Model selection in electromagnetic production of kaons

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Motivation for the work on kaon photo/electroproduction

- We aim at understanding the baryon spectrum and production dynamics of particles with strangeness at low energies.

- Constituent Quark Model predicts a lot more $N^*$ states than was observed in pion production experiments → “missing” resonance problem.

- Models for the description of elementary hyperon electroproduction are a suitable tool for hypernuclear physics calculations.

- New good-quality photoproduction data from BGOOD, LEPS, GRAAL, MAMI and (particularly) CLAS collaborations allow us to tune free parameters of the models.

- As the $\alpha_s$ increases with decreasing energy, we cannot use perturbative QCD at low energies → need for introducing effective theories and models.
Introduction

Photoproduction process

\[ p(p) + \gamma(k) \rightarrow K^+(p_K) + \Lambda(p_\Lambda) \]

- Threshold: \( E_{\gamma}^{lab} = 0.911 \text{ GeV}, \ W = 1.609 \text{ GeV} \)

- In the lowest order, the reaction is described by the exchange of hadrons.
  - *The 3rd nucleon-resonance region:* many resonant states and no dominant one in the kaon photoproduction 
    \( \rightarrow \) need to assume a large number of nucleon resonances with mass \(< 2.5 \text{ GeV} \)

- Resonance region: resonance contributions dominate \( (N^*) \)

- Background: a plenty of nonresonant contributions \( (p, K, \Lambda; K^* \text{ and } Y^*) \)
Isobar model

Single-channel approximation
• higher-order contributions (rescattering, FSI) included, to some extent, by means of effective values of the coupling constants

Use of effective hadron Lagrangian
• hadrons either in their ground or excited states
• amplitude constructed as a sum of tree-level Feynman diagrams
  • background and resonant part

• hadronic form factors account for the inner structure of hadrons

Free parameters (couplings, hff’s cutoffs) adjusted to experimental data.
Satisfactory agreement with the data in the energy range $W = 1.6 - 2.5 \text{ GeV}$. 
Isobar model
Calculation procedure

- Reaction amplitude: sum of $s$-, $t$-, and $u$-channel (non) Born amplitudes

$$
\mathcal{M} = \sum_{x} \mathcal{M}_{x}, \text{ where } x \equiv s, t, u, N^*, K^*, Y^* 
$$

- Each contribution can be rewritten in a compact form

$$
\mathcal{M}(p, p_\Lambda, k) = \bar{u}(p_\Lambda) \gamma_5 \left( \sum_{j=1}^{6} \mathcal{A}_j(s, t, u) \mathcal{M}_j \right) u(p), \\
\text{where } \mathcal{A}_j \text{ are scalar amplitudes and } \mathcal{M}_j \text{ are gauge-invariant operators, i.e. } k_\mu \mathcal{M}_j^\mu = 0,
$$

$$
\mathcal{M}_1 = \frac{1}{2} \left[ k \not\cdot \varepsilon - \varepsilon \not\cdot k \right], \quad \mathcal{M}_2 = (p \cdot \varepsilon) - (k \cdot p) \frac{(k \cdot \varepsilon)}{k^2}, \\
\mathcal{M}_3 = (p_\Lambda \cdot \varepsilon) - (k \cdot p_\Lambda) \frac{(k \cdot \varepsilon)}{k^2}, \quad \mathcal{M}_4 = \varepsilon(k \cdot p) - k(p \cdot \varepsilon), \\
\mathcal{M}_5 = \varepsilon(k \cdot p_\Lambda) - k(p_\Lambda \cdot \varepsilon), \quad \mathcal{M}_6 = k(k \cdot \varepsilon) - \varepsilon k^2.
$$
Isobar model
Calculation procedure

- CGLN amplitudes \( f_i(k^2, s, t) \)

\[
\mathbb{M} = \chi_\Lambda \dagger \mathcal{F} \chi_p; \quad \mathcal{F} = f_1(\vec{\sigma} \cdot \vec{\varepsilon}) - i f_2(\vec{\sigma} \cdot \hat{p}_K)[\vec{\sigma} \cdot (\hat{k} \times \vec{\varepsilon})] + f_3(\vec{\sigma} \cdot \hat{k})(\hat{p}_K \cdot \vec{\varepsilon}) \\
+ f_4(\vec{\sigma} \cdot \hat{p}_K)(\hat{p}_K \cdot \vec{\varepsilon}) + f_5(\vec{\sigma} \cdot \hat{k})(\hat{k} \cdot \vec{\varepsilon}) + f_6(\vec{\sigma} \cdot \hat{p}_K)(\hat{k} \cdot \vec{\varepsilon})
\]

where e.g.
\[
f_1 = N^*[-(W - m_p)A_1 + (k \cdot p)A_4 + (k \cdot p_\Lambda)A_5 - k^2A_6]
\]

