Explore N(1520) transition form factors with non-perturbative dispersion theory at low energy

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1. Introduction to the nucleon transition form factors (TFFs)
2. Dispersive formalism
3. $N^*(1520)$ TFFs’ results (preliminary)
4. Outlook
We try to understand the structure of the nucleon.

How large is $\langle 0 | \bar{q} q q | N \rangle$ and $\langle 0 | \text{Meson Baryon} | N \rangle$, quantitatively?
Nucleon electromagnetic structure

We try to understand the structure of the nucleon.

Nucleon transition form factors

QCD running coupling

How large is $\langle 0 | qqq | N \rangle$ and $\langle 0 | \text{Meson Baryon} | N \rangle$, quantitatively?

Need model-independent tool → Dispersion theory
**Dispersion theory in a nutshell**

**Axiomatic QFT**
→ Form factors are analytic functions in the complex plane.

**Unitarity + analyticity**
→ the location of cut, branch point, singularities...

**Cauchy integral Formula:**

\[
F(q^2) = \frac{1}{\pi} \int_{4m^2}^{\infty} \frac{\text{Im } F(s)}{s - q^2 - i\epsilon} \, ds.
\]

Unitarity cut \([4m^2_{\pi}, \infty)\)
Previous studies on TFFs by Uppsala group

\[ \Sigma(J^P = 1^+_2) \rightarrow \Lambda(J^P = 1^+_2) \] (Granados, Leupold, Perotti) [1]

Nucleon isovector form factors (Leupold) [2]

\[ \Sigma^*(J^P = 3^+_2) \rightarrow \Lambda(J^P = 1^+_2) \] (Junker, Leupold, Perotti, Vitos) [3]

\[ \Delta(J^P = 3^+_2) \rightarrow N(J^P = 1^+_2) \] (Aung, Leupold, Perotti) (in preparation)

Quark mass dependence of nucleon EMFF
(An, Alvarado, Leupold, Alvarez-Ruso) (in preparation)

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Nucleon excited states [4]

Branching ratios of \( N^* \) by PDG:

1. \( N\pi \)
   55 – 65%
2. \( \Delta(1232)\pi \), (S-wave)
   15 – 23%
3. \( \Delta(1232)\pi \), (D-wave)
   7 – 11%
4. \( N\rho \), \( S = \frac{3}{2} \), (S-wave)
   10 – 16%
5. \( N\rho \), \( S = \frac{1}{2} \), (D-wave)
   0.2 – 0.4%
6. \( N\rho \), \( S = \frac{3}{2} \), (D-wave)
   0
7. \( N\eta \)
   0.07 – 0.08%

**Our Strategy:** include low energy degrees of freedom (\( N, \Delta, \pi, \rho \)) as model-independent as possible.
$N^*(1520)$ TFFs

$N^*(1520)$ $I = 1/2$ and $J^P = 3/2^-$. 

$$\langle N|j_\mu|N^*\rangle = e \bar{u}_N(p_N) \Gamma_{\mu \nu}(q) u_{N^*}(p_{N^*})$$

with

$$\Gamma_{\mu \nu}(q) := i(\gamma^\mu q^\nu - q^\mu g_{\mu \nu}) m_N F_1(q^2) + \sigma^{\mu \alpha} q_\alpha q^\nu F_2(q^2) +$$

$$+ i(q^\mu q^\nu - q^2 g_{\mu \nu}) F_3(q^2),$$

where $q^\mu := p_{N^*}^\mu - p_N^\mu$. In this work, we focus on isovector TFFs:

**Imaginary part:**

1. Pion vector form factor: $F_\nu$
2. Baryon-meson exchanges: BM

Imaginary part $\xrightarrow{\text{Dispersion relation}}$ full amplitude
$N^*(1520)$ TFFs

$N^*(1520)$ TFFs

Diagram showing the process:

$N \rightarrow BM \rightarrow F_v \rightarrow \gamma^*$

$\approx N/\Delta \rightarrow \pi\pi \rightarrow F_v \rightarrow \gamma^*$

Pion p-wave phase shift $\delta_1$.

$\delta_1$ contains $\rho$ meson information

$f_1(s) = \frac{\sin \delta_1(s)}{\sqrt{1 - \frac{4m^2_\pi}{s}}} \frac{\sqrt{s}}{2}$

$F_v(s) \approx \Omega(s) = \exp \left[ \frac{s}{\pi} \int_{4m^2_\pi}^{\infty} ds' \frac{\delta_1(s')}{s'(s' - s)} \right]$
Fit to hadronic decay data provides $v_1$, $v_3$, $v_5$ and $v_7$.

