

## $\Lambda$ hyperon in covariant confined quark model

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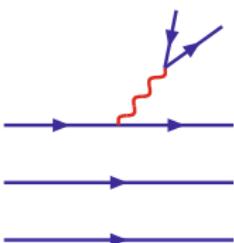
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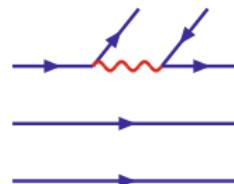
## Nonleptonic two-body weak decays of baryons

- Ground states of baryons with  $J^P = \frac{1}{2}^+$  can decay only weakly via the internal  $W$ -exchange.
- The nonleptonic two-body decays of baryons have five different color-flavor quark topologies.
- They can be divided into two groups:
  - reducible tree-diagrams
  - irreducible  $W$ -exchange diagrams
- The tree-diagrams are factorized into the lepton decay of the emitted meson and the baryon-baryon transition matrix elements of the weak currents.
- $W$ -exchange diagrams are more difficult to evaluate from first principles.
- First attempts to estimate the  $W$ -exchange contributions have been made by using a pole model approach.
- It was shown that  $W$ -exchange contributions are sizeable and cannot be neglected.

## Topology of nonleptonic weak decays

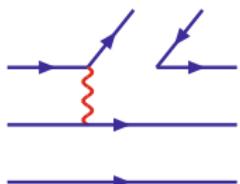


Ia

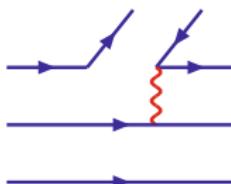


Ib

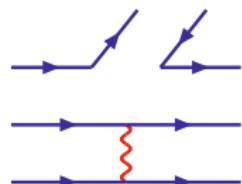
## Tree diagrams



IIa



IIb



III

## W-exchange diagrams

## Nonleptonic double charmed baryon decays

	$I_a$	$I_b$	$II_a$	$II_b$	$III$
$\Xi_{cc}^{++} \rightarrow \Sigma_c^{(*)++} + \bar{K}^{(*)0}$	-	✓	-	-	-
$\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime,*)+} + \pi^+(\rho^+)$	✓	-	-	✓	-
$\Xi_{cc}^{++} \rightarrow \Sigma^{(*)+} + D^{(*)+}$	-	-	-	✓	-
$\Xi_{cc}^+ \rightarrow \Xi_c^{(\prime,*)\bar{0}} + \pi^+(\rho^+)$	✓	-	✓	-	-
$\Xi_{cc}^+ \rightarrow \Lambda_c^+(\Sigma_c^{(*)+}) + \bar{K}^{(*)0}$	-	✓	✓	-	-
$\Xi_{cc}^+ \rightarrow \Sigma_c^{(*)++} + K^{(*)-}$	-	-	✓	-	-
$\Xi_{cc}^+ \rightarrow \Xi_c^{(\prime,*)+} + \pi^0(\rho^0)$	-	-	✓	✓	-
$\Xi_{cc}^+ \rightarrow \Xi_c^{(\prime,*)+} + \eta(\eta')$	-	-	✓	✓	-
$\Xi_{cc}^+ \rightarrow \Omega_c^{(*)0} + K^{(*)+}$	-	-	✓	-	-
$\Xi_{cc}^+ \rightarrow \Lambda^0(\Sigma^{(*)0}) + D^{(*)+}$	-	-	-	✓	✓
$\Xi_{cc}^+ \rightarrow \Sigma^{(*)+} + D^{(*)0}$	-	-	-	-	✓
$\Xi_{cc}^+ \rightarrow \Xi^{(*)0} + D_s^{(*)+}$	-	-	-	-	✓
$\Omega_{cc}^+ \rightarrow \Xi_c^{(\prime,*)+} + \bar{K}^{(*)0}$	-	✓	-	✓	-
$\Omega_{cc}^+ \rightarrow \Xi^{(*)0} + D^{(*)+}$	-	-	-	✓	-
$\Omega_{cc}^+ \rightarrow \Omega_c^{(*)0} + \pi^+(\rho^+)$	✓	-	-	-	-

## Nonleptonic double charmed baryon decays

We will consider the decays that belong to the same topology class:

$$\Xi_{cc}^{++} \rightarrow \Xi_c^+ (\Xi_c'^+) + \pi^+ (\rho^+) \quad \text{T-Ia and W-IIb}$$

$$\Omega_{cc}^+ \rightarrow \Xi_c^+ (\Xi_c'^+) + \bar{K}^0 (K^{*0}) \quad \text{T-Ib and W-IIb}$$

Quantum numbers and interpolating currents:

Baryon	$J^P$	Interpolating current	Mass (MeV)
$\Xi_{cc}^{++}$	$\frac{1}{2}^+$	$\varepsilon_{abc} \gamma^\mu \gamma_5 u^a (c^b C \gamma_\mu c^c)$	3620.6
$\Omega_{cc}^+$	$\frac{1}{2}^+$	$\varepsilon_{abc} \gamma^\mu \gamma_5 s^a (c^b C \gamma_\mu c^c)$	3710.0
$\Xi_c'^+$	$\frac{1}{2}^+$	$\varepsilon_{abc} \gamma^\mu \gamma_5 c^a (u^b C \gamma_\mu s^c)$	2577.4
$\Xi_c^+$	$\frac{1}{2}^+$	$\varepsilon_{abc} c^a (u^b C \gamma_5 s^c)$	2467.9

## Körner-Pati-Woo (KPW) theorem

J.G. Körner, Nucl. Phys. B25, 282 (1971); J.C. Pati and C.H. Woo, Phys. Rev. D3, 2920 (1971)

The  $W$ -exchange contributions to the above decays fall into two classes:

- The decays with a  $\Xi_c^{'+}$ -baryon containing a symmetric  $\{us\}$  diquark described by the interpolating current  $\varepsilon_{abc} (u^b C \gamma_\mu s^c)$ .
- The  $W$ -exchange contribution is strongly suppressed due to the KPW theorem which states that the contraction of the flavor antisymmetric current-current operator with a flavor symmetric final state configuration is zero in the  $SU(3)$  limit.
- The decays with a  $\Xi_c^+$ -baryon containing an antisymmetric  $[us]$  diquark described by the interpolating current  $\varepsilon_{abc} (u^b C \gamma_5 s^c)$ .
- In this case the  $W$ -exchange contribution is not a priori suppressed.

