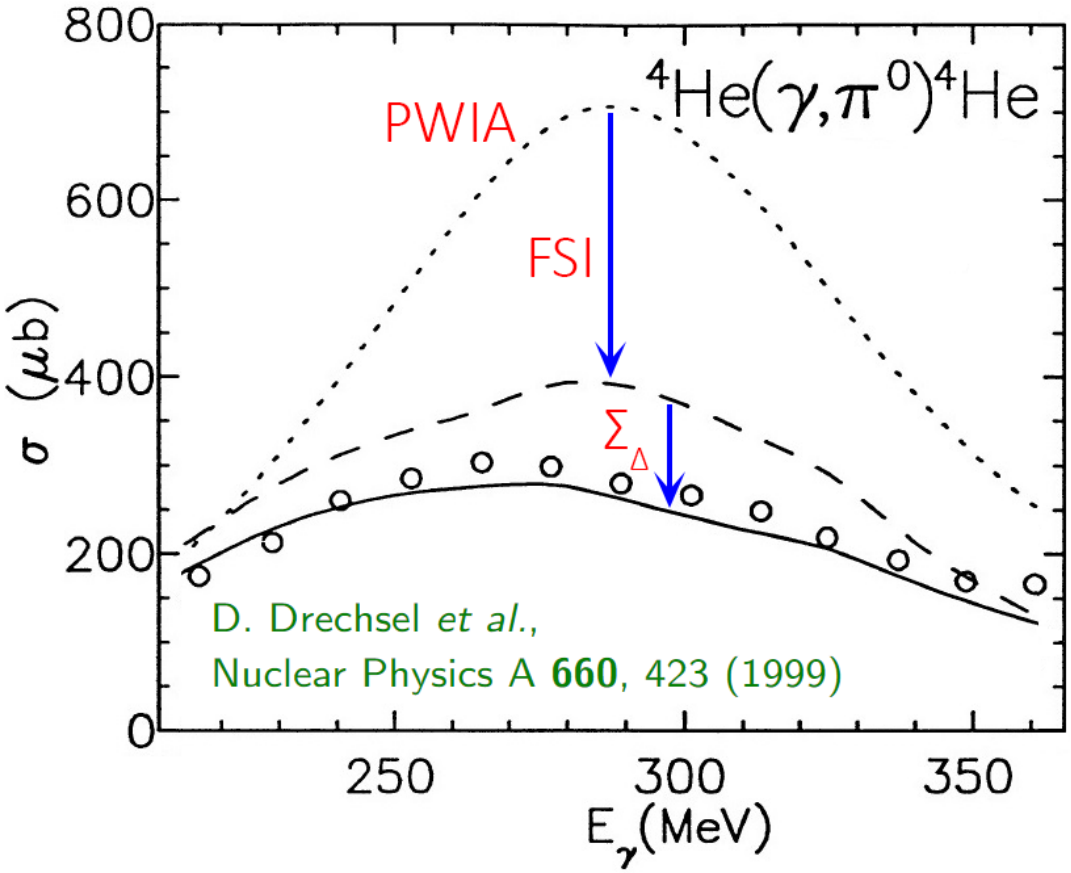
Neutron distribution can be extracted:

$$F_n(q) = F_N(q) - F_p(q)$$

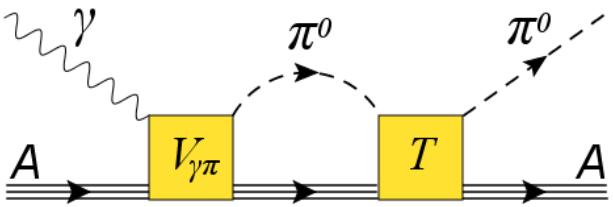
Precise theoretical model for photoproduction on nuclei is required

Elastic scattering amplitude is needed to describe pion production

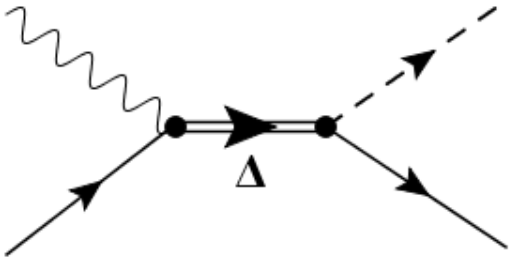
3 photoproduction components:



- $V_{\gamma\pi}$ for PWIA
- scattering amplitude T for FSI



- Effective $\Delta(1232)$ self-energy Σ_Δ



$$G_\Delta^{\text{bound}} = (W - m_\Delta + i\Gamma_\Delta/2 - \boldsymbol{\Sigma_\Delta})^{-1}$$

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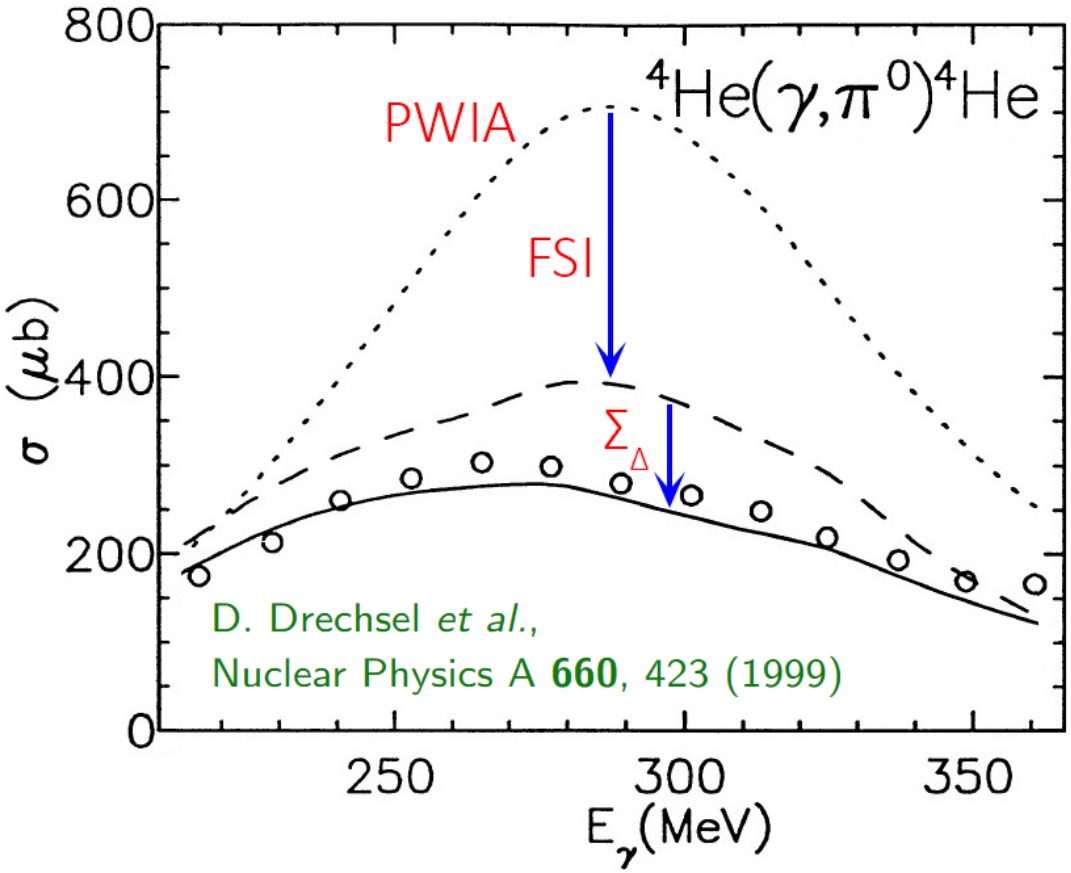

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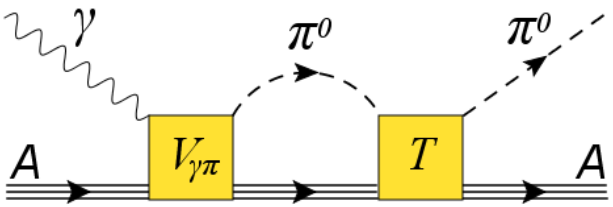

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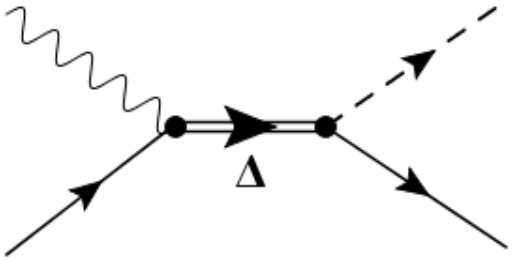
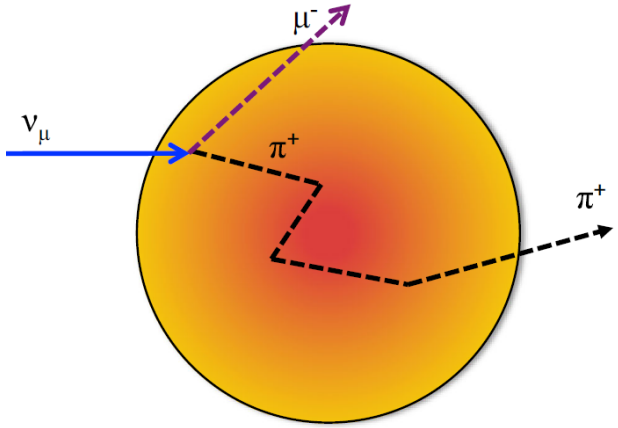


