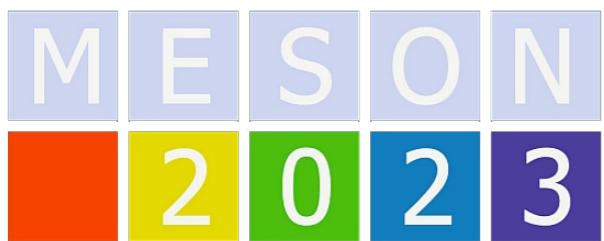


A dispersive estimate of $a_0(980)$ contribution in hadronic light-by-light scattering in $(g - 2)$

Oleksandra Deineka

in coll. with Igor Danilkin, Marc Vanderhaeghen

22.06.2023



Deutsche
Forschungsgemeinschaft



Magnetic moment of the muon $\vec{\mu} = \frac{Q}{2m} g \vec{S}$

Anomalous part: $a_\mu = \frac{(g - 2)_\mu}{2}$

**Ultra-precise measurements
Standard Model calculations**

$$a_\mu^{exp} = 116592061(41) \times 10^{-11}$$
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4.2 σ difference!

Motivation: anomalous magnetic moment of muon

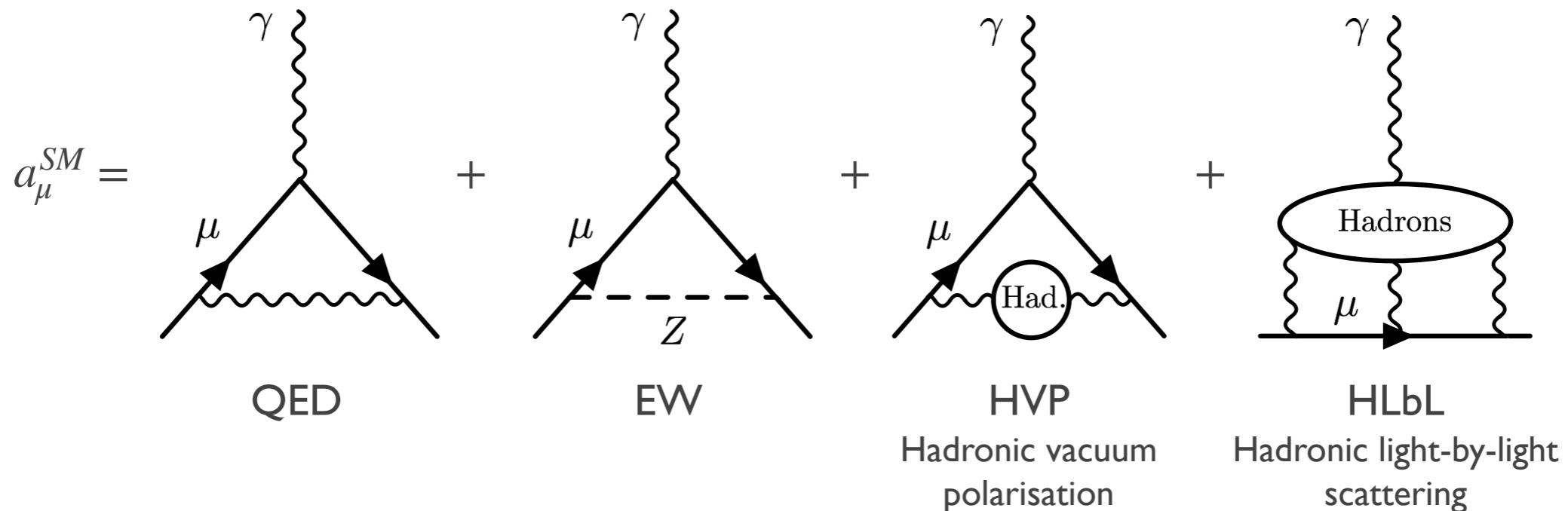
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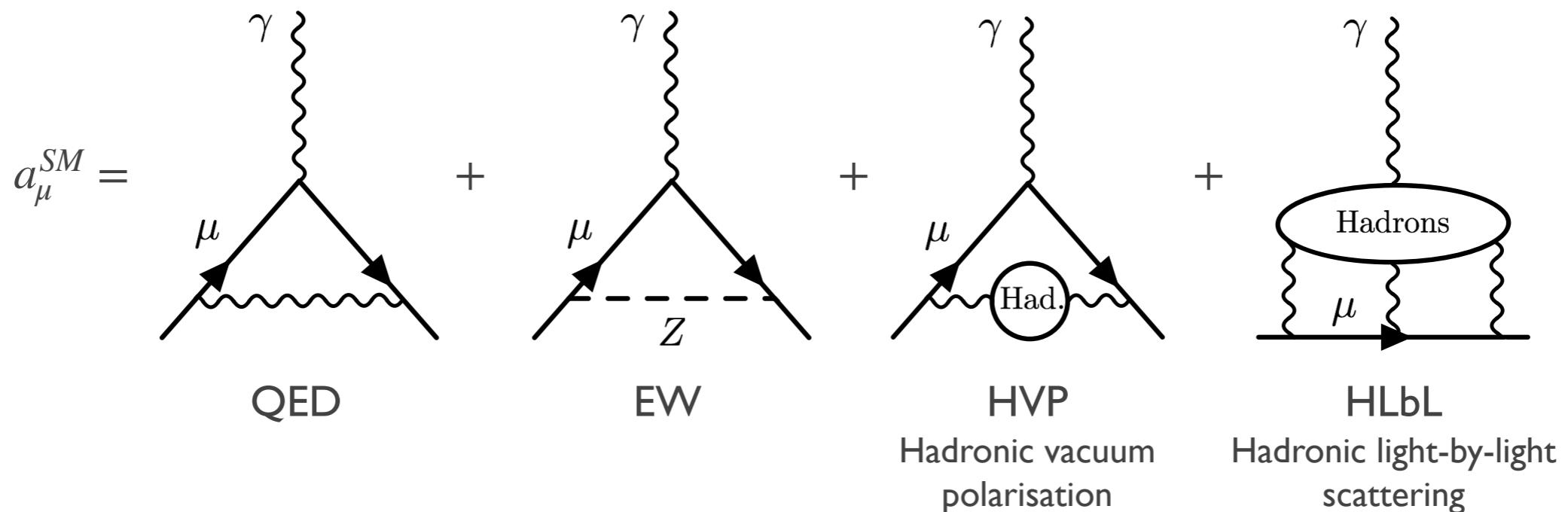
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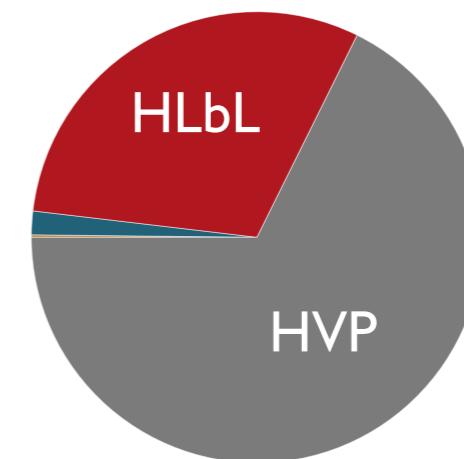
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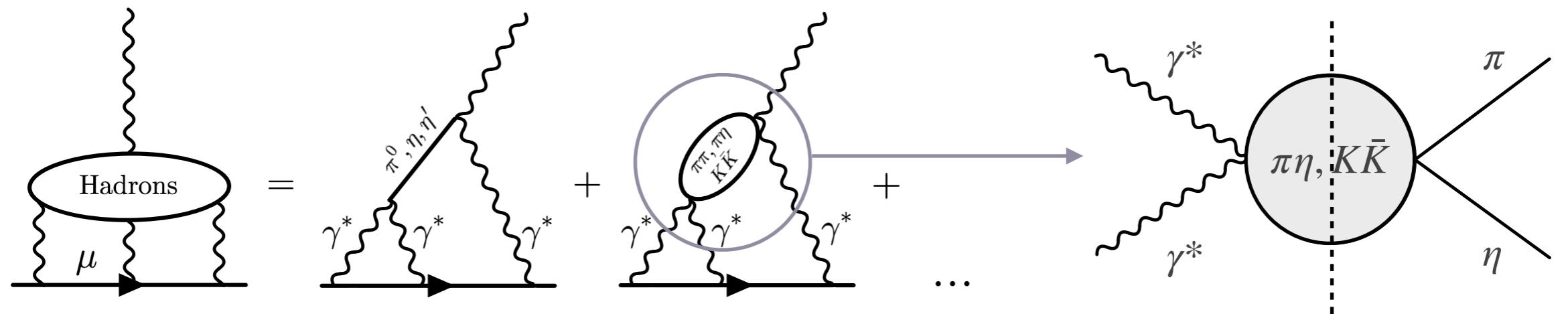
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To reduce HLbL contribution to the uncertainty one needs
data-driven model-independent approach

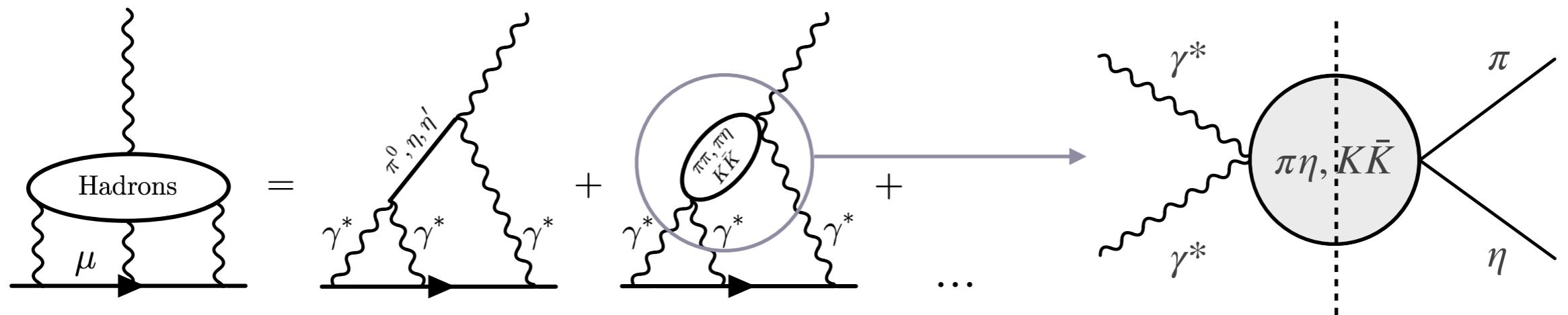


Scalar resonances contribution to (g-2)



Ingredients for HLbL: $\gamma^*\gamma^* \rightarrow \pi\pi, \pi\eta, K\bar{K} \dots$ for **spacelike** γ^* : $q = -Q^2 < 0$

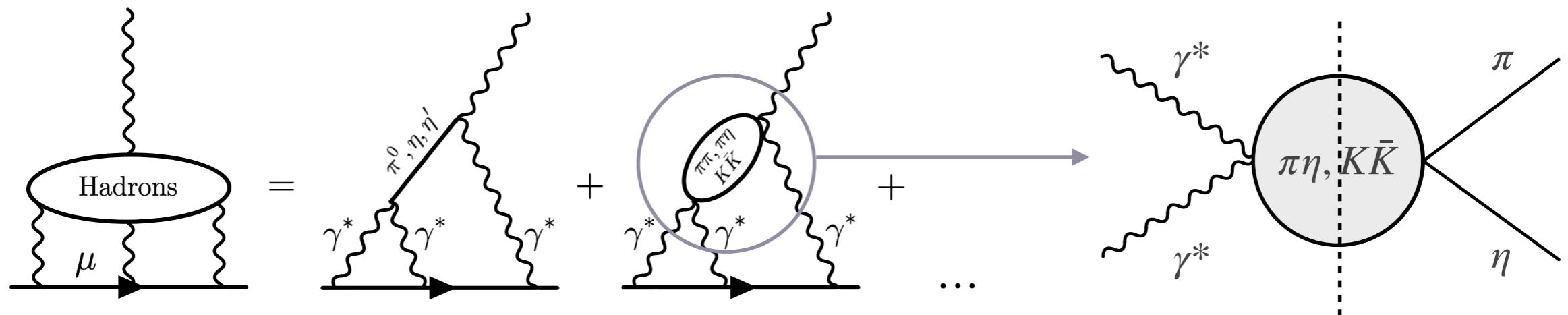
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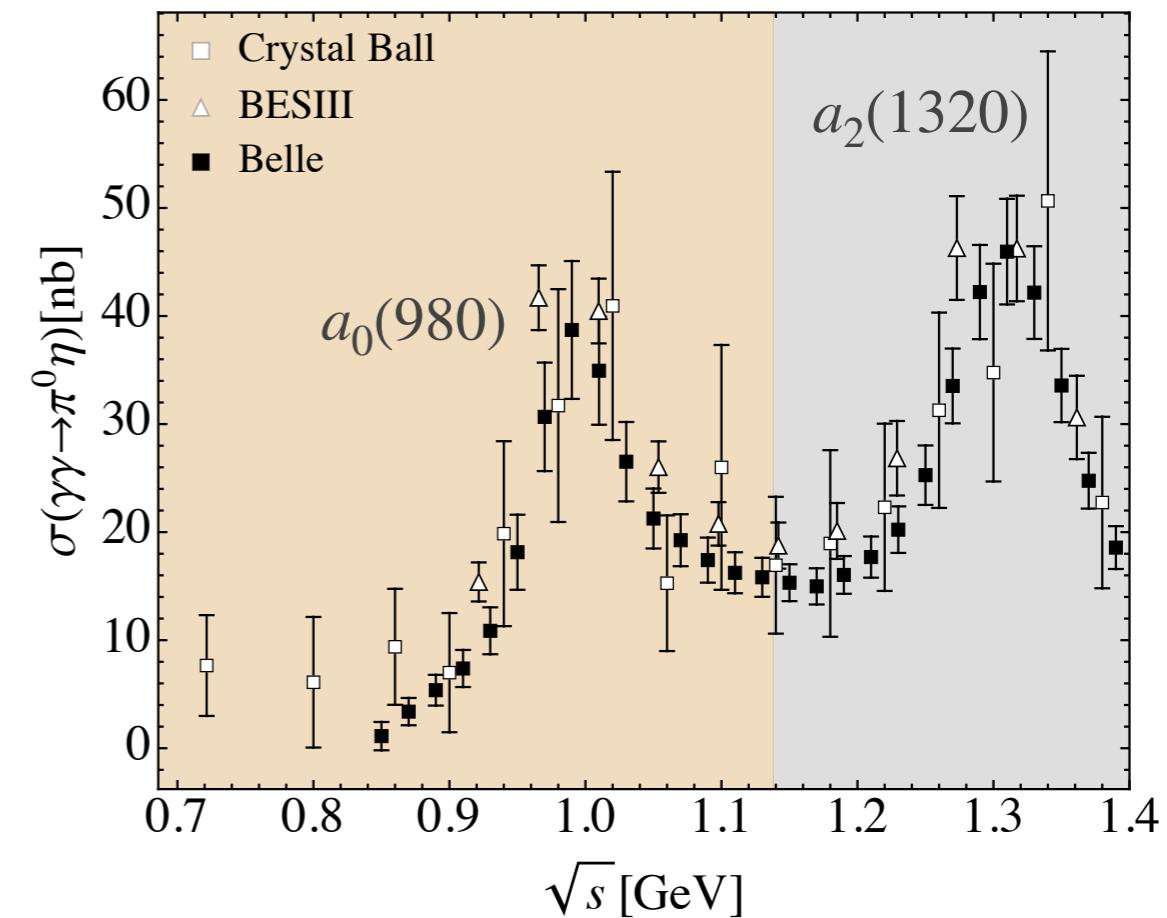
Contribution	Value $\times 10^{11}$
π^0, η, η' -poles	93.8(4.0)
π, K -loops/boxes	-16.4(2)
S -wave $\pi\pi$ rescattering	-8(1)
subtotal	69.4(4.1)
scalars	-1(3)
tensors	
axial vectors	6(6)
u, d, s -loops/short-distance	15(10)
c -loop	3(1)
total	92(19)

