Revealing violations of macrorealism in flavor oscillations

Leggett-Garg inequalities and no-signaling-in-time conditions
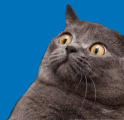
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Standard quantum mechanics

- linearity of Schrödinger equation allows superpositions:

 ψ_1, ψ_2 are solutions $\Rightarrow \psi = c_1\psi_1 + c_2\psi_2$ is also a solution

- evolution of quantum system due to Schrödinger equation is deterministic

- measurement destroys superposition with outcomes distributed due to Born rule:

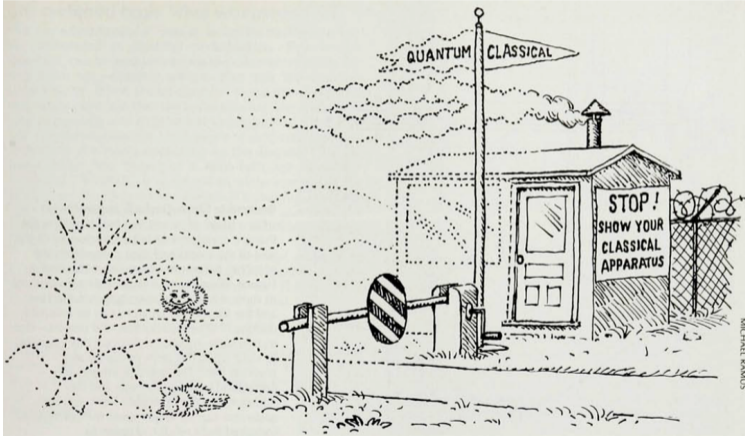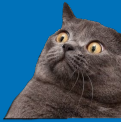$$P_1 = |c_1|^2, \ P_2 = |c_2|^2 \ (\langle\psi_1|\psi_2\rangle = 0).$$

Troubles with standard QM

Standard quantum mechanics exposes two different regimes:

1. Schrödinger evolution: linear, deterministic and reversible.
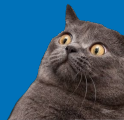2. Measurement: non-linear, stochastic and irreversible.
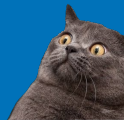
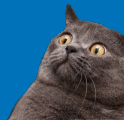# Is there a border between Q and C worlds?

Conditions of macrorealism

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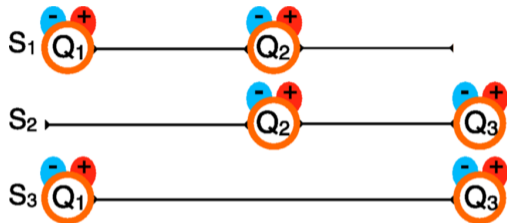
> **Scenario**
> If we perform a measurement of a dichotomic (Yes/No $= \pm 1$) observable Q of the macroscopic system, we find its well-defined pre-existing value without disturbing the dynamics of the system.

A. J. Leggett and A. Garg, Phys. Rev. Lett. **54**, 857 (1985)
C. Emary, N. Lambert, and F. Nori, Rep. Prog. Phys. **77**, 016001 (2014)
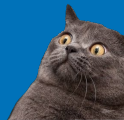
Three times case

Constraints from macrorealism

Macrorealism implies certain constraints on the measurement statistics in the considered scenario, e.g., on correlation functions $C_{ij} = \langle Q_i Q_j \rangle \equiv \langle Q(t_i) Q(t_j) \rangle$.
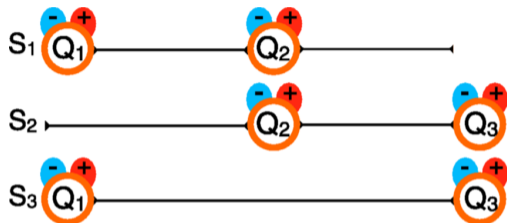
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Leggett-Garg inequalities (Stand.)

$$1 + C_{12} + C_{23} + C_{13} \geq 0,$$
$$1 - C_{12} - C_{23} + C_{13} \geq 0,$$
$$1 + C_{12} - C_{23} - C_{13} \geq 0,$$
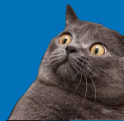$$1 - C_{12} + C_{23} - C_{13} \geq 0,$$

Leggett-Garg inequalities (Wigner)

$$P(Q_2, Q_3) - P(-Q_1, Q_2) - P(Q_1, Q_3) \leq 0,$$
$$P(Q_1, Q_3) - P(Q_1, -Q_2) - P(Q_2, Q_3) \leq 0,$$
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Testing LGI in flavor oscillations

