Phenomenology of the ligthest hybrid meson nonet

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Outline

- Conventional mesons: brief review
- Light hybrids: masses and decays
- Decay of the J/Psi into (isoscalar) hybrids
- The Sill distribution as a useful tool
- Conclusions

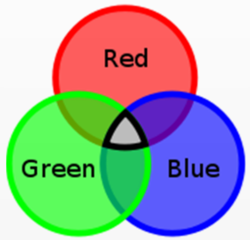
Symmetries of QCD

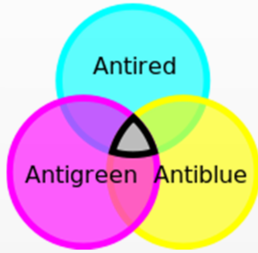
Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B



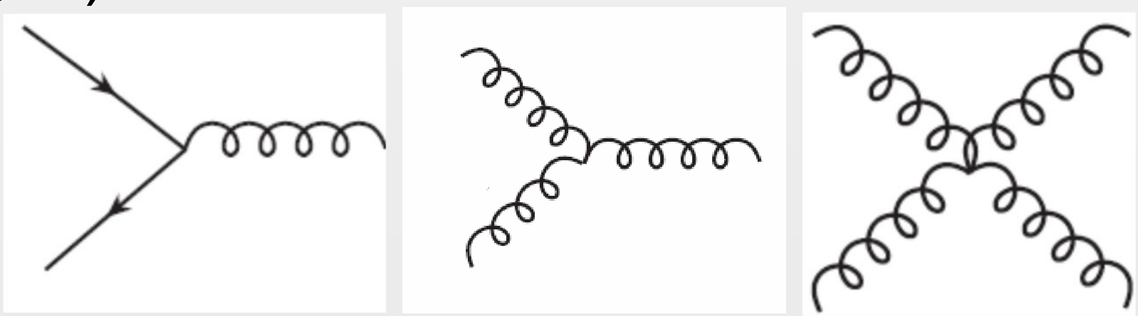


$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix};\;\; i = u,d,s\;,...$$

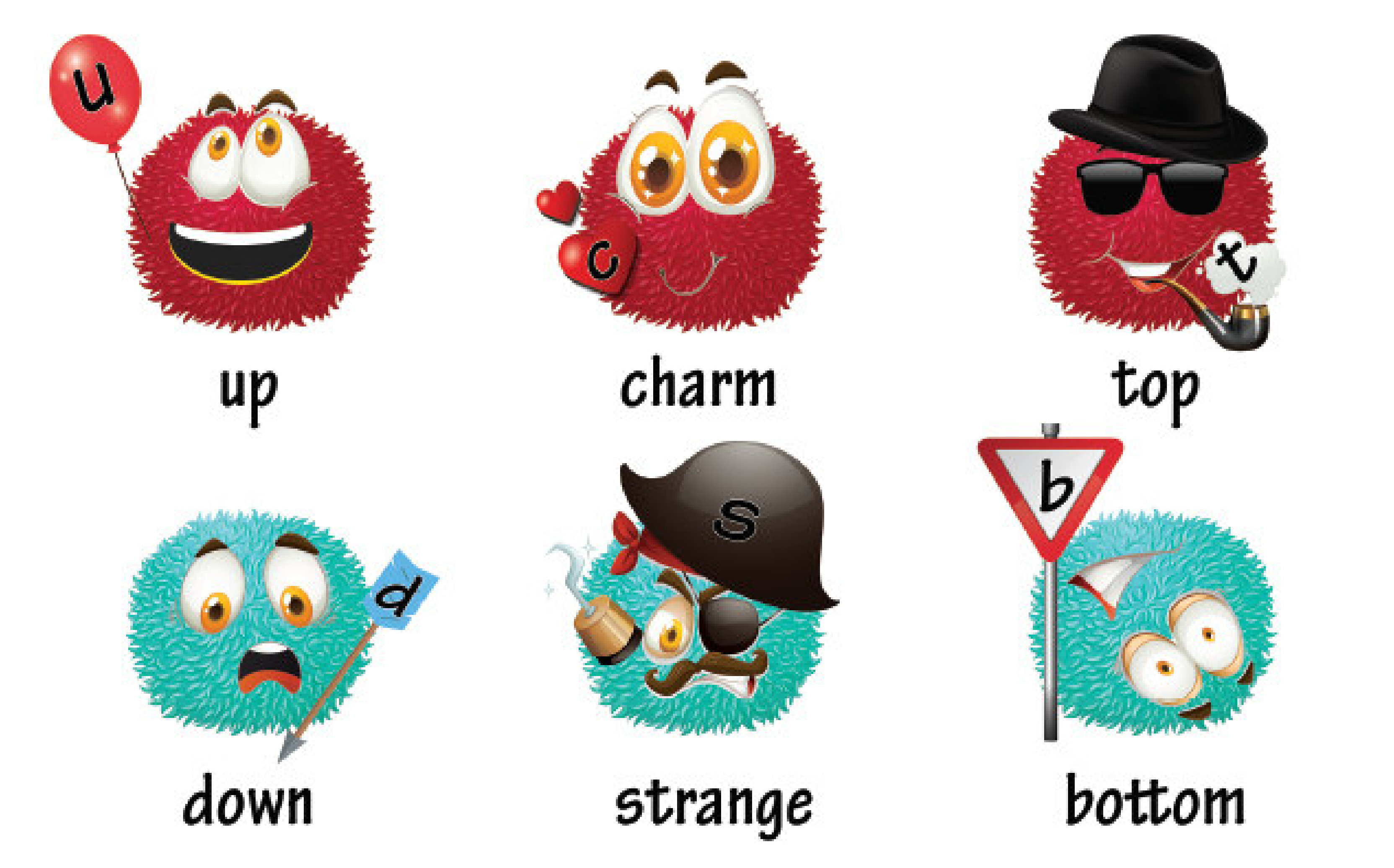
$$\boxed{\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}}$$

8 type of gluons (R~~G~~,B~~G~~,…)

$$A_\mu^a \;;\;\; a = 1,...,8$$

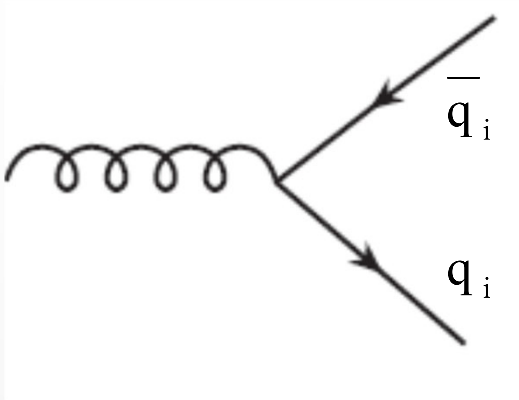


Confinement: quarks never ‘seen’ directly.
How they might look like ☺



Picture by Pawel Piotrowski

Flavor symmetry



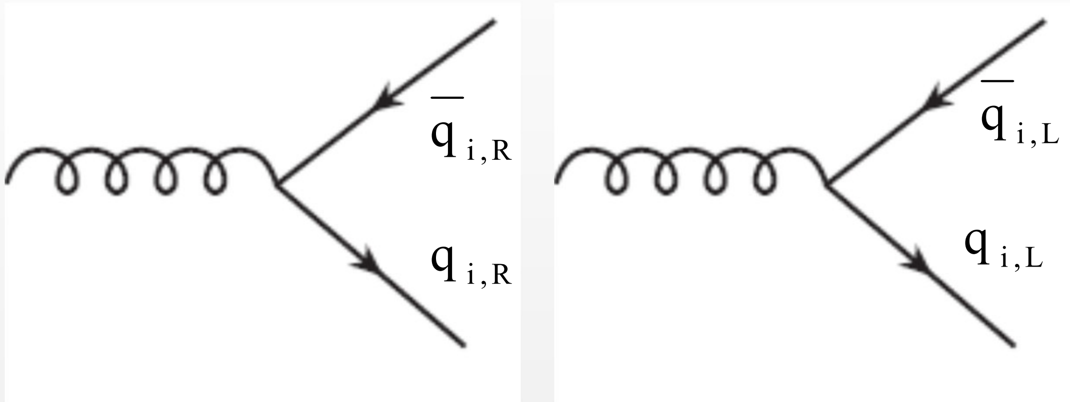
Gluon-quark-antiquark vertex

It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$\mathrm{U} \in \mathrm{U}(3)_{\mathrm{V}} \rightarrow \mathrm{U}^{+}\mathrm{U} = 1$$

Chiral symmetry



$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$\mathrm{q_i = q_{i,R} + q_{i,L} \rightarrow U^R_{ij} q_{j,R} + U^L_{ij} q_{j,L}}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

In the chiral limit (mi=0) chiral symmetry is exact, but is **spontaneously broken** by the QCD vacuum

Chiral (or axial) anomaly: explicitely broken by quantum fluctuations

$$\boxed{\partial^\mu(\bar{q}^i\gamma_\mu\gamma_5 q^i) = \frac{3g^2}{16\pi^2}\,\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}(G_{\mu\nu}G_{\rho\sigma})}$$

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are “white” and are called hadrons.

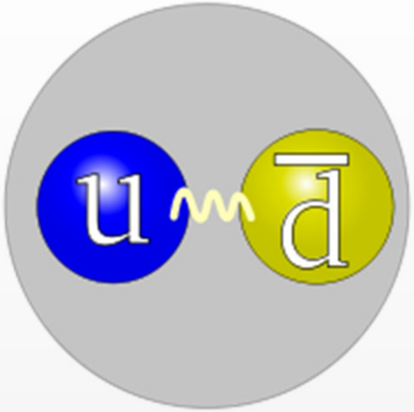
Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Example of conventional quark-antiquark states: the ρ and the π mesons



$$|\rho^+\rangle \propto |u\bar{d}\rangle + \frac{1}{N_c}\left(|\pi^+\pi^0\rangle + \ldots\right)$$

where

$$|u\bar{d}\rangle = |\text{valence } u + \text{valence } \bar{d} + \text{gluons}\rangle$$

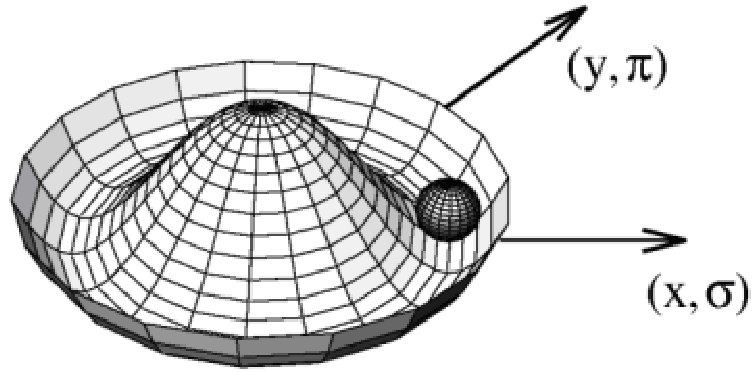
Rho-meson $\quad m_{\rho^+} = 775 \ \text{MeV}$

Pion $\quad m_{\pi^+} = 139 \ \text{MeV}$

$$m_u + m_d \approx 7 \ \text{MeV}$$

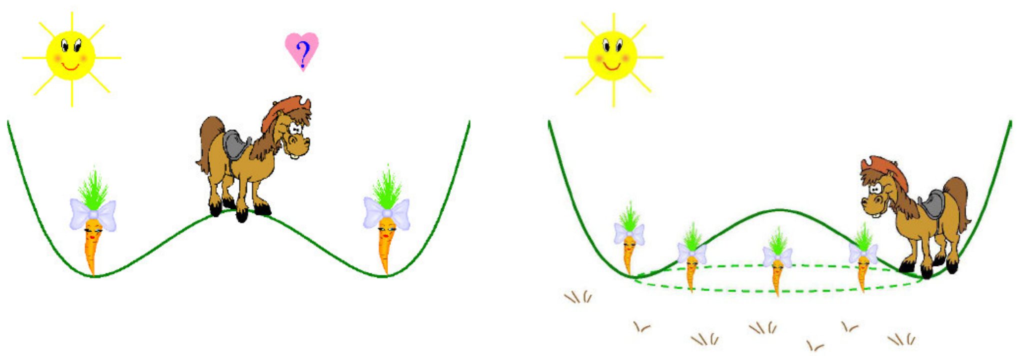
Mass generation in QCD is a nonpert. penomenon based on SSB
(mentioned previusly).