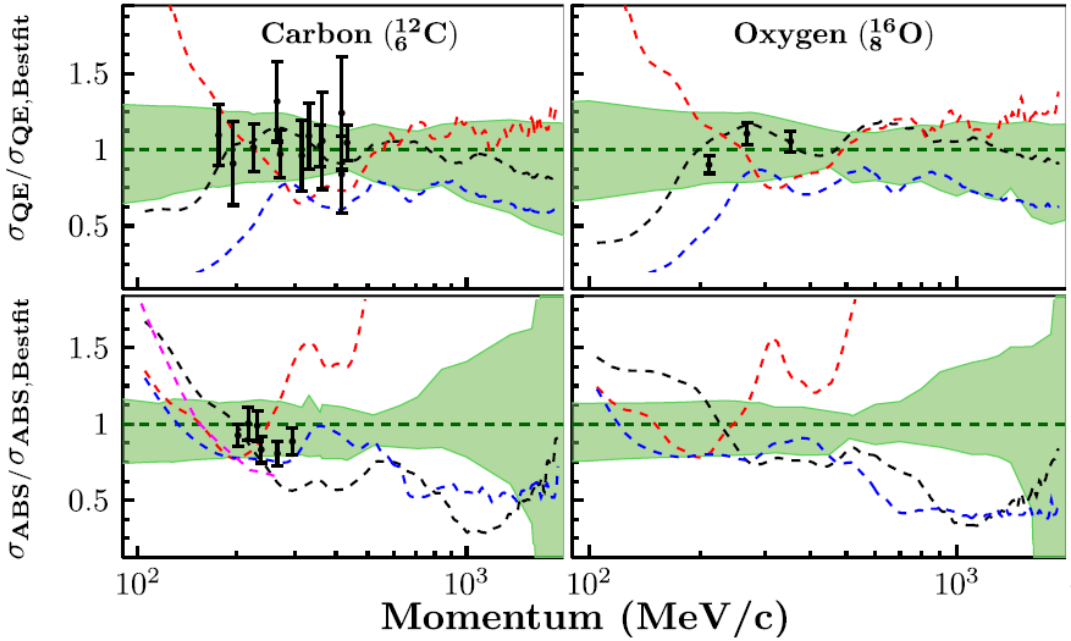
$$G_\Delta^{\text{bound}} = (W - m_\Delta + i\Gamma_\Delta/2 - \boldsymbol{\Sigma_\Delta})^{-1}$$

$$\hat{F}_{\gamma\pi} = \sum_i \hat{f}^{(i)}_{\gamma\pi} + \sum_i \sum_{j\neq i} \hat{t}^{(j)} \hat{G}(E) \hat{f}^{(i)}_{\gamma\pi} + \sum_i \sum_{j\neq i} \sum_{k\neq j} \hat{t}^{(k)} \hat{G}(E) \hat{t}^{(j)} \hat{G}(E) \hat{f}^{(i)}_{\gamma\pi} + \cdots ,$$

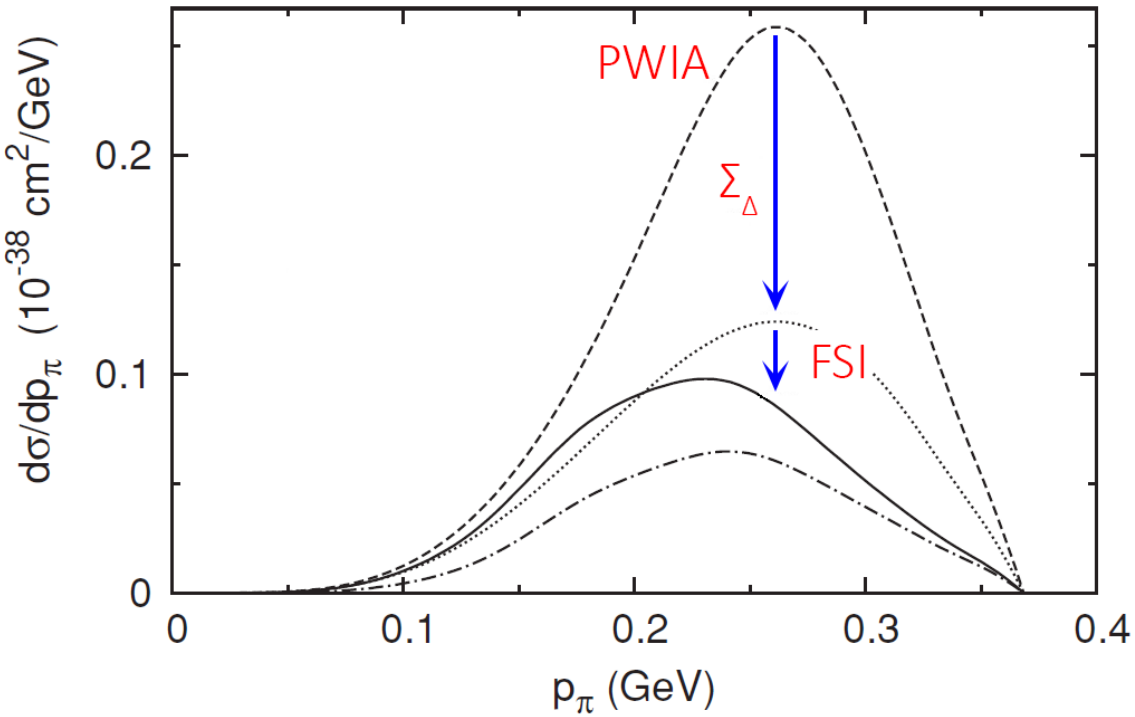
What is the role of the intermediate charge exchange and spin-flip?

Pion scattering and photoproduction help to study neutrinos





E.S. Pinzon Guerra *et al.*,
Phys. Rev. D **99**, 052007 (2019)



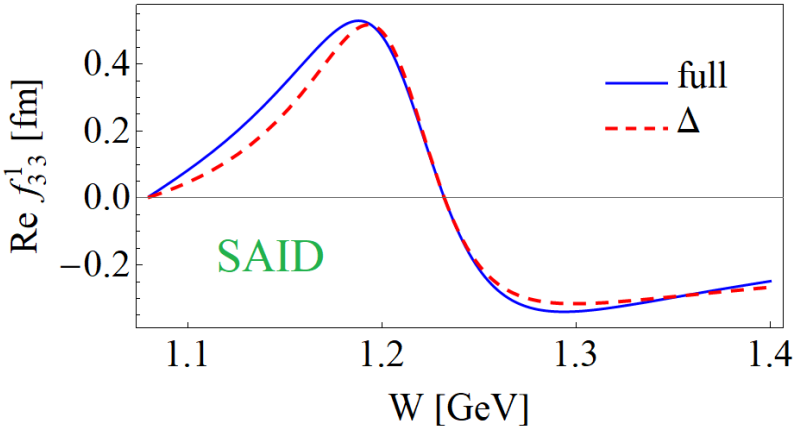
$$\nu_\mu + {}^{12}C \longrightarrow \mu^- + \pi^+ + {}^{12}C$$

S.X. Nakamura *et al.*,
Phys. Rev. C **81**, 035502 (2010)

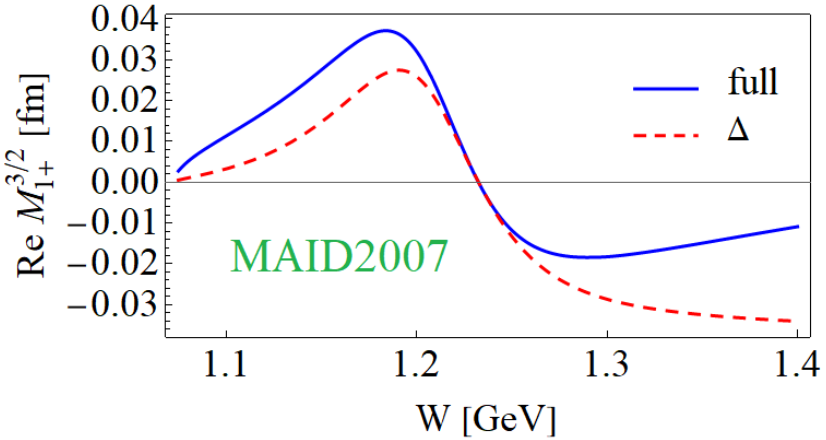
FSI is particularly important for T2K and T2HyperK

Common approach for scattering and photoproduction

scattering:



photoproduction:



$$f_{33}^{1(\Delta)} = \frac{\Gamma_\Delta/2}{m_\Delta - W - i\Gamma_\Delta/2 + \boldsymbol{\Sigma_\Delta}}$$

$$U \propto t\, F_N(q)$$

$$\arg M_{1+}^{3/2} = \arg f_{33}^1$$

$$V_{\gamma\pi} \propto f_{\gamma\pi} F_N(q)$$

Non-resonant contributions from SAID and MAID2007

Common approach for scattering and photoproduction

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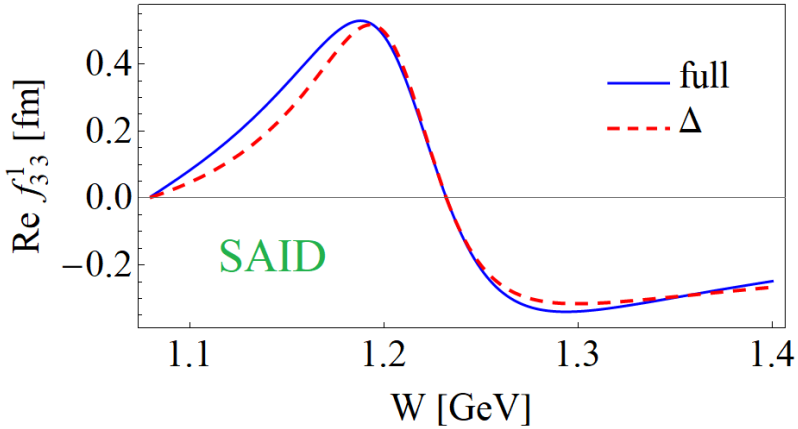
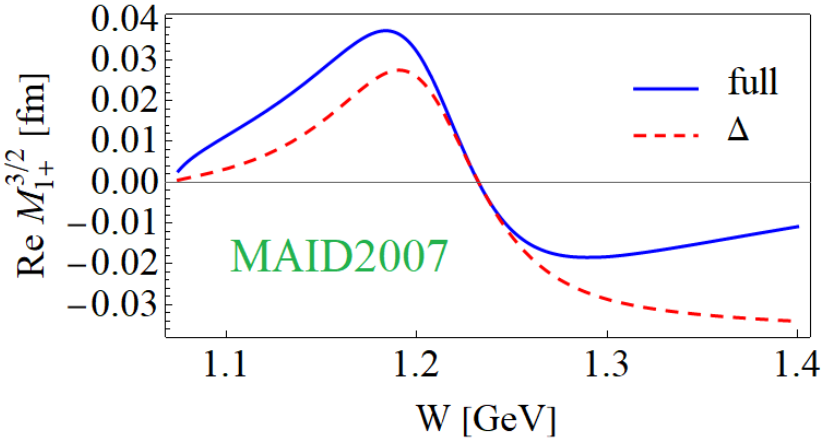


photoproduction:



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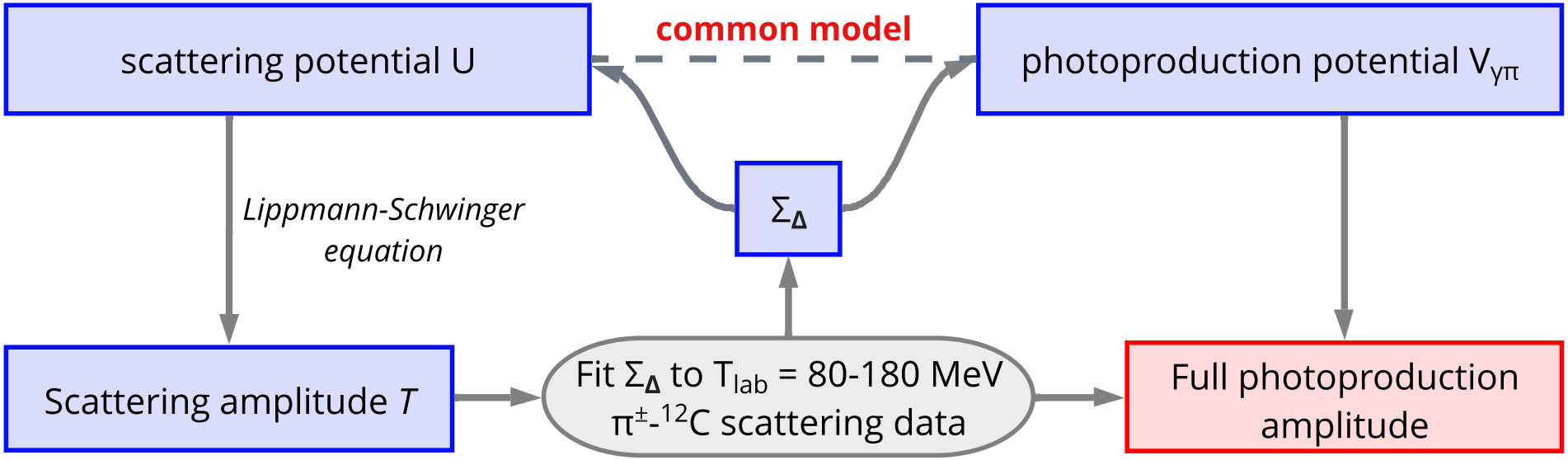
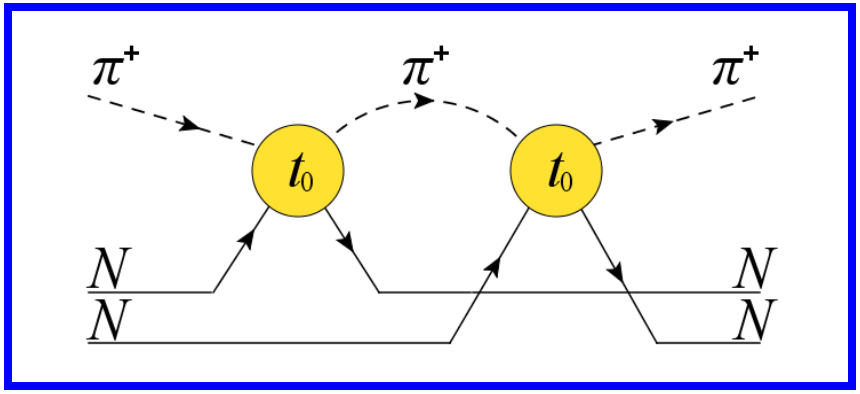


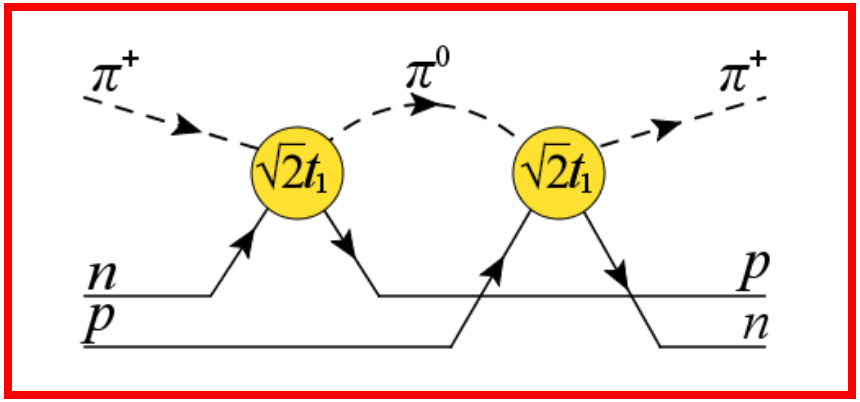
The second-order potential is respecting the Pauli principle

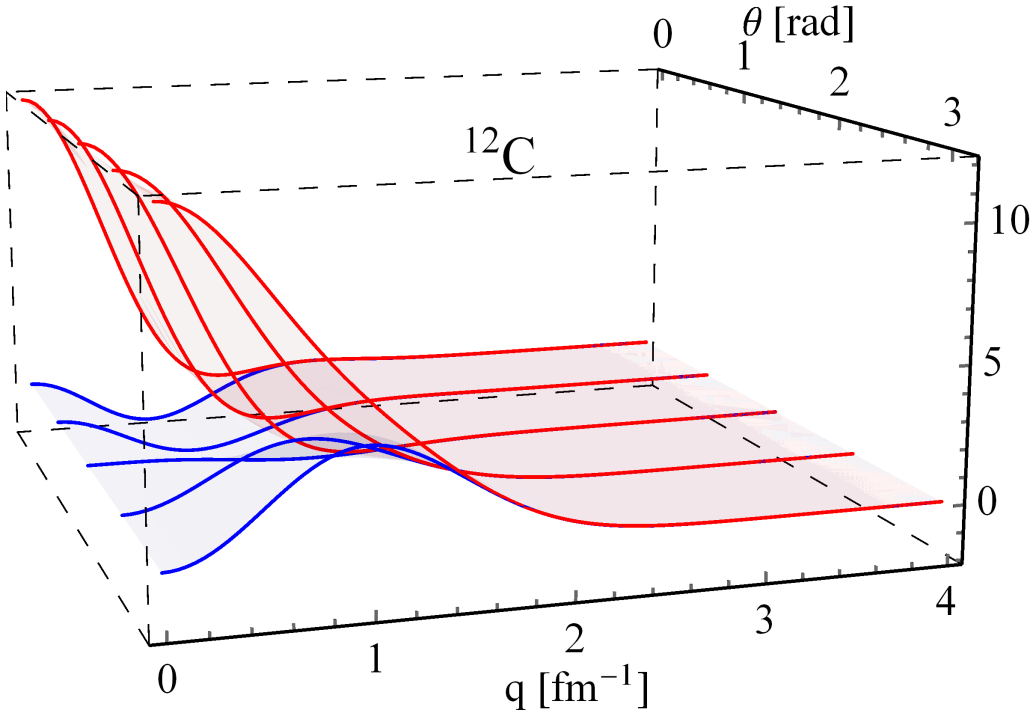
$$t = t_0 + t_1\, \hat{\boldsymbol{t}}_\pi \cdot \hat{\boldsymbol{\tau}}_N + (t_2 + t_3\, \hat{\boldsymbol{t}}_\pi \cdot \hat{\boldsymbol{\tau}}_N)\, \hat{\boldsymbol{\sigma}} \cdot \mathbf{n}$$

$$U^{(2)}(\boldsymbol{k}', \boldsymbol{k}) = -\int \frac{\mathrm{d}\boldsymbol{k}''}{(2\pi)^3} G_0(\boldsymbol{k}'') \Big[\boxed{t_0(\boldsymbol{k}', \boldsymbol{k}'') t_0(\boldsymbol{k}'', \boldsymbol{k}) C_0(\boldsymbol{k}' - \boldsymbol{k}'', \boldsymbol{k}'' - \boldsymbol{k})}$$

$$+ \boxed{2t_1(\boldsymbol{k}', \boldsymbol{k}'') t_1(\boldsymbol{k}'', \boldsymbol{k}) C_{\mathrm{ex}}(\boldsymbol{k}' - \boldsymbol{k}'', \boldsymbol{k}'' - \boldsymbol{k})} + (\text{spin-flip}) C_{\mathrm{ex}}(\ldots) \Big]$$