## Effective Hamiltonian and nonlocal quark currents

The effective Hamiltonian describing the  $\bar{q}_1 q_2 \rightarrow \bar{q}'_1 q'_2$  transition is given by

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{q_1 q_2} V_{q'_1 q'_2}^\dagger (C_1 Q_1 + C_2 Q_2)$$

$$Q_1 = (\bar{q}_{1\ a} O_L q_{2\ b})(\bar{q}'_{1\ b} O_L q'_{2\ a}) \quad Q_2 = (\bar{q}_{1\ a} O_L q_{2\ a})(\bar{q}'_{1\ b} O_L q'_{2\ b})$$

The notation is  $O_{L/R}^\mu = \gamma^\mu (1 \mp \gamma_5)$ .

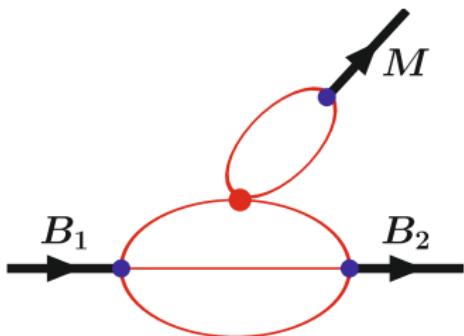
The nonlocal version of the interpolating currents:

$$J_B(x) = \int dx_1 \int dx_2 \int dx_3 F_B(x; x_1, x_2, x_3) \epsilon^{a_1 a_2 a_3} \Gamma_1 q_1^{a_1}(x_1) (q_2^{a_2}(x_2) C \Gamma_2 q_3^{a_3}(x_3))$$

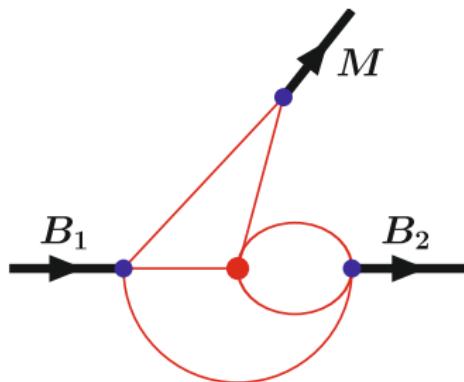
$$F_B = \delta^{(4)} \left( x - \sum_{i=1}^3 w_i x_i \right) \Phi_B \left( \sum_{i < j} (x_i - x_j)^2 \right)$$

where  $w_i = m_i / (\sum_{j=1}^3 m_j)$  and  $m_i$  is the quark mass.  $\Gamma_1, \Gamma_2$  are the Dirac strings of the initial and final baryons.

## Matrix elements



tree diagrams Ia, Ib



W-exchange diagram IIb

$$\langle B_2 | \mathcal{H}_{\text{eff}} | B_1 \rangle = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^\dagger \bar{u}(p_2) \left( 12 C_T M_T + 12 (C_1 - C_2) M_W \right) u(p_1).$$

$$C_T = \begin{cases} C_T = +(C_2 + \xi C_1) & \text{charged meson} \\ C_T = -(C_1 + \xi C_2) & \text{neutral meson} \end{cases}$$

The factor of  $\xi = 1/N_c$  is set to zero in the numerical calculations.

## Tree-diagram contribution: factorization

The contribution from the tree diagram factorizes into two pieces:

$$M_T = M_T^{(1)} \cdot M_T^{(2)}$$

$$M_T^{(1)} = N_c g_M \int \frac{d^4 k}{(2\pi)^4 i} \tilde{\Phi}_M(-k^2) \text{tr} [O_L S_d(k - w_d q) \Gamma_M S_{s(u)}(k + w_{s(u)} q)]$$

$$\begin{aligned} M_T^{(2)} &= g_{B_1} g_{B_2} \int \frac{d^4 k_1}{(2\pi)^4 i} \int \frac{d^4 k_2}{(2\pi)^4 i} \tilde{\Phi}_{B_1}(-\vec{\Omega}_1^2) \tilde{\Phi}_{B_2}(-\vec{\Omega}_2^2) \\ &\times \Gamma_1 S_c(k_2) \gamma^\mu S_c(k_1 - p_1) O_R S_{u(s)}(k_1 - p_2) \tilde{\Gamma}_2 S_{s(u)}(k_1 - k_2) \gamma_\mu \gamma_5 \end{aligned}$$

The  $M_T^{(1)}$  is related to the leptonic decay constants:

$$M_T^{(1)} = \begin{cases} -f_P \cdot q & \text{pseudoscalar meson} \\ +f_V m_V \cdot \epsilon_V & \text{vector meson} \end{cases}$$