SSB and the donkey of Buridan: hadronic approaches



$$\boxed{\sigma_N \rightarrow \sigma_N + \phi}$$

Jean Buridan (in Latin, *Johannes Buridanus*) (ca. 1300 – after 1358)



Although Nicolás likes the symmetric food configuration, he must break the symmetry deciding which carrot is more appealing. In three dimensions, there is a continuous valley where Nicolás can move from one carrot to the next without effort.

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	$\mathsf{I}=1$ $u\bar{d},\ \bar{u}d,\ \frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	$\mathsf{I}=\frac{1}{2}$ $u\bar{s},\ d\bar{s};\ \bar{d}s,\ -\bar{u}s$	$\mathsf{I}=0$ f'	$\mathsf{I}=0$ f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	$\boldsymbol{\pi}$	$\boldsymbol{K}$	$\boldsymbol{\eta}$	$\boldsymbol{\eta'(958)}$	-11.3	-24.5
$1\,^3S_1$	1^{--}	$\boldsymbol{\rho(770)}$	$\boldsymbol{K^*(892)}$	$\boldsymbol{\phi(1020)}$	$\boldsymbol{\omega(782)}$	39.2	36.5
$1\,^1P_1$	1^{+-}	$\boldsymbol{b_1(1235)}$	$\boldsymbol{K_{1B}}^\dagger$	$h_1(1380)$	$\boldsymbol{h_1(1170)}$		
$1\,^3P_0$	0^{++}	$\boldsymbol{a_0(1450)}$	$\boldsymbol{K_0^*(1430)}$	$\boldsymbol{f_0(1710)}$	$\boldsymbol{f_0(1370)}$		
$1\,^3P_1$	1^{++}	$\boldsymbol{a_1(1260)}$	$\boldsymbol{K_{1A}}^\dagger$	$\boldsymbol{f_1(1420)}$	$\boldsymbol{f_1(1285)}$		
$1\,^3P_2$	2^{++}	$\boldsymbol{a_2(1320)}$	$\boldsymbol{K_2^*(1430)}$	$\boldsymbol{f_2'(1525)}$	$\boldsymbol{f_2(1270)}$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\boldsymbol{\pi_2(1670)}$	$\boldsymbol{K_2(1770)}^\dagger$	$\eta_2(1870)$	$\boldsymbol{\eta_2(1645)}$		
$1\,^3D_1$	1^{--}	$\boldsymbol{\rho(1700)}$	$\boldsymbol{K^*(1680)}$		$\boldsymbol{\omega(1650)}$		
$1\,^3D_2$	2^{--}		$\boldsymbol{K_2(1820)}$				
$1\,^3D_3$	3^{--}	$\boldsymbol{\rho_3(1690)}$	$\boldsymbol{K_3^*(1780)}$	$\boldsymbol{\phi_3(1850)}$	$\boldsymbol{\omega_3(1670)}$	31.8	30.8
$1\,^3F_4$	4^{++}	$\boldsymbol{a_4(2040)}$	$\boldsymbol{K_4^*(2045)}$		$\boldsymbol{f_4(2050)}$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\boldsymbol{\pi(1300)}$	$K(1460)$	$\boldsymbol{\eta(1475)}$	$\boldsymbol{\eta(1295)}$		
$2\,^3S_1$	1^{--}	$\boldsymbol{\rho(1450)}$	$\boldsymbol{K^*(1410)}$	$\boldsymbol{\phi(1680)}$	$\boldsymbol{\omega(1420)}$		
$3\,^1S_0$	0^{-+}	$\boldsymbol{\pi(1800)}$			$\eta(1760)$		

Some selected nonets

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d},\ d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s},\ d\bar{s}$ $s\bar{d},\ s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Chiral partners

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Tensor and (axial-)tensors

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

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From well-known tensor mesons to yet unknown axial-tensor mesons

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While the ground-state tensor ($J^{PC} = 2^{++}$) mesons $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, and $f_2'(1525)$ are well known experimentally and form an almost ideal nonet of quark-antiquark states, their chiral partners, the ground-states axial-tensor ($J^{PC} = 2^{--}$) mesons are poorly settled: only the kaonic member $K_2(1820)$ of the nonet has been experimentally found, whereas the isovector state ρ_2 and two isoscalar states ω_2 and ϕ_2 are still missing. Here, we study masses, strong, and radiative decays of tensor and axial-tensor mesons within a chiral model that links them: the established tensor mesons are used to test the model and to determine its parameters, and subsequently various predictions for their chiral partners, the axial-tensor mesons, are obtained. The results are compared to current lattice QCD outcomes as well as to other theoretical approaches and show that the ground-state axial-tensor mesons are expected to be quite broad, the vector-pseudoscalar mode being the most prominent decay mode followed by the tensor-pseudoscalar one. Nonetheless, their experimental finding seems to be possible in ongoing and/or future experiments.

DOI: 10.1103/PhysRevD.106.036008

TABLE I. Chiral multiplets, their currents, and transformations up to $J=3$. [* and/or $f_0(1500)$; **a mix of.] The first two columns correspond to the assignment suggested in the Quark Model review of the PDG [8], to which we refer for further details and references (see also the discussion in the text).

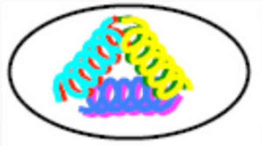
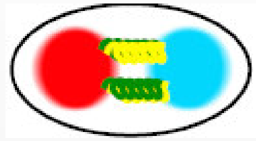
J^{PC}, $^{2S+1}L_J$	$I=1(\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}})$; $I=1(-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d)$; $I=0(\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)$**	Microscopic currents	Chiral multiplet	Transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times \times U(1)_{\mathrm{A}}$
0^{-+}, $^1S_0$	π; K; $\eta, \eta'(958)$	$P^{ij}=\frac{1}{2}\bar{q}^j \mathrm{i}\gamma^5 q^i$	$\Phi = S + \mathrm{i}P$ $(\Phi^{ij} = \bar{q}^j_{\mathrm{R}} q^i_{\mathrm{L}})$	$\Phi \to \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi U_{\mathrm{R}}^\dagger$
0^{++}, $^3P_0$	$a_0(1450)$; $K_0^*(1430)$; $f_0(1370), f_0(1710)$*	$S^{ij}=\frac{1}{2}\bar{q}^j q^i$		
1^{--}, $^1S_1$	$\rho(770)$; $K^*(892)$; $\omega(782), \phi(1020)$	$V_\mu^{ij}=\frac{1}{2}\bar{q}^j\gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L_\mu^{ij} = \bar{q}^j_{\mathrm{L}}\gamma_\mu q^i_{\mathrm{L}})$	$L_\mu \to U_{\mathrm{L}} L_\mu U_{\mathrm{L}}^\dagger$
1^{++}, $^3P_1$	$a_1(1260)$; $K_{1,A}$; $f_1(1285), f_1(1420)$	$A_\mu^{ij}=\frac{1}{2}\bar{q}^j\gamma^5\gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R_\mu^{ij} = \bar{q}^j_{\mathrm{R}}\gamma_\mu q^i_{\mathrm{R}})$	$R_\mu \to U_{\mathrm{R}} R_\mu U_{\mathrm{R}}^\dagger$
1^{+-}, $^1P_1$	$b_1(1235)$; $K_{1,B}$; $h_1(1170), h_1(1380)$	$P_\mu^{ij}=-\frac{1}{2}\bar{q}^j\gamma^5\overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + \mathrm{i}P_\mu$ $(\Phi_\mu^{ij} = \bar{q}^j_{\mathrm{R}}\mathrm{i}\overleftrightarrow{D}_\mu q^i_{\mathrm{L}})$	$\Phi_\mu \to \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi_\mu U_{\mathrm{R}}^\dagger$
1^{--}, $^3D_1$	$\rho(1700)$; $K^*(1680)$; $\omega(1650), \phi(?)$	$S_\mu^{ij}=\frac{1}{2}\bar{q}^j\mathrm{i}\overleftrightarrow{D}_\mu q^i$		
2^{++}, $^3P_2$	$a_2(1320)$; $K_2^*(1430)$; $f_2(1270), f_2'(1525)$	$V_{\mu\nu}^{ij}=\frac{1}{2}\bar{q}^j(\gamma_\mu\mathrm{i}\overleftrightarrow{D}_\mu+\cdots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L_{\mu\nu}^{ij} = \bar{q}^j_{\mathrm{L}}(\gamma_\mu\mathrm{i}\overleftrightarrow{D}_\nu+\cdots)q^i_{\mathrm{L}})$	$L_{\mu\nu} \to U_{\mathrm{L}} L_{\mu\nu} U_{\mathrm{L}}^\dagger$
2^{--}, $^3D_2$	$\rho_2(?)$; $K_2(1820)$; $\omega_2(?), \phi_2(?)$	$A_{\mu\nu}^{ij}=\frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu\mathrm{i}\overleftrightarrow{D}_\nu+\cdots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R_{\mu\nu}^{ij} = \bar{q}^j_{\mathrm{R}}(\gamma_\mu\overleftrightarrow{D}_\nu+\cdots)q^i_{\mathrm{R}})$	$R_{\mu\nu} \to U_{\mathrm{R}} R_{\mu\nu} U_{\mathrm{R}}^\dagger$
2^{-+}, $^1D_2$	$\pi_2(1670)$; $K_2(1770)$; $\eta_2(1645), \eta_2(1870)$	$P_{\mu\nu}^{ij}=-\frac{1}{2}\bar{q}^j(\mathrm{i}\gamma^5\overleftrightarrow{D}_\mu\overleftrightarrow{D}_\nu+\cdots)q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + \mathrm{i}P_{\mu\nu}$ $(\Phi_{\mu\nu}^{ij} = \bar{q}^j_{\mathrm{R}}(\overleftrightarrow{D}_\mu\overleftrightarrow{D}_\nu+\cdots)q^i_{\mathrm{L}})$	$\Phi_{\mu\nu} \to \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi_{\mu\nu} U_{\mathrm{R}}^\dagger$
2^{++}, $^3F_2$	$a_2(?)$; $K_2^*(?)$; $f_2(?), f_2'(?)$	$S_{\mu\nu}^{ij}=-\frac{1}{2}\bar{q}^j(\overleftrightarrow{D}_\mu\overleftrightarrow{D}_\nu+\cdots)q^i$		
3^{--}, $^3D_3$	$\rho_3(1690)$; $K_3^*(1780)$; $\omega_3(1670), \phi_3(1850)$	$\vdots$	$\vdots$	$\vdots$

Uniwersytet Jana Kochanowskiego w Kielcach

Table from:

F.G., R. Pisarski, A. Koenigstein Phys.Rev.D 97 (2018) 9, 091901 e-Print: 1709.07454

Non-conventional mesons: beyond qq

1) Glueballs  2) Hybrids 

3) Four-quark states

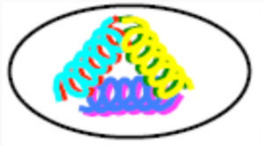
Compact diquark-antidiquark states 

Molecular states (a type of dynamical generation) 

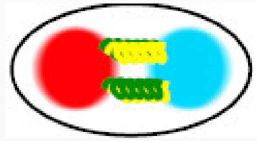
Companion poles (another type of dynamical generation)

Non-conventional mesons: beyond qq

1) Glueballs



2) Hybrids



Talks by:

Arthur Vereijken,
Tensor glueball in a chiral approach,
Fr. 22/6, 15:00 (B3)

Enrico Trotti,
Scattering of glueballs with J=0,2
Fr. 22/6, 15:40 (C3)

Toward a nonet of hybrid state/PDG

$\pi_1(1600)$

$I^G(J^{PC}) = 1^-(1^{-+})$

See the review on "Spectroscopy of Light Meson Resonances" and a note in PDG 06, Journal of Physics **G33** 1 (2006).

