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# $\eta_c$ production within light-front approach in $e^- A$, and $pp$ collisions

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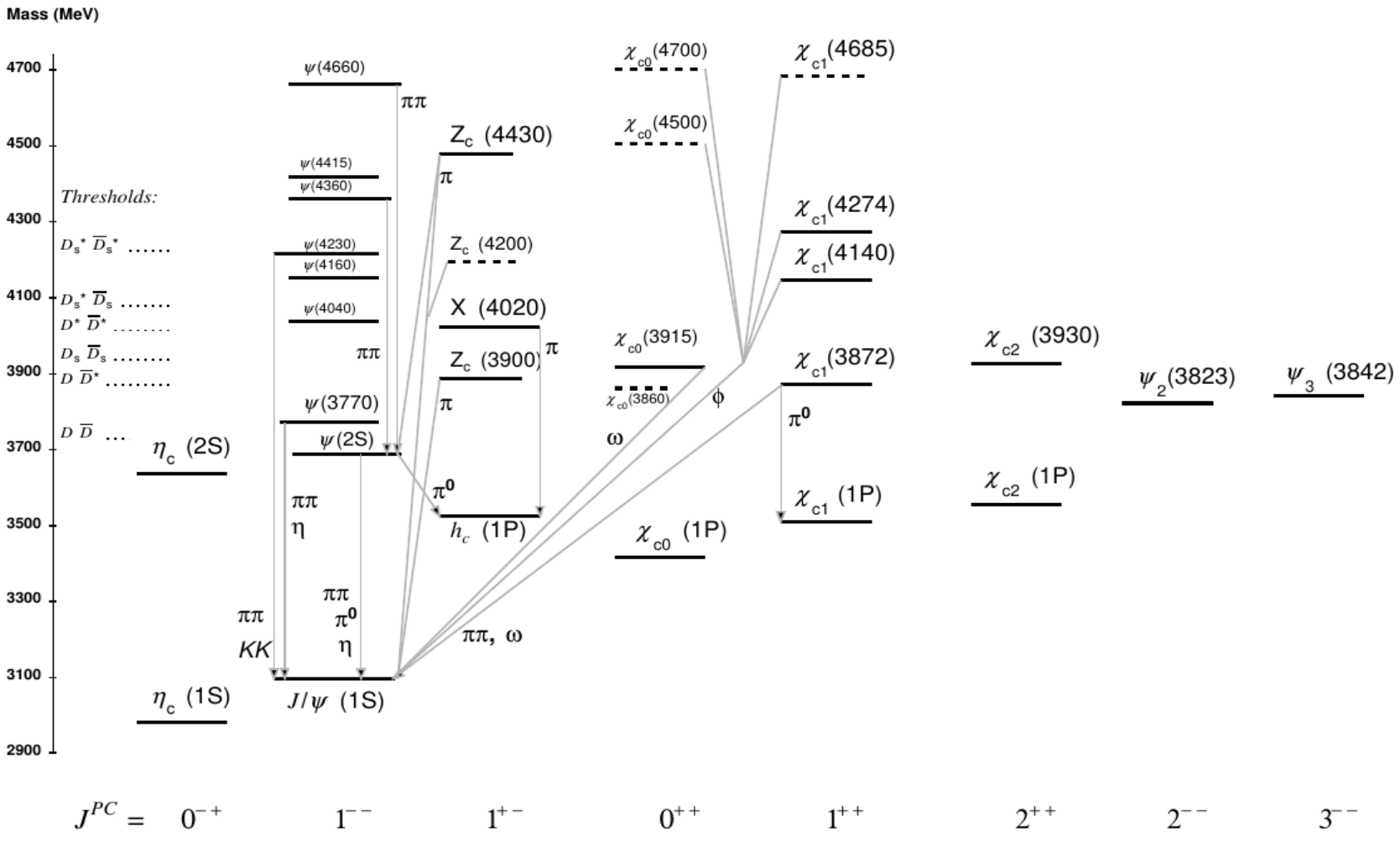
# Based on References:


I. Babiarz, V. P. Goncalves, R. Pasechnik, W. Schäfer, A. Szczurek,
*$\gamma^*\gamma^* \to \eta_c$ (1S,2S) transition form factors for spacelike photons*
Phys.Rev.D 100 (2019) 5, 054018

I. Babiarz, R. Pasechnik, W. Schäfer, A. Szczurek,
*Prompt hadroproduction of $\eta_c$(1S, 2S) in the $k_\perp$-factorization approach*
JHEP 02 (2020) 037

I. Babiarz, V. P. Goncalves, W. Schäfer, A. Szczurek, *Exclusive $\eta_c$ production by $\gamma^*\gamma$ interactions in electron-ion collisions*, e-Print: 2306.00754 [hep-ph]

