Light front approach to axial meson photon transition form factors: probing the structure of $\chi_{c1}(3872)$

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Outline

$\gamma^*\gamma^*$ transition form factors for the axial vector meson and spacelike photons

An exotic axial vector: $\chi_{c1}(3872)$, can one pin down its $c\bar{c}$ component?

I. Babiarz, R. Pasechnik, W. Schäfer and A. Szczurek, "Light-front approach to axial-vector quarkonium $\gamma^*\gamma^*$ form factors," [arXiv:2208.05377 [hep-ph]], JHEP09(2022)170.

I. Babiarz, R. Pasechnik, W. Schäfer and A. Szczurek, "Probing the structure of $\chi_{c1}(3872)$ with photon transition form factors," Phys. Rev. D **107** (2023) no.7, L071503 [arXiv:2303.09175 [hep-ph]].

$\gamma^*\gamma^*$-transition form factors for $J^{PC} = 1^{++}$ axial mesons

$$
\begin{aligned}
\frac{1}{4\pi\alpha_{\rm em}}\mathcal{M}_{\mu\nu\rho} &= i\left(q_1 - q_2 + \frac{Q_1^2 - Q_2^2}{(q_1+q_2)^2}(q_1+q_2)\right)_\rho \tilde{G}_{\mu\nu}\,\frac{M}{2X}F_{\rm TT}(Q_1^2,Q_2^2) \\
&+ \; ie^L_\mu(q_1)\tilde{G}_{\nu\rho}\,\frac{1}{\sqrt{X}}F_{\rm LT}(Q_1^2,Q_2^2) + ie^L_\nu(q_2)\tilde{G}_{\mu\rho}\,\frac{1}{\sqrt{X}}F_{\rm TL}(Q_1^2,Q_2^2)\,.
\end{aligned}
$$

- Above we introduced

$$
\tilde{G}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\,,\; X = (q_1\cdot q_2)^2 - q_1^2 q_2^2
$$

and the polarization vectors of longitudinal photons

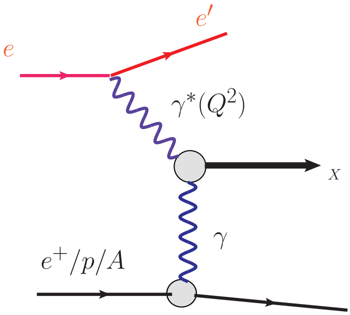
$$
e^L_\mu(q_1) = \sqrt{\frac{-q_1^2}{X}}\left(q_{2\mu} - \frac{q_1\cdot q_2}{q_1^2}q_{1\mu}\right), \qquad e^L_\nu(q_2) = \sqrt{\frac{-q_2^2}{X}}\left(q_{1\nu} - \frac{q_1\cdot q_2}{q_2^2}q_{2\nu}\right).
$$

- $F_{\rm TT}(0,0) = 0$, there is **no decay to two photons** (Landau-Yang).
- $F_{\rm LT}(Q^2,0) \propto Q$ (absence of kinematical singularities).

$$
f_{\rm LT}(Q^2) = \frac{F_{\rm LT}(Q^2,0)}{Q}
$$

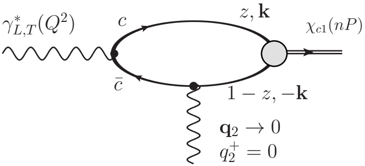
- $f_{\rm LT}(0)$ gives rise to so-called “reduced width” $\tilde{\Gamma}$.

Accessing transition form factors



- We need at least **one virtual photon** to produce an axial vector in photon photon collisions. This excludes ultraperipheral heavy ion collisions, where photons are quasi-real.
- Electron scattering gives us access to finite Q^2 and a whole polarization density matrix of virtual photons.
- Feasible options are:
  1. single tag e^+e^- collisions. Here the tagged lepton couples to the virtual photon, while photons from the lepton "lost in the beampipe" are quasireal.
  2. electron-proton or electron-ion scattering. Here especially heavy ions such as Gold which large charge $Z = 79$ give rise to a large quasireal photon flux enhanced by Z^2.