Chiral perturbation theory gives $v_2$, $v_4$, $v_6$. 
Cuts, Poles and Singularities

Anomalous threshold condition

\[
m^{2}_{\text{exc}} < \frac{1}{2}(m^{2}_{N^{*}} + m^{2}_{N} - 2m^{2}_{\pi})
\]

\[
m^{2}_{\text{exc}} = m^{2}_{N} \quad \text{(see back up slides for rigorous derivation)}
\]

\[
T(s) = \frac{1}{2\pi i} \int_{c_{R}}^{\infty} \frac{\text{disc}_{\text{UNI}} T(z)}{z-c} \, dz + \frac{1}{2\pi i} \int_{\gamma} \frac{d\gamma}{dt} \frac{\text{disc}_{\text{ANOM}} T((\gamma(t))}{\gamma(t)-s} \, dt
\]

Cutkosky cutting rules

\[N^* \]

\[N/\Delta\]

\[\text{Res}\]

\[N\]

\[\gamma_1\]

\[\gamma_2\]

\[\gamma_3\]

\[\gamma_4\]

\[\gamma_t\]

\[\gamma_s\]

\[\gamma_c\]

\[\gamma_d\]

Two singularities on the second Riemann Sheet

The first Riemann sheet includes an unitarity and an anomalous part.
Comparison with 1-loop scalar-triangle

How do we make sure we are right about the analytic structures?

→ use 1-loop scalar triangle (G. ’t Hooft, M. Veltman) as a toy calculation for double-check!

Our dispersive relation for the scalar triangle perfectly matches the analytic results:

\[
N^* = T(s) = \frac{1}{2\pi i} \int_{\Pi}^{\infty} \text{disc}_{\text{UNI}} T(z) \frac{dz}{z - c} + \frac{1}{2\pi i} \int_{\gamma} \frac{d\gamma}{dt} \text{disc}_{\text{ANOM}} T((\gamma(t))) \frac{dt}{\gamma(t) - s}
\]
Subtracted dispersion relations for TFFs:

\[
F_i(q^2) = F_i(0) + \frac{q^2}{12\pi} \int_{4m_i^2}^{\Lambda^2} \frac{ds}{s} \frac{T_i(s) p_{c.m.}^3(s) F_{V}^{*}(s)}{s^{3/2} (s - q^2 - i\epsilon)} + F_i^{anom}(q^2) \text{ for } i = 1, 2, 3.
\]

\[T_i \sim N^* N \to 2\pi \text{ amplitudes calculated from Muskhelishvili-Omnès formalism:}\]

\[
T_i(s) = K_i(s) + \Omega(s) P_i + T_i^{anom}(s)
\]

\[
+ \Omega(s) s \int_{4m_i^2}^{\Lambda^2} \frac{ds'}{s'} \frac{K_i(s') \sin \delta(s')}{|\Omega(s')| (s' - s - i\epsilon) s'}.
\]

\[P_{i=1,2,3} \text{ are fit parameters (contact term interactions).}\]

Fix \( P_i \) by matching to \( \Gamma_{N\rho} \) S, D-wave decay widths (backup slides).
Isovector TFFs := \( \frac{1}{2}(F_i^{\text{proton}} - F_i^{\text{neutron}}) \) \( i = 1, 2, 3 \).

Only quantitative input: subtraction constants \( P_{1,2,3} \)

\( \Rightarrow \) fix hadronic \( N^*N \rightarrow \pi\pi \) amplitudes

\( \Rightarrow \) \( N^*(1520) \rightarrow N\gamma^* \)

Red: Model dependent parametrization for isovector TFFs
(Jlab data for proton, MAID model for neutron)

Blue: this work

Full lines: Real part, Dashed lines: Imaginary part.
Theory predictions: Time-like TFFs (preliminary)

\[ N^* \rightarrow N e^+ e^- \]

(Assuming isovector dominance)

Our prediction:
\[ \Gamma_{N^* \rightarrow N e e} \approx (4.6, 5.2) \text{ keV} \]
\[ \Gamma_{N^* \rightarrow N e e}^{(\text{QED})} \approx 4.4 \text{ keV} \]

Results can be tested by HADES.
Theory predictions: Time-like TFFs (preliminary)

$N^* \rightarrow N\mu^+\mu^-$

$N^* \rightarrow Ne^+e^-$

Muonic Dalitz decay prediction:
$\Gamma_{N^*\rightarrow N\mu^+\mu^-} \approx (0.6, 1.1) \text{ keV}$
$\Gamma_{N^*\rightarrow N\mu^+\mu^-} (\text{QED}) \approx 0.4 \text{ keV}$