## W-exchange diagram contribution: no factorization

$$\begin{aligned} M_W &= g_{B_1} g_{B_2} g_M \int \frac{d^4 k_1}{(2\pi)^4 i} \int \frac{d^4 k_2}{(2\pi)^4 i} \int \frac{d^4 k_3}{(2\pi)^4 i} \tilde{\Phi}_{B_1}(-\vec{\Omega}_1^2) \tilde{\Phi}_{B_2}(-\vec{\Omega}_2^2) \tilde{\Phi}_M(-P^2) \\ &\times 2\Gamma_1 S_c(k_1) \gamma^\mu S_c(k_2) (1 - \gamma_5) S_d(k_2 - k_1 + p_2) \Gamma_M S_{s(u)}(k_2 - k_1 + p_1) \gamma_\mu \gamma_5 \\ &\times \text{tr} [S_{u(s)}(k_3) \tilde{\Gamma}_2 S_{s(u)}(k_3 - k_1 + p_2) (1 + \gamma_5)] \end{aligned}$$

Here  $\Gamma_1 \otimes \tilde{\Gamma}_2 = I \otimes \gamma_5$  for  $B_2 = \Xi_c^+$  and  $-\gamma_\nu \gamma_5 \otimes \gamma^\nu$  for  $B_2 = \Xi'_c +$ .

To verify the KPW theorem in the case of  $B_2 = \Xi'_c +$  we use the identity

$$\text{tr} [S_u(k_3) \gamma_\nu S_s(k_3 - k_1 + p_2)] = -\text{tr} [S_s(-k_3 + k_1 - p_2) \gamma_\nu S_u(-k_3)]$$

Then by shifting  $k_3 \rightarrow -k_3 + k_1 - p_2$  one gets the same expression with opposite sign and  $u \leftrightarrow s$  interchange. Thus, if  $m_u = m_s$  then  $M_W \equiv 0$ .

It directly confirms the KPW-theorem.

## Evaluation of the diagrams

- Use the Schwinger representation of the propagator:

$$\frac{m+k}{m^2 - k^2} = (m+k) \int_0^\infty d\alpha \exp[-\alpha(m^2 - k^2)]$$

- Choose a simple Gaussian form for the vertex function

$$\Phi(-K^2) = \exp(K^2/\Lambda^2)$$

where the parameter  $\Lambda$  characterizes the hadron size.

- We imply that the loop integration  $k$  proceed over Euclidean space:

$$k^0 \rightarrow e^{i\frac{\pi}{2}} k_4 = ik_4, \quad k^2 = (k^0)^2 - \vec{k}^2 \rightarrow -k_E^2 \leq 0.$$

- We also put all external momenta  $p$  to Euclidean space:

$$p^0 \rightarrow e^{i\frac{\pi}{2}} p_4 = ip_4, \quad p^2 = (p^0)^2 - \vec{p}^2 \rightarrow -p_E^2 \leq 0$$

so that the quadratic momentum form in the exponent becomes negative-definite and the loop integrals are absolutely convergent.

## Evaluation of the diagrams

- Convert the loop momenta in the numerator into derivatives over external momenta:

$$k_i^\mu e^{2kr} = \frac{1}{2} \frac{\partial}{\partial r_{i\mu}} e^{2kr},$$

- Move the derivatives outside of the loop integrals.
- Calculate the scalar loop integral:

$$\prod_{i=1}^n \int \frac{d^4 k_i}{i\pi^2} e^{kA_k + 2kr} = \prod_{i=1}^n \int \frac{d^4 k_i^E}{\pi^2} e^{-k_E A_k - 2k_E r_E} = \frac{1}{|A|^2} e^{-rA^{-1}r}$$

where a symmetric  $n \times n$  real matrix  $A$  is positive-definite.

- Use the identity

$$P \left( \frac{1}{2} \frac{\partial}{\partial r} \right) e^{-rA^{-1}r} = e^{-rA^{-1}r} P \left( \frac{1}{2} \frac{\partial}{\partial r} - A^{-1}r \right)$$

to move the exponent to the left.

## Evaluation of the diagrams

- Employ the commutator

$$[\frac{\partial}{\partial r_i{}_\mu}, r_j{}_\nu] = \delta_{ij} g_{\mu\nu}$$

to make differentiation in

$$P \left( \frac{1}{2} \frac{\partial}{\partial r} - A^{-1} r \right)$$

for any polynomial  $P$ . The necessary commutations of the differential operators are done by a FORM program.

- One obtains

$$\Pi = \int_0^\infty d^n \alpha F(\alpha_1, \dots, \alpha_n),$$

where  $F$  stands for the whole structure of a given diagram.

## Evaluation of the diagrams

The set of Schwinger parameters  $\alpha_i$  can be turned into a simplex by introducing an additional  $t$ -integration via the identity

$$1 = \int_0^\infty dt \delta(t - \sum_{i=1}^n \alpha_i)$$

leading to

$$\Pi = \int_0^\infty dt t^{n-1} \int_0^1 d^n \alpha \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, \dots, t\alpha_n).$$

## Infrared confinement

- Cut off the upper integration at  $1/\lambda^2$

$$\Pi^c = \int_0^{1/\lambda^2} dt t^{n-1} \int_0^1 d^n \alpha \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, \dots, t\alpha_n)$$

- The infrared cut-off has removed all possible thresholds in the quark loop diagram.
- We take the cut-off parameter  $\lambda$  to be the same in all physical processes.