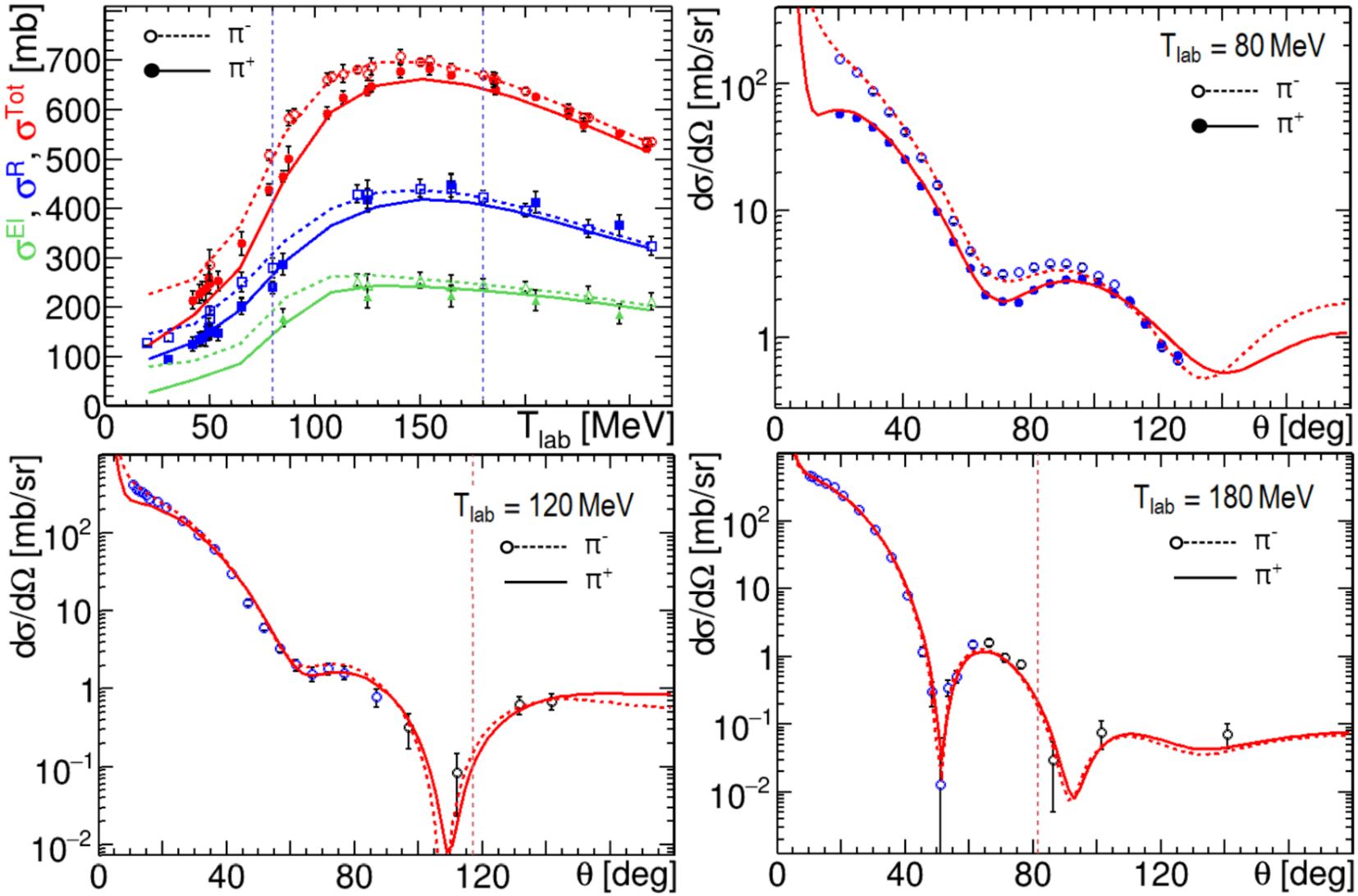


Harmonic oscillator shell model $C_{\mathrm{ex}}(\boldsymbol{q}_1, \boldsymbol{q}_2)$ and $C_0(\boldsymbol{q}_1, \boldsymbol{q}_2)$ for $|\boldsymbol{q}_{1,2}| = q$

Fit to $T_{\text{lab}} = 80 - 180$ MeV $\pi^{\pm}$-^{12}C scattering data

3 **energy-independent** parameters fitted to SIN, TRIUMF, LAMPF, RAL and CERN data

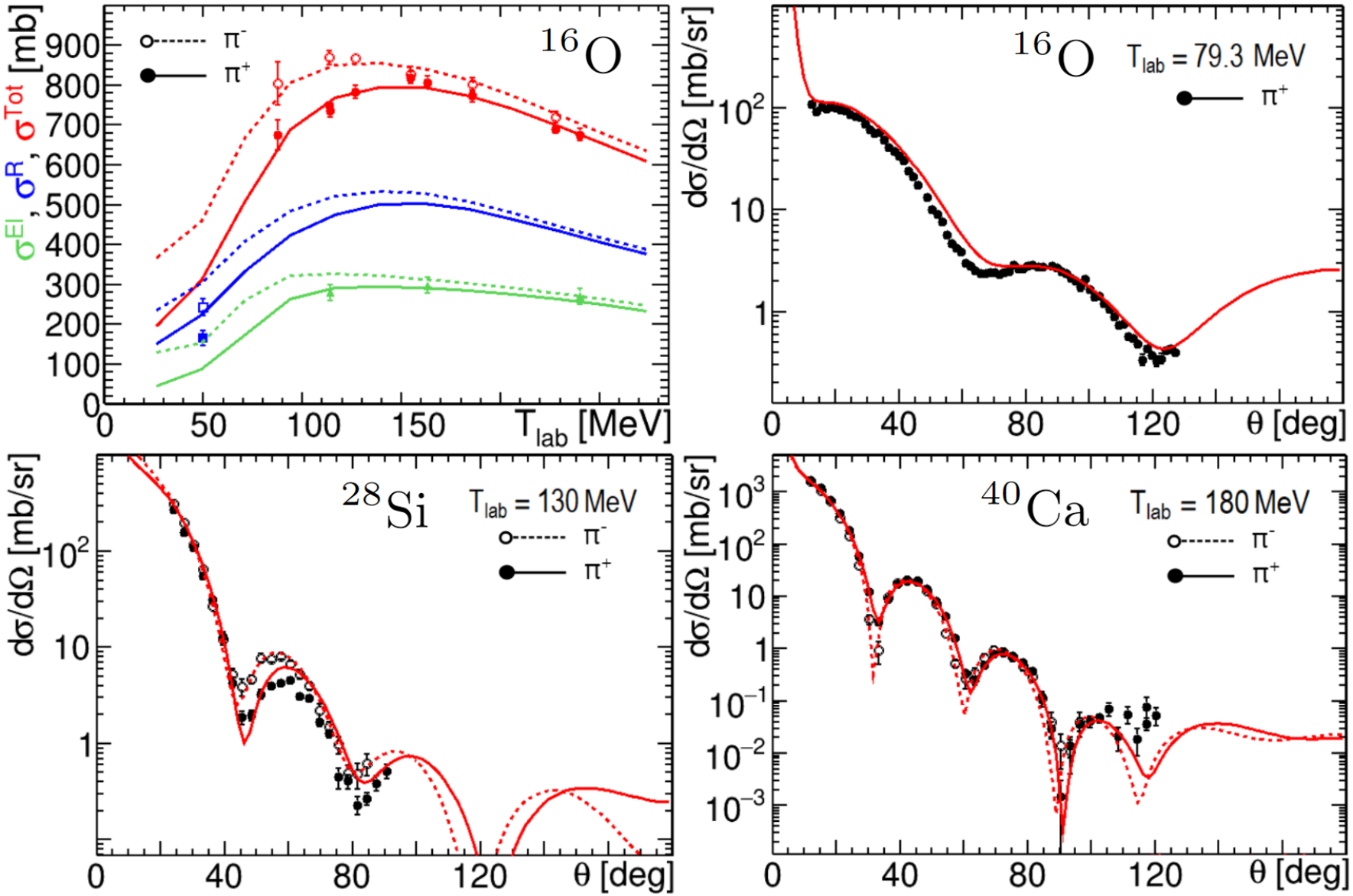
$\Sigma_{\Delta} = 12.9 \pm 1.3 - (33.2 \pm 0.8)i$ MeV $\qquad \chi^2/\text{ndf} = 1.5$



VT and M Vanderhaeghen, arXiv:2306.04913 [nucl-th]

Prediction for $\pi^{\pm}$-^{16}O, ^{28}Si and ^{40}Ca

Model parameters fixed to ^{12}C data fit



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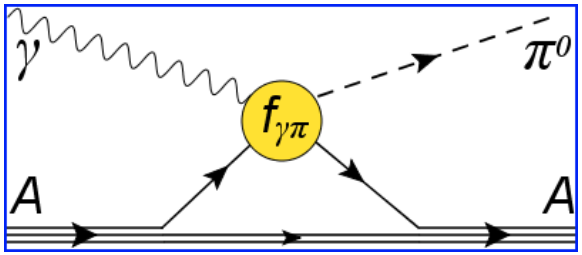
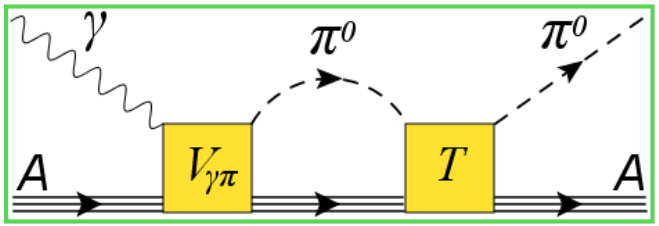
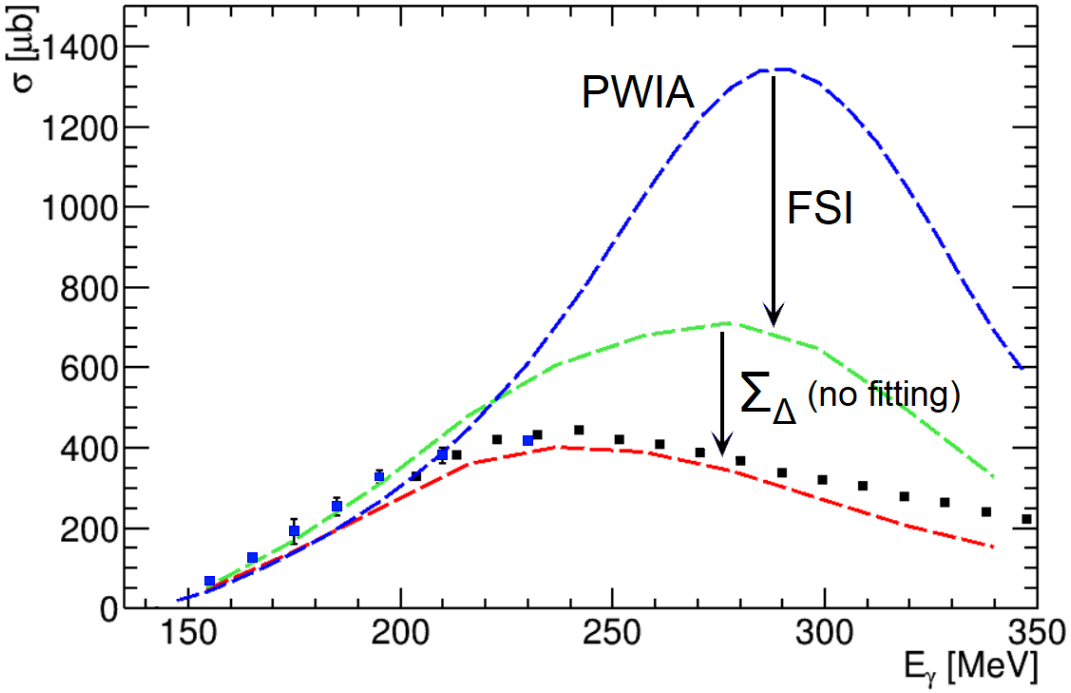
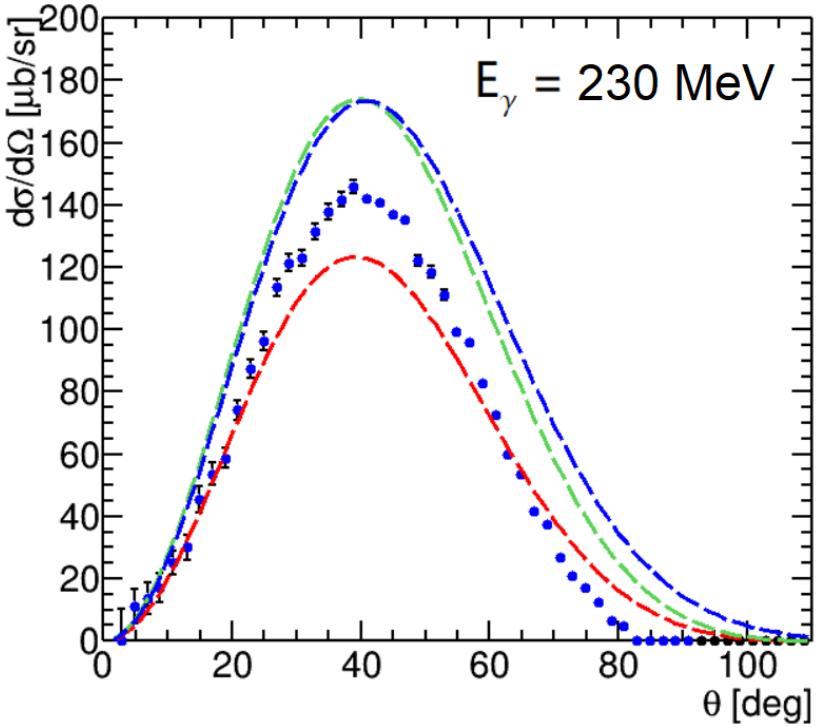
Prediction for coherent π^0 photoproduction on ^{12}C