Electronic Dalitz decay prediction:
$\Gamma_{N^*\rightarrow Ne^+e^-} \approx (4.6, 5.2) \text{ keV}$
$\Gamma_{N^*\rightarrow Ne^+e^-} (\text{QED}) \approx 4.4 \text{ keV}$
Our dispersive prediction $N^+(1520) \to n\pi^+\pi^0$ as an example can be used to test the quality of isobar model predictions.

\[
T_i(s) = K_i(s) + \Omega(s) P_i + T_i^\text{anom}(s)
\]

\[
+ \Omega(s) s \int_0^\infty \frac{ds'}{\pi} \frac{K_i(s') \sin \delta(s')}{|\Omega(s')|} \frac{1}{s' - s - i\epsilon}.
\]
Outlook

1. In progress: $N^+ \rightarrow p\pi^+\pi^-$, $N^+ \rightarrow p\pi^0\pi^0$ including pion s-wave re-scattering dispersively ($\sigma$ meson).
2. Cross-check the fit parameters (e.g. sign of $N^*N\pi=+0.655$) with isobar model experts (e.g. Jülich-Bonn-Washington model).
3. Determination of $P_{1,2,3}$ (short-distance physics) from QCD-based functional methods (Bethe-Salpeter equation and Dyson-Schwinger Eqs) → collaboration with Gernot Eichmann (Graz U.) and Christian S. Fischer (Giessen U.).
Tool box for non-perturbative QCD

Quark-gluon-based methods:

1. **Dyson-Schwinger Equations**
   - Infinitely many coupled non-perturbative Eqs. for quark gluon correlators.

2. **QCD light-cone sum rules**
   - Operator product expanding correlators around light-cone $x^2 \sim 0 \rightarrow \frac{\Lambda_{QCD}^2}{Q^2} \approx 0$.

3. **Lattice QCD**
   - Unable to calculate TFFs due to multi-particle state contamination and many other technical reasons.

Hadron-based methods:

1. **Chiral perturbation theory**
   - Works very well for Goldstone-bosons by non-linear realisation $SU_L(2) \otimes SU_R(2)/SU_V(2)$

2. **Dispersion theory**

Relativistic quark model and light cone sum rules calculation [5] for TFF of $N \rightarrow N(1535)$.

Dyson-Schwinger Equation prediction [5] TFF of $N \rightarrow N(1440)$.
Fixing coupling constants with isobar models

\( \gamma N \rightarrow \pi N \) s-channel scattering amplitude.

Two amplitudes are constructive (destructive) at \( s = (mN^*)^2 \) if \( h > 0 (h < 0) \).

\[ \gamma N \rightarrow \pi N \text{ amplitude} \]
Experimental data on space-like TFFs

We can only calculate isovector TFFs: \[ \frac{1}{2}(F^\text{proton}_i - F^\text{neutron}_i), \quad i = 1, 2, 3 \]

No error estimate from MAID data
F1 has a complicated structure at low and high energy.

F2 is not iso-vector dominant at low \( Q^2 \) and it probably has large uncertainty

F3 has simple behaviour at low and high energy and is dominant at low energy → not measurable at the photon point.

Orange: Proton data (J-lab).
Blue: Neutron estimates (MAID)
Fixing input parameters

\[ N^* N\pi \rightarrow h = 0.655 \]
\[ N^* N\Delta \rightarrow H_1 = 0.28, H_2 = -5.6 \]

How to fix subtraction constants \( P_{1,2,3} \)?

Method A: Fit \( P_{1,2,3} \) to slopes of TFF data

Pro: easy to do.

Con: no data exist in \( 0 < Q^2 \leq 0.25 \text{GeV}^2 \)!!

Method B: Fit to the \( N(1520) \rightarrow N\rho \) hadronic decay widths and make use of slopes of TFFs.

Contact terms: \( P_1, P_2, P_3 \)

\( \rightarrow 8 \) possible sign choices.

\[ dF_1(Q^2)/dQ^2 > 0 \rightarrow \text{choose } P_1 > 0 \]
\[ N(1520) \rightarrow N\rho \text{ fit suggests } P_2 \approx 0 \]
\[ dF_3(Q^2)/dQ^2 < 0 \rightarrow \text{choose } P_3 < 0 \]

\( \rightarrow \) predict \( F_{1,2,3}(Q^2) \)

Branching ratios of \( N^* \)

in PDG:

1. \( N\pi \) 55 – 65%
2. \( \Delta\pi \) (S-wave) 15 – 23%
3. \( \Delta\pi \) (D-wave) 7 – 11%
4. \( N\rho, (S = \frac{3}{2}, L = 0) \) 10 – 16%
5. \( N\rho, (S = \frac{1}{2}, L = 2) \) 0.2 – 0.4%
6. \( N\rho, (S = \frac{3}{2}, L = 2) \approx 0 \)
Anomalous singularity

