

# Decays of the tensor glueball in a chiral approach / Is $f_2(1950)$ the tensor glueball?

Arthur Vereijken

In collaboration with: Francesco Giacosa, Shahriyar Jafarzade, Milena Piotrowska,  
UJK, Kielce

June 23rd 2023

17th International Workshop on Meson Physics



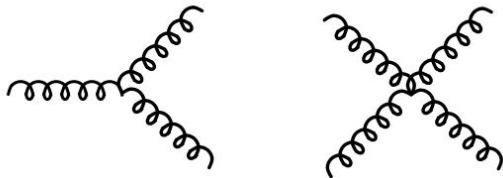
# Overview

- 1 Introduction
- 2 Tensor glueball in a chiral model
- 3 tensor glueball candidates
- 4 Summary

# Introduction

The QCD lagrangian contains gluon self-interaction due to its non-abelian SU(3) symmetry

$$\mathcal{L}_{QCD} = \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m_i \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$
$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$$



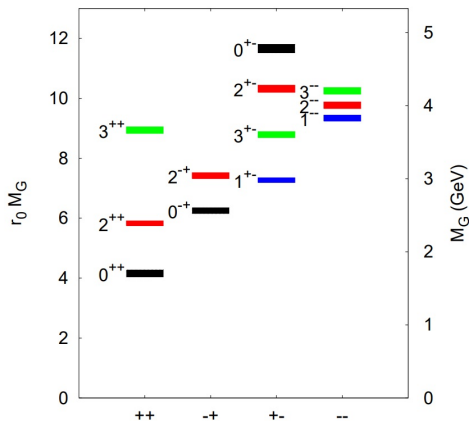
This begs the question: is there a bound state made of only gluons, a particle that does not contain any matter?

# Lattice QCD

Lattice calculations have found a large spectrum of pure gluon states.

The tensor ( $J^{PC} = 2^{++}$ ) is the second lightest glueball and so one of the best candidates for experimental verification.

Lattice calculations have some difficulties computing decay rates, so there is room for us to find new information using our chiral model.



Chen et al, 2005

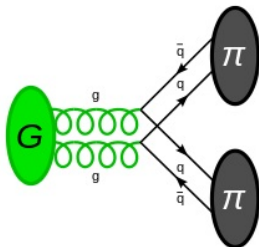
# Glueball width

Glueballs are expected to have relatively small decay widths, from large  $N_c$  scaling:

$$A_{gg \rightarrow \bar{q}q + \bar{q}q} \propto N_c^{-1}$$

$$A_{\bar{q}q \rightarrow \bar{q}q + \bar{q}q} \propto N_c^{-\frac{1}{2}}$$

All processes glueball  $\rightarrow$  hadrons are also suppressed because of the OZI rule



Numerous experiments are working on data related to glueballs

- BESIII
- LHCb
- GlueX
- Compass
- Clas 12
- PANDA

Experimentally  $J/\psi$  decays are one of the best places to search for glueballs.

The most important symmetry breaking patterns for the eLSM are:

- Breaking of dilatation symmetry by dilaton field  $G$  (scalar glueball), leading to gluon condensate

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left[ G^4 \log\left(\frac{G}{\Lambda_G}\right) - \frac{G^4}{4} \right]$$

- Spontaneous chiral symmetry breaking; QCD Lagrangian is (almost) invariant under chiral transformations, but the vacuum is not. This leads to a chiral condensate and pions as massless scalars
- The condensates lead to shifts e.g.  $G \rightarrow G + G_0, \Phi \rightarrow \Phi + \Phi_0$  which leads to mass terms similarly to the Higgs mechanism.
- Explicit chiral symmetry breaking gives pions a small mass compared to the other mesons

# Linear Sigma Model

The LSM was previously extended for tensor and axial tensor mesons and its decay products of vectors, axial vectors, etc.

$$\mathcal{L}_{\text{eLSM}} = \mathcal{L}_{\text{dil}} + \text{Tr} \left[ \left( D_\mu \Phi \right)^\dagger \left( D_\mu \Phi \right) \right] - m_0^2 \left( \frac{G}{G_0} \right)^2 \text{Tr} \left[ \Phi^\dagger \Phi \right] - \frac{1}{4} \text{Tr} \left[ \left( L_{\mu\nu}^2 + R_{\mu\nu}^2 \right) \right] + \dots ,$$

Gave us decent results for tensor mesons:

Decay process (in model)	eLSM (MeV)	PDG (MeV)
$a_2(1320) \rightarrow \rho(770) \pi$	$71.0 \pm 2.6$	$73.61 \pm 3.35 \leftrightarrow (70.1 \pm 2.7)\%$
$K_2^*(1430) \rightarrow K^*(892) \pi$	$27.9 \pm 1.0$	$26.92 \pm 2.14 \leftrightarrow (24.7 \pm 1.6)\%$
$K_2^*(1430) \rightarrow \rho(770) K$	$10.3 \pm 0.4$	$9.48 \pm 0.97 \leftrightarrow (8.7 \pm 0.8)\%$
$K_2^*(1430) \rightarrow \omega(782) K$	$3.5 \pm 0.1$	$3.16 \pm 0.88 \leftrightarrow (2.9 \pm 0.8)\%$
$f_2'(1525) \rightarrow K^*(892) K + \text{c.c.}$	$19.89 \pm 0.73$	



# Glueball chiral interactions

Compared to the work on tensor mesons, we need to replace the tensors to realize flavour blindness:

$$T_{\mu\nu} \longrightarrow G_{2,\mu\nu} \cdot \mathbf{1}$$

The lagrangian leading to tensor glueball decays involves solely left- and right-handed chiral fields:

$$\mathcal{L} = \lambda G_{\mu\nu} \left( \text{Tr} \left[ \{L^\mu, L^\nu\} \right] + \text{Tr} \left[ \{R^\mu, R^\nu\} \right] \right)$$

Left- and right-handed fields are in terms of the vector and axial vector nonets

$$L^\mu := V^\mu + A_1^\mu, \quad R^\mu := V^\mu - A_1^\mu.$$

With nice transformation rules  $L^\mu \rightarrow U_L L^\mu U_L^\dagger$ ,  $R^\mu \rightarrow U_R R^\mu U_R^\dagger$  under the chiral transformations of  $U_L(3) \times U_R(3)$

# Tensor glueball decays

The Lagrangian leads to three kinematically allowed decay channels

- Decaying of the tensor glueball to the two pseudoscalar mesons have the following decay rate formula

$$\Gamma_{G_2 \rightarrow P^{(1)} P^{(2)}} = \frac{\kappa_{gpp,i} \lambda^2 |\vec{k}_{p^{(1)},p^{(2)}}|^5}{60 \pi m_{G_2}^2};$$

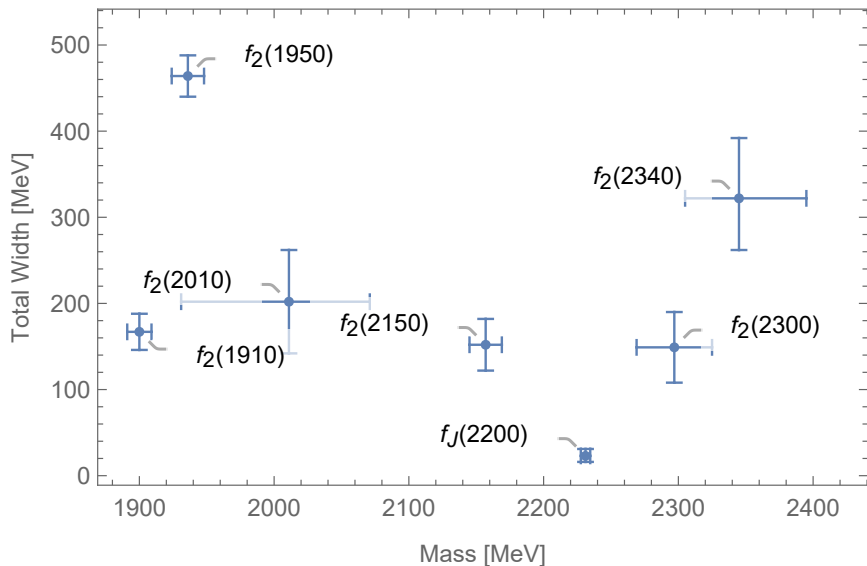
- while for the vector and pseudoscalar mesons

$$\Gamma_{G_2 \rightarrow V^{(1)} V^{(2)}} = \frac{\kappa_{gVV,i} \lambda^2 |\vec{k}_{V^{(1)},V^{(2)}}|}{120 \pi m_{G_2}^2} \left( 15 + \frac{5 |\vec{k}_{V^{(1)},V^{(2)}}|^2}{m_{V^{(1)}}^2} + \frac{5 |\vec{k}_{V^{(1)},V^{(2)}}|^2}{m_{V^{(2)}}^2} + \frac{2 |\vec{k}_{V^{(1)},V^{(2)}}|^4}{m_{V^{(1)}}^2 m_{V^{(2)}}^2} \right);$$