$\hat{V}_{\gamma\pi} \propto$





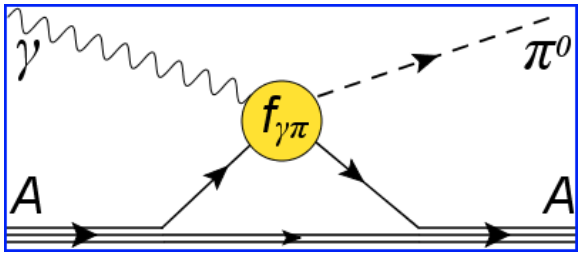


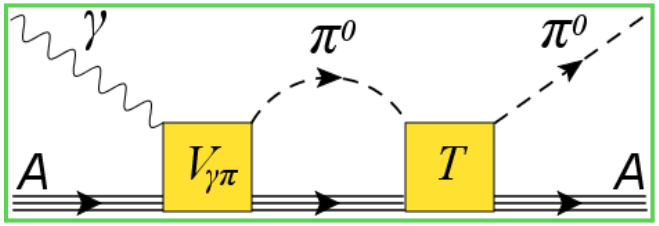


Data: B. Krusche *et al.*, Phys. Lett. B **526**, 287 (2002), C.M. Tarbert, PhD thesis (2007)

Prediction for coherent π^0 photoproduction on ^{12}C

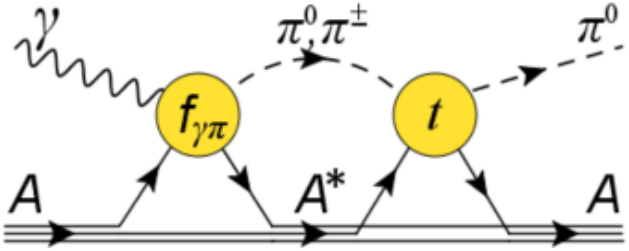
$\hat{V}^{(1)}_{\gamma\pi} \propto$

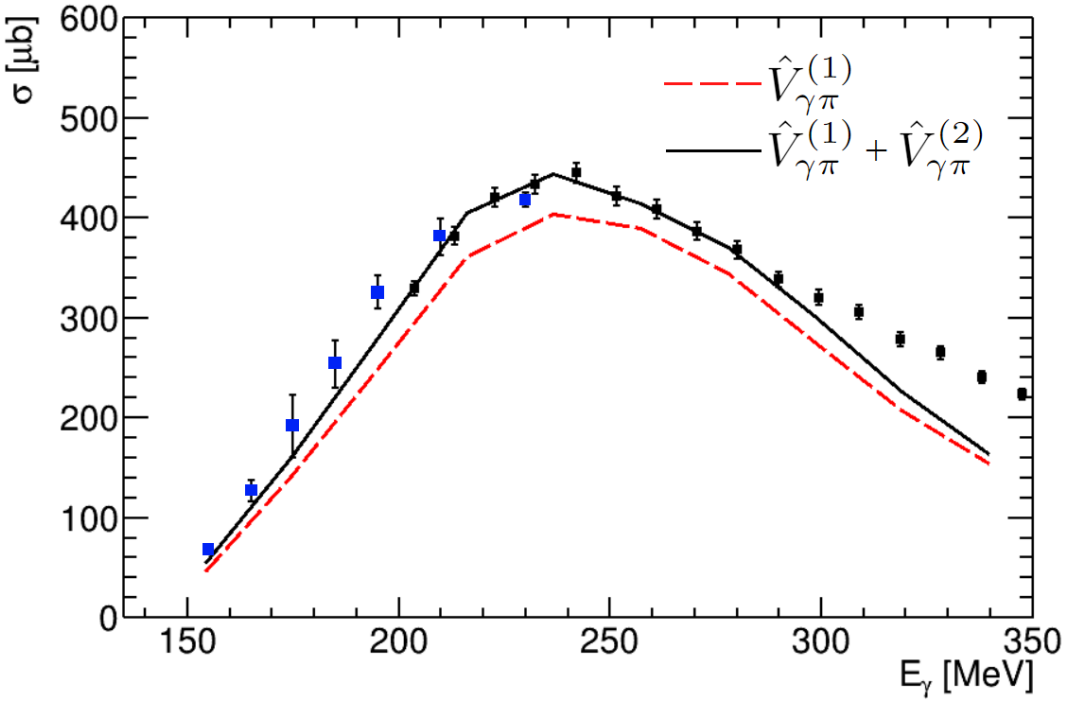


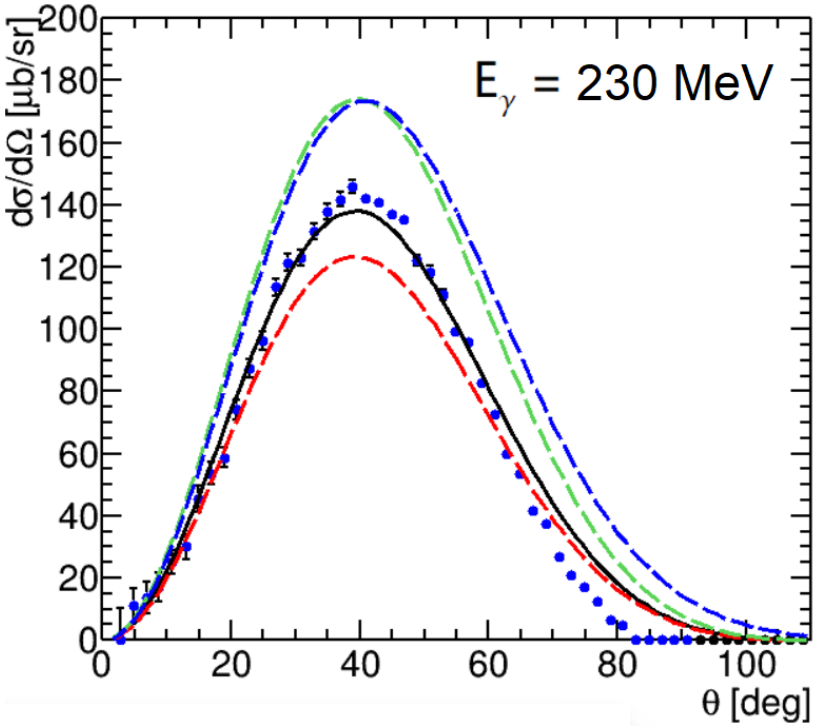


The 2nd-order part of photoproduction potential:

$\hat{V}^{(2)}_{\gamma\pi} \propto$







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Summary and Outlook

- The medium effects in $\pi^{\pm}$ scattering are described by introducing the effective Δ self-energy Σ_{Δ}
- Derived second-order potential provides adequate fits for $T_{\text{lab}} = 80 - 180$ MeV scattering
- Heavier nuclei are successfully described without fitting
- The model is successfully applied to π^0 photoproduction
- Intermediate charge exchange and nucleon spin flip cause a non-negligible shift in the cross section

- **Exploration**: sensitivities of the model
  theoretical error estimate
- **Extension**: application to heavy nuclei: ^{40,48}Ca, ^{116}Sn, ^{208}Pb

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Thank you for your attention!