- Response functions, e.g. transverse cross section

\[
\frac{d\sigma}{d\Omega} = \sigma_T = C \left\{ |f_1|^2 + |f_2|^2 - 2 \text{Re} \, f_1 f_2^* \cos \theta_K \right. \\
+ \sin^2 \theta_K \left[ \frac{1}{2}(|f_3|^2 + |f_4|^2) + \text{Re} \, (f_1 f_4^* + f_2 f_3^* + f_3 f_4^* \cos \theta_K) \right] \right\},
\]

(for other response functions see Z. Phys. A 352 (1995) 327 )
Isobar model

Novel features of our isobar model

Exchanges of high-spin resonant states

• non physical lower-spin components removed by appropriate choice of $\mathcal{L}_{int}$

$$V_S^{\mu} \mathcal{P}_{ij,\mu\nu}^{(1/2)} V_{EM}^\nu = 0$$

Energy-dependent decay widths of nucleon resonances $\rightarrow$ restoration of unitarity

$$\Gamma(\vec{q}) = \Gamma_{N^*} \frac{\sqrt{s}}{m_{N^*}} \sum_i x_i \left( \frac{||\vec{q}_i||}{||\vec{q}_i^{N^*}||} \right)^{2l+1} \frac{D(||\vec{q}_i||)}{D(||\vec{q}_i^{N^*}||)},$$

Extension from photoproduction to electroproduction

• Phenomenological form factors in the electromagnetic vertex

• Longitudinal couplings of $N^*$’s to $\gamma^*$ (crucial at small $Q^2$)

$$V_{EM}^{\mu} (N_{1/2}^* p\gamma) = -i \frac{g_{EM}^3}{(m_R + m_p)^2} \Gamma_{\gamma^*} \gamma_\beta \mathcal{F}^\beta,$$

$$V_{EM}^{\mu} (N_{3/2}^* p\gamma) = -i \frac{g_{EM}^3}{m_R(m_R + m_p)^2} \gamma_5 \Gamma_{\gamma^*} \left( \not{q} g_{\mu\beta} - q_{\beta} \gamma_\mu \right) \mathcal{F}^\beta,$$

$$V_{EM}^{\mu\nu} (N_{5/2}^* p\gamma) = -i \frac{g_{EM}^3}{(2m_p)^5} \Gamma_{\gamma^*} \left( q_{\alpha} q_{\beta} g_{\mu\nu} + q^2 g_{\alpha\mu} g_{\beta\nu} - q_{\alpha} q_{\nu} g_{\beta\mu} - q_{\beta} q_{\nu} g_{\alpha\mu} \right) p^\alpha \mathcal{F}^\beta.$$
Fitting the data in the $K^+\Lambda$ channel

Minimization of $\chi^2/n.d.f.$ with help of MINUIT library

Resonance selection

- **s channel**: spin-1/2, 3/2, and 5/2 $N^*$ with mass $< 2.5$ GeV;
- **t channel**: $K^*(892)$, $K_1(1272)$
- **u channel**: $Y^*(1/2)$ and $Y^*(3/2)$

Free parameters ($\approx 30 + 10$):

- **SU(3)$_f$**: $-4.4 \leq g_{K\Lambda N}/\sqrt{4\pi} \leq -3.0,$
  $0.8 \leq g_{K\Sigma N}/\sqrt{4\pi} \leq 1.3$
- **$K^*$’s have vector and tensor couplings**
- spin-1/2 resonance $\rightarrow$ 1 parameter;
  spin-3/2 and 5/2 resonance $\rightarrow$ 2 parameters
- 2 cut-off parameters for the hff
- 1 longitudinal coupling for each $N^*$
- 2 cut-off parameters for the emff of $K^*$ and $K_1$

Experimental data

3383 $p(\gamma, K^+)\Lambda$ data

- cross section for $W < 2.355$ GeV
  (CLAS 2005 & 2010; LEPS, Adelseck-Saghai)
- hyperon polarisation for $W < 2.225$ GeV
  (CLAS 2010)
- beam asymmetry (LEPS)

171 $p(e, e'K^+)\Lambda$ data

- $\sigma_U, \sigma_T, \sigma_L, \sigma_{LT'}, \sigma_K$
Resulting models for the $K^+\Lambda$ photo- and electroproduction