Transition amplitude in the Drell-Yan frame



LF-Fock state expansion

$$|\chi_{c1}; \lambda, P_+, \boldsymbol{P}\rangle = \sum_{i,j,\sigma,\bar\sigma} \frac{\delta^i_j}{\sqrt{N_c}} \int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \Psi^\lambda_{\sigma\bar\sigma}(z,\boldsymbol{k}) \times \Big| Q_{i\sigma}(zP_+, \mathbf{p}_Q) \bar{Q}^j_{\bar\sigma}((1-z)P_+, \mathbf{p}_{\bar Q}) \Big\rangle + \dots$$

$$\boldsymbol{k} = (1-z)\boldsymbol{p}_Q - z\boldsymbol{p}_{\bar Q} \qquad \boldsymbol{P} = \boldsymbol{p}_Q + \boldsymbol{p}_{\bar Q}$$

- We evaluate the $\gamma^*\gamma^* \to \chi_{c1}$ amplitude in the Drell-Yan frame where $q_{1\mu} = q_{1+}n^+_\mu + q_{1-}n^-_\mu$ and $q_{2\mu} = q_{2-}n^-_\mu + q^\perp_{2\mu}$, using the light front plus-component of the current:

$$\langle \chi_{c1}(\lambda)|J_+(0)|\gamma^*_L(Q^2)\rangle = 2q_{1+}\sqrt{N_c} \int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \times \sum_{\sigma,\bar\sigma} \Psi^{\lambda *}_{\sigma\bar\sigma}(z,\boldsymbol{k}) \, (\boldsymbol{q}_2 \cdot \nabla_{\boldsymbol{k}}) \Psi^{\gamma_L}_{\sigma\bar\sigma}(z,\boldsymbol{k},Q^2) \,.$$

- for *spacelike* photons, the plus component of the current is free from parton number changing or instantaneous fermion exchange contributions.

Quarkonium light front wave functions

- We adopt two different approaches to LFWFs:

Terentev substitution - LFWF from potential model

- Quark three-momentum in bound state rest frame

$$\vec{k} = (\boldsymbol{k}, k_z), \qquad k_z = \left(z - \frac{1}{2}\right) M_{c\bar{c}} \qquad M_{c\bar{c}}^2 = \frac{\boldsymbol{k}^2 + m_c^2}{z(1-z)}$$

- radial WF $u_{nP}(k)$ becomes (with appropriate Jacobian) radial LFWF $\psi(z, \boldsymbol{k})$
- canonical spin is substituted by LF helicity via **Melosh transform**

$$\xi_Q = R(z, \boldsymbol{k})\chi_Q\,, \qquad \xi_{\bar{Q}}^* = R^*(1-z, -\boldsymbol{k})\chi_{\bar{Q}}^* \qquad R(z, \boldsymbol{k}) = \frac{m_c + zM_{c\bar{c}} - i\vec{\sigma}\cdot(\vec{n}\times\boldsymbol{k})}{\sqrt{(m_c + zM_{c\bar{c}})^2 + \boldsymbol{k}^2}}$$

- We use a variety of interquark potentials summarized in J. Cepila et al., Eur. Phys. J. C **79** (2019) no.6, 495

Basis light front quantization (BLFQ)

- bound state WFs from effective LF-Hamiltonian

$$H_{\rm eff}|\chi_c; \lambda_A, P_+, \boldsymbol{P}\rangle = M_\chi^2 |\chi_c; \lambda_A, P_+, \boldsymbol{P}\rangle\,,$$

- we use LFWFs from Y. Li, P. Maris and J. P. Vary, Phys. Rev. D **96** (2017), 016022
- effective Hamiltonian which contains a term motivated by a "soft-wall" confinement from LF-holography, as well as a longitudinal confinement potential supplemented by one gluon exchange including the full spin-structure.