# Spectrum of charmonium system and beyond



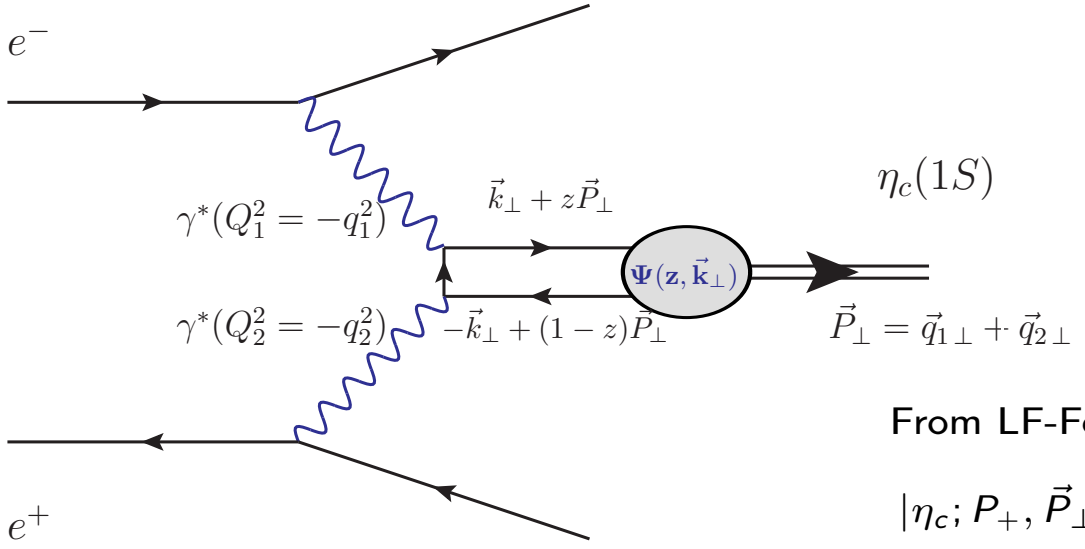

R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022)

$J^{PC} = 0^{-+}$ - pseudoscalar ; $0^{++}$ - scalar ; $1^{--}$ - vector ; $1^{-+}$ - axial vector

# Transition form factor $\gamma^*\gamma^*$ to S-wave $(c\bar{c})$ bound system

$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\,\pi\alpha_{\rm em}\,(-i)\varepsilon_{\mu\nu\alpha\beta}\,q_1^\alpha q_2^\beta\,F(Q_1^2, Q_2^2)$$

$F_{\gamma^*\gamma^*\to\mathcal{Q}}$ - provide information, how photons couple to $c\bar{c}$ state - $2\gamma$ can couple only to quarkonia with even charge parity




$\psi(z, \vec{k}_\perp)$ - $c\bar{c}$ light-cone wave function
z - the fraction of the longitudinal momentum carried by quark
$\vec{k}_\perp = (1-z)\vec{p}_{Q\perp} - z\vec{p}_{\bar{Q}\perp}$

From LF-Fock state expansion

$$|\eta_c; P_+, \vec{P}_\perp\rangle = \sum_{i,j,\lambda,\bar{\lambda}} \frac{\delta^i_j}{\sqrt{N_c}} \int \frac{dz d^2\vec{k}_\perp}{z(1-z)16\pi^3} \Psi_{\lambda\bar{\lambda}}(z, \vec{k}_\perp)$$

$$\times\, |Q_{i\lambda}(zP_+, \mathrm{p}_Q)\bar{Q}^j_{\bar{\lambda}}((1-z)P_+, \mathrm{p}_{\bar{Q}})\rangle + \ldots$$

space-like photons, their virtualities: $Q_i^2 > 0$

$$F(Q_1^2, Q_2^2) = e_c^2\sqrt{N_c}\cdot\int \frac{dz d^2\vec{k}_\perp}{z(1-z)16\pi^3}\psi(z, \vec{k}_\perp)$$

$$\times\left\{\frac{1-z}{(\vec{k}_\perp - (1-z)\vec{q}_{2\perp})^2 + z(1-z)\vec{q}_{1\perp}{}^2 + m_c^2} + \frac{z}{(\vec{k}_\perp + z\vec{q}_{2\perp})^2 + z(1-z)\vec{q}_{1\perp}{}^2 + m_c^2}\right\}.$$

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# Light-front wave functions from the rest-frame

Rest-frame wave functions for $J = 0$:

$$\Psi_{\tau\bar{\tau}}(\vec{p}) = \underbrace{\frac{1}{\sqrt{2}}\xi_Q^{\tau\dagger}\,\hat{\mathcal{O}}\,i\sigma_2\,\xi_{\bar{Q}}^{\bar{\tau}*}}_{\text{spin}-\text{orbit}}\;\underbrace{\frac{u(p)}{p}}_{\text{radial}}\;\frac{1}{\sqrt{4\pi}};$$