$\pi_1(1600)$ T-Matrix Pole $\sqrt{s}$

$\pi_1(1600)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1661^{+15}_{-11} OUR AVERAGE		Error includes scale factor of 1.2.		

$\pi_1(1600)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
240± 50 OUR AVERAGE		Error includes scale factor of 1.7. See the ideogram below.		

$\pi_1(1600)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi\pi$	seen
Γ_2	$\rho^0\pi^-$	seen
Γ_3	$f_2(1270)\pi^-$	not seen
Γ_4	$b_1(1235)\pi$	seen
Γ_5	$\eta'(958)\pi^-$	seen
Γ_6	$\eta\pi$	
Γ_7	$f_1(1285)\pi$	seen

$\pi_1(1400)$

$I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1400)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
1354 ±25 OUR AVERAGE		Error includes scale factor of 1.8. See the ideogram below.			

$\pi_1(1400)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
330 ±35 OUR AVERAGE					

$\pi_1(1400)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\eta\pi^0$	seen
Γ_2	$\eta\pi^-$	seen

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A unique I=1 hybrid state π1

PHYSICAL REVIEW LETTERS 122, 042002 (2019)

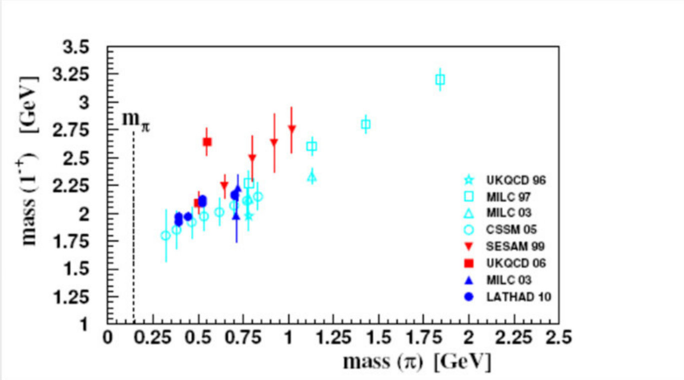
Determination of the Pole Position of the Lightest Hybrid Meson Candidate

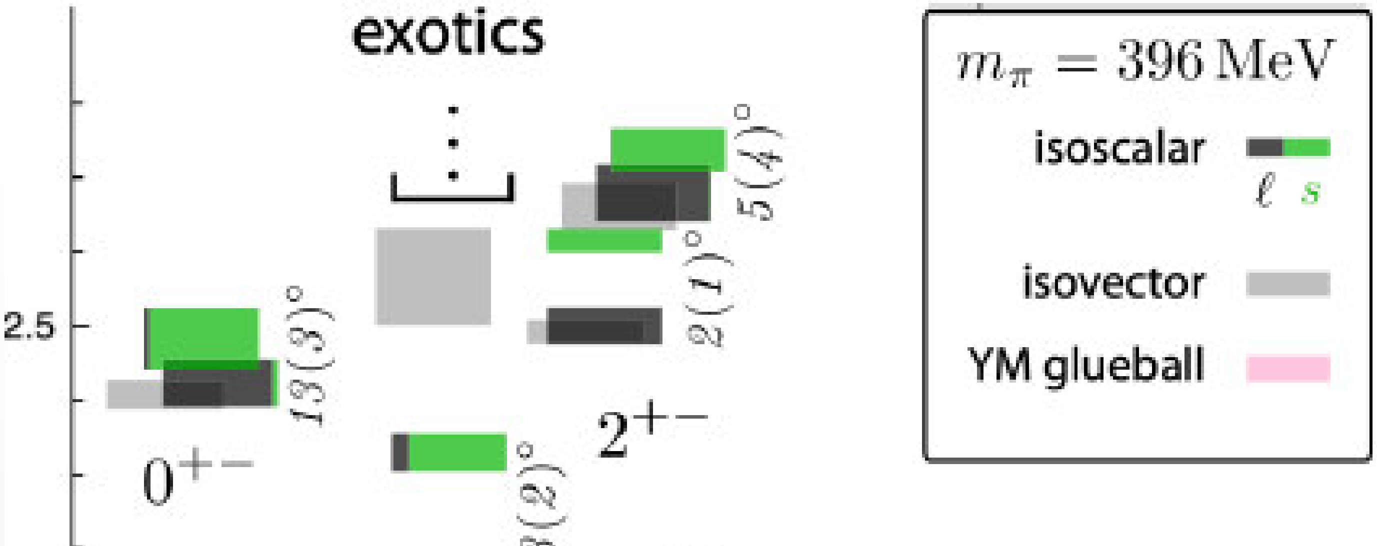
A. Rodas,[1,*] A. Pilloni,[2,3,†] M. Albaladejo,[2,4] C. Fernández-Ramírez,[5] A. Jackura,[6,7] V. Mathieu,[2] M. Mikhasenko,[8] J. Nys,[9] V. Pauk,[10] B. Ketzer,[8] and A. P. Szczepaniak[2,6,7]

Mapping states with explicit gluonic degrees of freedom in the light sector is a challenge, and has led to controversies in the past. In particular, the experiments have reported two different hybrid candidates with spin-exotic signature, $\pi_1(1400)$ and $\pi_1(1600)$, which couple separately to $\eta\pi$ and $\eta'\pi$. This picture is not compatible with recent Lattice QCD estimates for hybrid states, nor with most phenomenological models. We consider the recent partial wave analysis of the $\eta^{(\prime)}\pi$ system by the COMPASS Collaboration. We fit the extracted intensities and phases with a coupled-channel amplitude that enforces the unitarity and analyticity of the S matrix. We provide a robust extraction of a single exotic π_1 resonant pole, with mass and width $1564 \pm 24 \pm 86$ and $492 \pm 54 \pm 102$ MeV, which couples to both $\eta^{(\prime)}\pi$ channels. We find no evidence for a second exotic state. We also provide the resonance parameters of the $a_2(1320)$ and $a_2'(1700)$.

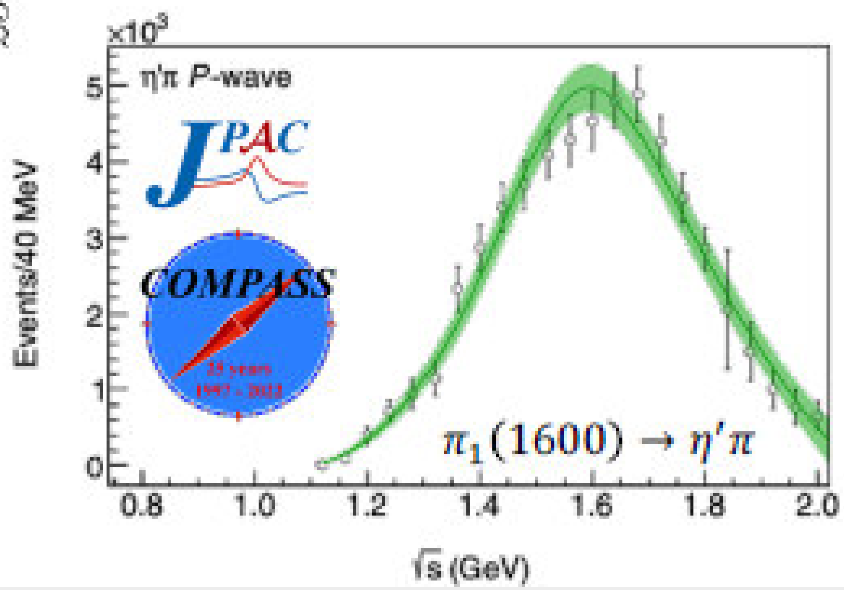
π1(1600) and π1(1400) are the same state
(in agreement with various models and lattice QCD)

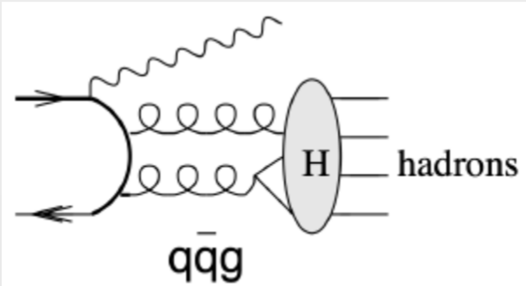
C. Meyer and E. Swanson,
Hybrid Mesons,
Prog. Part. Nucl. Phys. 82 (2015) 21
[arXiv:1502.07276 [hep-ph]].





PRD83 (2011) 111502





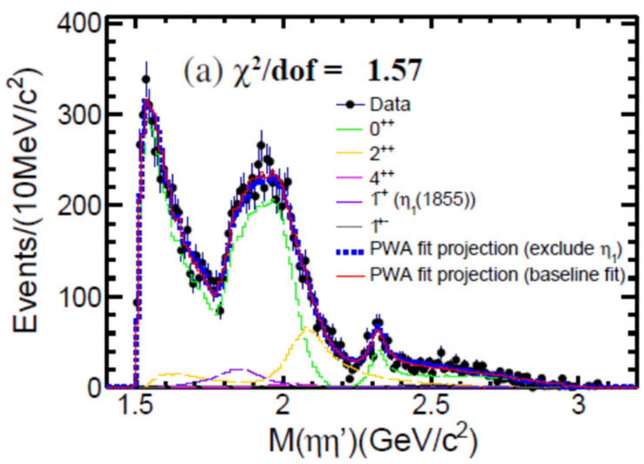
New experimental finding: $\eta_1(1855)$

Observation of an isoscalar resonance with exotic $J^{PC} = 1^{-+}$ quantum numbers in $J/\psi \to \gamma\eta\eta'$

M. Ablikim[1], M. N. Achasov[10,b], P. Adlarson[68], S. Ahmed[14], M. Albrecht[4], R. Aliberti[28], A. Amoroso[67A,67C], M. R. An[32],

Using a sample of $(10.09\pm0.04)\times10^9$ J/ψ events collected with the BESIII detector at the BEPCII storage ring, a partial wave analysis of the decay $J/\psi \to \gamma\eta\eta'$ is performed. The first observation of an isoscalar state with exotic quantum numbers $J^{PC} = 1^{-+}$, denoted as $\eta_1(1855)$, is reported in the process $J/\psi \to \gamma\eta_1(1855)$ with $\eta_1(1855) \to \eta\eta'$. Its mass and width are measured to be $(1855\pm9^{+6}_{-1})$ MeV/c^2 and $(188\pm18^{+3}_{-8})$ MeV, respectively, where the first uncertainties are statistical and the second are systematic, and its statistical significance is estimated to be larger than 19σ.