Light front wave functions from potential models

- For the weakly bound systems a procedure to obtain the LFWF from Schrödinger WFs has been proposed by Terentev. In this case the helicity dependent WF $\Psi^{(\lambda_A)}_{\lambda\bar\lambda}(z,\boldsymbol{k})$ factorizes into a "radial" part, and a spin-orbit part obtained by a Melosh-rotation $R(z,\boldsymbol{k})$.

- Rest frame:

$$\begin{aligned}\Psi^{(\lambda_A)}_{\tau\bar\tau}(\vec k) &= \sum_{L_z+S_z=\lambda_A} Y_{1L_z}(\hat k)\left\langle \frac{1}{2}\frac{1}{2}\tau\bar\tau \middle| 1S_z\right\rangle \langle 11L_zS_z|1\lambda_A\rangle \frac{u(k)}{k} \\ &= \frac{1}{2}\sqrt{\frac{3}{4\pi}}\,\xi_Q^{\tau\dagger}\,\left(\vec\sigma\cdot\frac{\vec k\times\vec E(\lambda_A)}{k}\right) i\sigma_2\,\xi_{\bar Q}^{\bar\tau *}\,\frac{u(k)}{k}\,.\end{aligned}$$

- Light front:

$$\Psi^{(\lambda_A)}_{\lambda\bar\lambda}(z,\boldsymbol{k}) = \chi_Q^{\lambda\dagger}\,\mathcal{O}'_{\lambda_A}\,i\sigma_2\,\chi_{\bar Q}^{\bar\lambda *}\;\psi(z,\boldsymbol{k})\,\sqrt{2(M^2_{Q\bar Q}-4m_Q^2)}\,,$$

with:

$$\mathcal{O}'_{\lambda_A} = \sqrt{\frac{3}{2}}\,R^\dagger(z,\boldsymbol{k})\left(\vec\sigma\cdot\frac{\vec k\times\vec E(\lambda_A)}{\sqrt{2}k}\right)R(1-z,-\boldsymbol{k})\,.$$

and

$$\psi(z,\boldsymbol{k}) = \frac{\pi\sqrt{M_{Q\bar Q}}}{2\sqrt{2}}\,\frac{u(k)}{k^2}\,,$$

Transition form factor from light front wave functions

- We use the well known perturbative LFWF of the longitudinal photon

$$\Psi^{\gamma_L}_{\sigma\bar{\sigma}}(z,\boldsymbol{k},Q^2) = ee_c\sqrt{z(1-z)}\,\frac{2z(1-z)Q}{\boldsymbol{k}^2+\epsilon^2}\,\delta_{\sigma,-\bar{\sigma}}\,,$$

  with $\epsilon^2 = m_c^2 + z(1-z)Q^2$.
- Only the $S_z = 0$ component with antiparallel quark helicities and one unit of orbital angular momentum contributes.

$$\frac{f_{\rm LT}(Q^2)}{Q^2+M_\chi^2} = -2\sqrt{2N_c}\,e_c^2\int\frac{dzd^2\boldsymbol{k}}{16\pi^3}\,\frac{k_x+ik_y}{[\boldsymbol{k}^2+\epsilon^2]^2}\sqrt{z(1-z)}\Big\{\Psi^{(+1)*}_{\uparrow\downarrow}(z,\boldsymbol{k}) + \Psi^{(+1)*}_{\downarrow\uparrow}(z,\boldsymbol{k})\Big\}$$