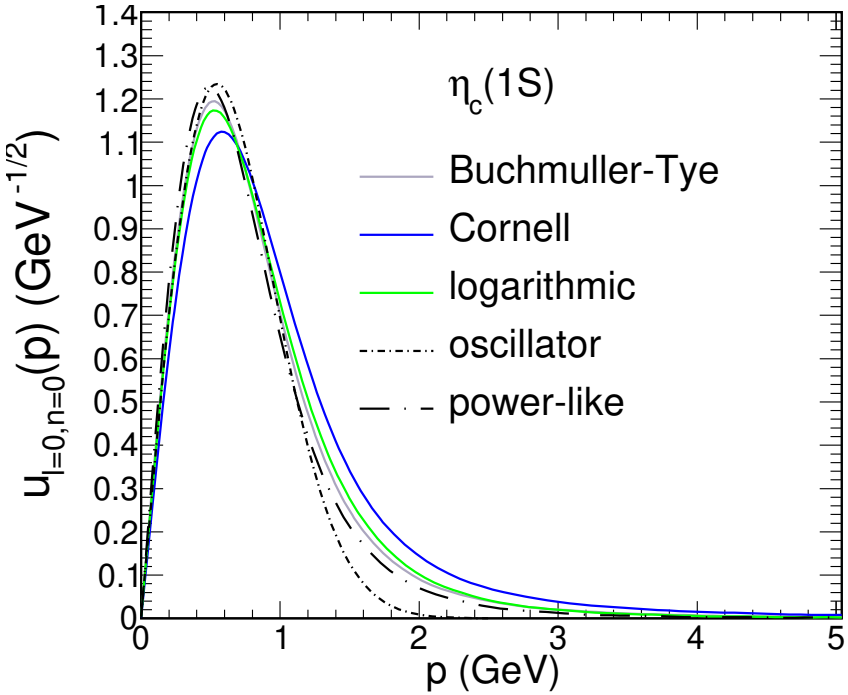
where $\hat{\mathcal{O}} =$

$$\begin{cases} \mathbb{I} & \text{spin-singlet, } S = 0, L = 0. \\ \frac{\vec{\sigma}\cdot\vec{k}}{k}, & \text{spin-triplet, } S = 1, L = 1. \end{cases}$$

mapping rest frame momentum to light-front representation:

$$\vec{p} = (\vec{k}_\perp, k_z) = (\vec{k}_\perp, \tfrac{1}{2}(2z-1)M_{c\bar{c}}),$$

$$M_{c\bar{c}}^2 = \frac{\vec{k}_\perp^2 + m_Q^2}{z(1-z)},$$




Melosh-transf. of spin-orbit part:

$$\xi_Q = R(z, \vec{k}_\perp)\chi_Q\,, \xi_{\bar{Q}}^* = R^*(1-z, -\vec{k}_\perp)\chi_{\bar{Q}}^*$$

$$R(z, \vec{k}_\perp) = \frac{m_Q + zM - i\vec{\sigma}\cdot(\vec{n}\times\vec{k}_\perp)}{\sqrt{(m_Q + zM)^2 + \vec{k}_\perp^2}}$$

$$\hat{\mathcal{O}}' = R^\dagger(z, \vec{k}_\perp)\hat{\mathcal{O}}\,i\sigma_2 R^*(1-z, -\vec{k}_\perp)(i\sigma_2)^{-1}$$

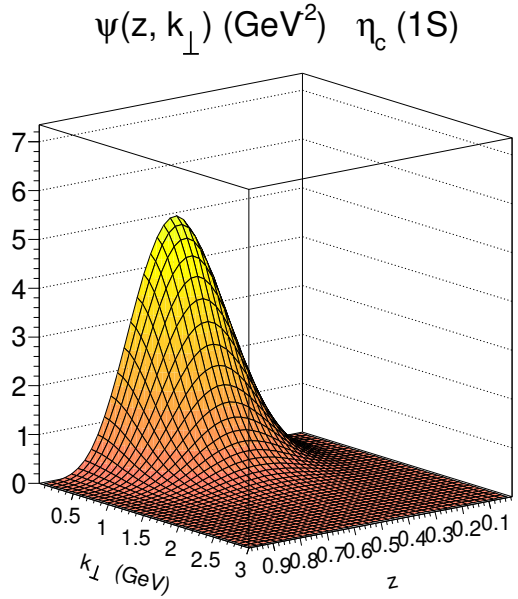
# S-wave light-front wave function for $J = 0$

$$\Psi_{\lambda\bar{\lambda}}(z, \vec{k}_\perp) = \begin{pmatrix} \Psi_{++}(z, \vec{k}_\perp) & \Psi_{+-}(z, \vec{k}_\perp) \\ \Psi_{-+}(z, \vec{k}_\perp) & \Psi_{--}(z, \vec{k}_\perp) \end{pmatrix}$$

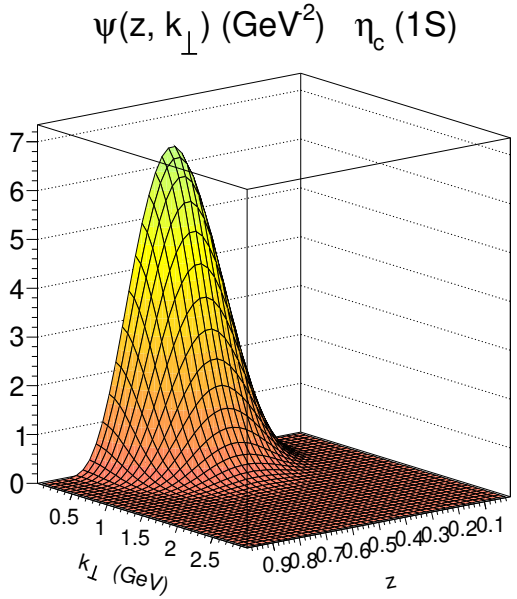
$$= \frac{1}{\sqrt{z(1-z)}} \begin{pmatrix} -k_x + ik_y & m_Q \\ -m_Q & -k_x - ik_y \end{pmatrix} \psi(z, \vec{k}_\perp)$$