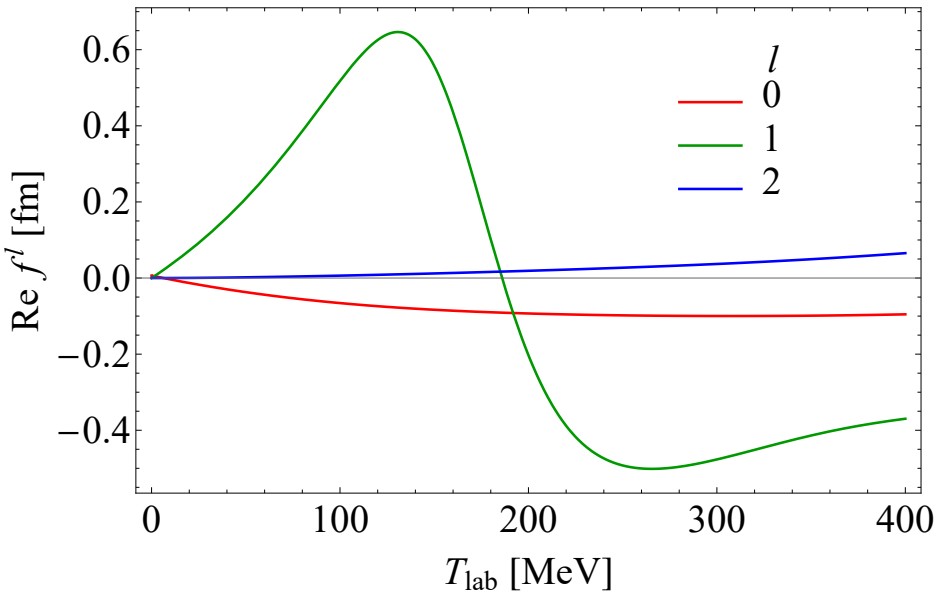
Spin-isospin-$\frac{3}{2}$ p-wave is dominant

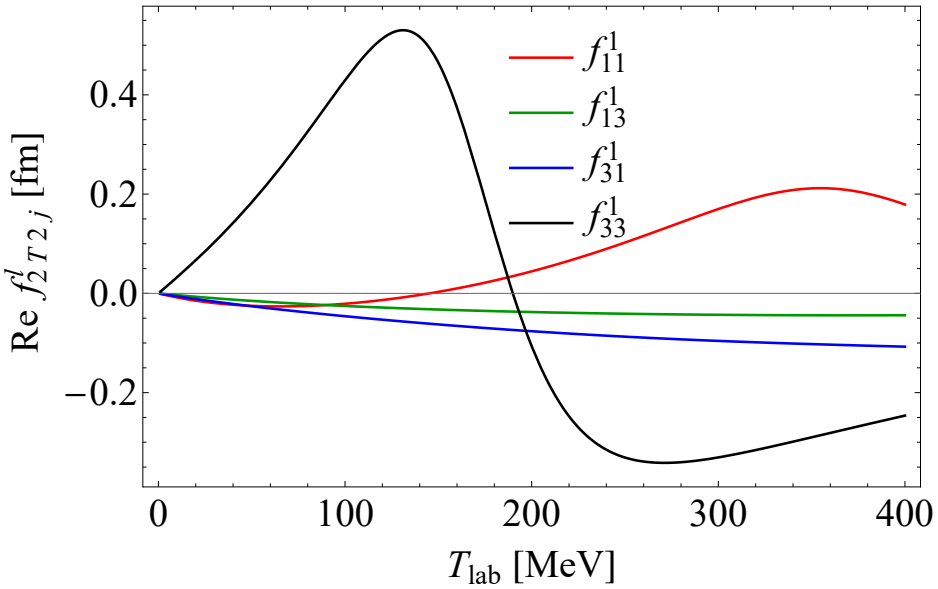
$$t = t_0 + t_1\, \hat{\boldsymbol{t}}_\pi \cdot \hat{\boldsymbol{\tau}}_N + (t_2 + t_3\, \hat{\boldsymbol{t}}_\pi \cdot \hat{\boldsymbol{\tau}}_N)\, \hat{\boldsymbol{\sigma}} \cdot \mathbf{n} \tag{1}$$

$$t_0 \propto \sum_l \left[l \left(f^l_{1\,2l-1} + 2 f^l_{3\,2l-1} \right) + (l+1) \left(f^l_{1\,2l+1} + 2 f^l_{3\,2l+1} \right) \right] P_l(\cos\theta) \tag{2a}$$

$$t_1 \propto \sum_l \left[l \left(f^l_{3\,2l-1} - f^l_{1\,2l-1} \right) + (l+1) \left(f^l_{3\,2l+1} - f^l_{1\,2l+1} \right) \right] P_l(\cos\theta) \tag{2b}$$

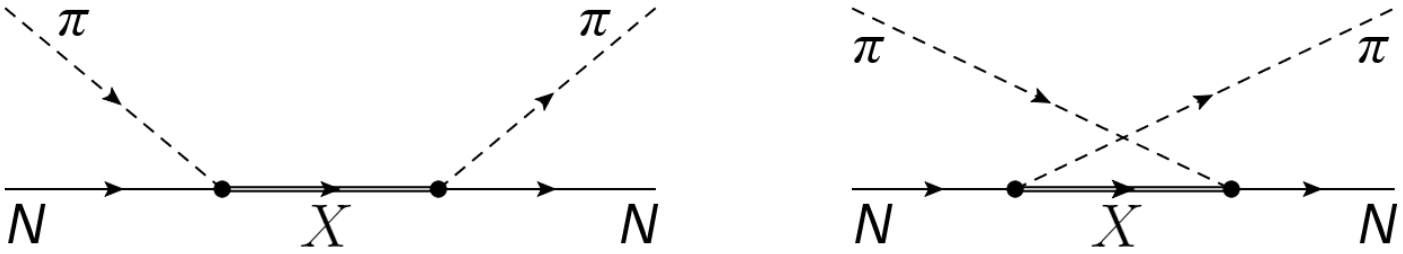
$f^l_{2T\,2j}$ from **SAID** R. L. Workman *et al.*, Phys. Rev. C **86**, 035202 (2012)





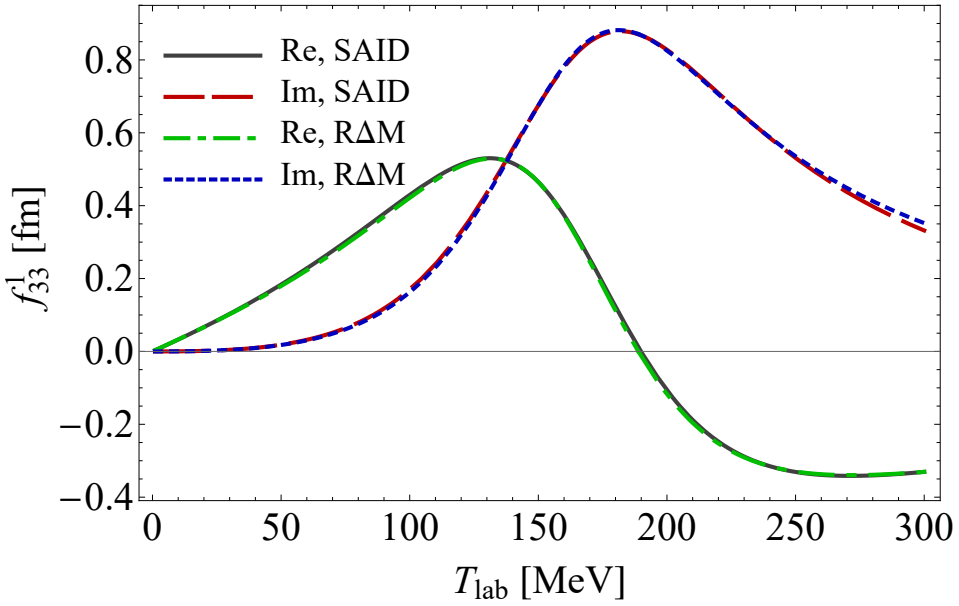
$$f \approx b_0 + b_1\, \hat{\boldsymbol{t}}_\pi \cdot \hat{\boldsymbol{\tau}}_N + (c_0 + c_1\, \hat{\boldsymbol{t}}_\pi \cdot \hat{\boldsymbol{\tau}}_N)\, \boldsymbol{k}' \cdot \boldsymbol{k} + i(s_0 + s_1\, \hat{\boldsymbol{t}}_\pi \cdot \hat{\boldsymbol{\tau}}_N)\, \hat{\boldsymbol{\sigma}} \cdot [\boldsymbol{k}' \times \boldsymbol{k}] \tag{3}$$

Relativistic Δ-isobar model



X refers to $N(939)$, $\Delta(1232)$ and $N^*(1440)$ intermediate states

E. Oset, H. Toki, W. Weise, Phys. Reports **83**, 281 (1982)

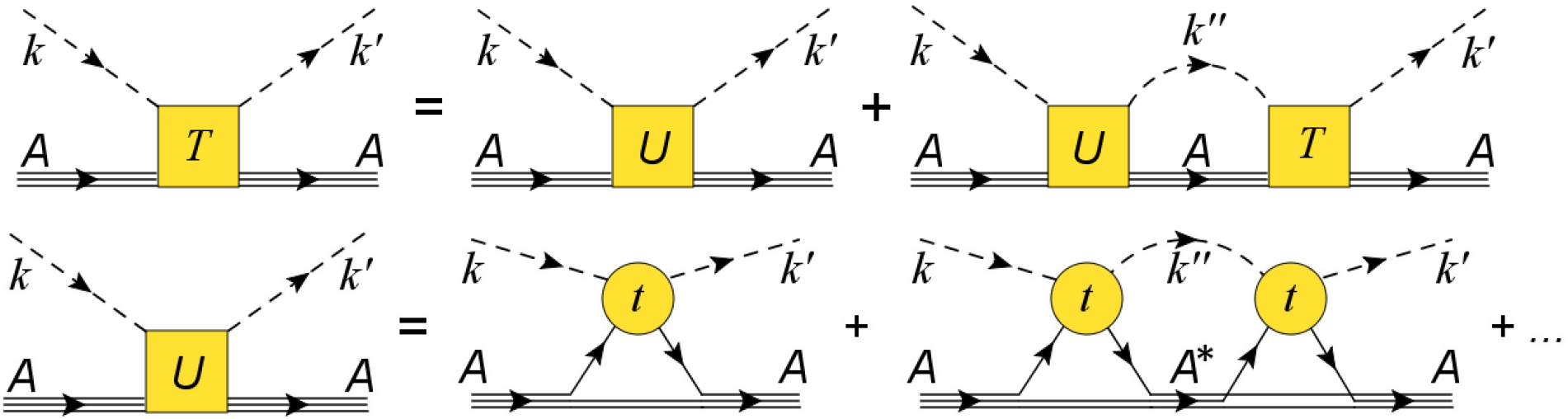


$$f^1_{33} = \frac{K^1_{33}}{1 - ikK^1_{33}}$$

$$K^1_{33} = \frac{1}{3}\frac{k^2}{4\pi m_\pi^2}\frac{m_N}{\sqrt{s}}\left[\frac{8f_N^2 m_N}{m_N^2 - \bar{u}} + \frac{8f_{N^*}^2 m_{N^*}}{m_{N^*}^2 - \bar{u}} + \left(\frac{2f_\Delta^2 m_\Delta}{m_\Delta^2 - s} + \frac{1}{9}\frac{2f_\Delta^2 m_\Delta}{m_\Delta^2 - \bar{u}}\right)\right]$$

P_{33} wave : **in nuclear medium** $m_\Delta \longrightarrow m_\Delta + \Sigma_\Delta$.
S_{11}, S_{31}, P_{11}, P_{31}, P_{13} partial waves from **SAID**