T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij,  
Phys. Rev. D81, 034010 (2010)

## Invariant and helicity amplitudes

The transition amplitudes in terms of invariant amplitudes:

$$\langle B_2 P | \mathcal{H}_{\text{eff}} | B_1 \rangle = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^\dagger \bar{u}(p_2) (\mathbf{A} + \gamma_5 \mathbf{B}) u(p_1)$$

$$\langle B_2 V | \mathcal{H}_{\text{eff}} | B_1 \rangle = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^\dagger$$

$$\times \bar{u}(p_2) \epsilon_{V\delta}^* \left( \gamma^\delta V_\gamma + p_1^\delta V_p + \gamma_5 \gamma^\delta V_{5\gamma} + \gamma_5 p_1^\delta V_{5p} \right) u(p_1)$$

The helicity amplitudes in terms of invariant amplitudes:

$$H_{\frac{1}{2} t}^V = \sqrt{Q_+} \mathbf{A} \quad H_{\frac{1}{2} t}^A = \sqrt{Q_-} \mathbf{B}$$

$$H_{\frac{1}{2} 0}^V = +\sqrt{Q_-/q^2} \left( m_+ V_\gamma + \frac{1}{2} Q_+ V_p \right) \quad H_{\frac{1}{2} 1}^V = -\sqrt{2Q_-} V_\gamma$$

$$H_{\frac{1}{2} 0}^A = +\sqrt{Q_+/q^2} \left( m_- V_{5\gamma} + \frac{1}{2} Q_- V_{5p} \right) \quad H_{\frac{1}{2} 1}^A = -\sqrt{2Q_+} V_{5\gamma}$$

Here  $m_\pm = m_1 \pm m_2$ ,  $Q_\pm = m_\pm^2 - q^2$  and  $|p_2| = \lambda^{1/2}(m_1^2, m_2^2, q^2)/(2m_1)$ .The parity relations:  $H_{-\lambda_2, -\lambda_M}^V = +H_{\lambda_2, \lambda_M}^V$ ,  $H_{-\lambda_2, -\lambda_M}^A = -H_{\lambda_2, \lambda_M}^A$

## Decay widths

The semileptonic decay widths read

$$\Gamma(B_1 \rightarrow B_2 + \ell^+ \nu_\ell) = \int_0^{(M_1 - M_2)^2} dq^2 \frac{d\Gamma(B_1 \rightarrow B_2 + \ell^+ \nu_\ell)}{dq^2},$$

$$\frac{d\Gamma(B_1 \rightarrow B_2 + \ell^+ \nu_\ell)}{dq^2} = \frac{1}{192\pi} G_F^2 \frac{|\mathbf{p}_2| q^2}{M_1^2} |V_{ij}|^2 \mathcal{H}_V.$$

The two-body decay widths read

$$\Gamma(B_1 \rightarrow B_2 + P(V)) = \frac{G_F^2}{32\pi} |V_{cs} V_{ud}^\dagger|^2 \frac{|\mathbf{p}_2|}{m_1^2} \mathcal{H}_{P(V)}$$

$$\mathcal{H}_P = \left| H_{\frac{1}{2}t} \right|^2 + \left| H_{-\frac{1}{2}t} \right|^2,$$

$$\mathcal{H}_V = \left| H_{\frac{1}{2}0} \right|^2 + \left| H_{-\frac{1}{2}0} \right|^2 + \left| H_{\frac{1}{2}1} \right|^2 + \left| H_{-\frac{1}{2}-1} \right|^2,$$

where  $H = H^V - H^A$ .

## Semileptonic decays

Cabibbo-favored semileptonic decays of double heavy charm baryons induced by the charm level  $c \rightarrow s$  transition ( $\ell = e^+, \mu^+$ ).

	$\Gamma [10^{-13} \text{ GeV}]$	$\Gamma [10^{-13} \text{ GeV}]$	$B [\%]$
$1/2^+ \rightarrow 1/2^+$			
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ + \ell^+ \nu_\ell$	0.70	$0.77 \pm 0.37$ [1]	2.72
$\Xi_{cc}^{++} \rightarrow \Xi_c'^+ + \ell^+ \nu_\ell$	0.97	$0.53 \pm 0.35$ [1]	3.76
$\Xi_{cc}^+ \rightarrow \Xi_c^0 + \ell^+ \nu_\ell$	0.69	$0.77 \pm 0.37$ [1]	2.00
$\Xi_{cc}^+ \rightarrow \Xi_c'^0 + \ell^+ \nu_\ell$	0.97	$0.53 \pm 0.35$ [1]	2.79
$\Omega_{cc}^+ \rightarrow \Omega_c^0 + \ell^+ \nu_\ell$	1.82	$1.25 \pm 0.80$ [1]	7.07
$1/2^+ \rightarrow 3/2^+$			
$\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} + \ell^+ \nu_\ell$	0.22	—	0.86
$\Xi_{cc}^+ \rightarrow \Xi_c^{*0} + \ell^+ \nu_\ell$	0.22	—	0.64
$\Omega_{cc}^+ \rightarrow \Omega_c^{*0} + \ell^+ \nu_\ell$	0.40	0.32 [2]	1.27

[1] Y. J. Shi, W. Wang and Z. X. Zhao, Eur. Phys. J. C 80, no.6, 568 (2020).

[2] Z. X. Zhao, Eur. Phys. J. C 78, no.9, 756 (2018).

