Revealing violations of macrorealism in flavor oscillations
Leggett-Garg inequalities and no-signaling-in-time conditions

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Standard quantum mechanics

- linearity of Schrödinger equation allows superpositions:

  \[ \psi_1, \psi_2 \text{ are solutions } \Rightarrow \psi = c_1\psi_1 + c_2\psi_2 \text{ is also a solution} \]

- evolution of quantum system due to Schrödinger equation is deterministic

- measurement destroys superposition with outcomes distributed due to Born rule:

  \[ P_1 = |c_1|^2, \quad P_2 = |c_2|^2 (\langle \psi_1 | \psi_2 \rangle = 0). \]
Troubles with standard QM

Standard quantum mechanics exposes two different regimes:

1. Schrödinger evolution: linear, deterministic and reversible.
Is there a border between Q and C worlds?
Conditions of macrorealism

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### Scenario
If we perform a measurement of a dichotomic (Yes/No $= \pm 1$) observable $Q$ of the macroscopic system, we find its well-defined pre-existing value without disturbing the dynamics of the system.

Three times case

Constraints from macrorealism

Macrorealism implies certain constraints on the measurement statistics in the considered scenario, e.g., on correlation functions $C_{ij} = \langle Q_i Q_j \rangle \equiv \langle Q(t_i)Q(t_j) \rangle$. 

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Leggett-Garg inequalities (Stand.)

\begin{align*}
1 + C_{12} + C_{23} + C_{13} & \geq 0, \\
1 - C_{12} - C_{23} + C_{13} & \geq 0, \\
1 + C_{12} - C_{23} - C_{13} & \geq 0, \\
1 - C_{12} + C_{23} - C_{13} & \geq 0,
\end{align*}


Leggett-Garg inequalities (Wigner)

\begin{align*}
P(Q_2, Q_3) - P(-Q_1, Q_2) - P(Q_1, Q_3) & \leq 0, \\
P(Q_1, Q_3) - P(Q_1, -Q_2) - P(Q_2, Q_3) & \leq 0, \\
P(Q_1, Q_2) - P(Q_2, -Q_3) - P(Q_1, Q_3) & \leq 0.
\end{align*}
Testing LGI in flavor oscillations

Dichotomic flavor observable
We can ask: Are you in flavor $F$ or not?

$$Q = 2 |F\rangle \langle F| - I,$$

We can take $F = \nu_e$ for neutrinos and $F = K^0$ for neutral kaons.
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Testing LGI in flavor oscillations: Neutrinos

Testing LGI in flavor oscillations: $K^0/\bar{K}^0$

Necessary and sufficient conditions

No Fine’s theorem!
In contrast to Bell inequalities, Leggett-Garg inequalities are not a necessary and sufficient condition for macrorealism: \textit{Macrorealism implies no violation of LGI, but satisfaction of LGI can still hide quantumness!}
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Alternative conditions for macrorealism

- *No signaling in time*: Past measurements do not influence the outcomes of future ones.
- *Arrow of time*: Future measurements do not influence the outcomes of past ones.

Necessary and sufficient conditions

NSIT\(^{(1)}\) : \( P(Q_3) = \sum_{Q_2} P(Q_2, Q_3) \),

NSIT\(^{(2)}\) : \( P(Q_1, Q_3) = \sum_{Q_2} P(Q_1, Q_2, Q_3) \),

NSIT\(^{(3)}\) : \( P(Q_2, Q_3) = \sum_{Q_1} P(Q_1, Q_2, Q_3) \),

AoT\(^{(1)}\) : \( P(Q_1, Q_2) = \sum_{Q_3} P(Q_1, Q_2, Q_3) \),

AoT\(^{(2)}\) : \( P(Q_1) = \sum_{Q_2} P(Q_1, Q_2) \),

AoT\(^{(3)}\) : \( P(Q_2) = \sum_{Q_3} P(Q_2, Q_3) \).

Tests of macrorealism with flavored particles

There is a single necessary and sufficient condition of macrorealism in two-flavor oscillations:

\[ \mathcal{N}(t) \equiv P_{F \rightarrow \tilde{F}}(2t) - 2P_{F \rightarrow \tilde{F}}(t) P_{F \rightarrow F}(t) = 0. \]

NSIT and AoT conditions: Neutrinos

NSIT and AoT conditions: Neutral kaons

![Graph showing NSIT conditions over time/mean proper lifetime]
Conclusions and outlook

- The set of necessary and sufficient NSIT/AoT conditions reduces to a single, non-trivial NSIT relation for macrorealism which can be potentially probed in two-flavor neutrino and neutral kaons experiments.

- The effect of decoherence for long detection times/distances allows for a net deviation from macrorealism. For this reason, neutrinos can never be described in a macrorealistic way, even when quantum coherence is apparently degraded because of the wave packet spreading.

- At late times, the LGIs are not faithful quantifiers of the macrorealistic description, since they are fulfilled whilst the NSIT condition is always violated.
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Thank you for your attention!