$$\psi(z, \vec{k}_\perp) = \frac{\pi}{\sqrt{2M_{c\bar{c}}}} \frac{u(p)}{p}$$

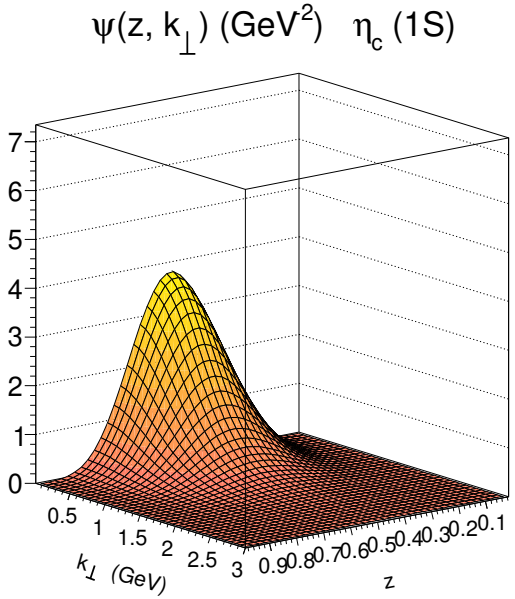
$$M^2_{c\bar{c}} = \frac{\vec{k}_\perp^2 + m_Q^2}{z(1-z)}$$




Buchmüller-Tye
$m_c = 1.48\,\mathrm{GeV}$



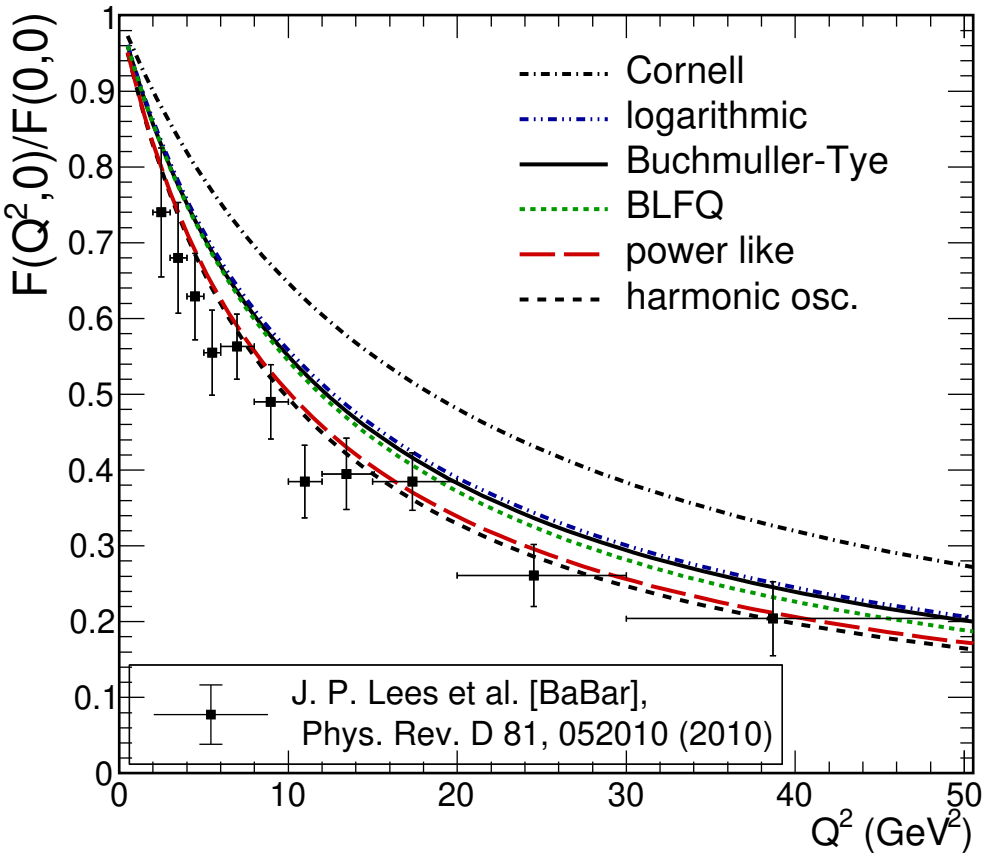

harmonic oscillator
$m_c = 1.4\mathrm{GeV}$




power-like
$m_c = 1.33\,\mathrm{GeV}$

# Normalized transition form factor at on-shell point

$$F(Q^2,0) = e_c^2\sqrt{N_c}\,4\int \frac{dz d^2\vec{k}_\perp}{\sqrt{z(1-z)}16\pi^3}\left\{\frac{1}{\vec{k}_\perp{}^2 + z(1-z)Q^2 + m_c^2}\tilde{\psi}_{\uparrow\downarrow}(z,k_\perp) + \frac{\vec{k}_\perp{}^2}{[\vec{k}_\perp{}^2 + z(1-z)Q^2 + m_c^2]^2}\left(\tilde{\psi}_{\uparrow\downarrow}(z,k_\perp) + \frac{m_c}{k_\perp}\tilde{\psi}_{\uparrow\uparrow}(z,k_\perp)\right)\right\}$$