Phys.Rev.Lett. 129 (2022) 19, 192002 2202.00621 [hep-ex]



A nonet of hybrid states?

Physics Letters B 834 (2022) 137478

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The phenomenology of the exotic hybrid nonet with $\pi_1(1600)$ and $\eta_1(1855)$

Vanamali Shastry[a,*], Christian S. Fischer[b,c], Francesco Giacosa[a,d]

arXiv:2203.04327

Beides π1(1600) and η1(1855), we expect also: K1(1750) and η1(1660). The last two not yet seen.

	M (MeV)
K_1^{hyb}	1761
η_1^L	1661
η_1^H	1855

Combined fit for the π1(1660) mesons (PDG values +latt)

$$m_{\pi_1} = 1661^{+15}_{-11}\ \text{MeV}$$

$$\Gamma_{\text{tot}} = 240 \pm 50\ \text{MeV}$$

$$D\text{-wave and } S\text{-wave: } \frac{BR(\pi_1 \to b_1\pi)_D}{BR(\pi_1 \to b_1\pi)_S} = 0.3 \pm 0.1$$

$$\frac{\Gamma_{f_1\pi}}{\Gamma_{\eta'\pi}} = 3.8 \pm 0.78$$

The following are the lattice estimates

1. $\Gamma_{b_1\pi} = 139\text{-}529$ MeV,
2. $\Gamma_{\rho\pi} = 0\text{-}20$ MeV,
3. $\Gamma_{K^*K} = 0\text{-}2$ MeV,
4. $\Gamma_{f_1\pi} = 0\text{-}24$ MeV,
5. $\Gamma_{f_1'\pi} = 0\text{-}2$ MeV,
6. $\Gamma_{\rho\omega} \leq 0.15$ MeV,
7. $\Gamma_{\eta\pi} = 0\text{-}1$ MeV,
8. $\Gamma_{\eta'\pi} = 0\text{-}12$ MeV.

A.J. Woss, et al., Hadron Spectrum, Decays of an exotic 1−+ hybrid meson resonance in QCD, Phys. Rev. D 103 (5) (2021) 054502, https://doi.org/10.1103/PhysRevD.103.054502, arXiv:2009.10034 [hep-lat].

Lagrangian for π1 and decays

$$\begin{aligned}\mathcal{L}_{hyb}^{\pi} &= g_{b_1\pi}^{c}\langle \pi_{1,\mu} b_1^{\mu}\pi\rangle + g_{b_1\pi}^{d}\langle \pi_{1,\mu\nu} b_1^{\mu\nu}\pi\rangle \\ &+ g_{f_1\pi}\langle \pi_{1,\mu} f_{1,N}^{\mu\nu}\partial_\nu \pi + \pi_{1,\mu} f_{1,S}^{\mu\nu}\partial_\nu\pi\rangle \\ &+ g_{\eta\pi}\langle \pi_{1,\mu}(\eta_N \partial^{\mu}\pi + \eta_S\partial^{\mu}\pi)\rangle + g_{\rho\pi}\langle \tilde{\pi}_{1,\mu\nu}\rho^{\mu\nu}\pi\rangle \\ &+ g_{\rho\omega}\langle \pi_{1,\mu}(\rho^{\mu\nu}\omega_\nu + \omega^{\mu\nu}\rho_\nu)\rangle .\end{aligned}$$

$$\Gamma_{b_1\pi} = \frac{1}{2}\frac{k_{b_1}}{24\pi m_{\pi_1}^2}\left(\frac{1}{m_{b_1}^2}\left(E_{b_1} g_{b_1\pi}^c + 2 g_{b_1\pi}^d m_{b_1}^2 m_{\pi_1}\right)^2 + 2(2E_{b_1} g_{b_1\pi}^2 m_{\pi_1} + g_{b_1\pi}^c)^2\right)$$

$$\frac{G_2}{G_0} = \sqrt{2}\frac{g_{b_1\pi}^c(-E_{b_1}+m_{b_1}) + 2g_{b_1\pi}^d m_{\pi_1}(-m_{b_1}^2 + E_{b_1}m_{b_1})}{g_{b_1\pi}^c(E_{b_1}+2m_{b_1}) + 2g_{b_1\pi}^d m_{\pi_1}(m_{b_1}^2 + 2E_{b_1}m_{b_1})}$$

$$\Gamma_{\rho\pi} = g_{\rho\pi}^2\frac{k_\rho^3}{6\pi}$$

$$\Gamma_{K^*K} = g_{\rho\pi}^2\frac{k_{K^*}^3}{12\pi}$$

….and other decay terms…

Results of the fit

$$\pi_1(1600)$$

Channel	Width (MeV)	Channel	Width (MeV)
$\Gamma_{b_1\pi}$	220 ± 34	$\Gamma_{f_1\pi}$	16.2 ± 3.1
$\Gamma_{\rho\pi}$	7.1 ± 1.8	$\Gamma_{f_1'\pi}$	0.83 ± 0.16
Γ_{K^*K}	1.2 ± 0.3	$\Gamma_{\eta\pi}$	0.37 ± 0.08
$\Gamma_{\rho\omega}$	0.08 ± 0.03	$\Gamma_{\eta'\pi}$	4.6 ± 1.0
		Γ_{tot}	250 ± 34

Predictions for other hybrids

$\eta_1^{hyb}(1660)$

Channel	Width (MeV)
	Set-1
$\Gamma_{a_1\pi}$	80 ± 15
Γ_{K^*K}	0.29 ± 0.075
$\Gamma_{\eta'\eta}$	0.41 ± 0.09
$\Gamma_{K_1(1270)K}$	0
$\Gamma_{\rho\rho}$	0.081 ± 0.028
$\Gamma_{K^*K^*}$	0
$\Gamma_{\omega\phi}$	0
$\Gamma_{f_1\eta}$	0
Γ_{tot}	81 ± 15

$\eta_1(1855)$

Channel	Width (MeV)
	Set-1
$\Gamma_{K_1(1270)K}$	253 ± 92
Γ_{K^*K}	1.45 ± 0.37
$\Gamma_{\eta'\eta}$	2.28 ± 0.51
$\Gamma_{a_1\pi}$	0
$\Gamma_{\rho\rho}$	0
$\Gamma_{K^*K^*}$	0.075 ± 0.027
$\Gamma_{\omega\phi}$	$\sim 10^{-4}$
$\Gamma_{f_1\eta}$	2.15 ± 0.56
Γ_{tot}	259 ± 92

$K_1^{hyb}(1750)$

Channel	Width (MeV)
	Set-1
$\Gamma_{K_1(1270)\pi}$	125 ± 42
$\Gamma_{K_1(1400)\pi}$	103 ± 45
$\Gamma_{h_1(1170)K}$	1.53 ± 0.28
$\Gamma_{\eta K}$	0.29 ± 0.07
$\Gamma_{\eta' K}$	2.77 ± 0.62
$\Gamma_{\rho K^*}$	0.045 ± 0.016
$\Gamma_{a_1 K}$	11.0 ± 2.32
$\Gamma_{\rho K}$	2.18 ± 0.56
$\Gamma_{\omega K}$	0.82 ± 0.21
$\Gamma_{\phi K}$	0.49 ± 0.12
$\Gamma_{K^*\pi}$	0.67 ± 0.17
$\Gamma_{K^*\eta}$	0.30 ± 0.08
$\Gamma_{\omega K^*}$	0.011 ± 0.004
$\Gamma_{b_1 K}$	64 ± 14
Γ_{tot}	312 ± 97

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Predictions for other hybrids

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Chiral partners of the hybrids

Eur. Phys. J. Plus (2020) 135:945
https://doi.org/10.1140/epjp/s13360-020-00900-z

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

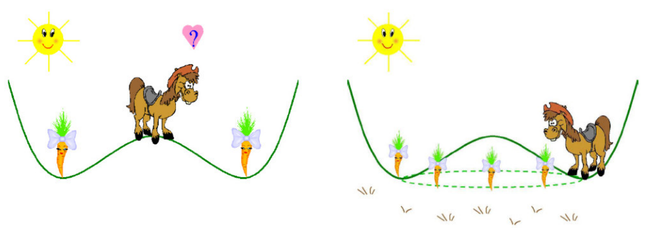
Hybrid phenomenology in a chiral approach

Walaa I. Eshraim[1,2], **Christian S. Fischer**[1,3], **Francesco Giacosa**[2,4,a], **Denis Parganlija**[5,6]

Resonance	Mass [MeV]
π_1^{hyb}	1660 [**input** using $\pi_1(1600)$ [9]]
$\eta_{1,N}^{hyb}$	1660
$\eta_{1,S}^{hyb}$	1751
K_1^{hyb}	1707
b_1^{hyb}	2000 [**input** set as an estimate]
$h_{1N,B}^{hyb}$	2000
$K_{1,B}^{hyb}$	2063
$h_{1S,B}^{hyb}$	2126

$\Gamma_{b_1^{hyb}\to\pi\omega(1650)}/\Gamma_{\pi_1^{hyb}\to\pi b_1}$	0.065
$\Gamma_{K_{1B}^{hyb}\to\pi K^*(1680)}/\Gamma_{\pi_1^{hyb}\to\pi b_1}$	0.19
$\Gamma_{h_{1,N}^{hyb}\to\pi\rho(1700)}/\Gamma_{\pi_1^{hyb}\to\pi b_1}$	0.16

Spontaneous Symmetry Breaking



Although Nicolás likes the symmetric food configuration, he must break the symmetry deciding which carrot is more appealing. In three dimensions, there is a continuous valley where Nicolás can move from one carrot to the next without effort.

Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics A 1037 (2023) 122683
NUCLEAR PHYSICS A
www.elsevier.com/locate/nuclphysa

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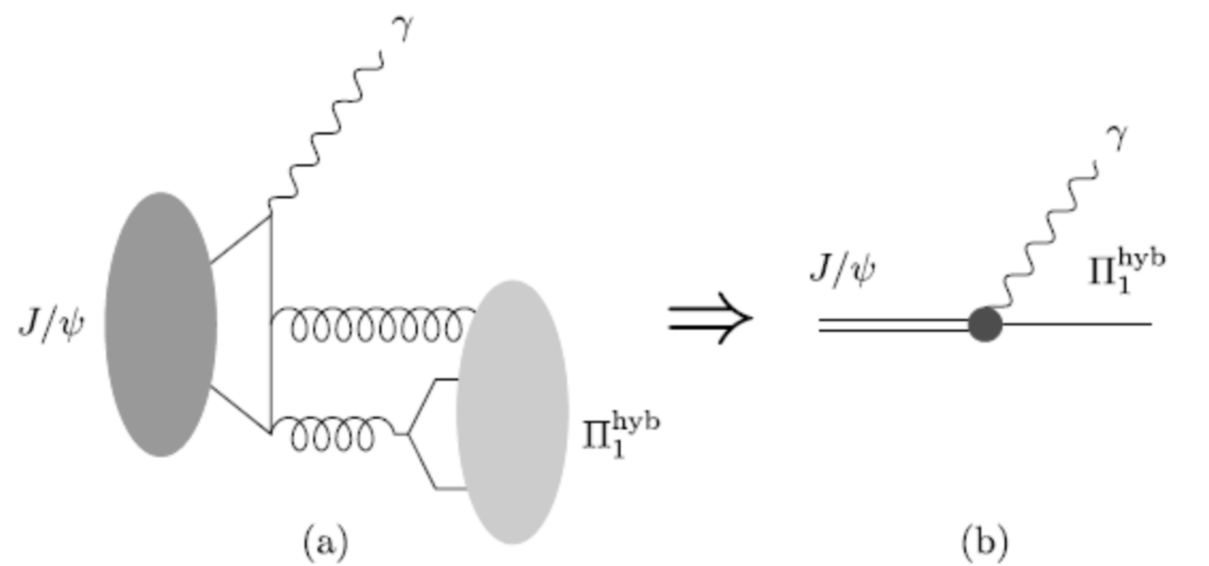
Radiative production and decays of the exotic $\eta_1'(1855)$ and its siblings

Vanamali Shastry [a,*], Francesco Giacosa [a,b]



J/Psi decay (via the so-called 'Sill' distribution)

Production Channel $(\phi_1\phi_2)$		Branching ratio (10^{-4}) Set-1 $\theta_h = 0^\circ$
$\eta_1^{hyb}(1660)$	$a_1\pi$	4.8 ± 1.4
	K^*K	$(1.73 \pm 0.49) \times 10^{-2}$
	$\eta'\eta$	$(2.28 \pm 0.65) \times 10^{-2}$
	$\rho\rho$	$(4.4 \pm 1.3) \times 10^{-3}$
$\eta_1(1855)$	$K_1(1270)K$	2.45 ± 0.70
	K^*K	$(1.86 \pm 0.53) \times 10^{-2}$
	K^*K^*	$(7.2 \pm 2.1) \times 10^{-4}$
	$f_1(1285)\eta$	$(27.6 \pm 7.9) \times 10^{-3}$
	$\eta\eta'$	$(2.70 \pm 0.76) \times 10^{-2}$ [10]

The branching ratios of the $J/\psi \to \gamma\eta_1^{hyb}(1660) \to \gamma\phi_1\phi_2$ and $J/\psi \to \gamma\eta_1'(1855) \to \gamma\phi_1\phi_2$

$$\Gamma_{A\to BC_1C_2} = \int_{s_{\rm th}}^{(\Delta M_{AC_2})^2} ds\, \Gamma_{A\to \mathcal{R}^*C_2}(s) d_s^i(s)$$

'Sill' implemented in all decays above

Beyond Breit-Wigner

Eur. Phys. J. A (2021) 57:336
https://doi.org/10.1140/epja/s10050-021-00641-2

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article - Theoretical Physics

A simple alternative to the relativistic Breit–Wigner distribution

Francesco Giacosa[1,2], Anna Okopińska[1], Vanamali Shastry[1,a]

ArXiv: 2106.03749

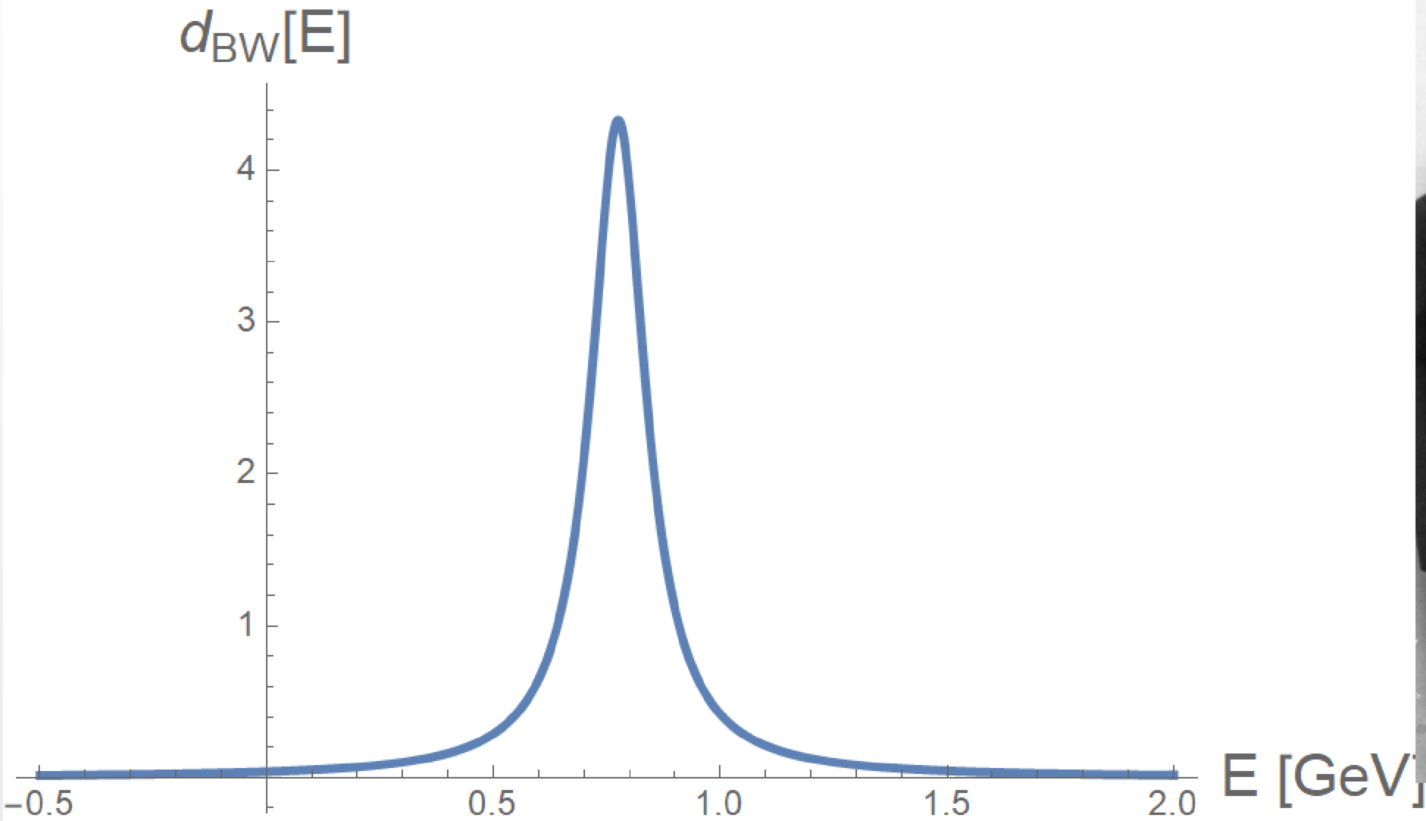
$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

$$d_S^{\mathrm{rBW}}(E) = \frac{2E}{\pi} \frac{M\Gamma}{(E^2-M^2)^2 + (M\Gamma)^2} \theta(E)$$

$$d_S^{\mathrm{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}}{(E^2-M^2)^2 + (\sqrt{E^2-E_{th}^2}\,\tilde{\Gamma})^2} \theta(E - E_{th})$$

Breit-Wigner distribution

$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$



Rho-meson as example.

BW extends from –inf to +inf. There is no left threshold.

Relativistic Breit-Wigner (rBW)

$$d_S^{\mathrm{rBW}}(E) = \frac{2E}{\pi} \frac{M\Gamma}{(E^2 - M^2)^2 + (M\Gamma)^2} \theta(E)$$

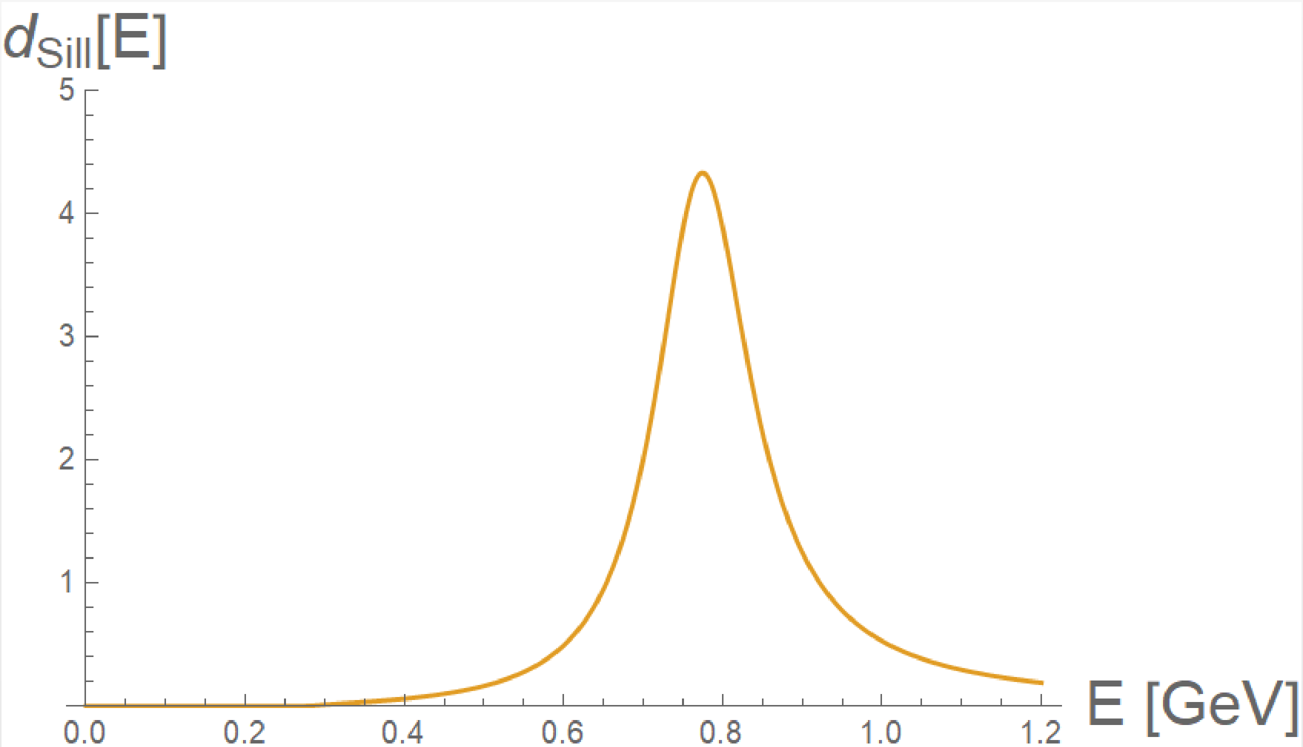
In a relativistic framework there is always a threshold! (eventually zero).

Function above not normalized as it stands.

From above often used in various applications.