**BS1 model ($\chi^2/n.d.f. = 1.64$)**
- $S_{11}(1535), S_{11}(1650), F_{15}(1680), P_{13}(1720), F_{15}(1860), D_{13}(1875), F_{15}(2000)$;
- $K^*(892), K_1(1272)$;
- $\Lambda(1520), \Lambda(1800), \Lambda(1890), \Sigma(1660), \Sigma(1750), \Sigma(1940)$;
- Multidipole form factor:
  $\Lambda_{bgr} = 1.88$ GeV, $\Lambda_{res} = 2.74$ GeV

**BS3 model ($\chi^2/n.d.f. = 1.74$)**
- $S_{11}(1535), S_{11}(1650), F_{15}(1680), P_{11}(1710), P_{13}(1720), F_{15}(1860), D_{13}(1875), P_{13}(1900), F_{15}(2000), D_{13}(2120)$;
- $K^*(892), K_1(1272)$;
- $\Lambda(1405), \Lambda(1600), \Lambda(1890), \Sigma(1670)$;
- Dipole form factor:
  $\Lambda_{bgr} = 1.24$ GeV, $\Lambda_{res} = 0.89$ GeV
Transverse, $\sigma_T$, and longitudinal, $\sigma_L$, cross sections of $p(e, e'K^+)\Lambda$

Extension from photo- to electroproduction

- **BS1**: naive extension by adding em. form factors only
- **BS3**: em. form factors and longitudinal couplings of $N^*$'s to $\gamma^*$ added
New fits for $K^+\Sigma^-$ channel

$\chi^2$ minimization and overfitting

Fitting procedure with MINUIT library: **minimizing the $\chi^2$**

$$
\chi^2 = \sum_{i=1}^{N} \frac{[d_i - p_i(c_1, \ldots, c_n)]^2}{\sigma_{d_i}^2},
$$

$(c_1, \ldots, c_n)$ - set of free parameters, $(d_1, \ldots, d_N)$ - set of data points, $p_i$ - theory, $\sigma_{d_i}$ - error

**Problem:** $\chi^2$ minimization cannot prevent overfitting

Example: polynomial curve fitting

- $f(x, w) = w_0 + w_1 x + w_2 x^2 + \cdots + w_k x^k$
- increasing order of polynomial $k$ fits the data well...
  
  ...but gives only poor description of the function which generated them...
  
  ...and may fail to generalize to new data

- **Occam’s razor** (law of parsimony): simpler models should be preferred

Model fits the noise in the sample
New fits of $K^+\Sigma^-$ channel
Least Absolute Shrinkage and Selection Operator (LASSO)

Remedy to the overfitting issue: regularization

- introduce a penalty term to the $\chi^2 \rightarrow$ penalization of large parameter values
- penalized $\chi^2_P$: $\chi^2_P = \chi^2 + P(\lambda)$
- penalty term: $P(\lambda) = \lambda^4 \sum_{i=1}^{N_{\text{res}}} |g_i|$
  $\lambda$ - regularization parameter, $g_i$ - resonances' couplings
- LASSO forces some of the parameters to zero
  $\rightarrow$ selection of a subset of the fit parameters
- $\lambda$ controls the strength of the penalty and thus the complexity of the model
  $\rightarrow$ higher powers of $\lambda$ allow fine sampling of the region of small $\lambda$
New fits of $K^+\Sigma^-$ channel

Information criteria:

- Akaike information criterion
  $$\text{AIC} = 2n_i + \chi_P^2$$

- Corrected Akaike information criterion
  $$\text{AICc} = \text{AIC} + \frac{2n_i(n_i+1)}{N-n_i-1}$$

- Bayesian information criterion
  $$\text{BIC} = n_i \ln(N) + \chi_P^2$$

  $n_i$ - no. of parameters corresponding to $\lambda_i$
  $N$ - number of data points

Applying the information criteria – forward selection

1. start with the full model: parameters initialized within $\langle -1; +1 \rangle$; use $\lambda_{\max}$
2. perform LASSO $\chi_P^2$ minimization and compute IC
3. in each run reduce $\lambda$ and run LASSO with the values of the previous run as starting values
4. repeat until $\lambda_{\min}$ is reached

Optimal $\lambda$ occurs at the minimum of the IC.
New fits of $K^+\Sigma^-$ channel