Second-order pion-nucleus optical potential



$$U^{(1)}(\boldsymbol{k}',\boldsymbol{k}) = t_0(\boldsymbol{k}',\boldsymbol{k})\rho(\boldsymbol{q})$$

$$U^{(2)}(\boldsymbol{k}',\boldsymbol{k}) = -\int \frac{\mathrm{d}\boldsymbol{k}''}{(2\pi)^3} G_0(\boldsymbol{k}'') \left[t_0(\boldsymbol{k}',\boldsymbol{k}'')t_0(\boldsymbol{k}'',\boldsymbol{k})C_0(\boldsymbol{k}'-\boldsymbol{k}'',\boldsymbol{k}''-\boldsymbol{k}) \right.$$
$$\left. +2t_1(\boldsymbol{k}',\boldsymbol{k}'')t_1(\boldsymbol{k}'',\boldsymbol{k})C_{\mathsf{ex}}(\boldsymbol{k}'-\boldsymbol{k}'',\boldsymbol{k}''-\boldsymbol{k}) + \ldots \right]$$

Correlation functions in the momentum space:

$$C_{0,\mathsf{ex}}(\boldsymbol{q}_1,\boldsymbol{q}_2) = \int \mathrm{d}\boldsymbol{r}_1\, \mathrm{d}\boldsymbol{r}_2\, e^{-i(\boldsymbol{q}_1\cdot\boldsymbol{r}_1+\boldsymbol{q}_2\cdot\boldsymbol{r}_2)} C_{0,\mathsf{ex}}(\boldsymbol{r}_1,\boldsymbol{r}_2)$$

$$C_{\mathsf{ex}}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \rho_2(\boldsymbol{r}_1,\boldsymbol{r}_2) - \rho(\boldsymbol{r}_1)\rho(\boldsymbol{r}_2) \qquad C_0(\boldsymbol{r}_1,\boldsymbol{r}_2) = C_{\mathsf{ex}}(\boldsymbol{r}_1,\boldsymbol{r}_2) - \frac{1}{A}\rho(\boldsymbol{r}_1)\rho(\boldsymbol{r}_2)$$

Explicit scattering potential

$$f \approx b_0 + b_1\, \hat{\boldsymbol{t}}_\pi \cdot \hat{\boldsymbol{\tau}}_N + (c_0 + c_1\, \hat{\boldsymbol{t}}_\pi \cdot \hat{\boldsymbol{\tau}}_N)\, \boldsymbol{k}' \cdot \boldsymbol{k} + i(s_0 + s_1\, \hat{\boldsymbol{t}}_\pi \cdot \hat{\boldsymbol{\tau}}_N)\, \hat{\boldsymbol{\sigma}} \cdot [\boldsymbol{k}' \times \boldsymbol{k}] \tag{1}$$

$$b_1^{\mathsf{bound}}(T_{\mathsf{lab}}) = b_1^{\mathsf{free}}(T_{\mathsf{lab}}) + b_1^{\mathsf{free}}(0)\, \frac{\sigma \rho_e / m_\pi^2 f_\pi^2}{1 - \sigma \rho_e / m_\pi^2 f_\pi^2} \tag{2}$$

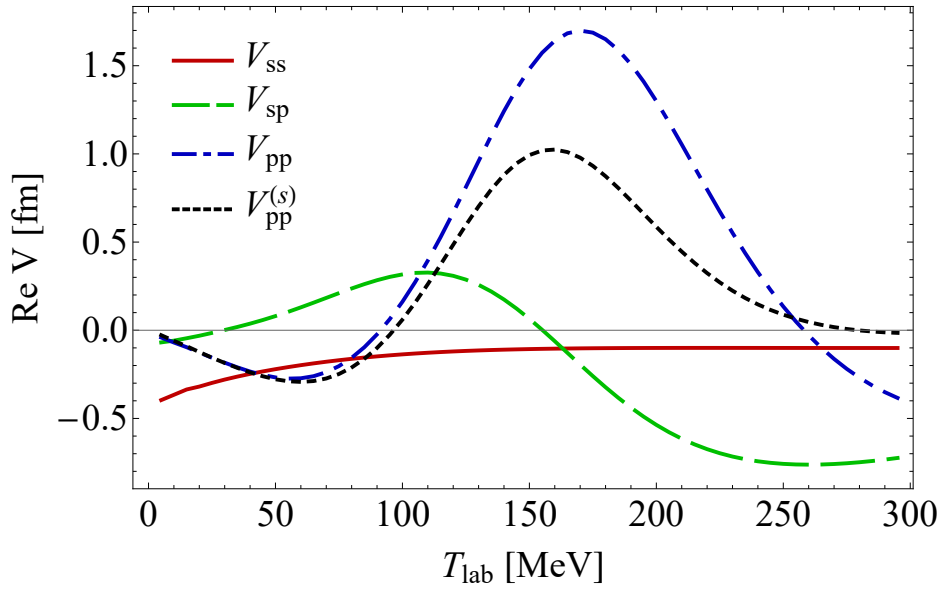
$$\mathrm{Im}\, \Delta b_0(T_{\mathsf{lab}}) = \mathrm{Im}\, \Delta b_0(0) + \alpha_{b_0} k_{0,\mathsf{2cm}}(T_{\mathsf{lab}}) \tag{3}$$

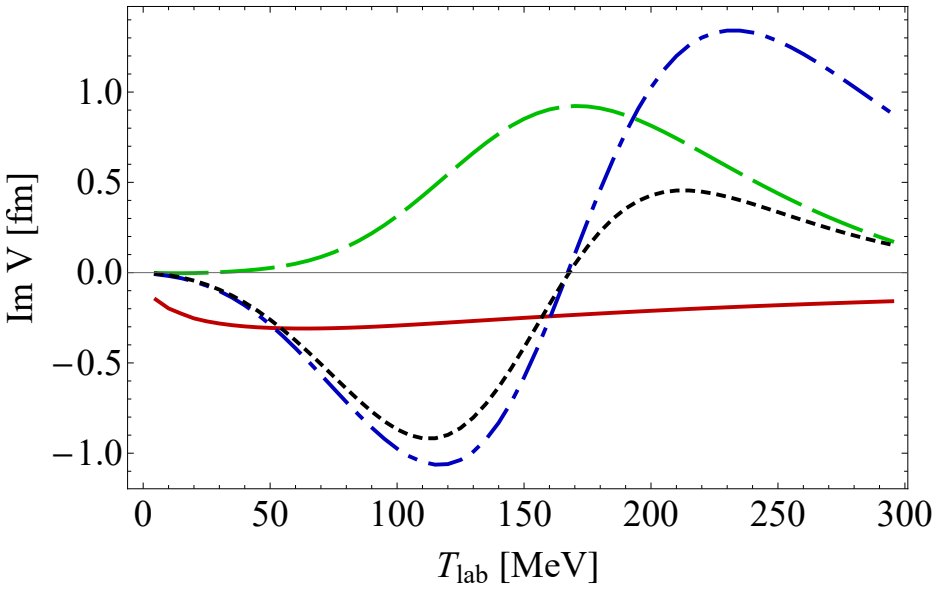
$$\hat{U} \approx \hat{U}^{(1)} + \hat{U}^{(2)} \tag{4}$$

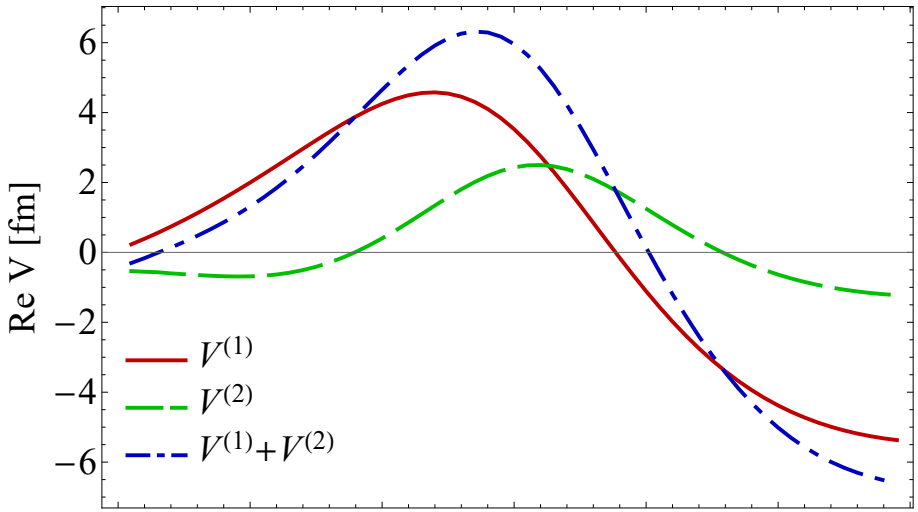
$$U^{(1)}(\boldsymbol{k}', \boldsymbol{k}) = \gamma\, \mathrm{Tr}\left[\rho(\boldsymbol{q}) t_{\mathsf{2cm}}(\boldsymbol{k}'_{\mathsf{2cm}}(\boldsymbol{k}', \boldsymbol{p}'_{\mathsf{eff}}), \boldsymbol{k}_{\mathsf{2cm}}(\boldsymbol{k}, \boldsymbol{p}_{\mathsf{eff}})\right] \tag{5}$$