- For the $Q\bar{Q}$ state,

$$F_{\rm TT}(Q^2,0) = -\frac{Q^2}{M}f_{\rm LT}(Q^2)\,.$$

- In the Melosh transform formalism for the n^3P_1 state, we have

$$\sqrt{z(1-z)}\left\{\Psi^{(+1)*}_{\uparrow\downarrow}(z,\boldsymbol{k}) + \Psi^{(+1)*}_{\downarrow\uparrow}(z,\boldsymbol{k})\right\} = (k_x - ik_y)\sqrt{\frac{3}{2}\,\frac{\pi\sqrt{M_{c\bar{c}}}}{2}}\,\frac{u_{\rm nP}(k)}{k^2}\,,$$

  Here, $k = \sqrt{M_{c\bar{c}}^2 - 4m_c^2}/2$, with $M_{c\bar{c}}^2 = (\boldsymbol{k}^2+m_c^2)/z(1-z)$.

$\gamma^*\gamma^*$ cross sections and the reduced width

- photon-photon cross sections:

$$\sigma_{ij} = \frac{32\pi(2J+1)}{N_i N_j} \frac{\hat{s}}{2M\sqrt{X}} \frac{M\Gamma}{(\hat{s}-M^2)^2 + M^2\Gamma^2} \Gamma^{ij}_{\gamma^*\gamma^*}(Q_1^2, Q_2^2, \hat{s}),$$

where $\{i,j\} \in \{\mathrm{T},\mathrm{L}\}$, and $N_\mathrm{T} = 2, N_\mathrm{L} = 1$ are the numbers of polarization states of photons. In terms of our helicity form factor, we obtain for the LT configuration, putting at the resonance pole $\hat{s} \to M^2$, and $J = 1$ for the axial-vector meson:

reduced width

$$\tilde{\Gamma}(A) = \lim_{Q^2\to 0} \frac{M^2}{Q^2} \Gamma^{\mathrm{LT}}_{\gamma^*\gamma^*}(Q^2, 0, M^2) = \frac{\pi\alpha_{\mathrm{em}}^2 M}{3} f_{\mathrm{LT}}^2(0),$$

- provides a useful measure of size of the relevant e^+e^- cross section in the $\gamma\gamma$ mode. For a $c\bar{c}$ state:

$$f_{\mathrm{LT}} = -e_f^2 M^2 \frac{\sqrt{3N_c}}{8\pi} \int_0^\infty \frac{dk\, k^2 u(k)}{(k^2+m_c^2)^2} \frac{1}{\sqrt{M_{c\bar{c}}}} \left\{ \frac{2}{\beta^2} - \frac{1-\beta^2}{\beta^3} \log\left(\frac{1+\beta}{1-\beta}\right) \right\}, \quad \beta = \frac{k}{\sqrt{k^2+m_c^2}}$$

- nonrelativistiv limit:

$$\tilde{\Gamma}(A) = \frac{2^5 \alpha_{\mathrm{em}}^2 e_c^2 N_c}{M_\chi^4} |R'(0)|^2$$

$\gamma_L^*\gamma$-transition form factors for $\chi_{c1}(1P)$ axial meson

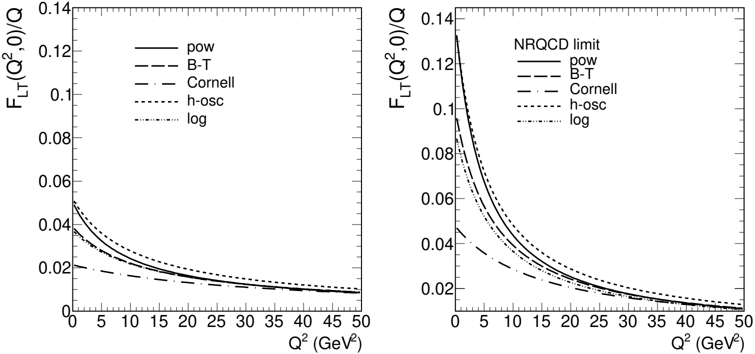


Figure: Form factor $f_{\rm LT}(Q^2) = F_{\rm LT}(Q^2, 0)/Q$ for one virtual photon.

- substantial reduction of reduced width when relativistic corrections are included.