$$\Gamma(\gamma\gamma \to \eta_c) = \frac{\pi}{4}\alpha_{em}^2 M_{\eta_c}^3 |F(0,0)|^2$$

$$\Psi_{\lambda\bar{\lambda}}(z,\vec{k}_\perp) = e^{im\phi}\,\tilde{\psi}_{\lambda\bar{\lambda}}(z,k_\perp)$$

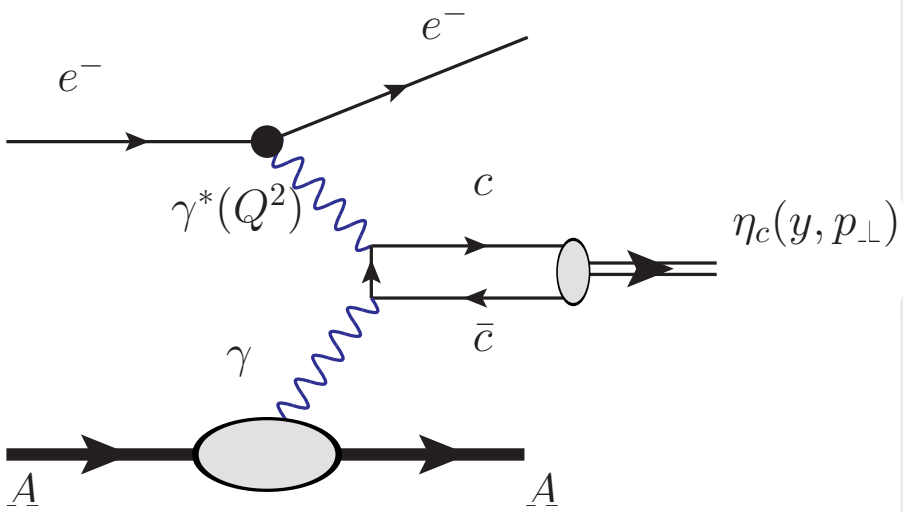
BLFQ: Phys.Rev.D 96 (2017) 016022

$$\vec{k}_\perp = k_\perp(\cos\phi, \sin\phi),\ m = |\lambda + \bar{\lambda}|$$

$$\tilde{\psi}_{\uparrow\downarrow}(z,k_\perp) \to \frac{m_c}{\sqrt{z(1-z)}}\psi(z,k_\perp)$$

$$\tilde{\psi}_{\uparrow\uparrow}(z,k_\perp) \to \frac{-|\vec{k}_\perp|}{\sqrt{z(1-z)}}\psi(z,k_\perp)$$

# Electron-ion collisions




the nuclear radius: $R_A = r_0 A^{1/3}$, with $r_0 = 1.1\,\mathrm{fm}$

$$\sigma(eA \to e\eta_c A) = \int d\omega_e dQ^2 \frac{d^2 N_e}{d\omega_e dQ^2} \times \sigma(\gamma^* A \to \eta_c A)$$

$$\sigma(\gamma^* A \to \eta_c A) = \int d\omega_A \frac{dN_A}{d\omega_A} \times \sigma_{\mathrm{TT}}(\gamma^*\gamma \to \eta_c; W_{\gamma\gamma}, Q^2, 0)$$

$$W_{\gamma\gamma} = \sqrt{4\omega_e\omega_A - p_\perp^2}$$

$$\omega_e = \frac{\sqrt{M^2 + p_\perp^2}}{2} e^{+y}$$

$$\omega_A = \frac{\sqrt{M^2 + p_\perp^2}}{2} e^{-y}$$

$$p_\perp^2 = \left(1 - \frac{\omega_e}{E_e}\right) Q^2$$

$$\frac{dN_A}{d\omega_A} = \frac{2Z^2\alpha_{em}}{\pi\omega_A}\left[\xi K_0(\xi)K_1(\xi) - \frac{\xi^2}{2}(K_1^2(\xi) - K_0^2(\xi))\right]$$

$\xi = R_A\omega_A/\gamma_L$, $K_0$ and $K_1$ -modified Bessel functions

i.e.: Ann. Rev. Nucl. Part. Sci. 55, 271 (2005)

$$\frac{d^2 N_e}{d\omega_e dQ^2} = \frac{\alpha_{em}}{\pi\omega_e Q^2}\left[\left(1 - \frac{\omega_e}{E_e}\right)\left(1 - \frac{Q^2_{min}}{Q^2}\right) + \frac{\omega_e^2}{2E_e^2}\right]$$

$Q^2_{min} = m_e^2\omega_e^2/[E_e(E_e - \omega_e)]$ and $Q^2_{max} = 4E_e(E_e - \omega_e)$