Trajectory of a singularity in the complex plane[6]
Calculating the couplings in quark di-quark approach in Dyson-Schwinger equations and Bethe-Salpeter equations
State of art method: Rainbow ladder + quark-di quark approximation (Valentin Mader et al.).

\[ \rho \rightarrow \pi\pi \text{ transition matrix elements} \]

\[ \Delta \rightarrow N\pi \text{ transition matrix elements} \]

\[ G_{\Delta N \pi}(Q^2) \]

\[ g_{\rho \pi \pi} \]
Match the hadrons with quarks and gluons

Compton scattering amplitudes (Gernot Eichmann et al):

\[ \text{Born terms} \]

\[ \text{s/u-channel } N^* \text{ resonances} \]

\[ \text{t-channel mesons} \]

\[ \text{pion loops} \]

\[ \text{(a) reproduces Born terms and } N^* \text{ resonances} \]

\[ \text{(b) reproduces handbag diagrams and t-channel meson poles} \]

\[ \text{(c) cat's ears diagrams} \]

+ \ldots
Dispersion theory in a nutshell

Example: Pion vector form factor

\[ S = 1 + iT \]

Unitarity \( SS^\dagger = 1 + i(T - T^\dagger) + |T|^2 = 1 \)

\[ \rightarrow 2ImT = |T|^2 \]

\[ \rightarrow ImT_{A\rightarrow B} = \frac{1}{2} \sum_x T_{A\rightarrow x} T^\dagger_{x\rightarrow B} \]

Simplest example: \( A = |\gamma^*\rangle, B = |\pi^- (p_1)\pi^+ (p_2)\rangle. \)

\[ \Rightarrow T_{\gamma^* \rightarrow \pi^- \pi^+} = eA^\mu \langle \pi^- (p_1)\pi^+ (p_2) |j^\mu |0 \rangle \]

\[ (p_1^\mu - p_2^\mu)F_\nu(s) \]

\[ T_{\gamma^* \rightarrow x} = eA^\mu \langle x |j^\mu |0 \rangle \]

\[ ImF_\nu(s)(p_1^\mu - p_2^\mu) = \frac{1}{2} \sum_x \langle \pi^- (p_1)\pi^+ (p_2) |x\rangle^* \langle x |j^\mu |0 \rangle \]

\[ |x\rangle = 2\text{pions}(s \geq 4m^2_\pi), 4\text{pions}(s \geq 16m^2_\pi), 2\text{kaons}(s \geq 4m^2_K), ... \]
Dispersion relation

Cauchy integral formula:

\[ F_v(s) = \frac{1}{2\pi i} \lim_{\epsilon \to 0} \int_{s_0=4m^2_\pi}^{\infty} dz \frac{F_v(z + i\epsilon) - F_v(z - i\epsilon)}{z - s} \]  

\( \text{(5)} \)

Schwarz Reflection Principle: \( F_v(z - i\epsilon) = F_v(z + i\epsilon)^* \)

\[ F_v(s) = \frac{1}{\pi} \int_{s_0=4m^2_\pi}^{\infty} dz \frac{\text{Im}[F_v(z + i\epsilon)]}{z - s} \]  

\( \text{Dispersion relation} \)  

\( \text{(6)} \)

Consider only the 2 pion contribution

\[ 2\text{Im}F_v(q^2)(p_1^\mu - p_2^\mu) \approx \int d\tau_2^{\pi}\langle\pi^- (p_1)\pi^+ (p_2)|\pi^- (p_1')\pi^+ (p_2') \rangle^* \langle\pi^- (p_1')\pi^+ (p_2')|j^\mu|0\rangle \]  

\( \text{Pion rescattering amplitude} \)

\( \text{F}_v(q^2)(p_1'^\mu - p_2'^\mu) \)  

\( \text{(7)} \)

Only pion p-wave re-scattering amp.

\[ f_1(s) = \frac{\sin\delta_1(s)}{\sqrt{1 - \frac{4m^2_\pi}{s}}} \frac{\sqrt{s}}{2} \text{ contributes!} \]

\[ \Rightarrow f_1(s) \text{ parametrized by phase-shift } \delta_1 \]

\( \delta_1 \Rightarrow \text{well measured by experiments!} \)
Dispersion relation

\[ \delta_1 \text{ contains } \rho \text{ meson information} \rightarrow \]

\[ F_v(s) \approx \Omega(s) = \exp \left[ \frac{s}{\pi} \int_{4m^2}^{\infty} ds' \frac{\delta_1(s')}{s'(s'-s)} \right] \]

Pion p-wave phase shift [7]