Scalar resonances contribution to (g-2)



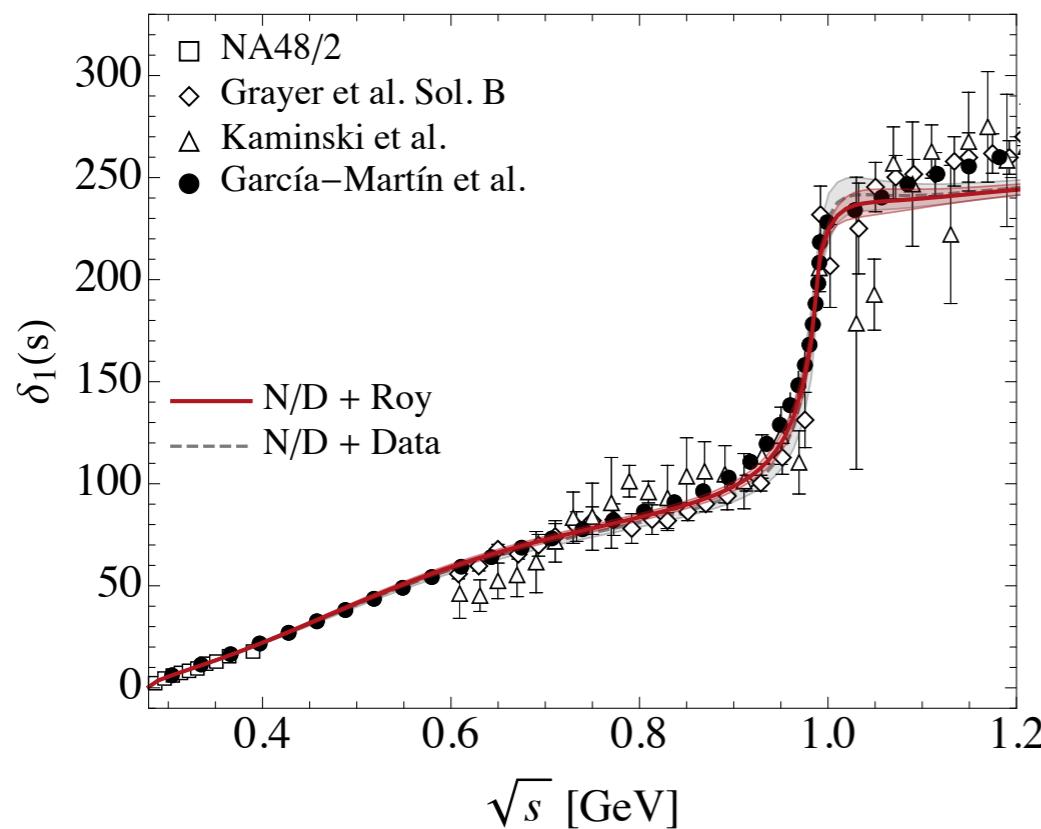
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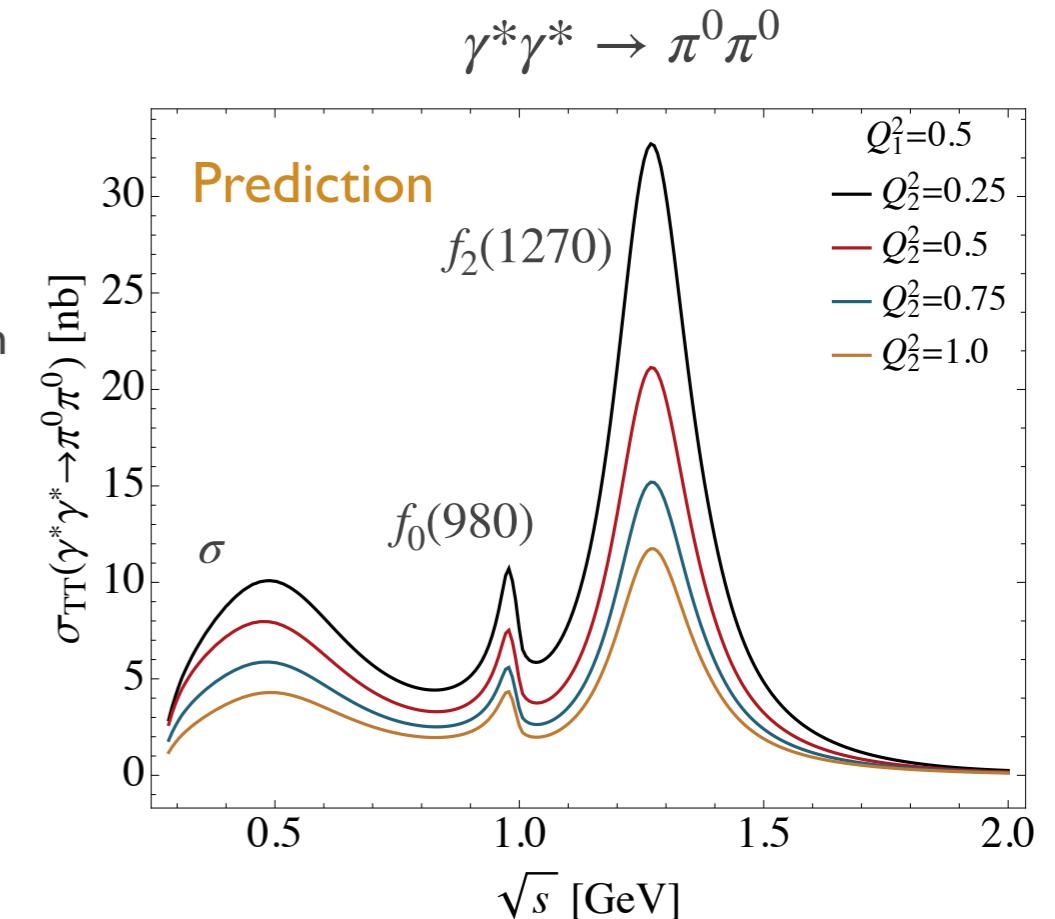


T. Aoyama et al. [g-2 White Paper] (2020)

Scalar resonances contribution to (g-2)

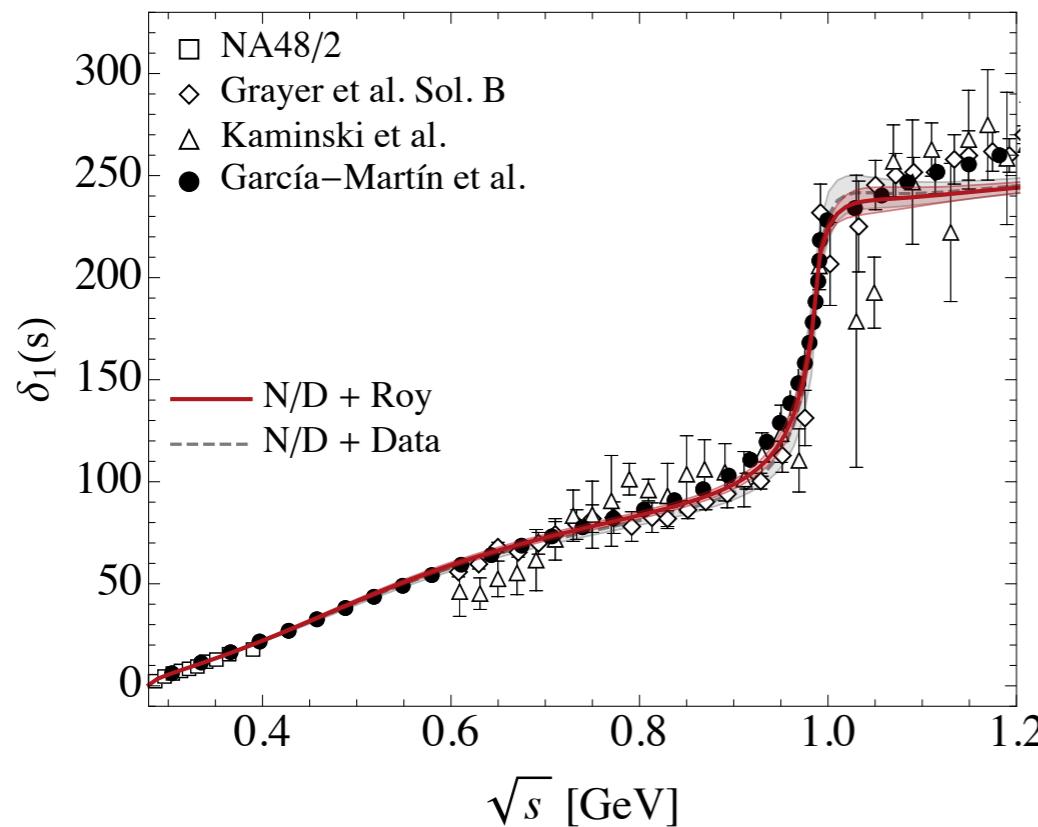


data-driven
dispersive approach

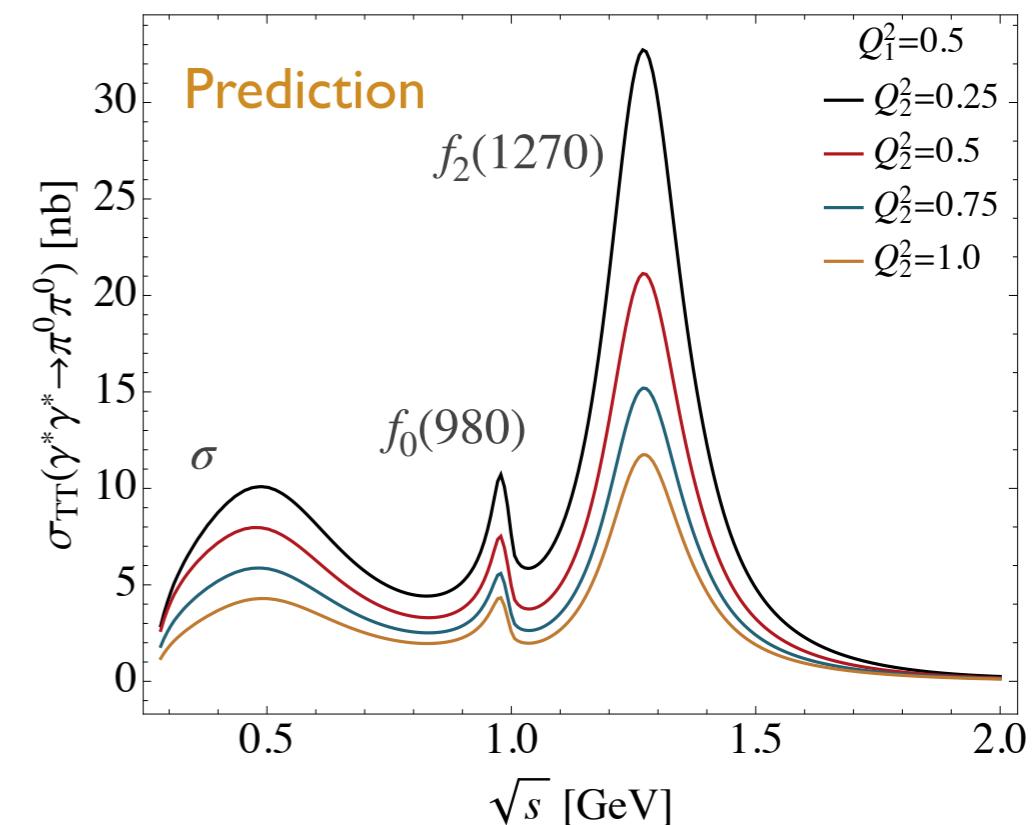
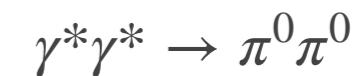


Danilkin, Deineka, Vanderhaeghen (2020)