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and Z. Tyulemissov, Phys. Rev. D 100, no.11, 114037 (2019)

$$\Omega_{cc}^+ \rightarrow \Xi_c'{}^+ + \bar{K}^0 (\bar{K}^{*0})$$

Helicity	Tree diagram	$W$ diagram	total
$H_{\frac{1}{2} t}^V$	0.20	-0.01	0.19
$H_{\frac{1}{2} t}^A$	0.25	-0.01	0.24
$\Gamma(\Omega_{cc}^+ \rightarrow \Xi_c'{}^+ + \bar{K}^0) = 0.15 \cdot 10^{-13} \text{ GeV}$			
$H_{\frac{1}{2} 0}^V$	-0.25	$0.04 \times 10^{-1}$	-0.25
$H_{\frac{1}{2} 0}^A$	-0.50	0.01	-0.49
$H_{\frac{1}{2} 1}^V$	0.27	-0.01	0.26
$H_{\frac{1}{2} 1}^A$	0.56	$0.04 \times 10^{-2}$	0.56
$\Gamma(\Omega_{cc}^+ \rightarrow \Xi_c'{}^+ + \bar{K}^{*0}) = 0.74 \cdot 10^{-13} \text{ GeV}$			

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and Z. Tyulemissov, Phys. Rev. D 99, no.5, 056013 (2019)

$$\Omega_{cc}^+ \rightarrow \Xi_c^+ + \bar{K}^0 (\bar{K}^{*0})$$

Helicity	Tree diagram	$W$ diagram	total
$H_{\frac{1}{2} t}^V$	-0.35	1.06	0.71
$H_{\frac{1}{2} t}^A$	-0.10	0.31	0.21
$\Gamma(\Omega_{cc}^+ \rightarrow \Xi_c^+ + \bar{K}^0) = 0.95 \cdot 10^{-13} \text{ GeV}$			
$H_{\frac{1}{2} 0}^V$	0.50	-0.69	-0.19
$H_{\frac{1}{2} 0}^A$	0.18	-0.45	-0.27
$H_{\frac{1}{2} 1}^V$	-0.11	-0.24	-0.35
$H_{\frac{1}{2} 1}^A$	-0.18	0.66	0.48
$\Gamma(\Omega_{cc}^+ \rightarrow \Xi_c^+ + \bar{K}^{*0}) = 0.62 \cdot 10^{-13} \text{ GeV}$			

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and Z. Tyulemissov, Phys. Rev. D 99, no.5, 056013 (2019)

$$\Xi_{cc}^{++} \rightarrow \Xi_c'{}^+ + \pi^+(\rho^+)$$

Helicity	Tree diagram	$W$ diagram	total
$H_{\frac{1}{2}t}^V$	-0.38	-0.01	-0.39
$H_{\frac{1}{2}t}^A$	-0.55	-0.02	-0.57
$\Gamma(\Xi_{cc}^{++} \rightarrow \Xi_c'{}^+ + \pi^+) = 0.82 \cdot 10^{-13} \text{ GeV}$			
$H_{\frac{1}{2}0}^V$	0.60	$0.04 \times 10^{-1}$	0.61
$H_{\frac{1}{2}0}^A$	1.20	0.01	1.21
$H_{\frac{1}{2}1}^V$	-0.49	-0.01	-0.50
$H_{\frac{1}{2}1}^A$	-1.27	$0.01 \times 10^{-1}$	-1.27
$\Gamma(\Xi_{cc}^{++} \rightarrow \Xi_c'{}^+ + \rho^+) = 4.27 \cdot 10^{-13} \text{ GeV}$			

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and Z. Tyulemissov, Phys. Rev. D 99, no.5, 056013 (2019)

$$\Xi_{cc}^{++} \rightarrow \Xi_c^+ + \pi^+(\rho^+)$$

Helicity	Tree diagram	$W$ diagram	total
$H_{\frac{1}{2}\,t}^V$	-0.70	0.99	0.29
$H_{\frac{1}{2}\,t}^A$	-0.21	0.30	0.09
$\Gamma(\Xi_{cc}^{++} \rightarrow \Xi_c^+ + \pi^+) = 0.18 \cdot 10^{-13} \text{ GeV}$			
$H_{\frac{1}{2}\,0}^V$	1.17	-0.70	0.47
$H_{\frac{1}{2}\,0}^A$	0.45	-0.44	0.003
$H_{\frac{1}{2}\,1}^V$	-0.20	-0.23	-0.43
$H_{\frac{1}{2}\,1}^A$	-0.41	0.62	0.21
$\Gamma(\Xi_{cc}^{++} \rightarrow \Xi_c^+ + \rho^+) = 0.63 \cdot 10^{-13} \text{ GeV}$			

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and Z. Tyulemissov, Phys. Rev. D 99, no.5, 056013 (2019)

## Comparison with other approaches. Abbr.: M=NRQM, T=HQET

Mode	Width (in $10^{-13}$ GeV)					
	our	Dhir	Jiang	Wang	Yu	Likhoded
$\Omega_{cc}^+ \rightarrow \Xi_c' + \bar{K}^0$	0.15	0.31 (M) 0.59 (T)				
$\Omega_{cc}^+ \rightarrow \Xi_c^+ + \bar{K}^0$	0.95	0.68 (M) 1.08 (T)				
$\Omega_{cc}^+ \rightarrow \Xi_c' + \bar{K}^{*0}$	0.74		2.64 $^{+2.72}_{-1.79}$			
$\Omega_{cc}^+ \rightarrow \Xi_c^+ + \bar{K}^{*0}$	0.62		1.38 $^{+1.49}_{-0.95}$			
$\Xi_{cc}^{++} \rightarrow \Xi_c' + \pi^+$	0.82	1.40 (M) 1.93 (T)		1.10		
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ + \pi^+$	0.18	1.71 (M) 2.39 (T)		1.57	1.58	2.25
$\Xi_{cc}^{++} \rightarrow \Xi_c' + \rho^+$	4.27		4.25 $^{+0.32}_{-0.19}$	4.12	3.82	
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ + \rho^+$	0.63		4.11 $^{+1.37}_{-0.86}$	3.03	2.76	6.70

## References

-  **N. Sharma and R. Dhir**, Phys. Rev. D 96, 113006 (2017).
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## Estimating uncertainties in the decay widths

- The only free parameter in our approach is the size parameter  $\Lambda_{cc}$ .
- We have chosen  $\Lambda_{cc} = \Lambda_c = 0.8675$  GeV.
- To estimate the uncertainty caused by the choice of the size parameter we allow the size parameter to vary from 0.6 to 1.135 GeV.
- We evaluate the mean  $\bar{\Gamma} = \sum \Gamma_i / N$  and the mean square deviation  $\sigma^2 = \sum (\Gamma_i - \bar{\Gamma})^2 / N$ .
- The rate errors amount to 6 – 15%.