‘Sill’ distribution

$$d_S(E) = d_S^{\mathrm{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}}{(E^2 - M^2)^2 + \left(\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}\right)^2} \theta(E - E_{th})$$



Sill for the rho-meson

Comments

$$s_{pole} = M^2 - \frac{\tilde{\Gamma}^2}{2} - i\sqrt{(M^2 - s_{th})\tilde{\Gamma}^2 + \frac{\tilde{\Gamma}^4}{4}} \; .$$

Note, for $\tilde{\Gamma}^2$ sufficiently smaller than $M^2 - s_{th}$, the pole of s can be approximated as

$$s_{pole} \simeq M^2 - i\sqrt{(M^2 - s_{th})}\tilde{\Gamma} = M^2 - iM\Gamma \; ,$$

The normalization

$$\int_{E_{th}}^{+\infty} \mathrm{dE}\, d_S^{\mathrm{Sill}}(E) = 1$$

for any E_{th}, M, and $\tilde{\Gamma}$ is a consequence of the proper treatment of the real part of the loop

Sill extension to multi-channel case

The extension to the N channels is straightforward::

$$G_S(s) = \frac{1}{s - M^2 + i\sum_{k=1}^{N} \tilde{\Gamma}_k \sqrt{s - s_{k,th}} + i\varepsilon}$$

with

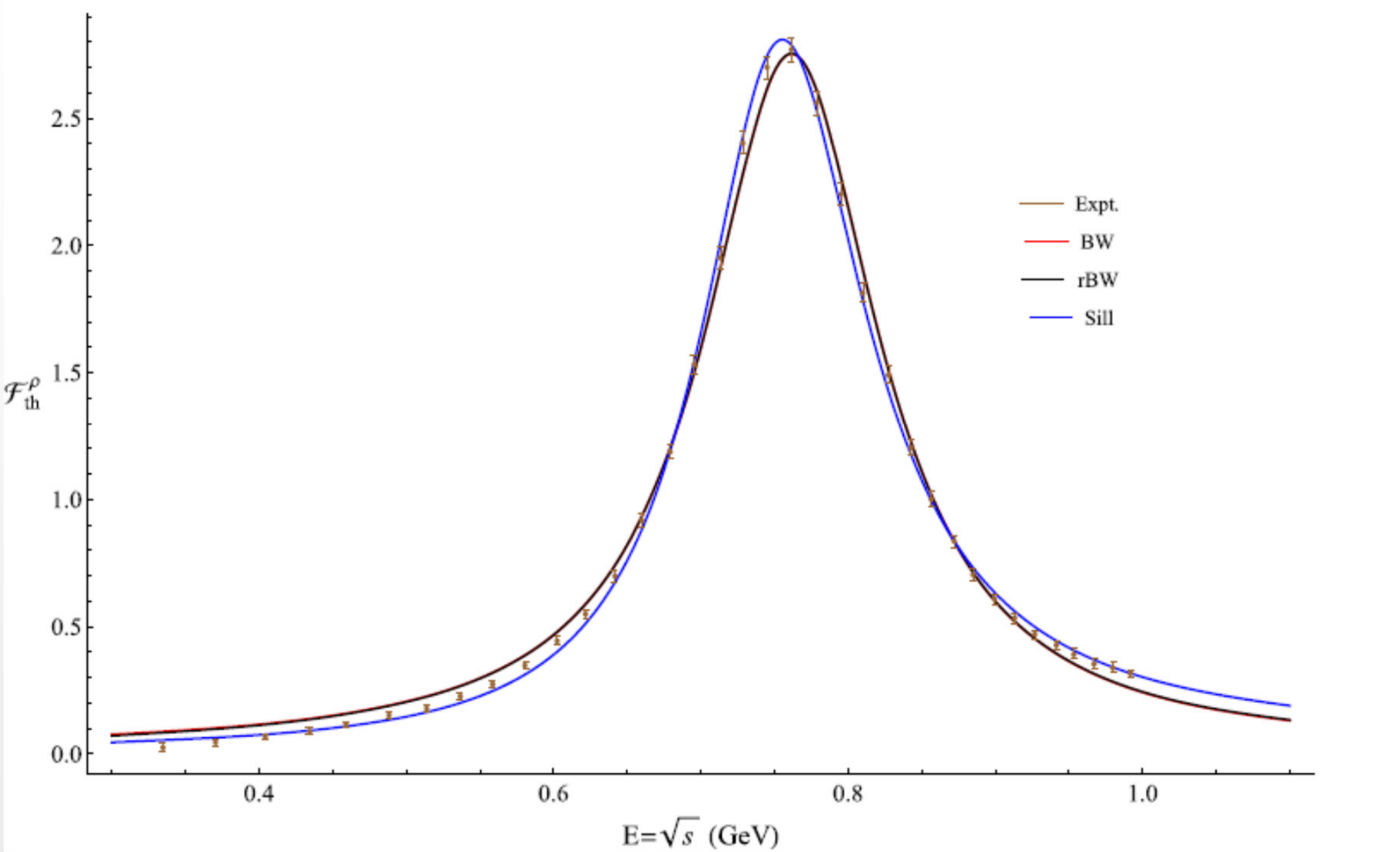
$$\tilde{\Gamma}_k = \Gamma_k \frac{M}{\sqrt{M^2 - E_{k,th}^2}} \text{ and}$$

$$s_{1,th} = E_{1,th}^2 \leq s_{2,th} \leq \ldots \leq_{N,th} = E_{N,th}^2 \,.$$

$$d_s^k(s) = \frac{1}{\pi} \frac{\sqrt{s - s_{\mathrm{th,k}}}\,\tilde{\Gamma}_k}{(s - M^2 - \sum_{i=1}^{Q}\sqrt{s_{\mathrm{th,i}} - s}\,\tilde{\Gamma}_i)^2 + \sum_{i=Q+1}^{N}(\sqrt{s - s_{\mathrm{th,i}}}\,\tilde{\Gamma}_i)^2}\theta(s - s_{\mathrm{th,k}})$$

where, $s_{\mathrm{th,k}}$ is the k^{th} threshold, and the integer Q is such that, for all $i < Q$, $s_{th,i} < s_{th,k}$

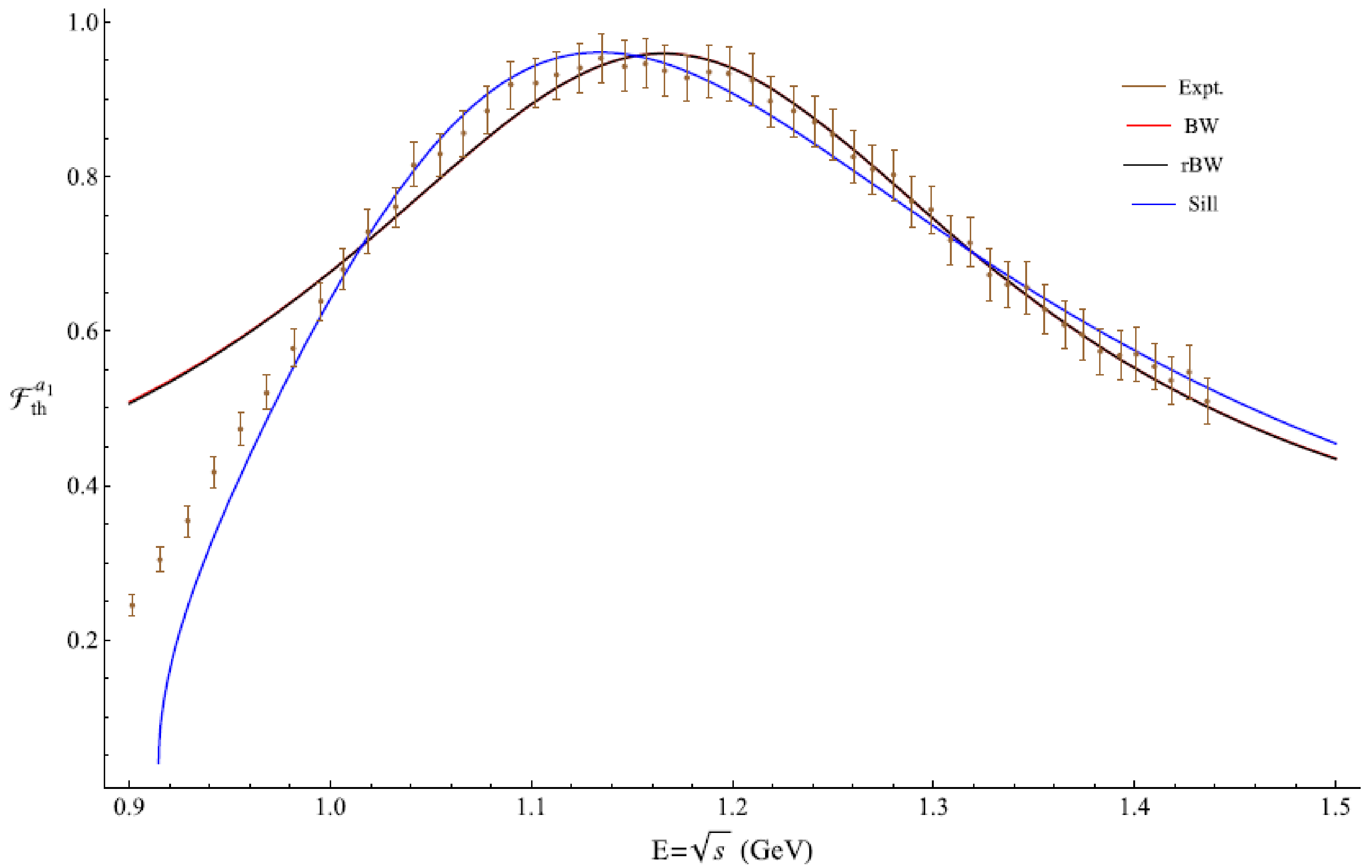
ρ meson



Aleph data for tau decay

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	761.64 ± 0.32	144.6 ± 1.3	10.16	$761.6 - i\,72.3$
Relativistic BW	758.1 ± 0.33	145.2 ± 1.3	9.42	$761.5 - i\,72.3$
Sill	755.82 ± 0.33	137.3 ± 1.1	3.52	$751.7 - i\,68.6$

a1 meson



Aleph data for tau decay

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	1165.6 ± 1.2	415 ± 15	4.31	$1166 - i\,208$
Relativistic BW	1146.5 ± 1.6	424 ± 16	4.25	$1165 - i\,209$
Sill	1181.3 ± 3.4	539 ± 27	3.52	$1046 - i\,250$

The Delta baryon

Fig. 6 The spectral function for the $\Delta(1232)$. Experimental data from [82]. It is visible that the Sill fairs marginally better than the (r)BW distributions