Fitting procedure

- resonance selection: motivation from previous analysis of $K^+\Lambda$ channel
- non resonant part: Born terms and exchanges of $K^*$ and $K_1$ and $\Sigma^*$’s
- resonant part: exchanges of $N^*$’s and $\Delta^*$’s in the $s$ channel
- around 600 data utilized to fit $\leq 25$ parameters
- result with the smallest $\chi^2/\text{ndf} = 2.3 \rightarrow$ fit $M$ (25 parameters, 14 resonances)
- LASSO applied at fit $M$: $\chi^2_P/\text{ndf} = 3.4 \rightarrow$ fit $L$ (17 parameters, 9 resonances)

Characteristics of models

- only one $\Delta$ resonance introduced
- no hyperon resonances needed for reliable data description
- results in very good agreement with the cross-section and beam-asymmetry data
- fit $L$ is very economical
New fits of $K^+\Sigma^-$ channel

Differential cross section in dependence on the photon lab energy
New fits of $K^+\Sigma^-\hspace{0.1cm}$ channel
Differential cross section in dependence on the photon lab energy - fit L w/o individual resonances

Notation: N7: N(1720)3/2^+, M4: N(2060)5/2^−
New fits of $K^+\Sigma^- \text{ channel}$

Beam asymmetry in dependence on the kaon center-of-mass angle - fit L w/o individual resonances
LASSO in the $K^+\Lambda$ channel

Selecting a subset of resonances (results very preliminary!)

Resonances in BS1 and BS1L:

- $S_{11}(1535), S_{11}(1650), F_{15}(1680), P_{13}(1720), F_{15}(1860), D_{13}(1875), F_{15}(2000)$;
- $K^*(892), K_1(1272)$;
- $\Lambda(1520), \Lambda(1800), \Lambda(1890), \Sigma(1660), \Sigma(1750), \Sigma(1940)$

<table>
<thead>
<tr>
<th>Model</th>
<th>no. of resonances</th>
<th>no. of parameters</th>
<th>$\chi^2$/n.d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS1</td>
<td>16</td>
<td>31</td>
<td>1.6</td>
</tr>
<tr>
<td>BS1L</td>
<td>9</td>
<td>20</td>
<td>3.6</td>
</tr>
</tbody>
</table>
Refitting the model’s parameters in the $K^+\Lambda$ channel

Ridge regression and cross validation for suppressing hyperon couplings

Why refit?

- include recent measurements of polarization observables (PR C 93, (2016) 065201)
- need to investigate more the role of hyperon resonances in $KY$ photoproduction
- large values of hyperon couplings: ridge regression to suppress them during the fitting procedure

Ridge regularization

- penalized $\chi^2_P$: $\chi^2_P = \chi^2 + \lambda^4 \sum_{i=1}^{n_\Lambda} g_i^2$, ($n_\Lambda =$ no. of $Y$ couplings)
- parameter values reduced but they are not reduced to zero

Cross validation

Bias-Variance trade-off
"One standard-error rule" for selecting appropriate model

Errors on the validation set help us determine the optimal value of $\lambda$

- average of validation errors over all $k$ runs
  \[ \overline{CV}(\lambda) = \frac{1}{k} \sum_{j=1}^{k} CV_j(\lambda) \]

- optimal $\lambda^*$ is then:
  \[ \lambda^* = \arg\min_{\lambda \in \{\lambda_{\min}, \ldots, \lambda_{\max}\}} \overline{CV}(\lambda) \]

"One standard-error rule": the most parsimonious model within one standard error from the minimum of the validation error should be chosen

- new optimal $\tilde{\lambda}$ is such that
  \[ \overline{CV}(\tilde{\lambda}) = \overline{CV}(\lambda^*) + SE(\lambda^*) \]
Relative percentage reduction of the resonance couplings

\[ 100 \times \left( \frac{|g_j| - |\tilde{g}_j|}{|g_j|} \right) \]

- \( g_j \) - values from the unregularized fitting
- \( \tilde{g}_j \) - values after performing Ridge regularization
$K^+\Lambda$ channel: beam asymmetry $\Sigma$
$K^+\Lambda$ channel: target asymmetry $T$
Summary

New version of isobar model for the $K^+\Lambda$ channel

- available for calculations online at:
  

Description extended from the $K^+\Lambda$ channel to the $K^+\Sigma^-$ channel.

Regularization methods introduced as a remedy for overfitting and as model selection tools.

Outlook

- testing the models in the DWIA calculations for hypernucleus production
- performing an analysis of $\Sigma$ photoproduction channels
- extending the analysis of electroproduction beyond $Q^2 = 1 \text{ GeV}^2$
- studying the production of $\Xi$ hypernuclei
- etc., etc....

Thank you for your attention!