Mode	Width (in $10^{-13}$ GeV)
$\Omega_{cc}^+ \rightarrow \Xi_c' + \bar{K}^0$	$0.14 \pm 0.01$
$\Omega_{cc}^+ \rightarrow \Xi_c' + \bar{K}^{*0}$	$0.72 \pm 0.06$
$\Omega_{cc}^+ \rightarrow \Xi_c^+ + \bar{K}^0$	$0.87 \pm 0.13$
$\Omega_{cc}^+ \rightarrow \Xi_c^+ + \bar{K}^{*0}$	$0.58 \pm 0.07$
$\Xi_{cc}^{++} \rightarrow \Xi_c' + \pi^+$	$0.77 \pm 0.05$
$\Xi_{cc}^{++} \rightarrow \Xi_c' + \rho^+$	$4.08 \pm 0.29$
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ + \pi^+$	$0.16 \pm 0.02$
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ + \rho^+$	$0.59 \pm 0.04$

## $\Lambda$ -hyperon

- **Mass of  $\Lambda$ -hyperon**  $m_\Lambda = 1.115683 \pm 0.000006$  GeV.
- **Mean life of  $\Lambda$ -hyperon**  $\tau_\Lambda = 2.632 \pm 0.020 \cdot 10^{-10}$  s.
- **Modes**

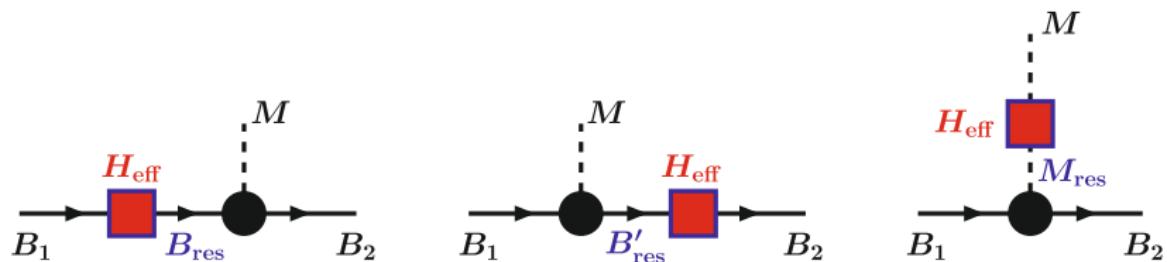
$$Br(\Lambda \rightarrow p\pi^-) = (63.9 \pm 0.5)\%,$$

$$Br(\Lambda \rightarrow n\pi^0) = (35.8 \pm 0.5)\%$$

M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and Z. Tyulemissov, Phys. Rev. D 104, no.7, 074004 (2021)

There are two classes of the Feynman diagrams generating matrix elements of these processes:

- short-distance (SD) diagrams,
- long-distance (LD) or pole diagrams.



## Matrix element

**Matrix element of  $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + 0^-$  decay reads**

$$M(B_1 \rightarrow B_2 + M) = M_{\text{SD}}(B_1 \rightarrow B_2 + M)$$

$$+ M_{\text{LD}_1}(B_1 \rightarrow B_{\text{res}} \rightarrow B_2 + M) + M_{\text{LD}_2}(B_1 \rightarrow B'_{\text{res}} + M \rightarrow B_2 + M),$$

$$M_{\text{SD}} = i^4 \bar{u}(p_2) \Gamma_{B_1 B_2 M}(p_1, p_2, q) u(p_1),$$

$$M_{\text{LD}_1} = i^6 \int \frac{d^4 k}{(2\pi)^4 i} \bar{u}(p_2) \Gamma_{B_{\text{res}} M B_2}(k, p_2, q) S_{B_{\text{res}}}(k) \Gamma_{B_1 B_{\text{res}}}(p_1, k) u(p_1),$$

$$M_{\text{LD}_2} = i^6 \int \frac{d^4 k}{(2\pi)^4 i} \bar{u}(p_2) \Gamma_{B_{\text{res}} B_2}(k, p_2) S_{B_{\text{res}}}(k) \Gamma_{B_1 M B_{\text{res}}}(p_1, k, q) u(p_1),$$

The propagator of the  $\frac{1}{2}^+$  resonances is the ordinary Dirac propagator,

$$S(p) = \frac{1}{m_{\text{res}} - \not{p}} = \frac{m_{\text{res}} + \not{p}}{m_{\text{res}}^2 - p^2}.$$