Scalar resonances contribution to ($g-2$)



data-driven
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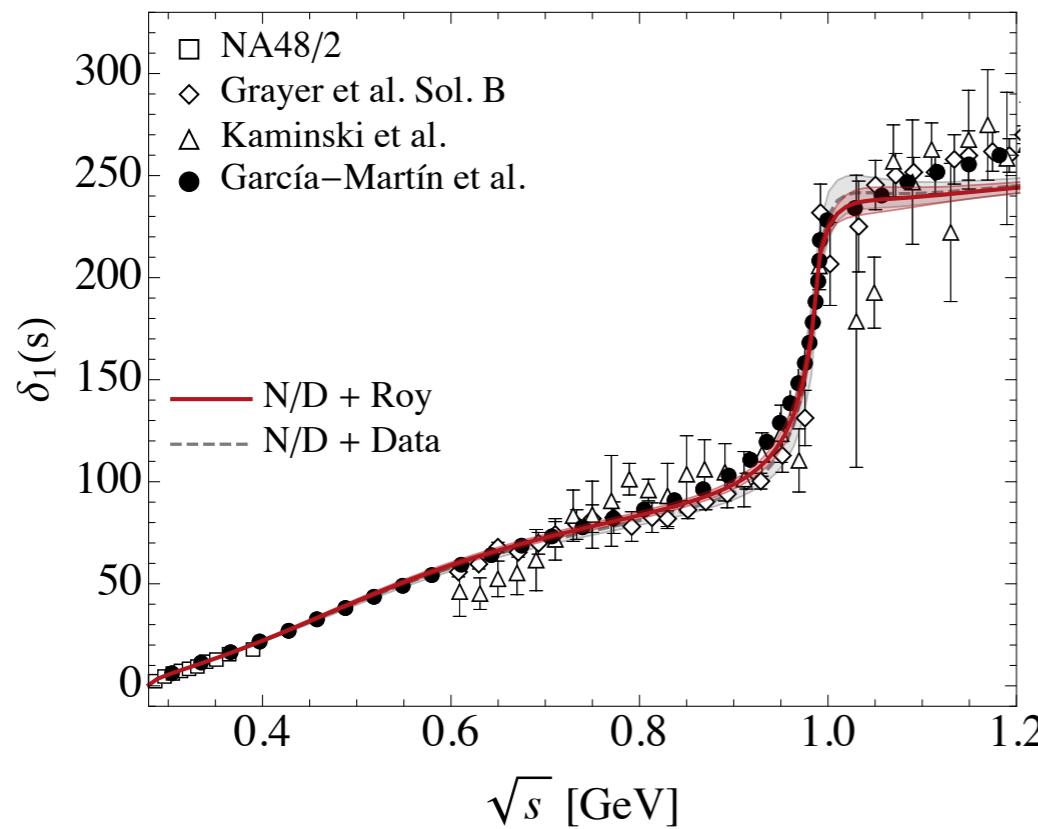


Danilkin, Deineka, Vanderhaeghen (2020)

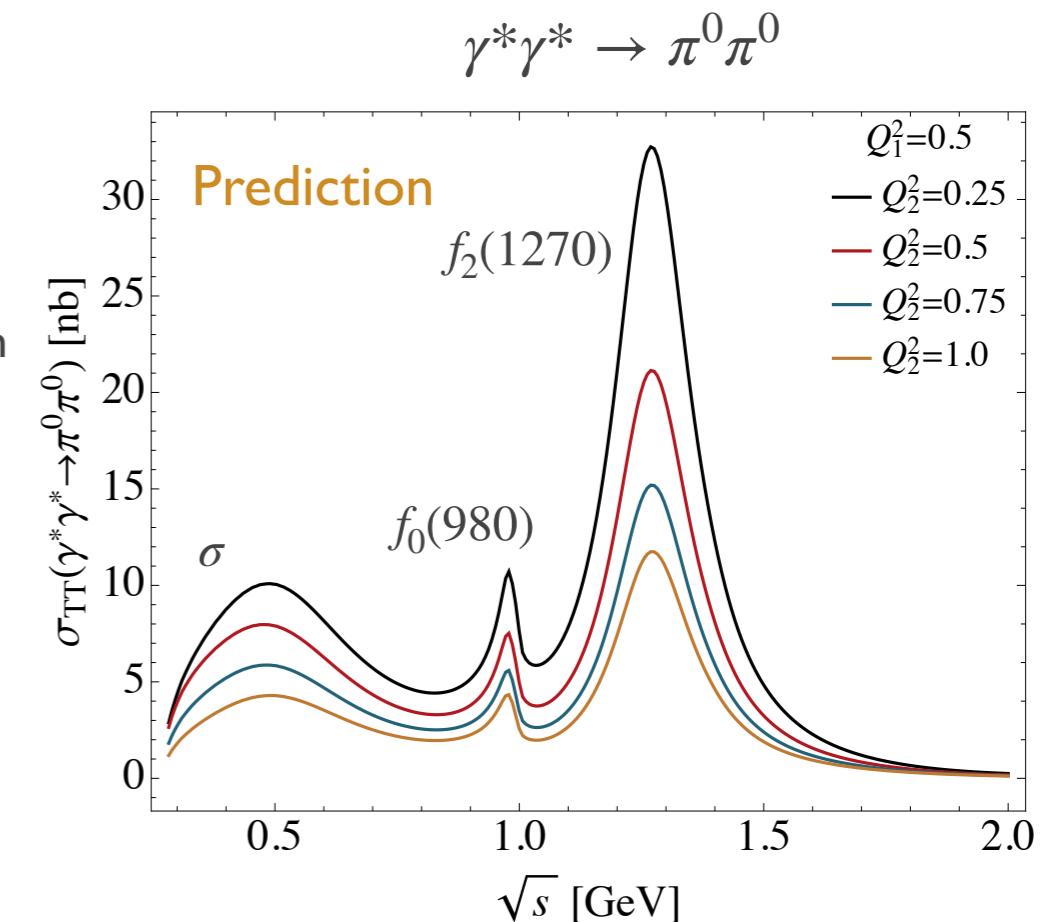
Very precise data for the $\{\pi\pi, KK\}$ system allowed to calculate the contribution from the $f_0(980)$ resonance

$$a_\mu^{\text{HLbL}}[f_0(980)] = -0.2(2) \times 10^{-11} \quad \text{Danilkin, Hoferichter, Stoffer (2021)}$$

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Problem: for precise determination of the $a_0(980)$ resonance the experimental input for the $\{\pi\eta, KK\}$ is needed, but there is **no hadronic information** in $\pi\eta$ channel

Resonance with definite J : need **partial wave** (p.w.) decomposition

$$T_{ab} = \sum_{J=0}^{\infty} (2J+1) t_{ab}^J(s) P_J(\cos \theta)$$

↑
coupled-channel indices

Unitarity relation: p.w. amplitude behaves asymptotically no worse than a constant

$$\text{Disc } t_{ab}(s) = \sum_c t_{ac}(s) \rho_c(s) t_{cb}^*(s)$$

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phase-space factor

Approach: partial wave dispersion relations

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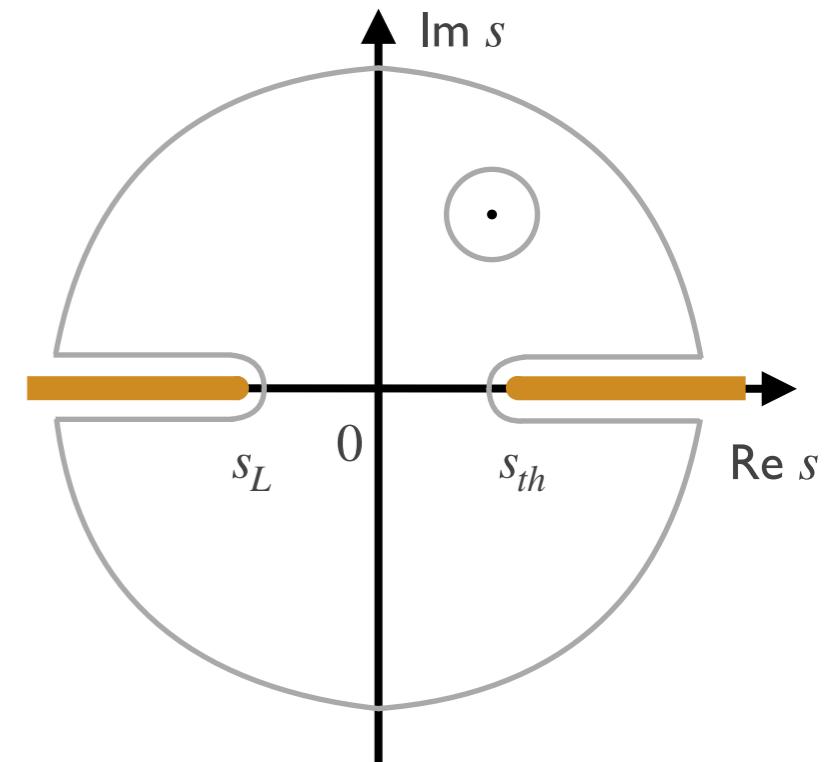
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Once-subtracted p.w. dispersion relation

$$t_{ab}(s) = t_{ab}(0) + \underbrace{\frac{s}{\pi} \int_{-\infty}^{s_L} \frac{ds'}{s'} \frac{\text{Disc } t_{ab}(s')}{s' - s}}_{U_{ab}(s)} + \frac{s}{\pi} \sum_c \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{t_{ac}(s') \rho_c(s') t_{cb}^*(s')}{s' - s}$$

↑ subtraction constant and
left-hand cut contribution



Once-subtracted p.w. dispersion relation

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Can be solved by means of **N/D ansatz**

$$t_{ab}(s) = \left(\frac{N(s)}{D(s)} \right)_{ab}$$

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contributions from the
left-hand cuts (lhc)

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contributions from the
right-hand cuts (rhc)

$$N_{ab}(s) = U_{ab}(s) + \frac{s}{\pi} \sum_c \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{N_{ac}(s') \rho_c(s') (U_{cb}(s') - U_{cb}(s))}{s' - s}$$

$$D_{ab}(s) = \delta_{ab} - \frac{s}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{N_{ab}(s') \rho_b(s')}{s' - s} = \Omega_{ab}^{-1}(s)$$

Chew, Mandelstam (1960)
Luming (1964)
Johnson, Warnock (1981)

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[Chew, Mandelstam \(1960\)](#)
[Luming \(1964\)](#)
[Johnson, Warnock \(1981\)](#)

Using the known analytical structure of lhc one can approximate **hadronic** $U_{ab}(s)$ as an expansion in a **conformal mapping variable** $\xi(s)$

[Gasparyan, Lutz \(2010\)](#)

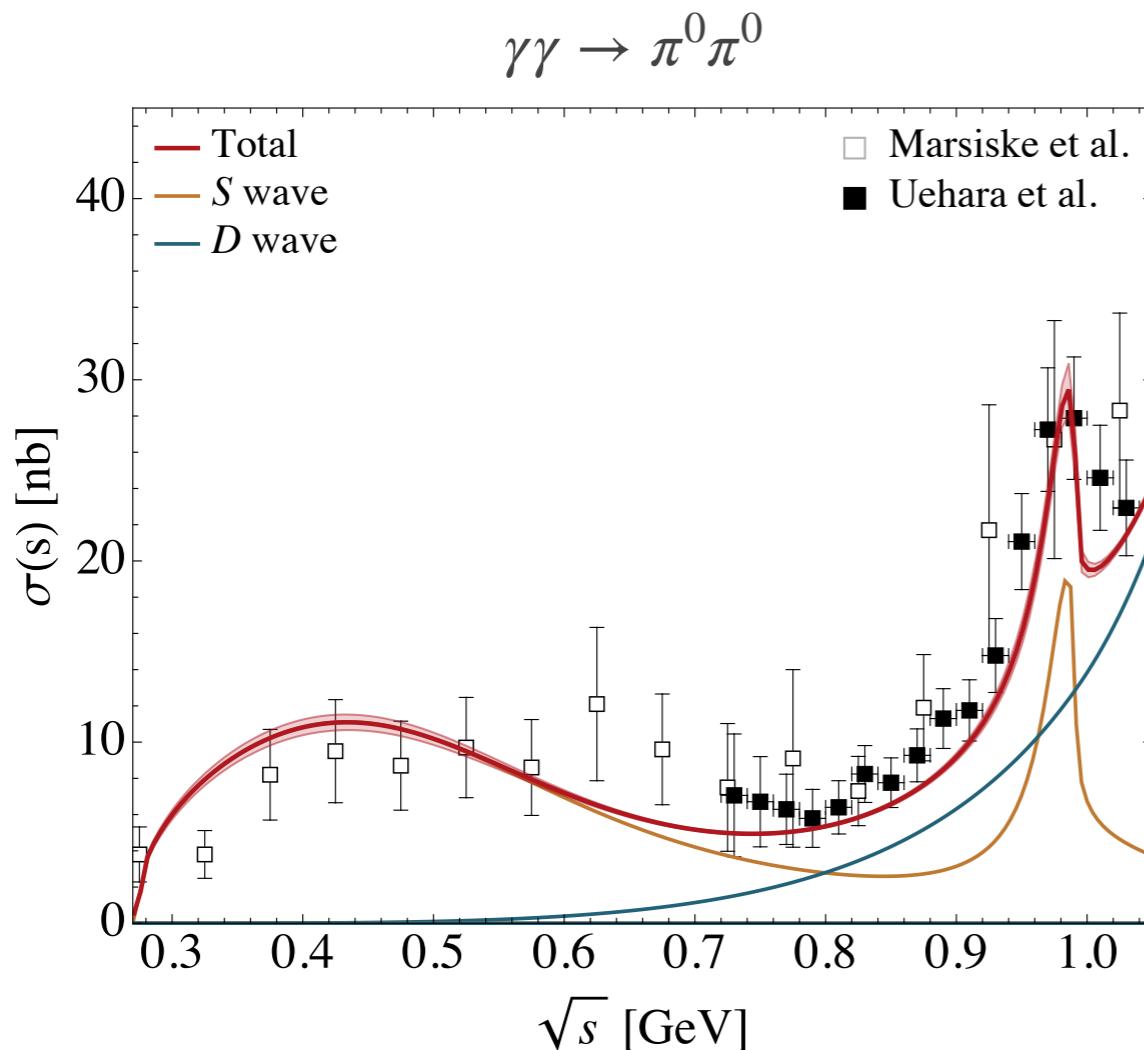
$$U(s) = \sum_{n=0}^{\infty} C_n (\xi(s))^n$$