Dichotomic flavor observable

We can ask: *Are you in flavor F or not?*

$$Q = 2|F\rangle\langle F| - I,$$

We can take $F = \nu_e$ for neutrinos and $F = K^0$ for neutral kaons.
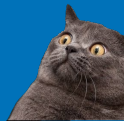
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- J. A. Formaggio, D. I. Kaiser, M. M. Murskyj, and T. E. Weiss, Phys. Rev. Lett. **117**, 050402 (2016).
- J. Naikoo, A. K. Alok, and S. Banerjee, Phys. Rev. D **97**, 053008 (2018).
- J. Naikoo, A. K. Alok, S. Banerjee, and S. U. Sankar, Phys. Rev. D **99**, 095001 (2019).
- J. Naikoo, S. Kumari, S. Banerjee, and A. K. Pan, J. Phys. G: Nucl. Part. Phys. **47**, 095004 (2020).
- J. Naikoo, A. K. Alok, S. Banerjee, S. U. Sankar, G. Guarnieri, C. Schultze, and B. C. Hiesmayr, Nucl. Phys. B **951**, 114872 (2020).
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- M. Blasone, F. Illuminati, L. Petruzziello, and L. Smaldone, arXiv:2111.09979 (2021).

Testing LGI in flavor oscillations: Neutrinos
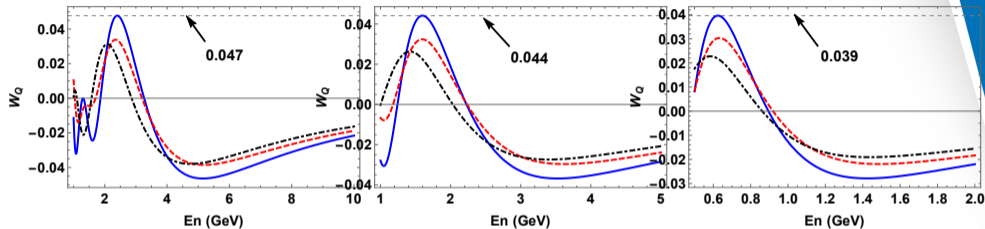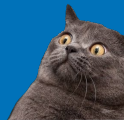
J. Naikoo, S. Kumari, S. Banerjee, and A. K. Pan, J. Phys. G: Nucl. Part. Phys. **47**, 095004 (2020).

Testing LGI in flavor oscillations: $K^0/\bar{K}^0$
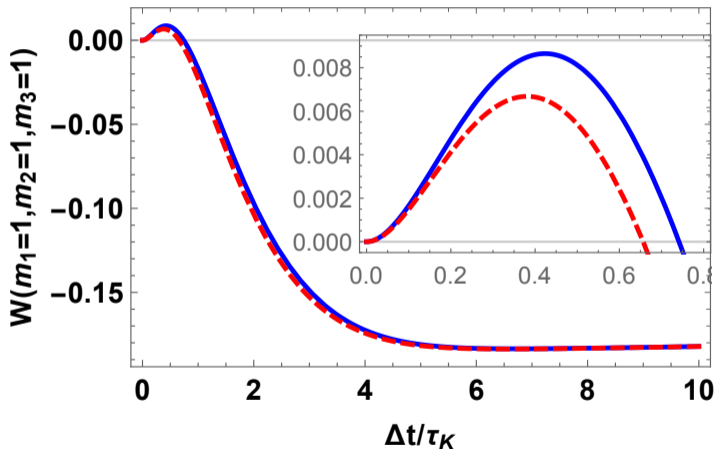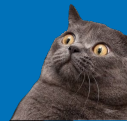
J. Naikoo, S. Kumari, S. Banerjee, and A. K. Pan, J. Phys. G: Nucl. Part. Phys. **47**, 095004 (2020).

Necessary and sufficient conditions

No Fine's theorem!
In contrast to Bell inequalities, Leggett-Garg inequalities are not a necessary and sufficient condition for macrorealism: *Macrorealism implies no violation of LGI, but satisfaction of LGI can still hide quantumness!*
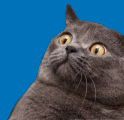
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Alternative conditions for macrorealism
- *No signaling in time*: Past measurements do not influence the outcomes of future ones.
- *Arrow of time*: Future measurements do not influence the outcomes of past ones.