## The invariant matrix elements for the pole diagrams with intermediate $\frac{1}{2}^+$ resonances

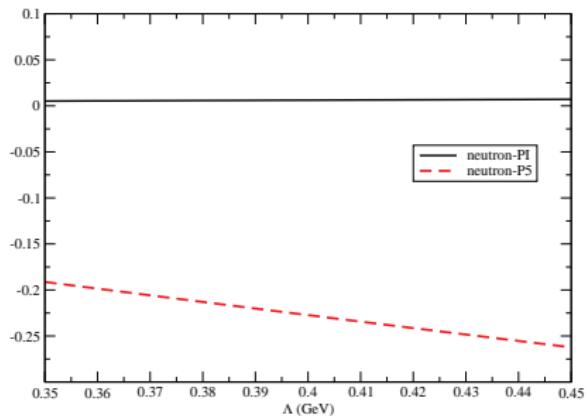
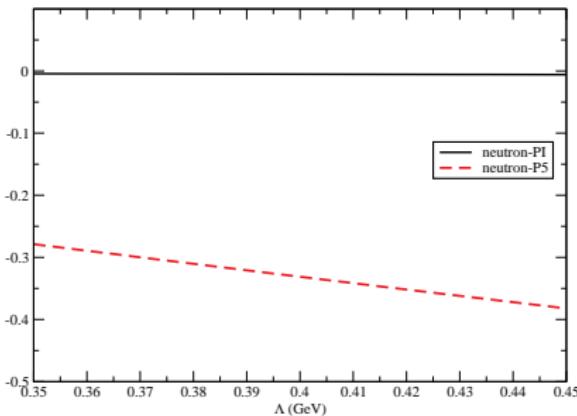
$$\tilde{M}_{LD_1} \equiv \tilde{M}_n = \bar{u}(p_2) \left( A_n + \gamma_5 B_n \right) u(p_1),$$

$$A_n = -\frac{B_{\Lambda n}(\textcolor{teal}{C}_{n\pi p} - m_\Lambda D_{n\pi p})}{m_n + m_\Lambda}, \quad B_n = -\frac{A_{\Lambda n}(\textcolor{teal}{C}_{n\pi p} + m_\Lambda D_{n\pi p})}{m_n - m_\Lambda},$$

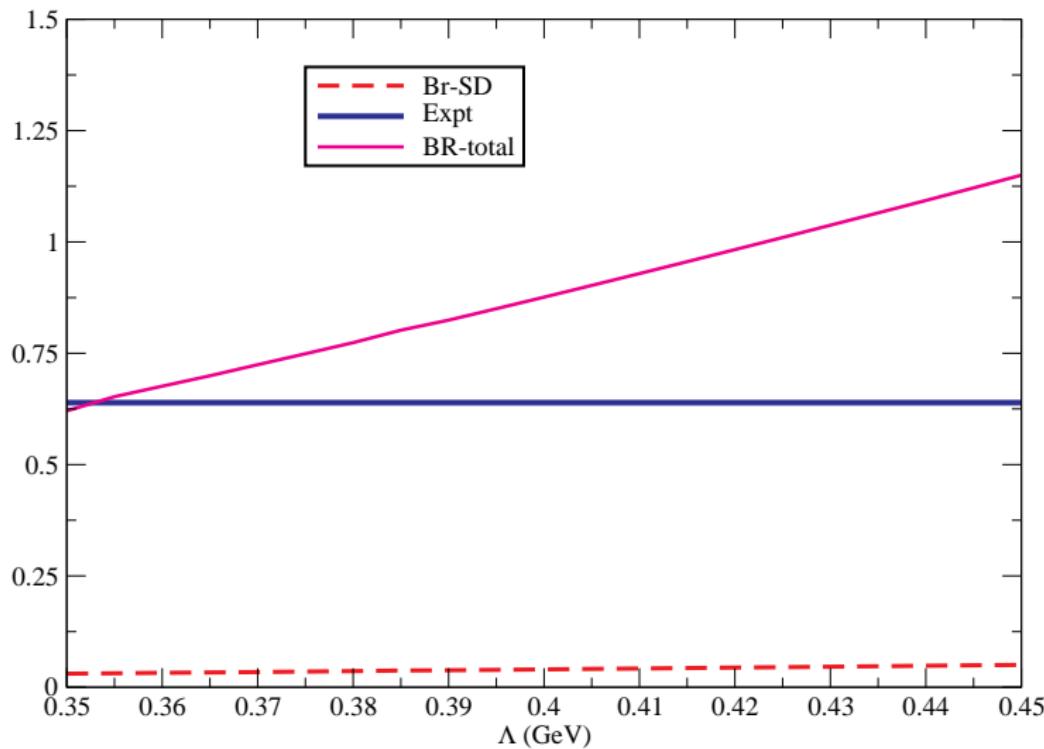
$$\tilde{M}_{LD_2} \equiv \tilde{M}_\Sigma = \bar{u}(p_2) \left( A_\Sigma + \gamma_5 B_\Sigma \right) u(p_1),$$

$$A_\Sigma = -\frac{B_{\Sigma^+ p}(\textcolor{teal}{C}_{\Lambda\pi\Sigma^+} - m_p D_{\Lambda\pi\Sigma^+})}{m_\Sigma + m_p}, \quad B_\Sigma = -\frac{A_{\Sigma^+ p}(\textcolor{teal}{C}_{\Lambda\pi\Sigma^+} + m_p D_{\Lambda\pi\Sigma^+})}{m_\Sigma - m_p},$$