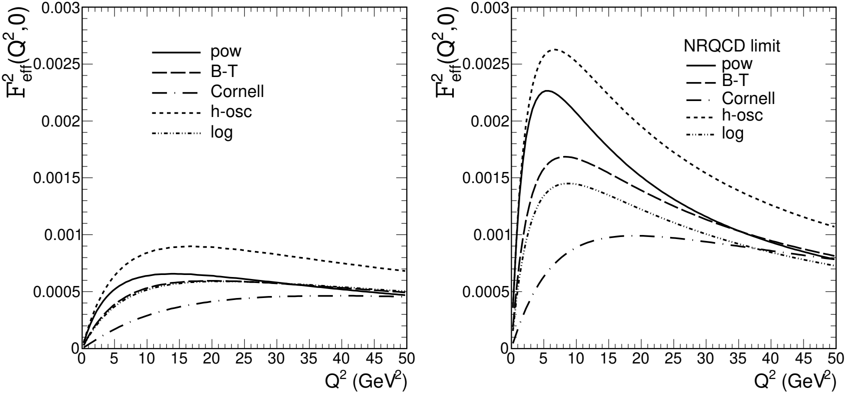


Figure: The square of the effective form factor as a function of photon virtuality within LFWF approach (on the l.h.s.) and in the nonrelativistic limit (on the r.h.s.).

$$
\begin{aligned}
\sigma_{\rm tot}^{\gamma^*\gamma}(Q^2,0) &= 16\pi^3\alpha_{\rm em}^2\delta(\hat{s}-M^2)\,\frac{Q^2}{Q^2+M^2}\left(1+\frac{Q^2}{2M^2}\right)\left(\frac{F_{\rm LT}(Q^2,0)}{Q}\right)^2 \\
&\equiv 16\pi^3\alpha_{\rm em}^2\delta(\hat{s}-M^2)\,\mathrm{F}^2_{\rm eff}(Q^2)\,.
\end{aligned}
$$

Reduced width of $\chi_{c1}(1P)$

Table: Reduced width

potential model	m_c (GeV)	$\lvert R'(0)\rvert$ (GeV$^{5/2}$)	$\tilde{\Gamma}(\chi_{c1})_{\mathrm{NRQCD}}$ (keV)	$\tilde{\Gamma}(\chi_{c1})$ (keV)
power-law	1.33	0.22	0.97	0.50
Buchmüller-Tye	1.48	0.25	0.82	0.30
Cornell	1.84	0.32	0.56	0.09
harmonic oscillator	1.4	0.27	1.20	0.53
logarithmic	1.5	0.24	0.72	0.27

- Considerably larger values of $\tilde{\Gamma}(\chi_{c1})$ are quoted in the literature. For example Danilkin & Vanderhaeghen (2017) report a value of $\tilde{\Gamma}(\chi_{c1}) \approx 1.6\,\mathrm{keV}$ from a sum rule analysis. Li et al. (2022) obtain $\tilde{\Gamma}(\chi_{c1}) \approx 3\,\mathrm{keV}$ from a LFWF approach.
- A measurement of the reduced width would therefore be very valuable.

$\chi_{c1}(3872)$ – the $[c\bar{c}]\, 2^3P_1$ component

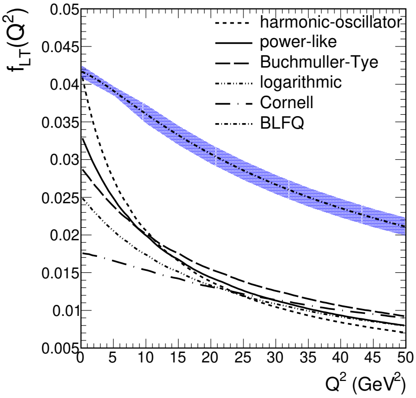


Figure: The dimensionless $\gamma_L^* \gamma \to \chi_{c1}(2P)$ transition form factor $f_{\rm LT}(Q^2)$.