L. Clemente and J. Kofler, Phys. Rev. A **91**, 062103 (2015)
L. Clemente and J. Kofler, Phys. Rev. Lett. **116**, 150401 (2016)

Necessary and sufficient conditions

$\mathsf{NSIT}^{(1)}:\ P(Q_3)\ =\ \sum_{Q_2} P(Q_2, Q_3)\,,$

$\mathsf{NSIT}^{(2)}:\ P(Q_1, Q_3)\ =\ \sum_{Q_2} P(Q_1, Q_2, Q_3)\,,$

$\mathsf{NSIT}^{(3)}:\ P(Q_2, Q_3)\ =\ \sum_{Q_1} P(Q_1, Q_2, Q_3)\,,$

$\mathsf{AoT}^{(1)}:\ P(Q_1, Q_2)\ =\ \sum_{Q_3} P(Q_1, Q_2, Q_3)\,,$

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Tests of macrorealism with flavored particles
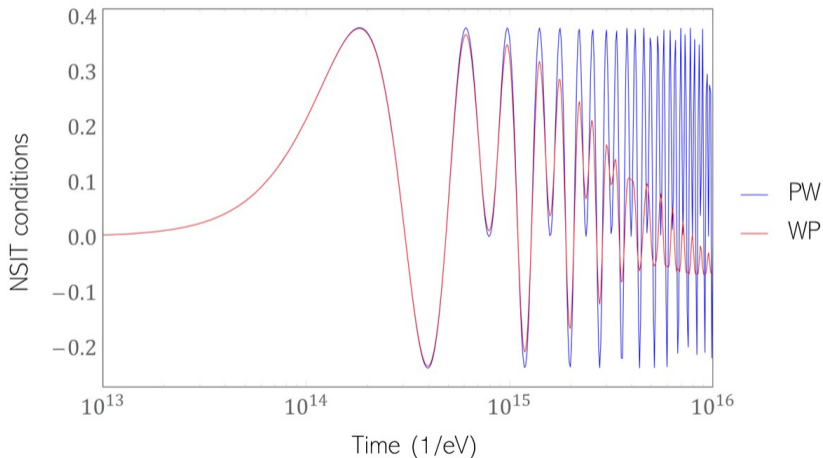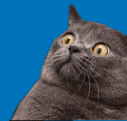
There is a single necessary and sufficient condition of macrorealism in two-flavor oscillations:

$$\mathcal{N}(t) \equiv P_{F \to \tilde{F}}(2t) - 2P_{F \to \tilde{F}}(t)\,P_{F \to F}(t) = 0.$$

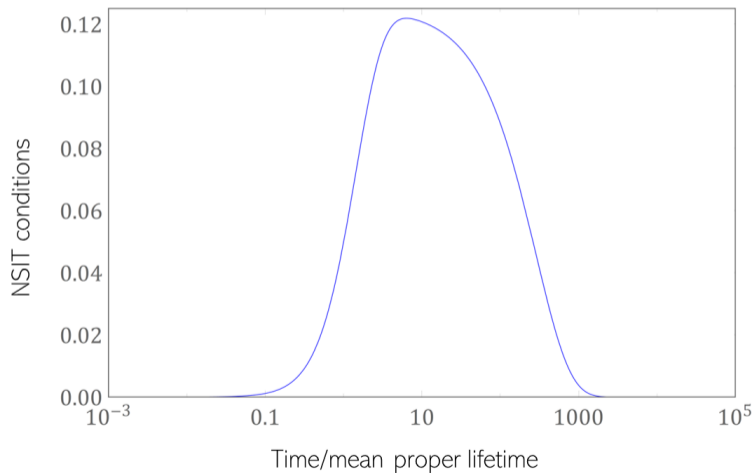M. Blasone, F. Illuminati, L. Petruzziello, K. Simonov, and L. Smaldone, arXiv:2211.16931 (2022).

NSIT and AoT conditions: Neutrinos

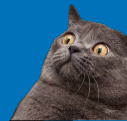M. Blasone, F. Illuminati, L. Petruzziello, K. Simonov, and L. Smaldone, arXiv:2211.16931 (2022).

NSIT and AoT conditions: Neutral kaons

Conclusions and outlook

- The set of necessary and sufficient NSIT/AoT conditions reduces to a single, non-trivial NSIT relation for macrorealism which can be potentially probed in two-flavor neutrino and neutral kaons experiments.
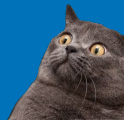
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- The effect of decoherence for long detection times/distances allows for a net deviation from macrorealism. For this reason, neutrinos can never be described in a macrorealistic way, even when quantum coherence is apparently degraded because of the wave packet spreading.
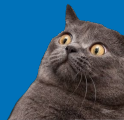
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- At late times, the LGIs are not faithful quantifiers of the macrorealistic description, since they are fulfilled whilst the NSIT condition is always violated.

Thank you for your attention!