coefficients fitted to data



physical region
 s_{th} s_E
 $\xi(s_E) = 0$

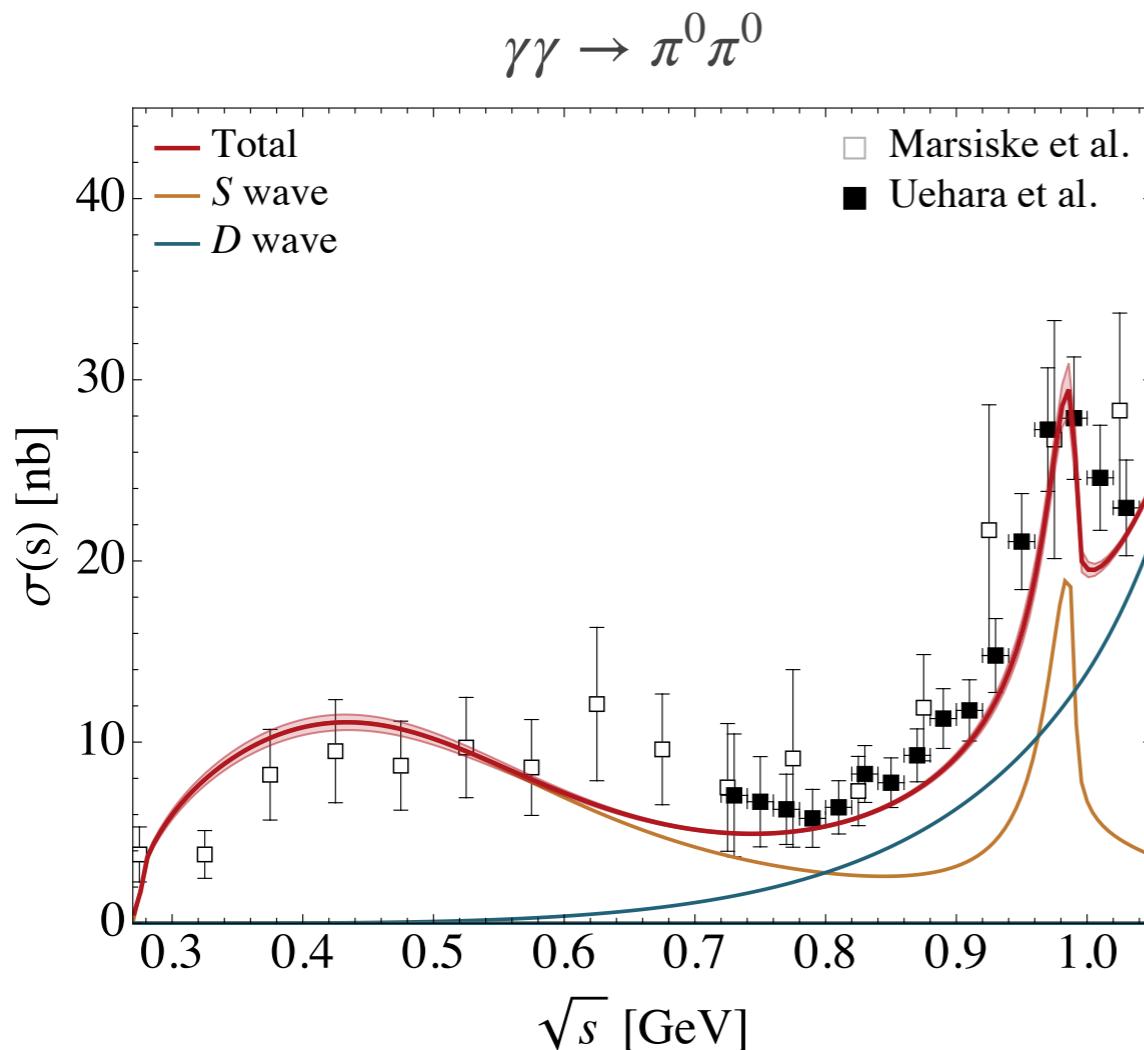
Dispersion relation: $\{\gamma\gamma, \pi\pi, KK\}$ system



$\gamma\gamma \rightarrow \{\pi\pi, KK\}, \quad I = 0,2$

$$\begin{pmatrix} h_{0,++}^{(0)}(s) \\ k_{0,++}^{(0)}(s) \end{pmatrix} = \begin{pmatrix} h_{0,++}^{(0),Born}(s) \\ k_{0,++}^{(0),Born}(s) \end{pmatrix} + s \Omega_0(s) \left[- \int_{s_{th}}^{\infty} \frac{ds'}{\pi} \frac{\text{Im } \Omega_0^{-1}(s')}{s'(s' - s)} \begin{pmatrix} h_{0,++}^{(0),Born}(s') \\ k_{0,++}^{(0),Born}(s') \end{pmatrix} + \text{heavier lhc} \right]$$

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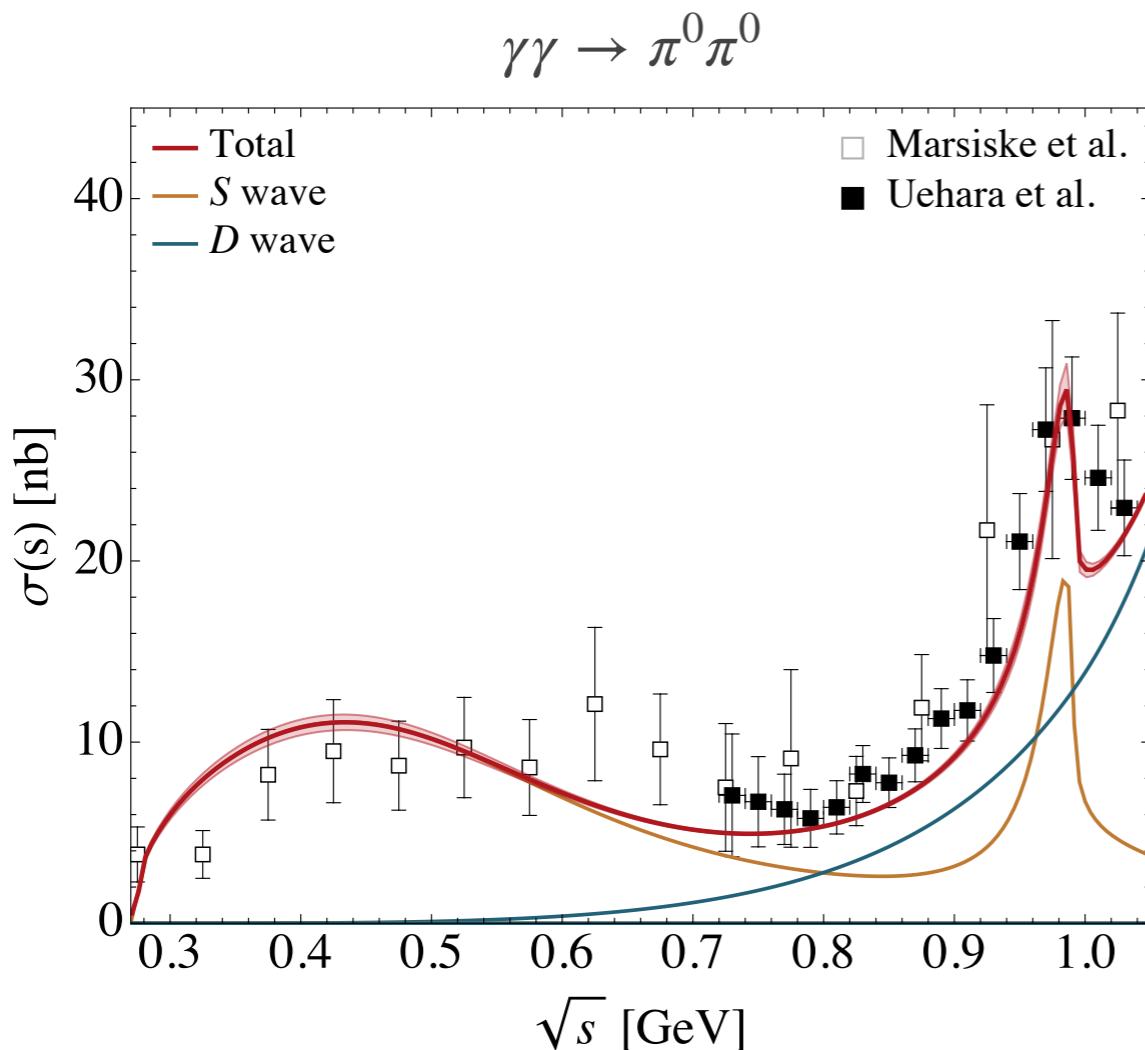


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X take into account only Born lhc

Dispersion relation: $\{\gamma\gamma, \pi\pi, KK\}$ system



$$\Gamma_{\gamma\gamma}[f_0(500)] = 1.37 \pm 0.13^{+0.09}_{-0.06} \text{ keV}$$

$$\Gamma_{\gamma\gamma}[f_0(980)] = 0.33 \pm 0.16^{+0.04}_{-0.16} \text{ keV}$$

Danilkin, Deineka, Vanderhaeghen (2020)

consistent with

$$\Gamma_{\gamma\gamma}^{Roy}[f_0(500)] = 1.7 \pm 0.4 \text{ keV}$$

$$\Gamma_{\gamma\gamma}^{MO}[f_0(980)] = 0.29 \pm 0.21^{+0.02}_{-0.07} \text{ keV}$$

$$\Gamma_{\gamma\gamma}^{Ampl. an.}[f_0(980)] = 0.32 \pm 0.05 \text{ keV}$$

Hoferichter et al. (2021)

Moussallam (2011)

Dai et al. (2014)