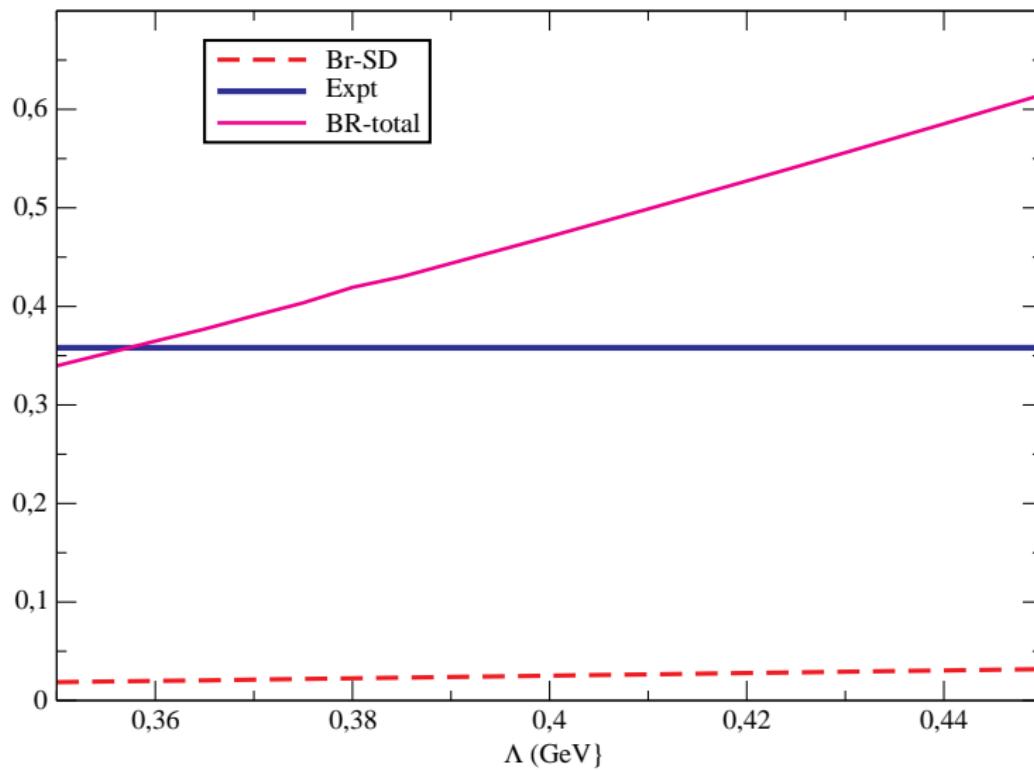
Dependence of the helicities  $P1 \equiv H_{1/2,t}^V$  and  $P5 \equiv H_{1/2,t}^A$  on the size parameter in the case of the neutron resonance. Left panel: the decay  $\Lambda \rightarrow p + \pi$ ; right panel: the decay  $\Lambda \rightarrow n + \pi$ .



$$\Lambda^0 \rightarrow p + \pi^-$$



$$\Lambda^0 \rightarrow n + \pi^0$$



$$\Lambda \rightarrow p\pi^-$$

**SD contributions** to the amplitudes  $A$  and  $B$  of the decay  $\Lambda \rightarrow p\pi^-$  in units of  $\text{GeV}^2$ .

	Ia	IIa	IIb	III	Sum(SD)
$A_{\text{SD}}$	$-0.372 \cdot 10^{-1}$	$0.269 \cdot 10^{-3}$	$0.300 \cdot 10^{-1}$	$0.213 \cdot 10^{-1}$	$0.144 \cdot 10^{-1}$
$B_{\text{SD}}$	$-0.345$	$-0.116$	$0.167$	$-0.452$	$-0.746$

**LD contributions** to the amplitudes  $A$  and  $B$  of the decay  $\Lambda \rightarrow p\pi^-$  in units of  $\text{GeV}^2$ .

	$n$	$\Sigma^+$	$K$	$K^*$	$\frac{1}{2}^-$	Sum(LD)
$A_{\text{LD}}$	$-2.1 \cdot 10^{-3}$	$-9.5 \cdot 10^{-3}$	0	$2.6 \cdot 10^{-2}$	$0.9 \cdot 10^{-1}$	$1.1 \cdot 10^{-1}$
$B_{\text{LD}}$	$-2.55$	$2.3 \cdot 10^{-1}$	$2.8 \cdot 10^{-2}$	0	0	$-2.3$

$$\Lambda \rightarrow n\pi^0$$

**SD contributions** to the amplitudes  $A$  and  $B$  of the decay  $\Lambda \rightarrow n\pi^0$  in units of  $\text{GeV}^2$ .

	Ib	IIa	IIb	III	Sum(SD)
$A_{\text{SD}}$	$-0.120 \cdot 10^{-1}$	$0.190 \cdot 10^{-3}$	$0.211 \cdot 10^{-1}$	$0.150 \cdot 10^{-1}$	$0.243 \cdot 10^{-1}$
$B_{\text{SD}}$	$-0.112$	$-0.82 \cdot 10^{-1}$	$0.119$	$-0.319$	$-0.394$

**LD contributions** to the amplitudes  $A$  and  $B$  of the decay  $\Lambda \rightarrow n\pi^0$  in units of  $\text{GeV}^2$ .

	$n$	$\Sigma^0$	$K$	$K^*$	$\frac{1}{2}^-$	Sum
$A_{\text{LD}}$	$-1.5 \cdot 10^{-3}$	$-6.6 \cdot 10^{-3}$	$0$	$8.4 \cdot 10^{-3}$	$6.2 \cdot 10^{-2}$	$0.6 \cdot 10^{-1}$
$B_{\text{LD}}$	$-1.83$	$1.6 \cdot 10^{-1}$	$0.9 \cdot 10^{-2}$	$0$	$0$	$-1.67$

## Asymmetry in $\Lambda$ decays

- $\alpha = 0.87 \pm 0.09$
- $\alpha = 0.642 \pm 0.013$
- $\alpha = 0.750 \pm 0.009 \pm 0.004$
- $\alpha = 0.7542 \pm 0.0010 \pm 0.0020$

CCQM

Particle Data Group 2016.

BESIII Collab. 1808.08917.

BESIII Collab. 2204.11058.

# Thanks a lot for your attention