# $\sigma_{\rm TT}$ cross-section for one virtual photon

$$\sigma_{\rm TT}(W_{\gamma\gamma}, Q_1^2, Q_2^2) = \frac{1}{4\sqrt{X}} \frac{M_{\eta_c}\Gamma_{\rm tot}}{(W_{\gamma\gamma}^2 - M_{\eta_c}^2)^2 + M_{\eta_c}^2\Gamma_{\rm tot}^2} \mathcal{M}^*(++)\mathcal{M}(++)$$

the helicity amplitude $\mathcal{M}(\lambda_1, \lambda_2) = e_\mu^1(\lambda_1) e_\nu^2(\lambda_2) \mathcal{M}^{\mu\nu}$

$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\pi\alpha_{\rm em}\,(-i)\varepsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta\, F(Q_1^2, Q_2^2)\,.$$

$X = (q_1 \cdot q_2)^2 - q_1^2 q_2^2$, in the limit $Q_2^2 \to 0$, $\sqrt{X} = q_1 \cdot q_2 = (M_{\eta_c}^2 + Q^2)/2$

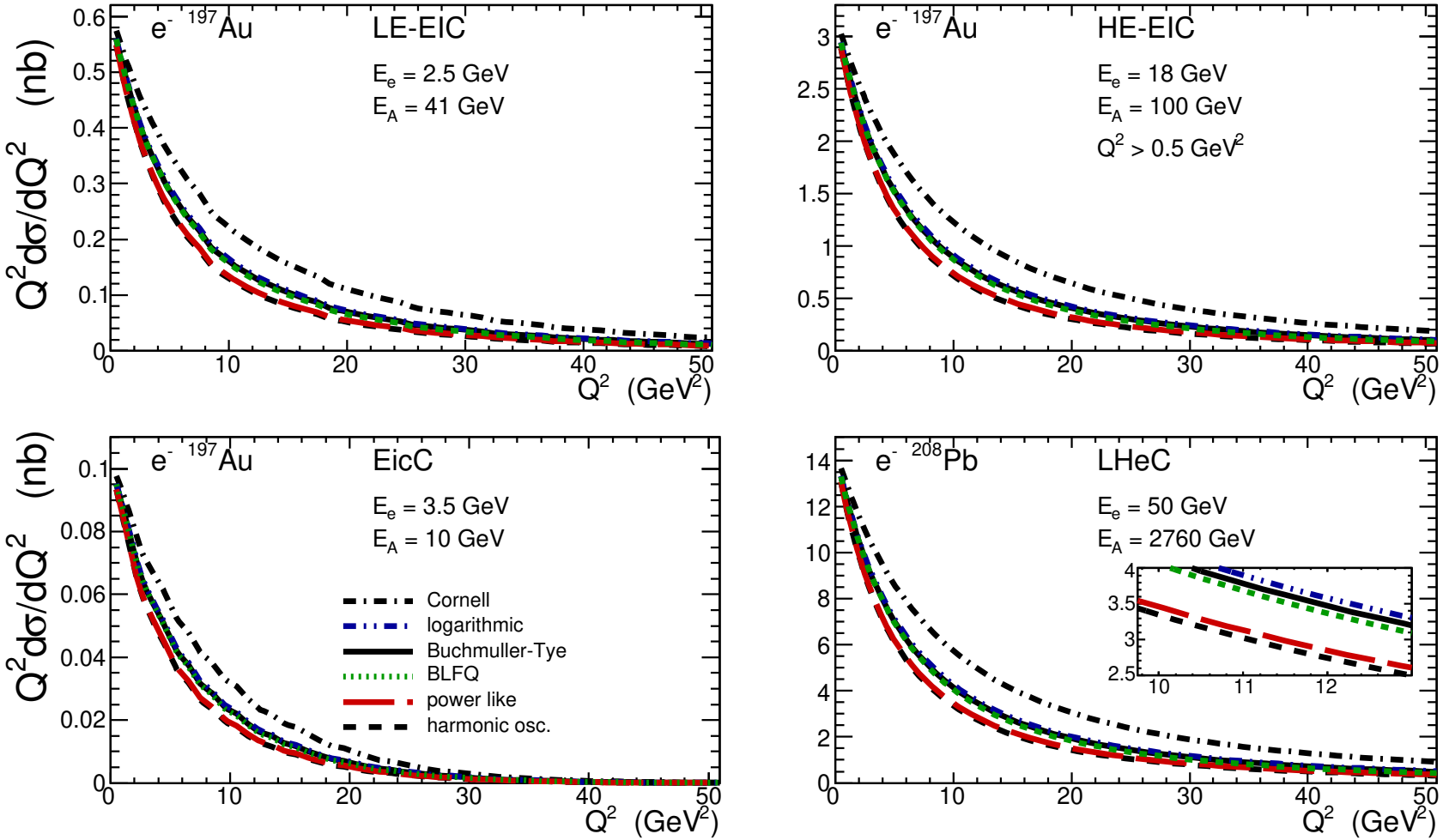
$$\sigma_{\rm TT}(W_{\gamma\gamma}, Q^2, 0) = 2\pi^2\alpha_{\rm em}^2 \frac{M_{\eta_c}\Gamma_{\rm tot}}{(W_{\gamma\gamma}^2 - M_{\eta_c}^2)^2 + M_{\eta_c}^2\Gamma_{\rm tot}^2} (M_{\eta_c}^2 + Q^2)\, F^2(Q^2, 0)\,.$$