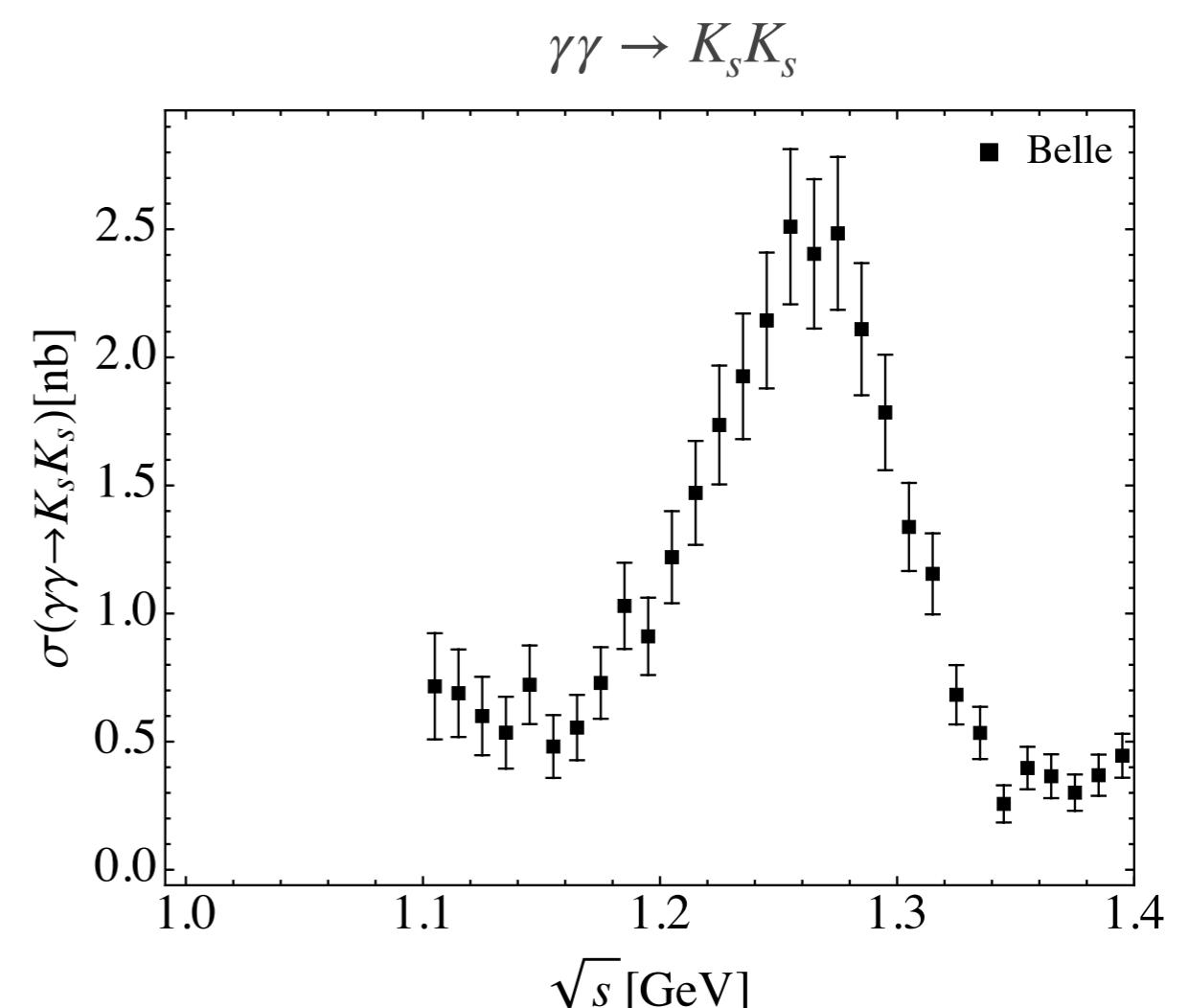
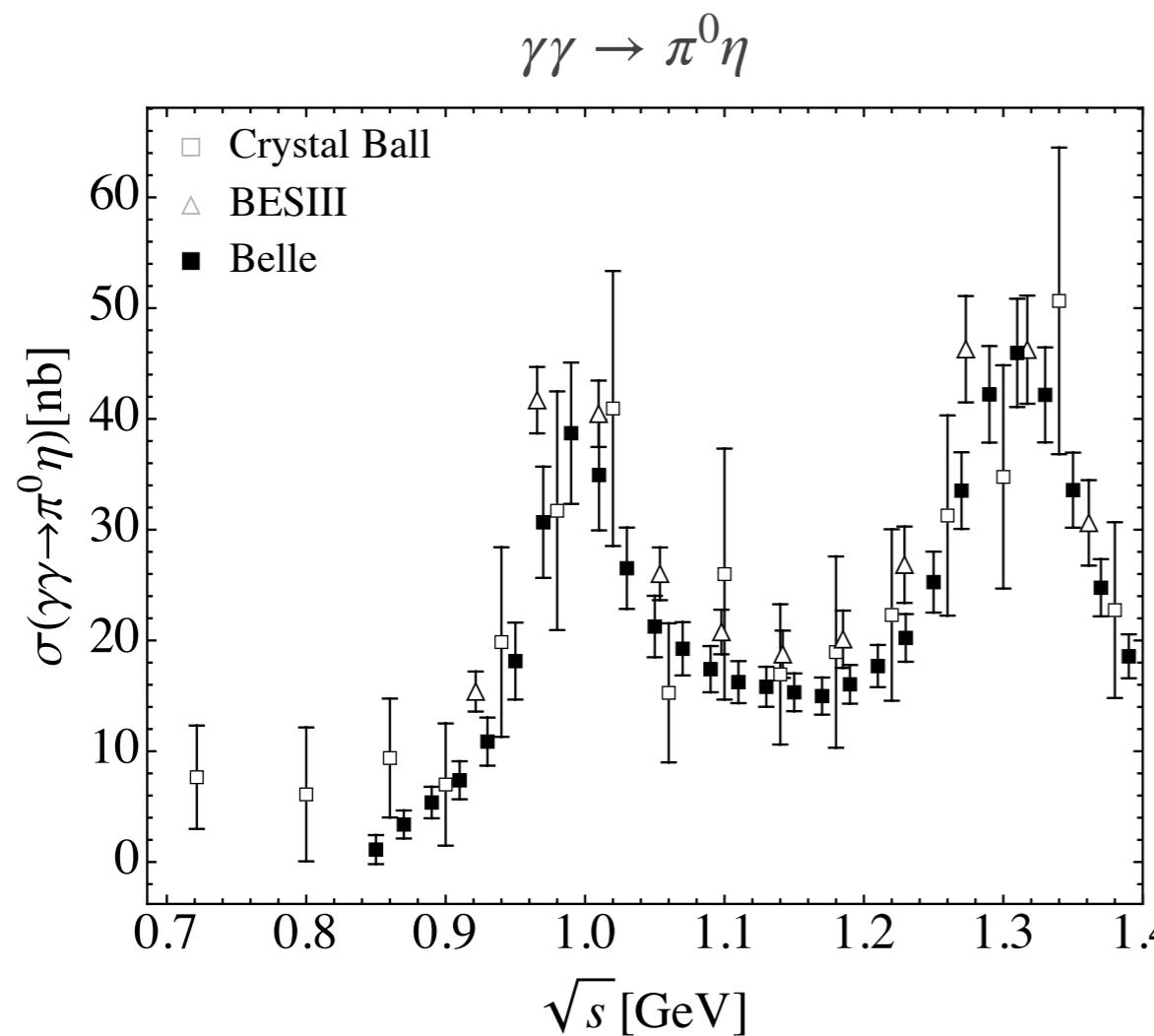
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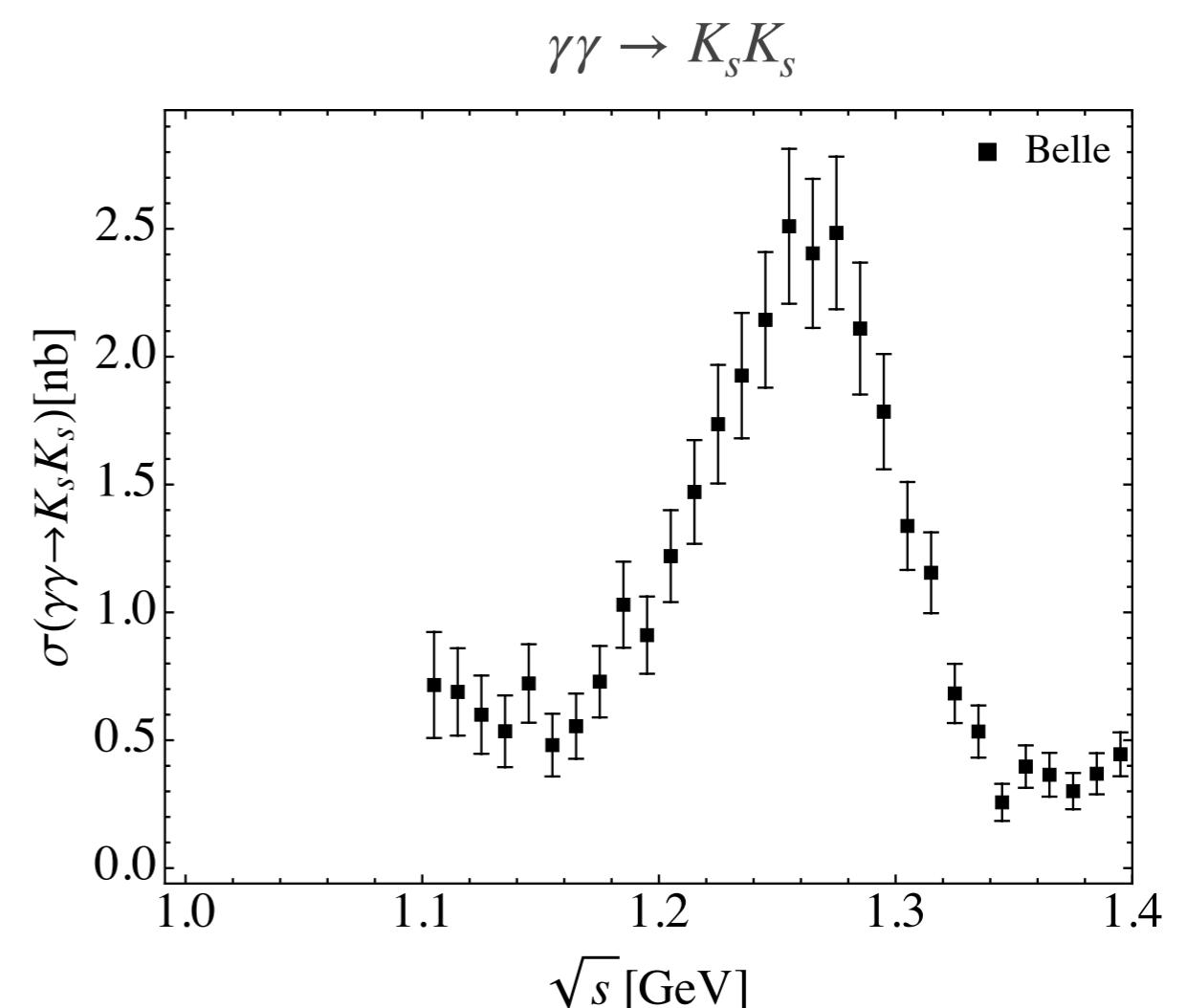
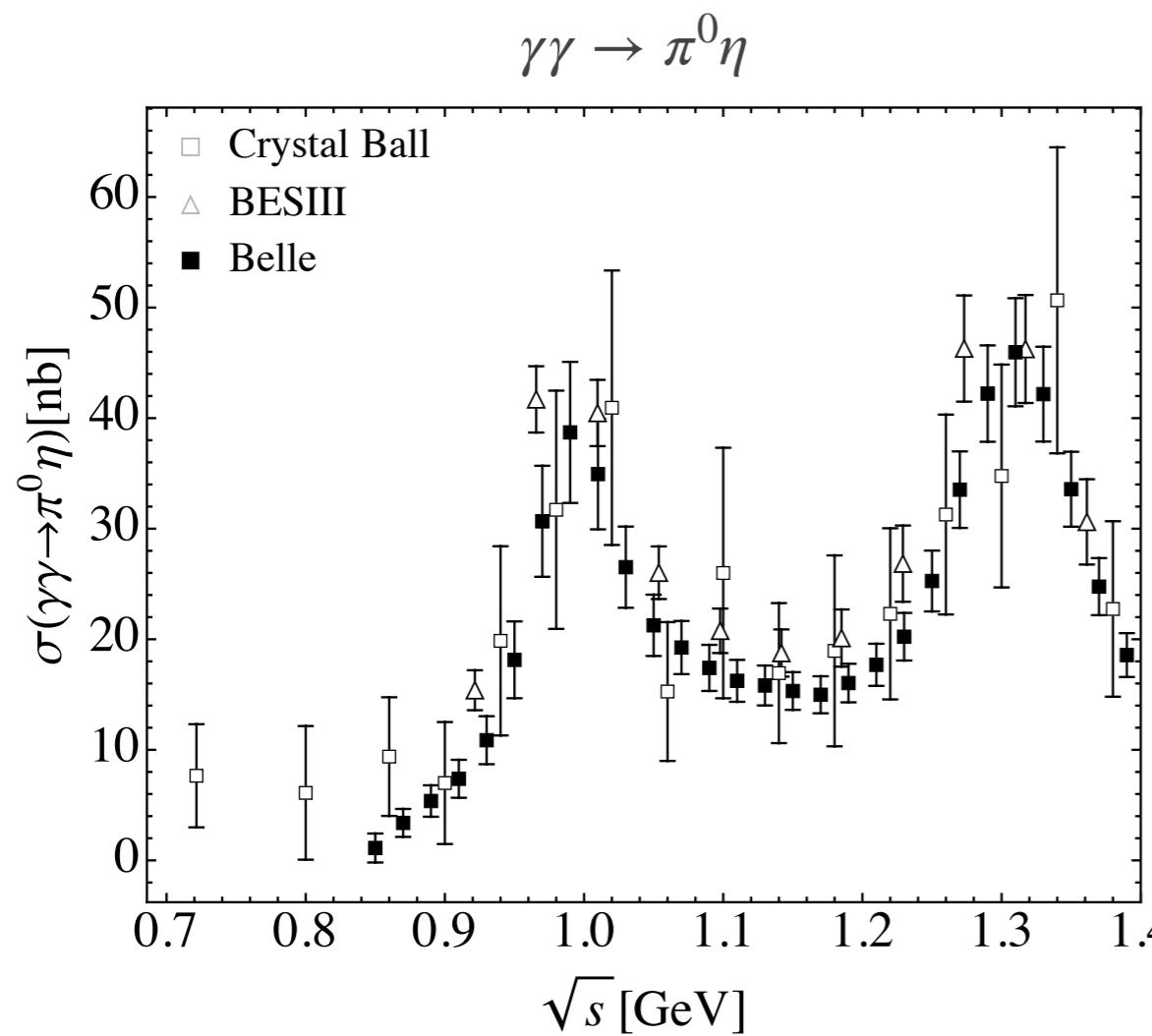
Dispersion relation: $\{\gamma\gamma, \pi\eta, KK\}$ system



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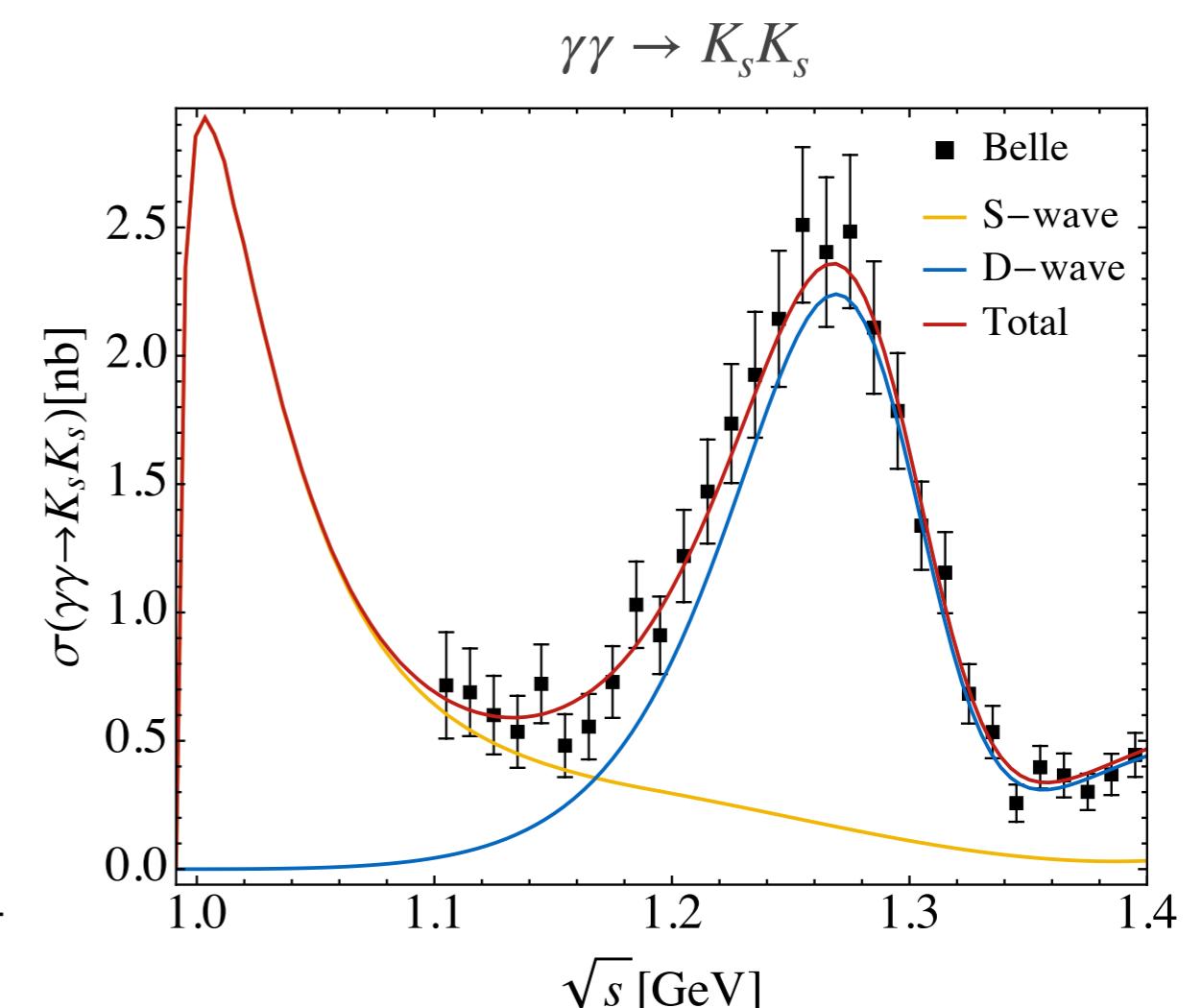
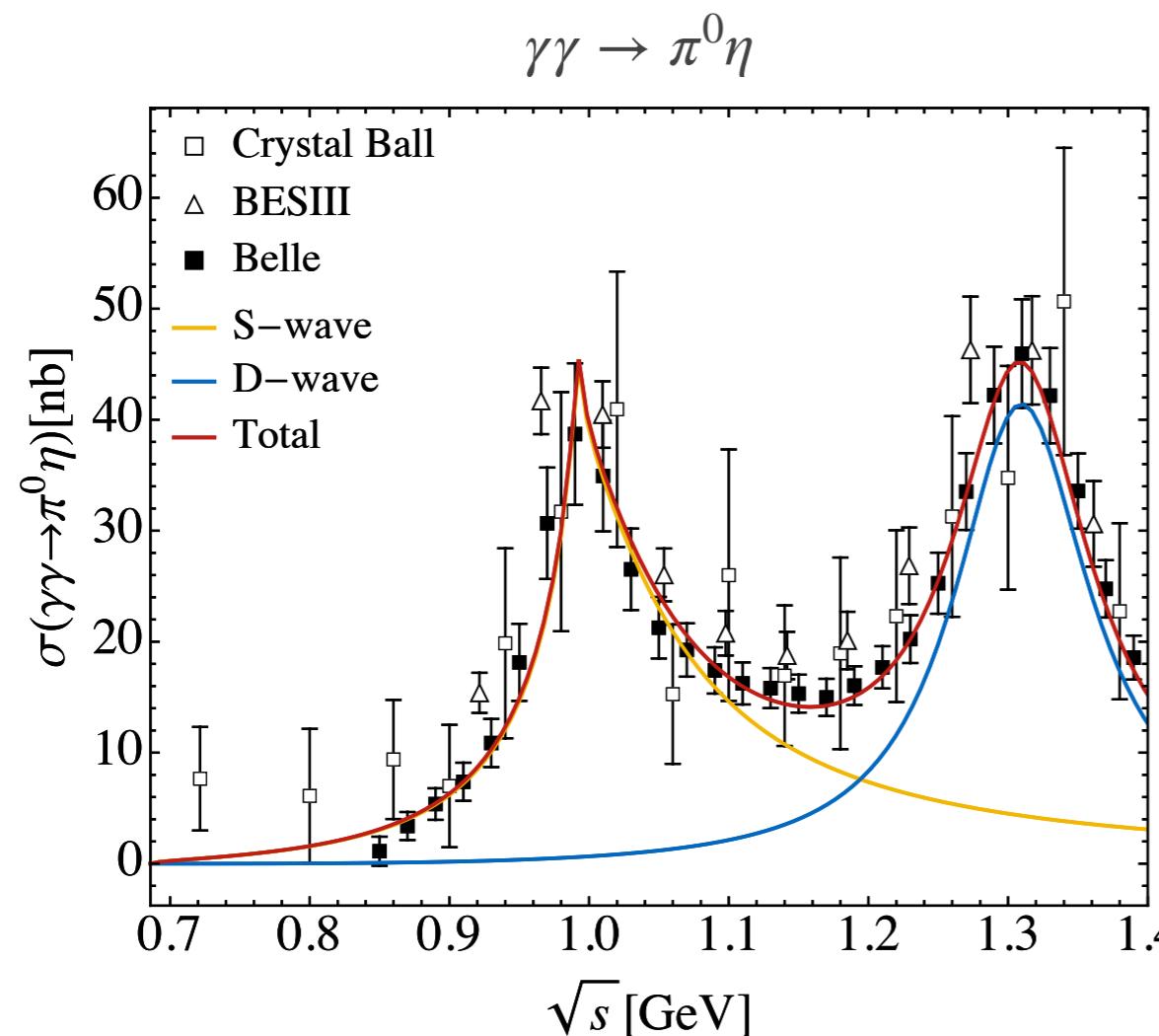