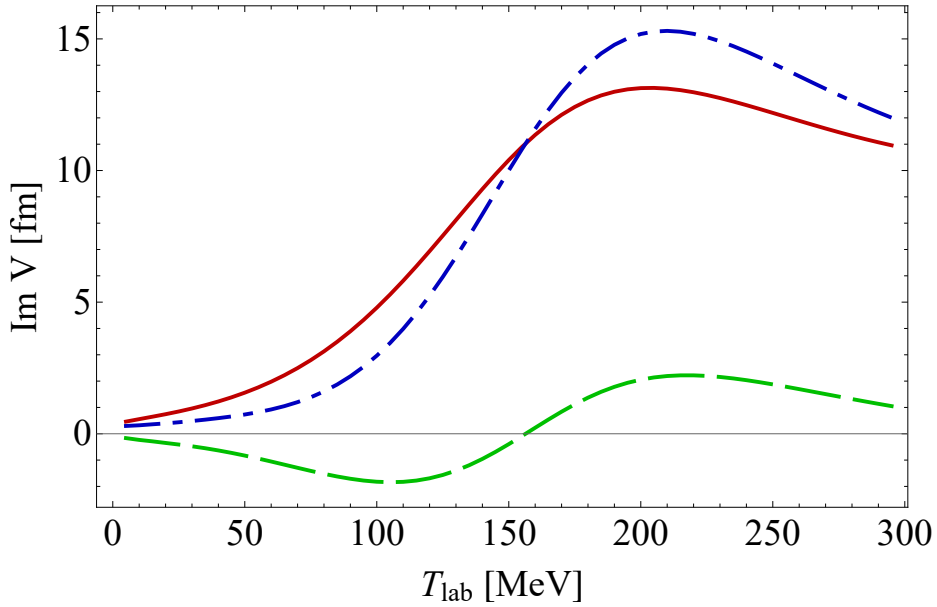
$$\begin{aligned} U^{(2)}(\boldsymbol{k}', \boldsymbol{k}) = -\int \frac{\mathrm{d}\boldsymbol{k}''}{(2\pi)^3} G_0(\boldsymbol{k}'') \Big[& t^{(0)}(\boldsymbol{k}', \boldsymbol{k}'') t^{(0)}(\boldsymbol{k}'', \boldsymbol{k}) C_0(\boldsymbol{k}' - \boldsymbol{k}'', \boldsymbol{k}'' - \boldsymbol{k}) \\ & + \Big(2t^{(1)}(\boldsymbol{k}', \boldsymbol{k}'') t^{(1)}(\boldsymbol{k}'', \boldsymbol{k}) \\ & + \Big(t^{(2)}(\boldsymbol{k}', \boldsymbol{k}'') t^{(2)}(\boldsymbol{k}'', \boldsymbol{k}) + 2t^{(3)}(\boldsymbol{k}', \boldsymbol{k}'') t^{(3)}(\boldsymbol{k}'', \boldsymbol{k}) \Big)\, \boldsymbol{n}_1 \cdot \boldsymbol{n}_2 \Big) C_{\mathsf{ex}}(\boldsymbol{k}' - \boldsymbol{k}'', \boldsymbol{k}'' - \boldsymbol{k}) \Big] \end{aligned} \tag{6}$$

Components of the scattering potential

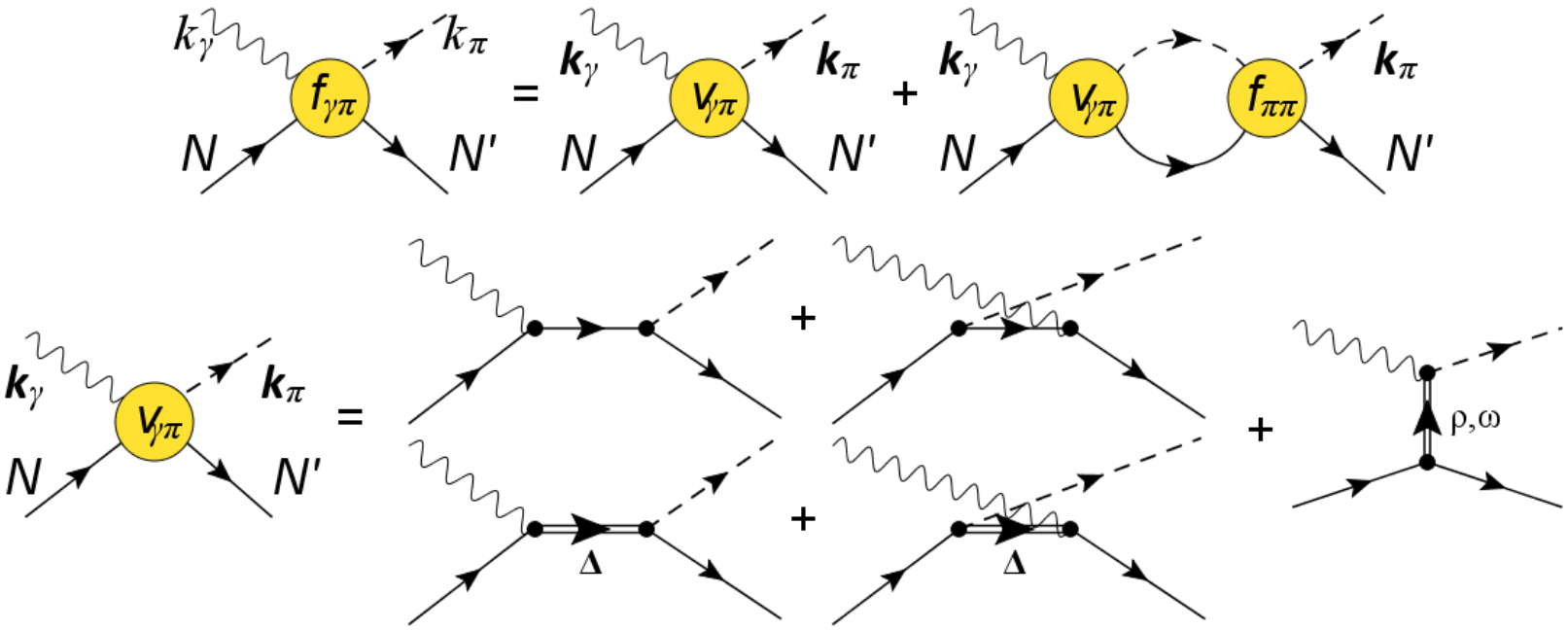








Elementary photoproduction amplitude



$$f^\gamma = i\,\tilde{\boldsymbol{\sigma}}\cdot\boldsymbol{\epsilon}_\lambda F_1 + (\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}})\boldsymbol{\sigma}\cdot(\hat{\boldsymbol{q}}\times\boldsymbol{\epsilon}_\lambda)F_2 + i(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{q}})(\tilde{\boldsymbol{k}}\cdot\boldsymbol{\epsilon}_\lambda)F_3 + i(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}})(\tilde{\boldsymbol{k}}\cdot\boldsymbol{\epsilon}_\lambda)F_4,$$

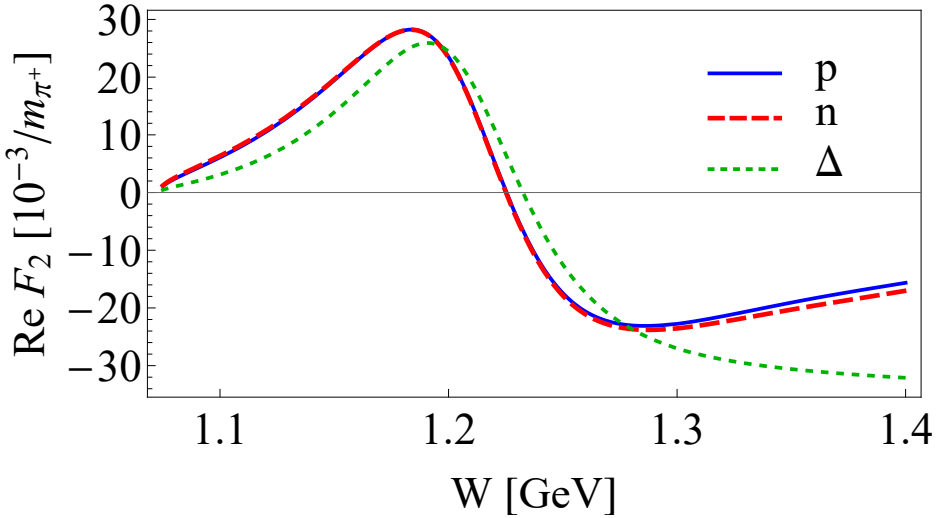
$$F_1 = \sum_{l\geq 0}\left[(lM_{l+} + E_{l+})\,P'_{l+1} + ((l+1)M_{l-} + E_{l-})\,P'_{l-1}\right] \tag{1a}$$

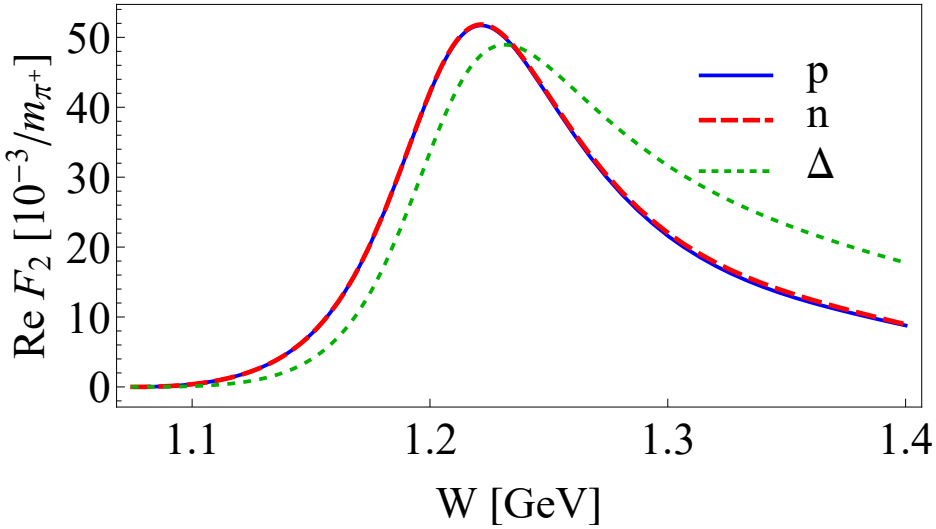
$$F_2 = \sum_{l\geq 1}\left[(l+1)M_{l+} + lM_{l-}\right]P'_l \tag{1b}$$

$$F_3 = \sum_{l\geq 1}\left[(E_{l+} - M_{l+})\,P''_{l+1} + (E_{l-} + M_{l-})\,P''_{l-1}\right] \tag{1c}$$

$$F_4 = \sum_{l\geq 2}\left[M_{l+} - E_{l+} - M_{l-} - E_{l-}\right]P''_l \tag{1d}$$

Elementary photoproduction amplitude





$$F_2 = F_2^{(B)} + 2M_{1+}^{(\Delta)}$$

$$M_{1+}^{(\Delta)} = F_{\gamma N\Delta} \frac{\Gamma_\Delta m_\Delta e^{i\phi}}{m_\Delta^2 - W^2 - i m_\Delta \Gamma_\Delta} F_{\pi N\Delta}$$