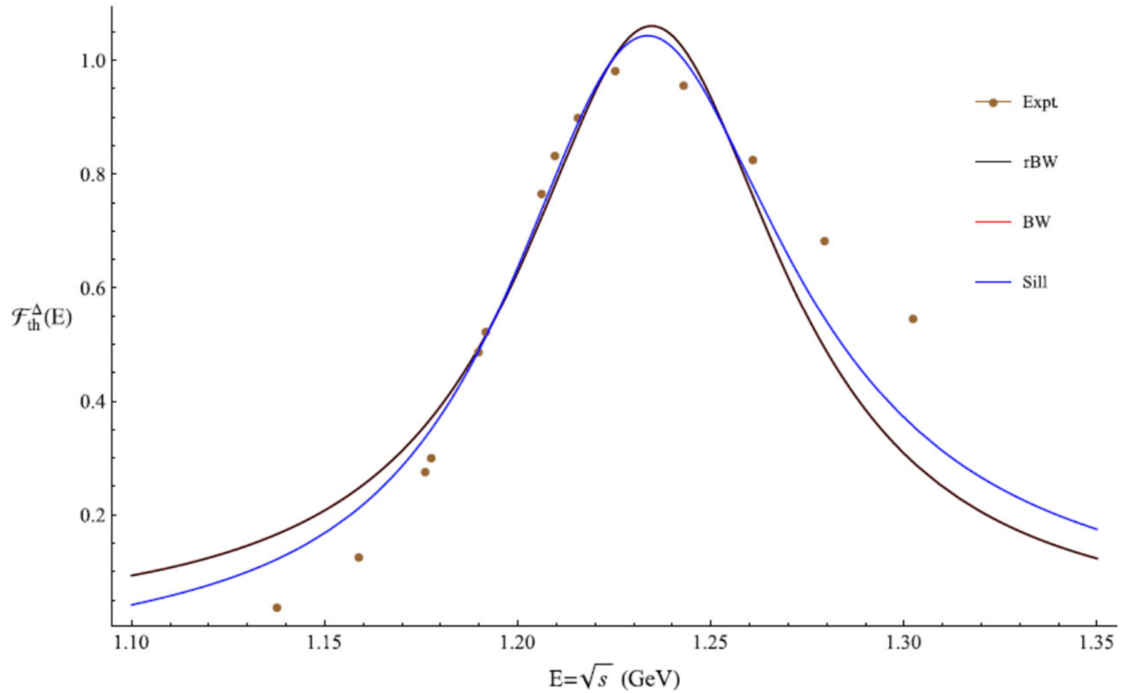


Table 5 Mass and width of $\Delta(1232)$ fitted using the three distributions discussed in the text, their error estimates, and the poles (as described in the text)

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	1234.6 ± 1.3	83.6 ± 4.1	2.92	$1234.7 - i\,41.8$
Relativistic BW	1233.9 ± 1.2	83.7 ± 4.2	2.92	$1234.7 - i\,41.8$
Sill	1236.2 ± 1.5	90.4 ± 4.9	1.53	$1235.4 - i\,45.2$

Data from: J.R. Haskins, Am. J. Phys. **53**, 988–991 (1985)

Recent Sill application/JPAC and CLAS

PHYSICAL REVIEW D **106,** 094009 (2022)

XYZ spectroscopy at electron-hadron facilities. II. Semi-inclusive processes with pion exchange

D. Winney,[1,2,*] A. Pilloni,[3,4,†] V. Mathieu,[5,‡] A. N. Hiller Blin,[6,7] M. Albaladejo,[8] W. A. Smith,[9,10] and A. Szczepaniak[9,10,11]

(Joint Physics Analysis Center)

description of the πp mass distribution in the Δ mass region:

$$d_{\Delta\to\pi p}(M^2) = \frac{1}{\pi}\frac{\rho(M^2)\tilde{\Gamma}_\Delta}{[M^2 - m_\Delta^2]^2 + [\rho(M^2)\tilde{\Gamma}_\Delta]^2}, \quad (39)$$

with $\rho(M^2) = \sqrt{M^2 - M_{\rm min}^2}$ and $\tilde{\Gamma}_\Delta = \Gamma_\Delta m_\Delta/\rho(m_\Delta^2)$. Interestingly, this function is normalized across the mass

First measurement of hard exclusive $\pi^-\Delta^{++}$ electroproduction beam-spin asymmetries off the proton

(The CLAS Collaboration)

ArXiv: 2303.11762

As a second completely independent method, a bin-by-bin background subtraction was performed based on a fit of the complete distribution (signal + background) with a so-called "Sill" function, which is a Breit-Wigner distribution including threshold effects [28] plus a fifth-order polynomial background in each Q^2, x_B, $-t$ and ϕ bin and for each helicity state. After the combined fit, the signal and background contributions were separated and the asymmetry was calculated based on the pure signal events. It was found that both methods provided consistent results for the signal asymmetry within the statistical uncertainty.

Recent Sill application/2

First measurement of hard exclusive $\pi^-\Delta^{++}$ electroproduction beam-spin asymmetries off the proton

(The CLAS Collaboration)

ArXiv: 2303.11762

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Recent Sill application/3: Xi(1620)

ArXiv: 2305.19093

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

ALICE

CERN

CERN-EP-2023-106
30 May 2023

Accessing the strong interaction between Λ baryons and charged kaons with the femtoscopy technique at the LHC

ALICE Collaboration*

$$f(k^*) = \frac{-2\tilde{\Gamma}_{\Lambda K^-}}{E^2 - M^2 + i\tilde{\Gamma}_{\Xi\pi}\sqrt{E^2 - E^2_{\mathrm{thr},\Xi\pi}} + i\tilde{\Gamma}_{\Lambda K^-}\sqrt{E^2 - E^2_{\mathrm{thr},\Lambda K^-}}}$$

$\boxed{\Xi(1620)}$ $I(J^P) = \frac{1}{2}(?^?)$ Status: $*$

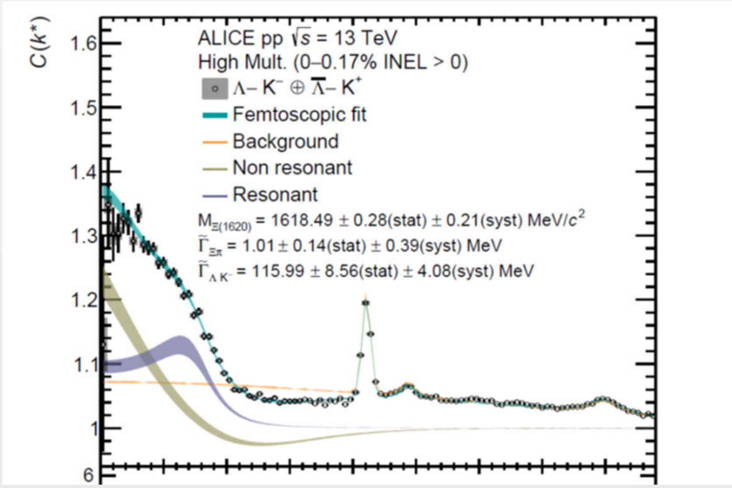
J, P need confirmation.

OMITTED FROM SUMMARY TABLE

What little evidence there is consists of weak signals in the $\Xi\pi$ channel. A number of other experiments (e.g., BORENSTEIN 72 and HASSALL 81) have looked for but not seen any effect.

$\Xi(1620)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
≈ 1620 OUR ESTIMATE				



$\Xi(1620)$ DECAY MODES

	Mode
Γ_1	$\Xi\pi$

Conclusions and outlook

Lightest hybrids

- Nonet of light hybrid states: two resonances still missing
- Postdictions/Predictions
- Search for the missing resonances promising

Sill distribution

- Simple Flatte-like relativistic implementation of threshold(s)
- Normalization, simple propagator
- J/Psi decay into hybrids

Thanks!

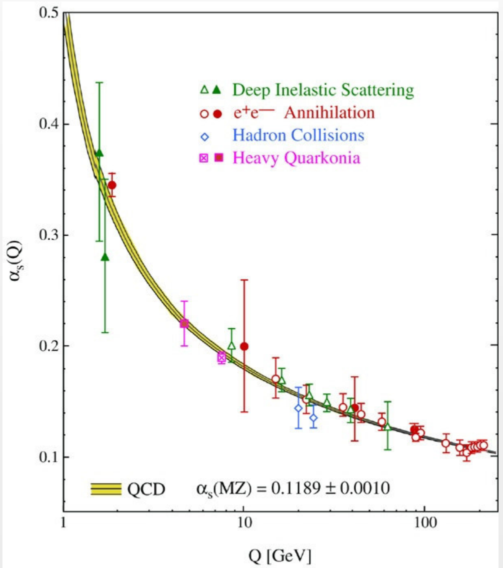
Trace anomaly:
the emergence of a dimension

Chiral limit: $m_q = 0$

$x^\mu \rightarrow x'^\mu = \lambda^{-1} x^\mu$ is a classical symmetry broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\Lambda_{YM} \approx 250 \text{ MeV}$

$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass: $m_{gluon} = 0 \rightarrow m^*_{gluon} \approx 500 - 800 \text{ MeV}$

Gluon condensate: $\left\langle G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

Comments

The Sill is Flatte-like, but not equal.

PHYSICAL REVIEW D **99**, 093007 (2019)

Isovector scalar $a_0(980)$ and $a_0(1450)$ resonances in the $B \to \psi(K\bar{K}, \pi\eta)$ decays

Zhou Rui,* Ya-Qian Li, and Jie Zhang

$$M_{a_0(980)}(\omega^2) = \frac{m_0^2}{m_0^2 - \omega^2 - i(g_{\pi\eta}^2 \rho_{\pi\eta} + g_{KK}^2 \rho_{KK})}$$

$$\rho_{\pi\eta} = \sqrt{\left[1 - \left(\frac{m_\eta - m_\pi}{\omega}\right)^2\right]\left[1 - \left(\frac{m_\eta + m_\pi}{\omega}\right)^2\right]},$$

$$\rho_{K\bar{K}} = \frac{1}{2}\sqrt{1 - \frac{4m_{K^\pm}^2}{\omega^2}} + \frac{1}{2}\sqrt{1 - \frac{4m_{K^0}^2}{\omega^2}}.$$

It does not reduce to Flatte (even not in the KK channel)

Comment

The Sill is Flatte-like, but not equal.