$\gamma\gamma \rightarrow \{\pi\eta, KK\}, \quad I = 1$

coefficients C_n fitted to the cross-section data + χPT constraints

$$\begin{pmatrix} h_{1,++}^{(0)}(s) \\ k_{1,++}^{(0)}(s) \end{pmatrix} = \begin{pmatrix} 0 \\ k_{1,++}^{(0),\text{Born}}(s) \end{pmatrix} + s \Omega_1(s) \left[- \int_{s_{th}}^{\infty} \frac{ds'}{\pi} \frac{\text{Im } \Omega_1^{-1}(s')}{s'(s'-s)} \begin{pmatrix} 0 \\ k_{1,++}^{(0),\text{Born}}(s') \end{pmatrix} + \text{heavier lhc} \right]$$

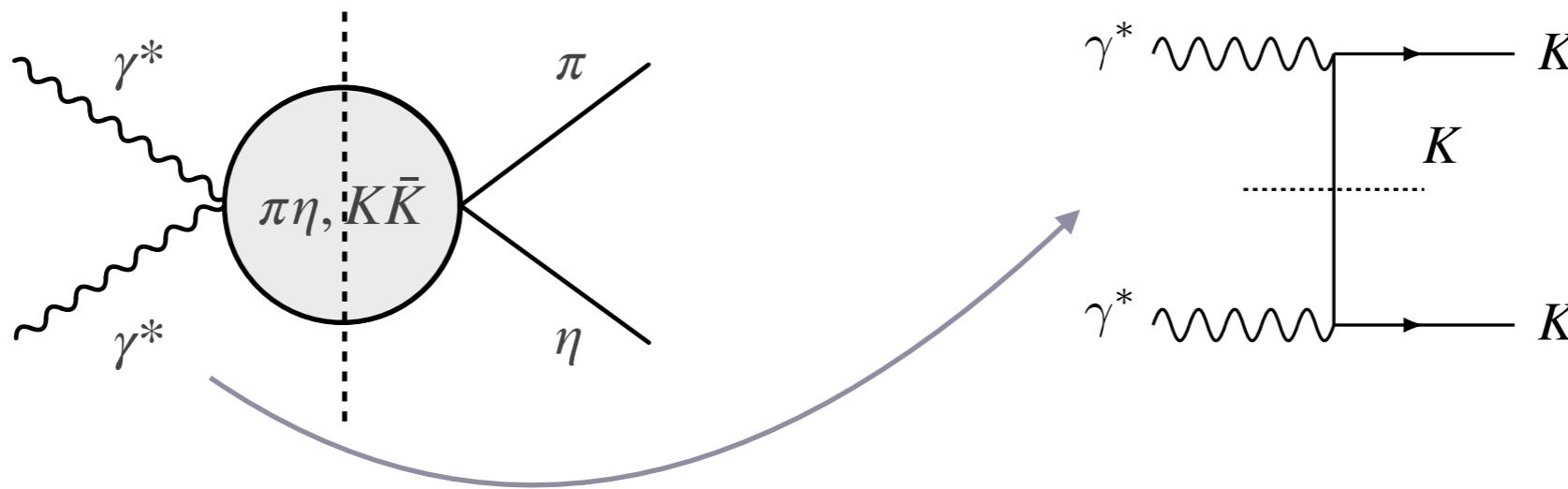
D-wave: Breit-Wigner parametrisation for $a_2(1320)$ and $f_2(1270)$



$a_0(980)$ is located on **the II Riemann sheet**:

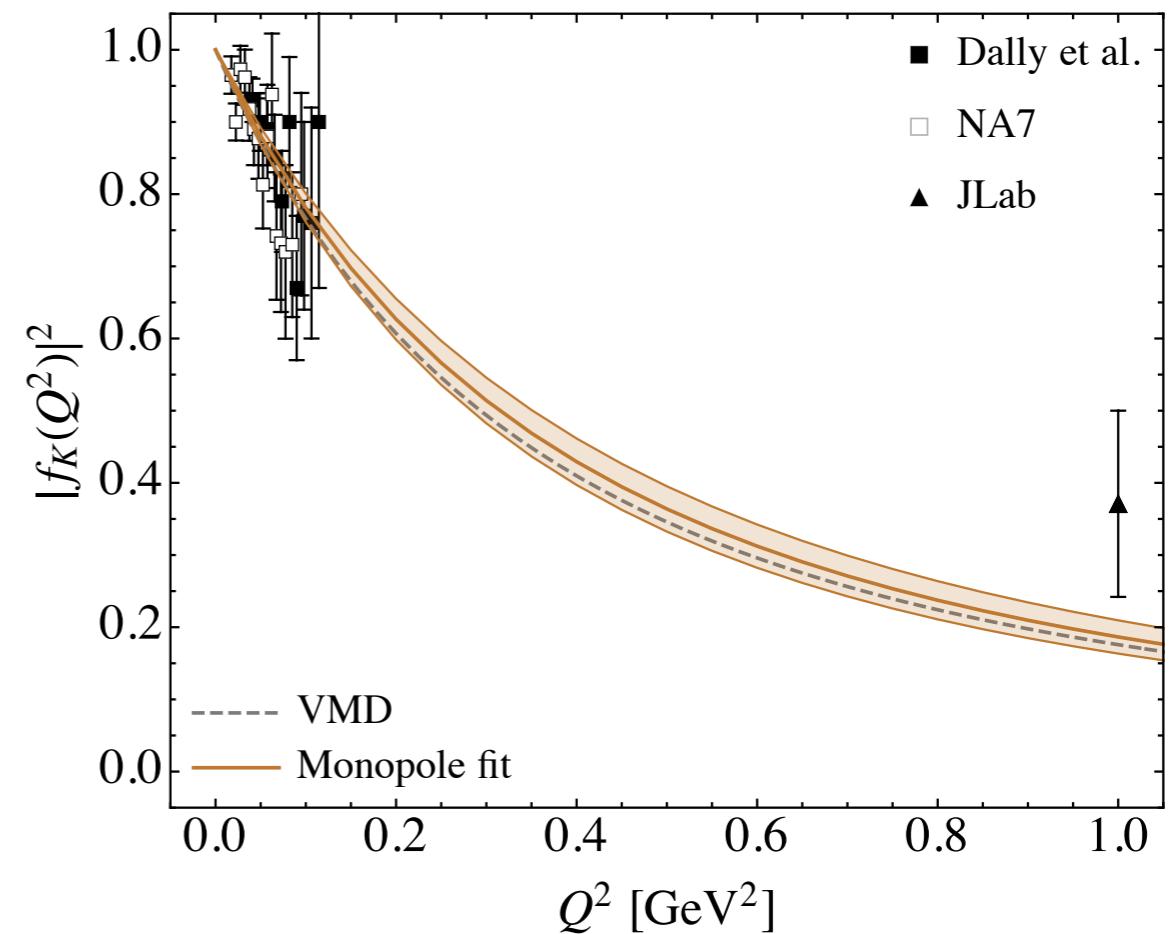
$$\sqrt{s_{a_0(980)}^{II}} = 1064 - i59 \text{ MeV}$$

Extension to the double-virtual case

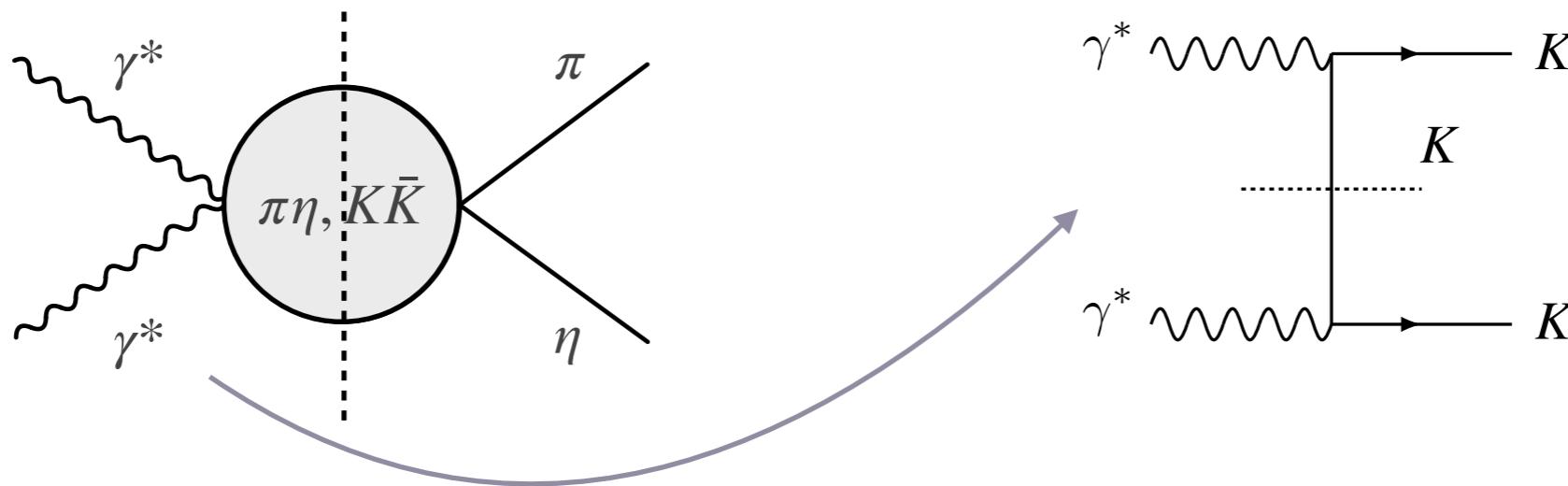


Left-hand cut require knowledge from γ^*KK form factor

$$\text{Disc} \left[\begin{array}{c} \gamma^* \\ \text{---} \\ K \\ \text{---} \\ K \end{array} \right] = \gamma^* \text{---} \left[\begin{array}{c} \dots \\ \text{had} \\ \dots \end{array} \right] \text{---} K$$