- and for the axial-vector and pseudoscalar mesons

$$\Gamma_{G_2 \rightarrow A_1 P} = \frac{\kappa_{gap,i} \lambda^2 |\vec{k}_{a_1,p}|^3}{120 \pi m_{G_2}^2} \left( 5 + \frac{2 |\vec{k}_{a_1,p}|^2}{m_{a_1}^2} \right).$$

# Isoscalar-tensor resonances



# Decay ratios

- Coupling constant is not known so we can only compute decay ratios
- Computation is done for a tensor glueball mass of 2210 MeV (later other masses)
- Vector channels are dominant, in particular  $\rho\rho$  and  $K^*K^*$

Decay Ratio	theory
$\frac{G_2(2210) \rightarrow \bar{K} K}{G_2(2210) \rightarrow \pi \pi}$	0.4
$\frac{G_2(2210) \rightarrow \eta \eta}{G_2(2210) \rightarrow \pi \pi}$	0.1
$\frac{G_2(2210) \rightarrow \eta \eta'}{G_2(2210) \rightarrow \pi \pi}$	0.004
$\frac{G_2(2210) \rightarrow \eta' \eta'}{G_2(2210) \rightarrow \pi \pi}$	0.006
$\frac{G_2(2210) \rightarrow \rho(770) \rho(770)}{G_2(2210) \rightarrow \pi \pi}$	55
$\frac{G_2(2210) \rightarrow K^*(892) K^*(892)}{G_2(2210) \rightarrow \pi \pi}$	46
$\frac{G_2(2210) \rightarrow \omega(782) \omega(782)}{G_2(2210) \rightarrow \pi \pi}$	18
$\frac{G_2(2210) \rightarrow \phi(1020) \phi(1020)}{G_2(2210) \rightarrow \pi \pi}$	6
$\frac{G_2(2210) \rightarrow a_1(1260) \pi}{G_2(2210) \rightarrow \pi \pi}$	0.24
$\frac{G_2(2210) \rightarrow K_{1,A} K}{G_2(2210) \rightarrow \pi \pi}$	0.08
$\frac{G_2(2210) \rightarrow f_1(1285) \eta}{G_2(2210) \rightarrow \pi \pi}$	0.02
$\frac{G_2(2210) \rightarrow f_1(1420) \eta}{G_2(2210) \rightarrow \pi \pi}$	0.01

# Data Comparison

Resonance	Decay Ratio	PDG	Model Prediction
$f_2(1910)$	$\rho\rho/\omega\omega$	$2.6 \pm 0.4$	3.1
$f_2(1910)$	$f_2(1270)\eta/a_2(1320)\pi$	$0.09 \pm 0.05$	0.07
$f_2(1910)$	$\eta\eta/\eta\eta'$	$< 0.05$	$\sim 8$
$f_2(1910)$	$\omega\omega/\eta\eta'$	$2.6 \pm 0.6$	$\sim 200$
<b><math>f_2(1950)</math></b>	$\eta\eta/\pi\pi$	<b><math>0.14 \pm 0.05</math></b>	<b>0.081</b>
<b><math>f_2(1950)</math></b>	<b><math>K\bar{K}/\pi\pi</math></b>	<b><math>\sim 0.8</math></b>	<b>0.32</b>
<b><math>f_2(1950)</math></b>	<b><math>4\pi/\eta\eta</math></b>	<b><math>&gt; 200</math></b>	<b><math>&gt; 700</math></b>
$f_2(2150)$	$f_2(1270)\eta/a_2(1320)\pi$	$0.79 \pm 0.11$	0.1
$f_2(2150)$	$K\bar{K}/\eta\eta$	$1.28 \pm 0.23$	$\sim 4$
$f_2(2150)$	$\pi\pi/\eta\eta$	$< 0.33$	$\sim 10$

Decay ratios for the decay channels with available data.