Flatté-like distributions and the $a_0(980)/f_0(980)$ mesons

V. Baru[1], J. Haidenbauer[2], C. Hanhart[2], A. Kudryavtsev[1], Ulf-G. Meißner[2,3]

Eur.Phys.J.A 23 (2005) 523-533e-Print: nuclth/0410099 [nucl-th]

$$\frac{d\sigma_i}{dm} \propto \left| \frac{m_R\sqrt{\Gamma_{\pi\eta}\Gamma_i}}{m_R^2 - m^2 - i m_R(\Gamma_{\pi\eta} + \Gamma_{K\bar{K}})} \right|^2 ,$$

with the partial widths $\Gamma_{\pi\eta} = \bar{g}_\eta q_\eta$ and

$$\Gamma_{K\bar{K}} = \bar{g}_K \sqrt{m^2/4 - m_K^2}$$

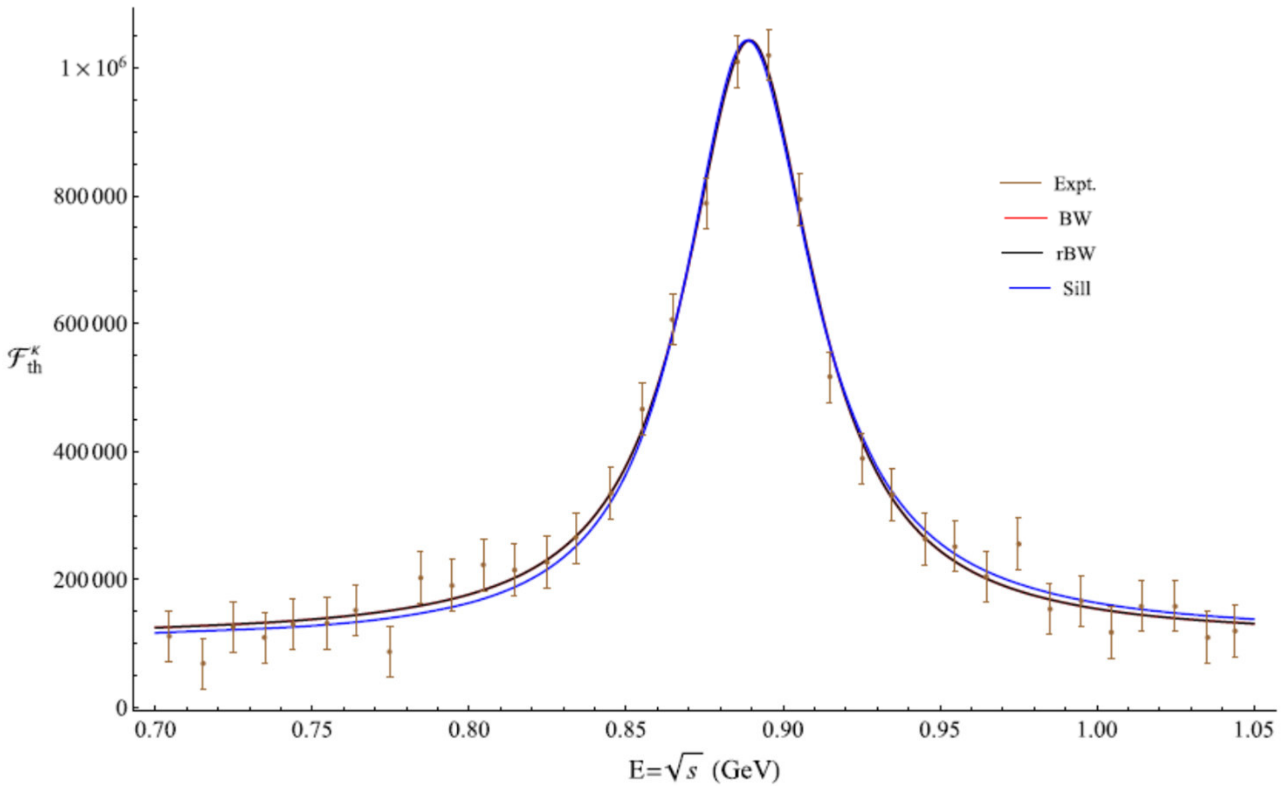
above threshold and

$$\Gamma_{K\bar{K}} = i\bar{g}_K \sqrt{m_K^2 - m^2/4}$$

The Sill is as Flatte along KK
(but not along pion-eta)

The K*(892) meson: basically no difference

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	889.37 ± 0.43	50.1 ± 1.6	1.78	$889.4 - i\,25.0$
Relativistic BW	889.01 ± 0.43	50.1 ± 1.6	1.78	$890.1 - i\,24.9$
Sill	889.06 ± 0.43	49.9 ± 1.6	2.08	$888.0 - i\,25.0$



J. Adam *et al.* [ALICE], arXiv:1601.07868

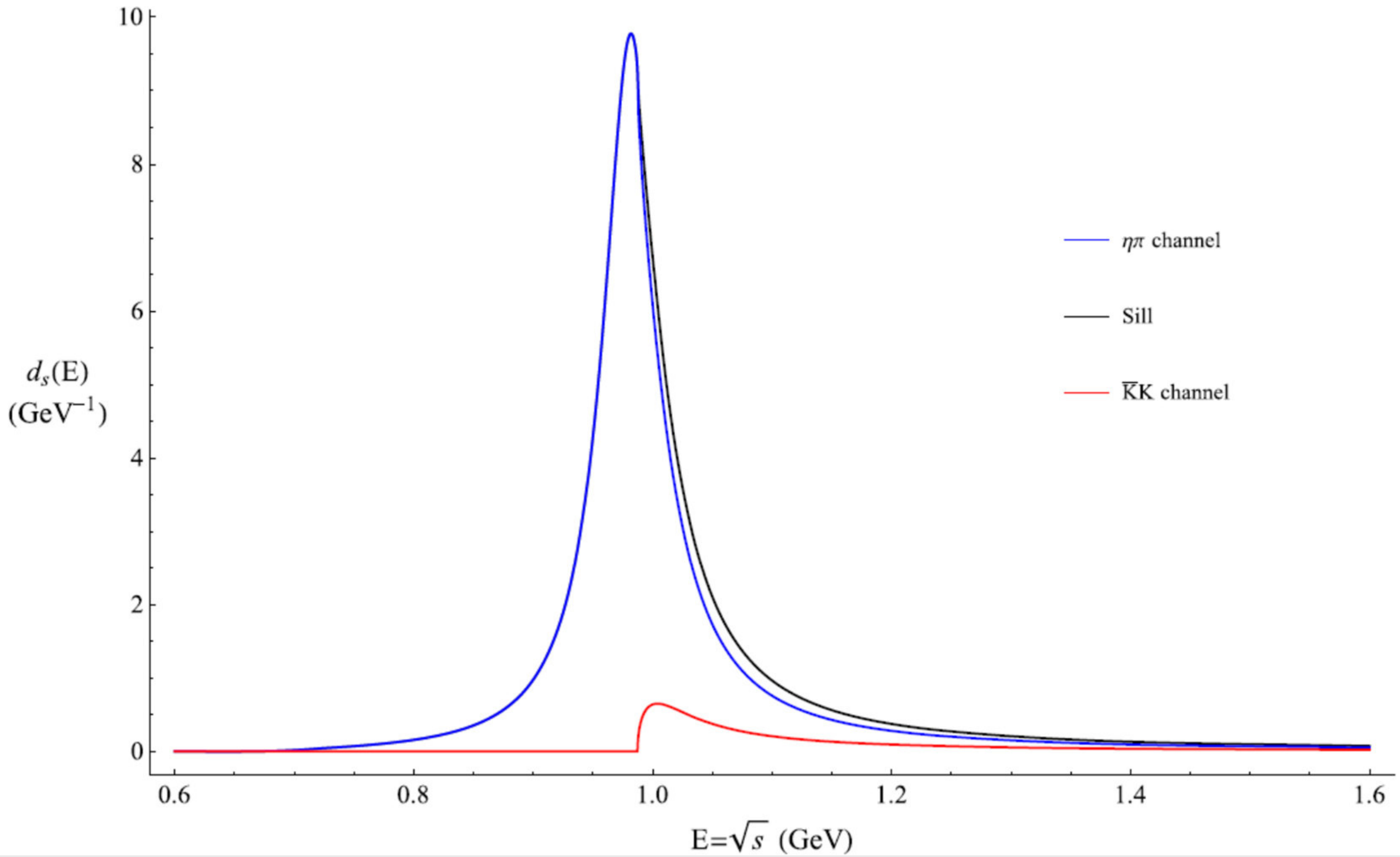
Sill: two-channel case

$$G_S(s) = \frac{1}{s - M^2 + i\tilde{\Gamma}_1\sqrt{s - s_{1,th}} + i\tilde{\Gamma}_2\sqrt{s - s_{2,th}} + i\varepsilon}.$$

$$d_S(s) = -\frac{1}{\pi}\,\mathrm{Im}[G_S(s)] = \begin{cases} \frac{1}{\pi}\frac{\tilde{\Gamma}_1\sqrt{s-s_{1,th}}+\tilde{\Gamma}_2\sqrt{s-s_{2,th}}}{(s-M^2)^2+(\tilde{\Gamma}_1\sqrt{s-s_{1,th}}+\tilde{\Gamma}_2\sqrt{s-s_{2,th}})^2} & \text{for } s > s_{2,th} \\ \frac{1}{\pi}\frac{\tilde{\Gamma}_1\sqrt{s-s_{1,th}}}{\left(s-M^2-\tilde{\Gamma}_2\sqrt{s_{2,th}-s}\right)^2+(\tilde{\Gamma}_1\sqrt{s-s_{1,th}})^2} & \text{for } s_{1,th} \le s \le s_{2,th} \\ 0 & \text{for } s < s_{1,th} \end{cases}$$

a0(980) example

Fig. 8 The Sill distribution of the $a_0(980)$ and the $\eta\pi$ and $\bar{K}K$ channels. The non-BW form due to the KK threshold is evident



Multichannel decay law

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Multichannel decay law

Francesco Giacosa [a,b,*]

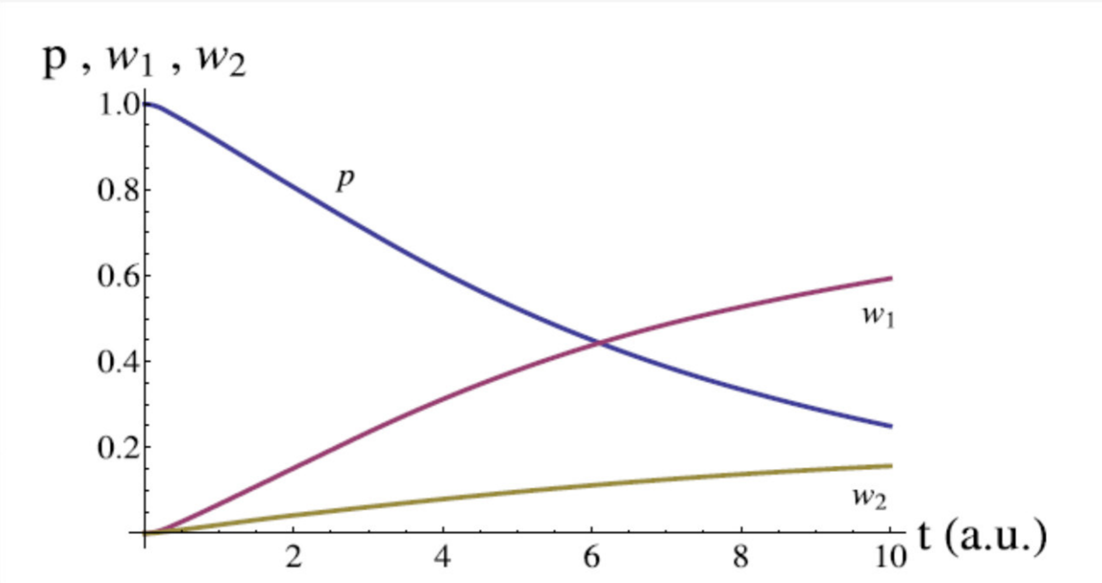

Fig. 1. The survival probability $p(t)$ of Eq. (1) and the decay probabilities $w_1(t)$ and $w_2(t)$ of Eq. (14) are plotted as function of t. The constraint $p+w_1+w_2=1$ holds. Note, t is expressed in a.u. of $[M^{-1}]$.

w1(t) is the probability that the decay has occurred in the first channel between (0,t)

$$\sum_{i=1}^{N} w_i = 1 - p(t)$$

$$w_i(t) = \int_{E_{th,i}}^{\infty} \mathrm{dE} \frac{2E^2 \Gamma_i(E)}{\pi} \left| \int_{E_{th,1}}^{\infty} \mathrm{dE}' d_S(E') \frac{e^{-iE't} - e^{-iEt}}{E'^2 - E^2} \right|^2$$

w1/w2 is not a constant