we can take advantage of the relation $\Gamma(\gamma\gamma \to \eta_c) = \frac{\pi}{4}\alpha_{em}^2 M_{\eta_c}^3 |F(0,0)|^2$

$$\begin{aligned}
\sigma_{\rm TT}(W_{\gamma\gamma}, Q^2, 0) &= 8\pi \frac{\Gamma_{\gamma\gamma}\Gamma_{\rm tot}}{(W_{\gamma\gamma}^2 - M_{\eta_c}^2)^2 + M_{\eta_c}^2\Gamma_{\rm tot}^2} \left(1 + \frac{Q^2}{M_{\eta_c}^2}\right)\left(\frac{F(Q^2,0)}{F(0,0)}\right)^2 \\
&\approx 8\pi^2\, \delta(W_{\gamma\gamma}^2 - M_{\eta_c}^2)\, \frac{\Gamma_{\gamma\gamma}}{M_{\eta_c}} \left(1 + \frac{Q^2}{M_{\eta_c}^2}\right)\left(\frac{F(Q^2,0)}{F(0,0)}\right)^2
\end{aligned}$$

e-Print: 2306.00754 [hep-ph]

# Differential distribution in photon virtuality




$Q^2 > 0.5\,\mathrm{GeV}^2$

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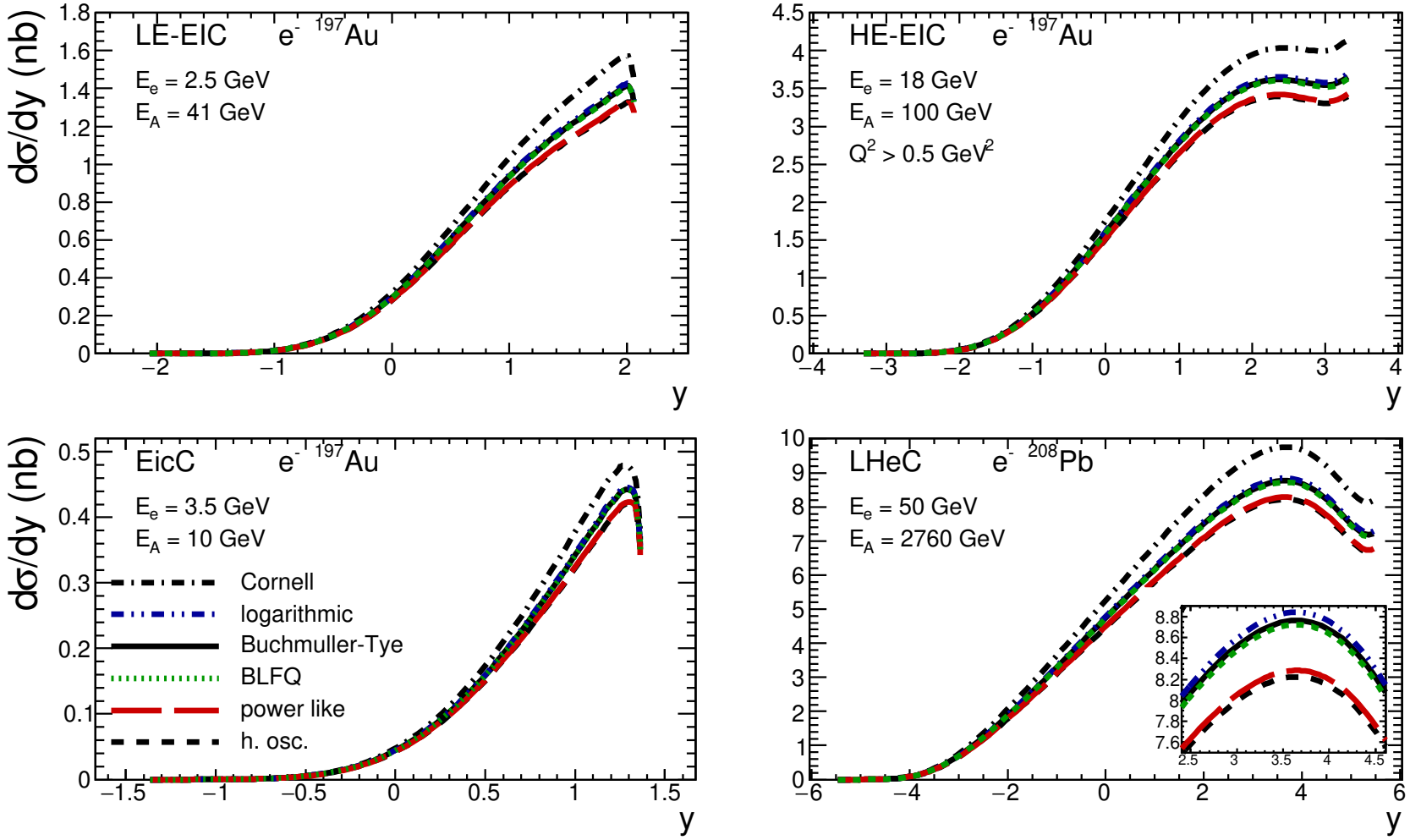
# Differential distribution in $\eta_c$ rapidity