- We use LFWFs for $n = 1$ radial excitation of the p-wave charmonium.
- We trace the different Q^2–dependences to differences of the z–dependence and constituent c-quark mass used in different models.
- error band for BLFQ reflects dependence on basis-size as proposed by its authors.

Reduced $\gamma_L^*\gamma$ width for $\chi_{c1}(3872)$

Table: The reduced width of the $\chi_{c1}(2P)$ state for several models of the charmonium wave functions with specific c-quark mass.

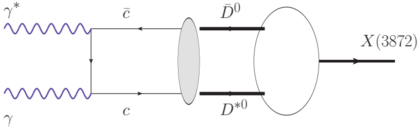
$c\bar{c}$ potential	m_c (GeV)	$f_{\rm LT}(0)$	$\tilde{\Gamma}_{\gamma\gamma}$ (keV)
harmonic oscillator	1.4	0.041	0.36
power-law	1.334	0.033	0.24
Buchmüller-Tye	1.48	0.029	0.18
logarithmic	1.5	0.025	0.14
Cornell	1.84	0.018	0.07
BLFQ	1.6	0.044	0.42

- First evidence for the production of $\chi_{c1}(3872)$ in single-tag e^+e^- collisions was reported by Belle Phys. Rev. Lett. **126** (2021) no.12, 122001 From three measured events, they provided a range for its reduced width, $0.02\,\mathrm{keV} < \tilde{\Gamma}_{\gamma\gamma} < 0.5\,\mathrm{keV}$. Recent update by Achasov et al. Phys. Rev. D **106** (2022) no.9, 093012 using a corrected value for the branching ratio $\mathrm{Br}(\chi_{c1}(3872) \to \pi^+\pi^- J/\psi)$ and reads

$$0.024\,\mathrm{keV} < \tilde{\Gamma}_{\gamma\gamma}(\chi_{c1}(3872)) < 0.615\,\mathrm{keV}$$

- all our results, including the BLFQ approach, lie **well within the experimentally allowed range**. Therefore, $\gamma\gamma$ data do not exclude the $c\bar{c}$ option, although there is certainly some room for a contribution from an additional meson-meson component.

Possible molecule contribution to $\tilde{\Gamma}$?



- apparently nothing (?) is known about the molecular contribution to the reduced width.
- What about the analogous contribution to the one we adopted in the hadronic case? Say $\gamma^*\gamma \to c\bar{c} \to \bar{D}D^*$, and FSI of $D\bar{D}^*$ generates the $X(3872)$.
- Spins of heavy quarks in $\chi_{c1}(3872)$ are entangled to be in the spin-triplet state (M. Voloshin, 2004). But near threshold the $c\bar{c}$ state produced via $\gamma\gamma$-fusion is in the $^1S_0$ state. (It's different for gluons, where color octet populates $^3S_1$!)
- $\to$ "handbag mechanism" suppressed in heavy quark limit.
- Purely hadronic models?

Summary

- We have derived the LFWF representation of axial quarkonia $\gamma^*\gamma^*$ transition form factors.
- These FFs contain valuable information on the structure of the meson.
- The reduced width of the ground state $\chi_{c1}(1P)$, for one longitudinal and one real photon $\tilde{\Gamma}$ is obtained in the ballpark of ~ 0.5 keV.
- In the case of $\chi_{c1}(3872)$, the values obtained for a 2^3P_1 charmonium are well within the range of the first Belle data. This suggests an important role of the $c\bar{c}$ Fock state for production in the $\gamma^*\gamma$ mode. (Of course there is still room for additional contributions.)
- Electroproduction of $\chi_{c1}(1P), \chi_{c1}(3872)$ in the Coulomb field of a heavy nucleus may give access to form factor $f_{\rm LT}(Q^2)$. This is additional information on the structure. We know how to calculate it for $c\bar{c}$ states.