Extension to the double-virtual case

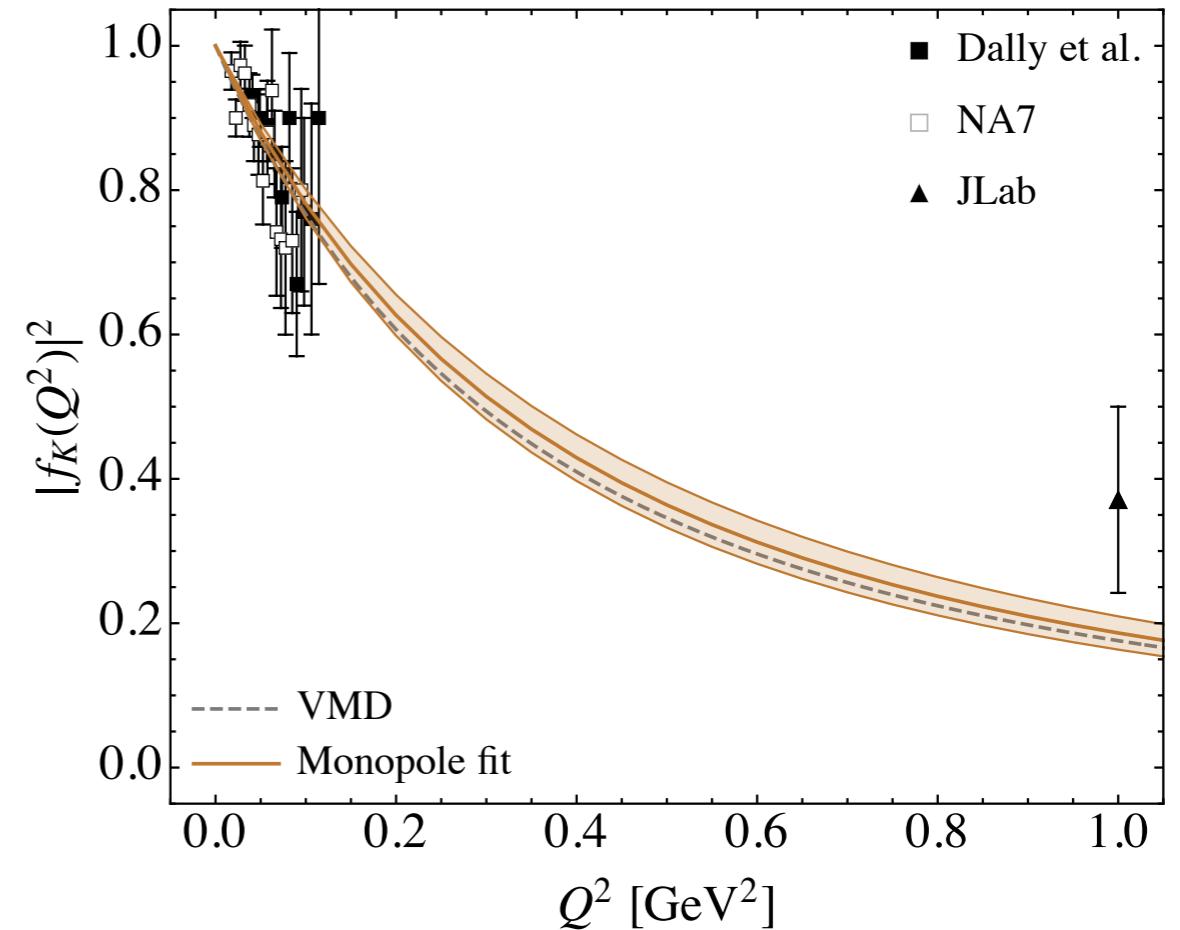


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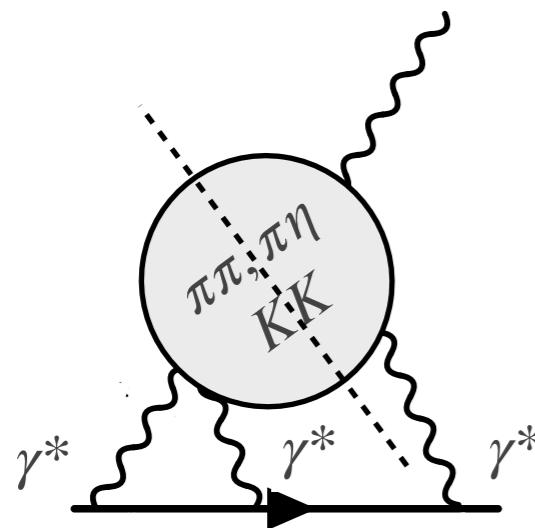
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p.w. helicity amplitudes suffer from kinematic constraints. For S -wave ($\bar{h} \equiv h - h^{\text{Born}}$)

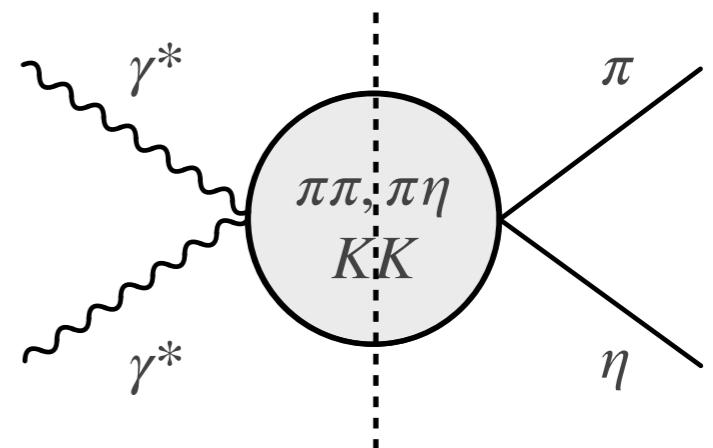
$$\bar{h}_{++} \pm \bar{h}_{00} \sim (s + (Q_1 \pm Q_2)^2)$$



Contribution to HLbL in ($g-2$)



Important ingredients
 $\gamma^* \gamma^* \rightarrow \pi\pi, \pi\eta, K\bar{K} \dots$
 $q = -Q^2 < 0$ spacelike γ^*

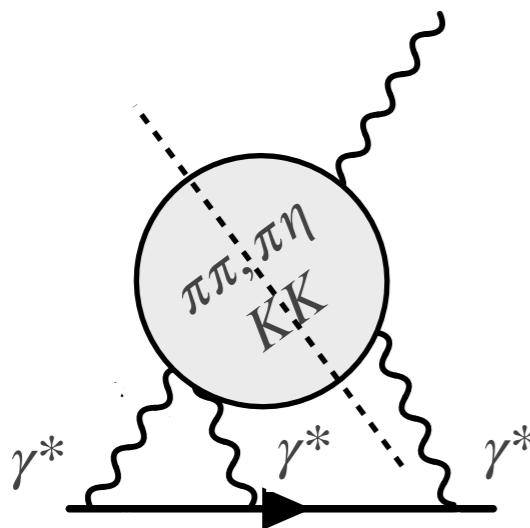


$$a_\mu^{HLbL} = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, Q_3) \bar{\Pi}_i(Q_1, Q_2, Q_3)$$

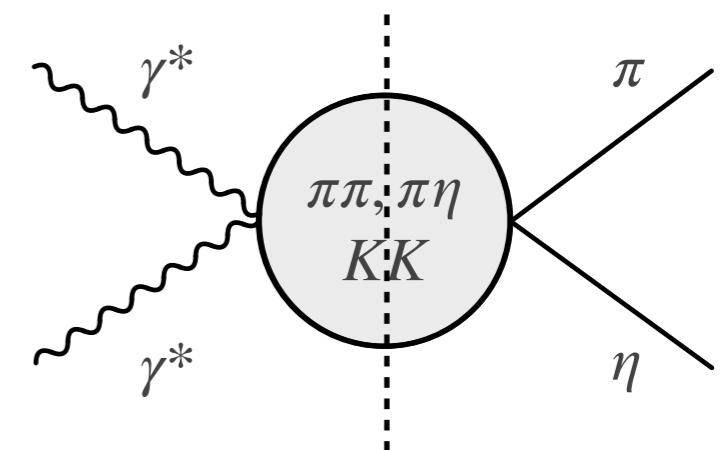
Colangelo et al. (2014-2017)

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i \quad \bar{\Pi}_i \text{ linear combination of } \Pi_i$$

Contribution to HLbL in $(g-2)$



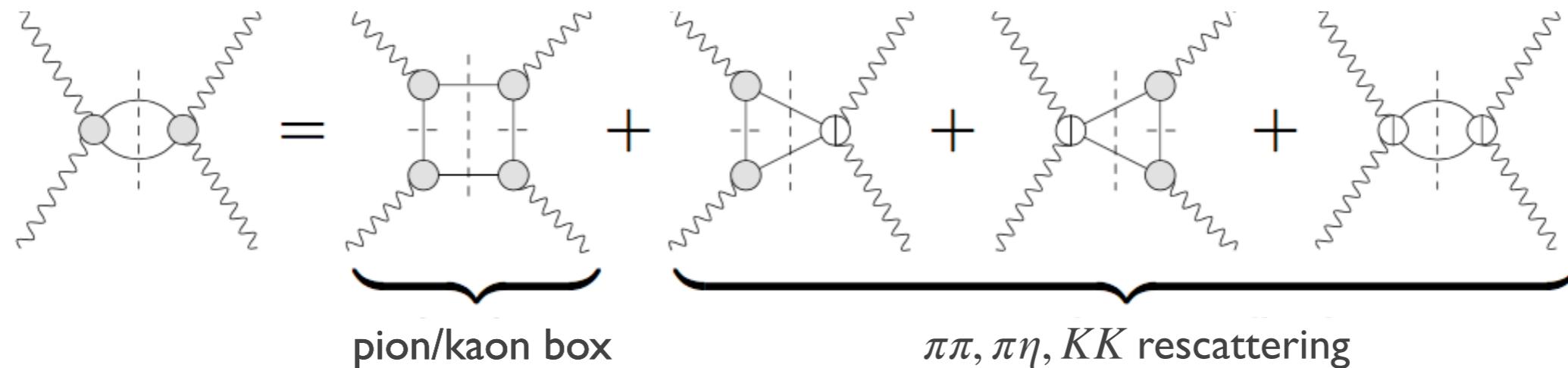
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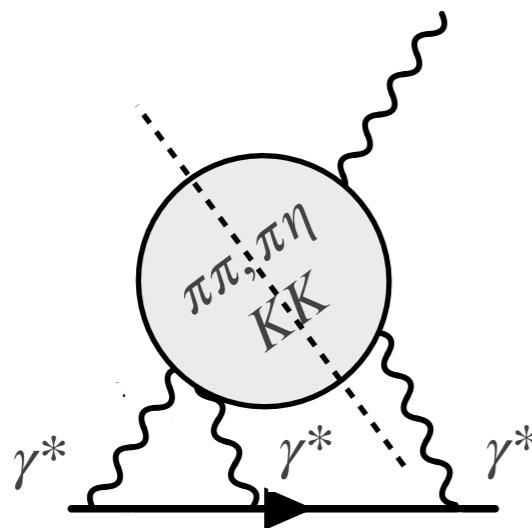
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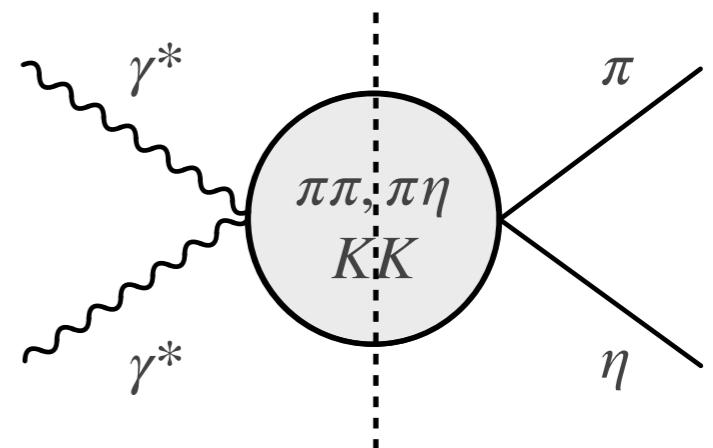
$$a_\mu^{\pi\pi, KK}[\text{box}] = -16.4(2) \times 10^{-11}$$

$$a_\mu^{HLbL}[\text{S-wave}, I=0] = -9.8(1) \times 10^{-11}$$

Contribution to HLbL in ($g-2$)



Important ingredients
 $\gamma^*\gamma^* \rightarrow \pi\pi, \pi\eta, K\bar{K} \dots$
 $q = -Q^2 < 0$ spacelike γ^*



$$a_\mu^{HLbL} = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, Q_3) \bar{\Pi}_i(Q_1, Q_2, Q_3)$$

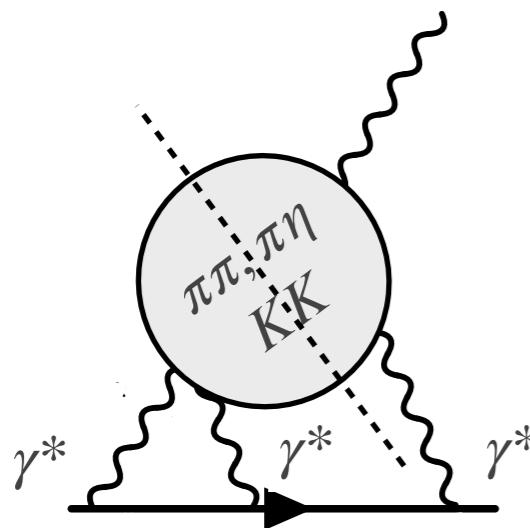
Rescattering contribution ($\bar{h} \equiv h - h^{Born}$) in the S -wave

$$\bar{\Pi}_3^{J=0} = \frac{1}{\pi} \int_{s_{th}}^\infty ds' \frac{-2}{\lambda_{12}(s')(s'+Q_3^2)^2} \left(4s' \text{Im} \bar{h}_{++}^{(0)}(s') - (s' - Q_1^2 + Q_2^2)(s' + Q_1^2 - Q_2^2) \text{Im} \bar{h}_{00,++}^{(0)}(s') \right)$$

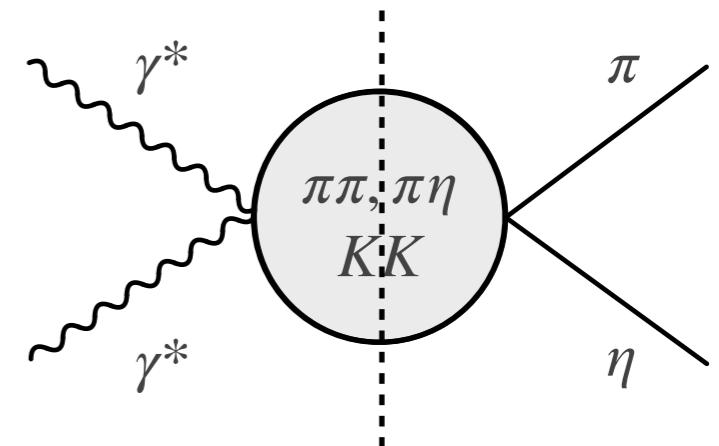
$$\bar{\Pi}_9^{J=0} = \frac{1}{\pi} \int_{s_{th}}^\infty ds' \frac{4}{\lambda_{12}(s')(s'+Q_3^2)^2} \left(2 \text{Im} \bar{h}_{++}^{(0)}(s') - (s' + Q_1^2 + Q_2^2) \text{Im} \bar{h}_{00,++}^{(0)}(s') \right)$$