$f_J(2220)$  is historically seen as a good candidate for the tensor glueball

### $f_J(2220)$ DECAY MODES

	Mode	Fraction ( $\Gamma_i/\Gamma$ )
$\Gamma_1$	$\pi\pi$	not seen
$\Gamma_2$	$\pi^+\pi^-$	not seen
$\Gamma_3$	$K\bar{K}$	not seen
$\Gamma_4$	$\rho\bar{\rho}$	not seen
$\Gamma_5$	$\gamma\gamma$	not seen
$\Gamma_6$	$\eta\eta'(958)$	seen
$\Gamma_7$	$\phi\phi$	not seen
$\Gamma_8$	$\eta\eta$	not seen

- Only  $\eta\eta'$  is seen, but we find it is  $\sim 10^{-3}$  times  $\pi\pi$  mode.
- PDG lists decay ratio  $\pi\pi/\bar{K}K = 1.0 \pm 0.5$ , we find  $\pi\pi/\bar{K}K \sim 2.5$

# Estimating glueball width

- A rough guess on the width of the tensor glueball can be made.
- Consider  $f_2 \equiv f_2(1270) \simeq \sqrt{1/2}(\bar{u}u + \bar{d}d)$  and  $f'_2 \equiv f'_2(1525) \simeq \bar{s}s$ , with  $\Gamma_{f_2 \rightarrow \pi\pi} = 157.2$  MeV and  $\Gamma_{f'_2 \rightarrow \pi\pi} = 0.71$  MeV.
- The amplitude for  $f_2 \rightarrow \pi\pi$  requires the creation of a single  $\bar{q}q$  pair from the vacuum and scales as  $1/\sqrt{N_c}$ , where  $N_c$  is the number of colors. On the other hand, the amplitude for  $f'_2 \rightarrow \pi\pi$  scales as  $1/N_c^{3/2}$  and goes schematically like

$$\bar{s}s \rightarrow gg \rightarrow \sqrt{1/2}(\bar{u}u + \bar{d}d)$$

# Estimating glueball width

- Consider a transition Hamiltonian

$$H_{int} = \lambda (|\bar{u}u\rangle \langle gg| + |\bar{d}d\rangle \langle gg| + |\bar{s}s\rangle \langle gg| + h.c.), \quad \lambda \propto 1/\sqrt{N_c}.$$

Then:  $A_{f'_2 \rightarrow \pi\pi} \simeq \sqrt{2}\lambda^2 A_{f_2 \rightarrow \pi\pi}$ , hence  $\Gamma_{f'_2 \rightarrow \pi\pi} \simeq 2\lambda^4 \Gamma_{f_2 \rightarrow \pi\pi}$ ,

- Tensor glueball decay into  $\pi\pi$  intuitively speaking, is at an 'intermediate stage', since it starts with a  $gg$  pair. One has:

$$A_{G_2 \rightarrow \pi\pi} \simeq \sqrt{2}\lambda A_{f_2 \rightarrow \pi\pi},$$

$$\Gamma_{G_2 \rightarrow \pi\pi} \simeq 2\lambda^2 \Gamma_{f_2 \rightarrow \pi\pi} \simeq \sqrt{2} \sqrt{\Gamma_{f_2 \rightarrow \pi\pi} \Gamma_{f'_2 \rightarrow \pi\pi}} \simeq 15 \text{MeV}.$$

- We emphasize that this is a **rough estimate**, based on large  $N_c$  scaling.
- Similar results to some holographic models: very large decay widths in vector modes.



# Glueball candidates

Resonances	Interpretation status
$f_2(1910)$	Agreement with some data, but large discrepancies in $\eta\eta'$ mode
<b><math>f_2(1950)</math></b>	<b><math>\eta\eta'/\pi\pi</math> agrees with data, no contradictions to data but broad tensor glueball Best fit as predominantly glueball</b>
$f_2(2010)$	Likely primarily strange-antistrange content
$f_2(2150)$	All available data contradicts theoretical prediction
$f_J(2220)$	Data on $\pi\pi/K\bar{K}$ disagrees with theory Only smallest predicted decay channels are seen
$f_2(2300)$	Likely primarily strange-antistrange content
$f_2(2340)$	Likely primarily strange-antistrange content would also imply a broad glueball

Spin 2 resonances and their status as the tensor glueball.

# Summary

- Glueballs are a yet undiscovered prediction of QCD and an active research topic of both theoretical models and experimental efforts
- We have adapted the eLSM for tensor mesons to describe the tensor glueball
- We obtain decay ratios; vector channels are dominant, in particular  $\rho\rho$  and  $K^*K^*$
- The  $f_2(1950)$  is clearly favored as a candidate by the eLSM.
- Sometimes data is limited, in particular, the analysis for the states  $f_J(2220)$ ,  $f_2(2300)$ , and  $f_2(2340)$  would benefit from more experimental data.
- Preliminary estimate for the decay widths gives 15 MeV for the  $\pi\pi$  channel, which implies a very broad glueball in the vector channels.

**Thank you for your attention**