+crossed

Contribution to HLbL in ($g-2$)



Important ingredients
 $\gamma^* \gamma^* \rightarrow \pi\pi, \pi\eta, K\bar{K} \dots$
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+crossed

Unitarity:

$$\gamma^* \gamma^* \rightarrow \gamma^* \gamma^* \quad \begin{array}{c} \curvearrowleft \\ \gamma^* \gamma^* \rightarrow \pi\eta \end{array} \quad \begin{array}{c} \curvearrowleft \\ \gamma^* \gamma^* \rightarrow K\bar{K} \end{array}$$

$$\text{Im} \bar{h}_{\lambda_1 \lambda_2, \lambda_3 \lambda_4}^{(0)}(s) = \bar{h}_{\lambda_1 \lambda_2}^{(0)}(s) \rho_{\pi\eta}(s) \bar{h}_{\lambda_3 \lambda_4}^{(0)*}(s) + \bar{k}_{\lambda_1 \lambda_2}^{(0)}(s) \rho_{K\bar{K}}(s) \bar{k}_{\lambda_3 \lambda_4}^{(0)*}(s)$$

For $I = 0$, the contributions from $f_0(500) + f_0(980)$ was calculated previously:

$$a_{\mu}^{HLbL}[S\text{-wave}, I = 0]_{resc.} = -9.8(1) \times 10^{-11}$$

$$a_{\mu}^{HLbL}[f_0(980)]_{resc.} = -0.2(1) \times 10^{-11}$$

Colangelo et al. (2014-2017)

Danilkin, Hofferichter, Stoffer (2011)

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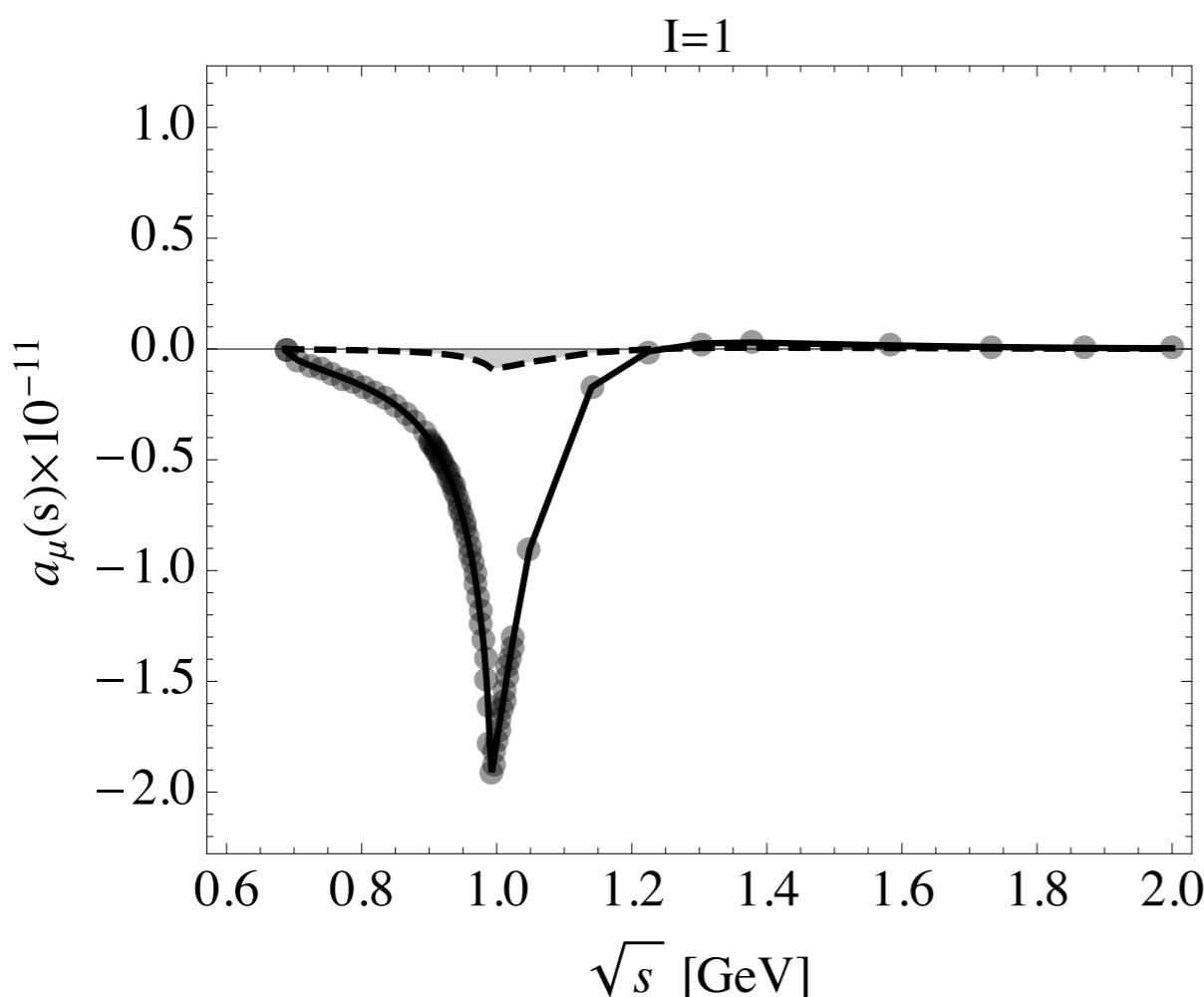
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$$a_{\mu}^{HLbL}[a_0(980)]_{resc.} = -0.46(2) \times 10^{-11}$$

$$a_{\mu}^{HLbL}[a_0(980)]_{NWA} = -([0.3, 0.6]_{-0.1}^{+0.2}) \times 10^{-11}$$

Schuler, Berends, van Gulik (1998)



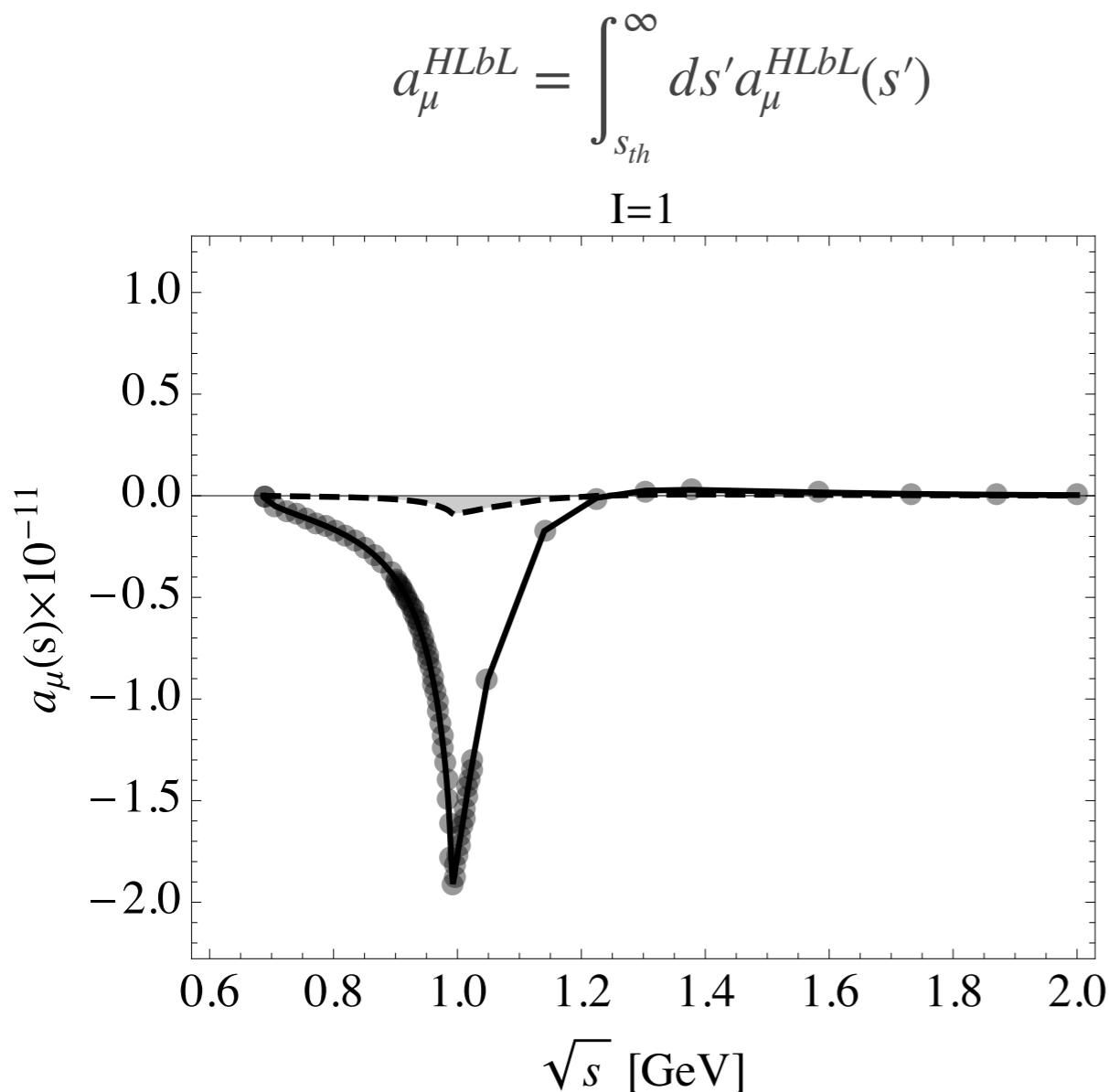
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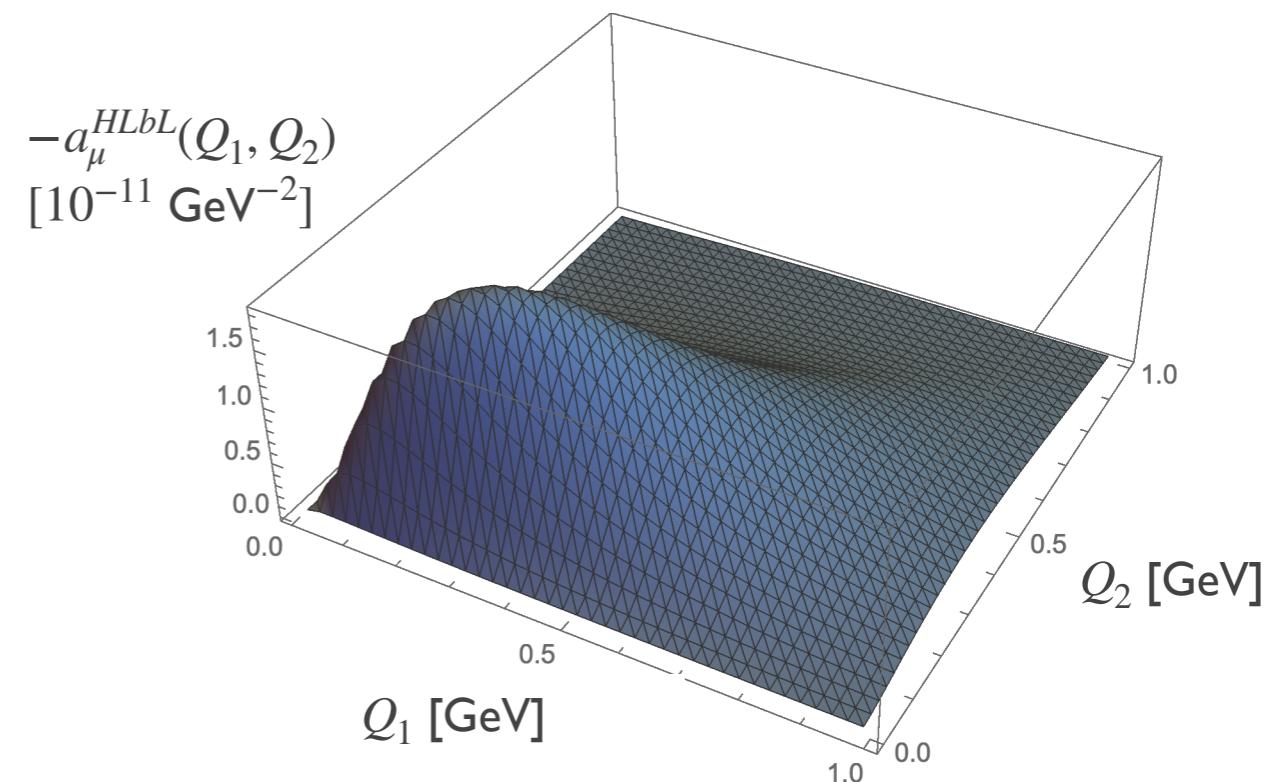
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Summary and outlook

- Using the partial-wave dispersive approach we described the $\gamma\gamma \rightarrow \pi^0\eta$ and $\gamma\gamma \rightarrow K_s\bar{K}_s$ cross sections simultaneously
- We found a pole corresponding to the $a_0(980)$ resonance on the II Riemann sheet with $\sqrt{s_{a_0(980)}^{II}} = 1064 - i59$ MeV (PRELIMINARY)
- We estimated the contribution from the $a_0(980)$ resonance to HLbL part of muon g-2 to be $a_\mu^{HLbL}[a_0(980)]_{resc.} = -0.46(2) \times 10^{-11}$ (PRELIMINARY)

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- The hadronic $\pi\eta$ part can be further constrained by including the data from $\phi \rightarrow \gamma\pi\eta$, $\eta' \rightarrow \pi\pi\eta$ and $\eta \rightarrow \pi^0\gamma\gamma$
- It is crucial to have the precise data in the $\gamma\gamma \rightarrow K^+K^-$ channel from BESIII
- The $\gamma\gamma \rightarrow \pi^0\eta$ Adler zero can be only included by introducing one more subtraction and including heavier left-hand cuts
- Using the once-subtracted dispersion relation for the single-virtual case relies on the availability of the $\gamma\gamma^* \rightarrow \pi^0\eta$ data from BESIII