$Q^2 > 0.5\,\mathrm{GeV}^2$

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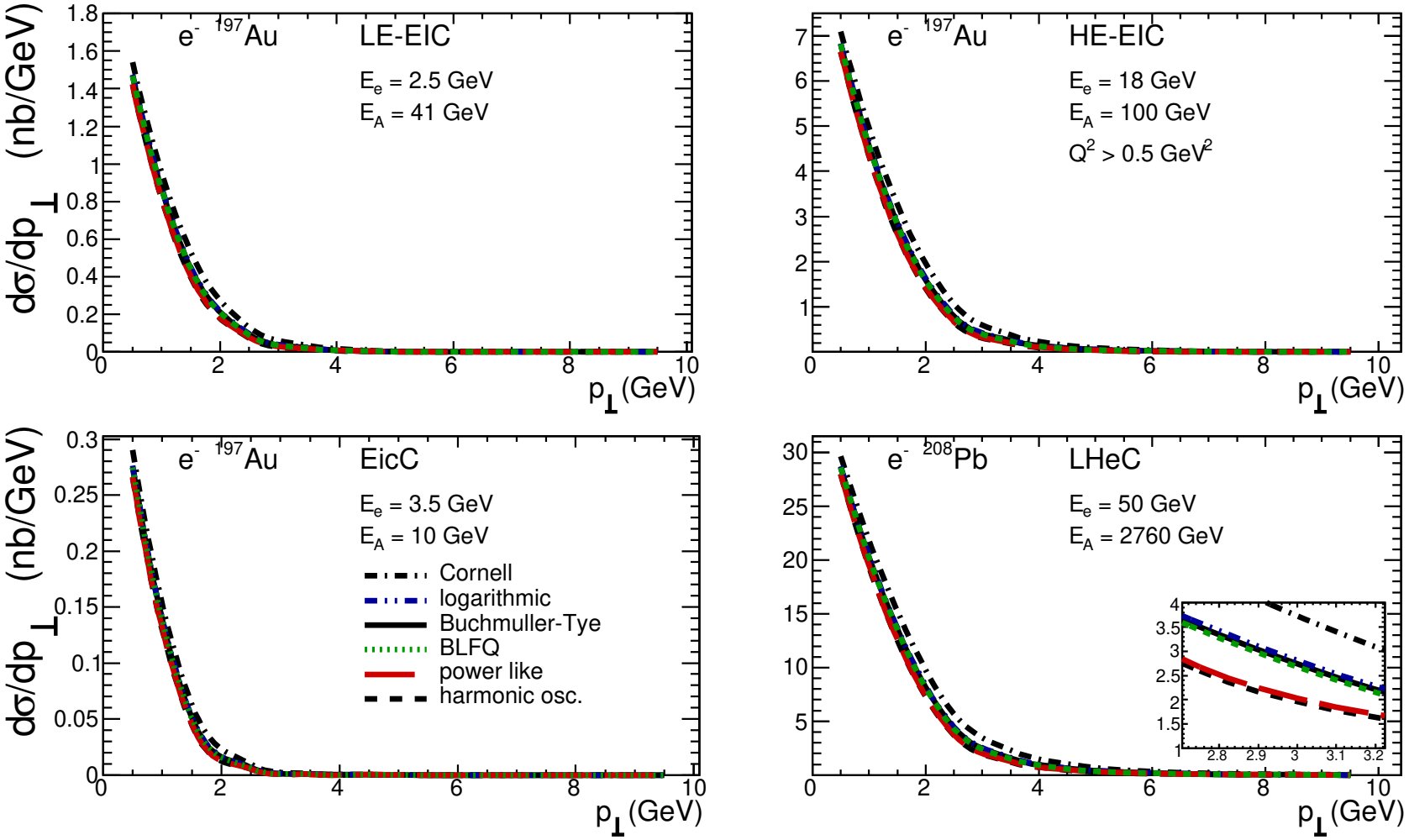
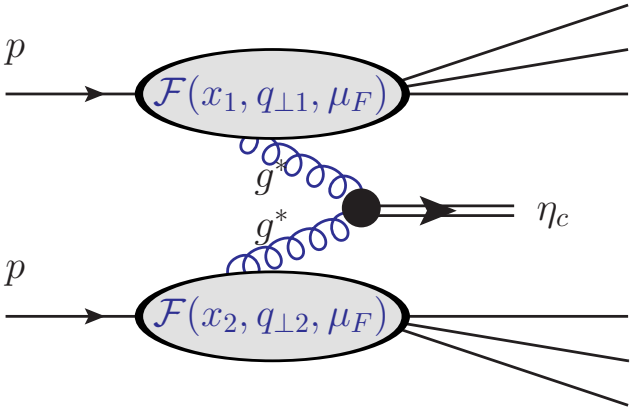
# Differential distribution in $\eta_c$ transverse momentum

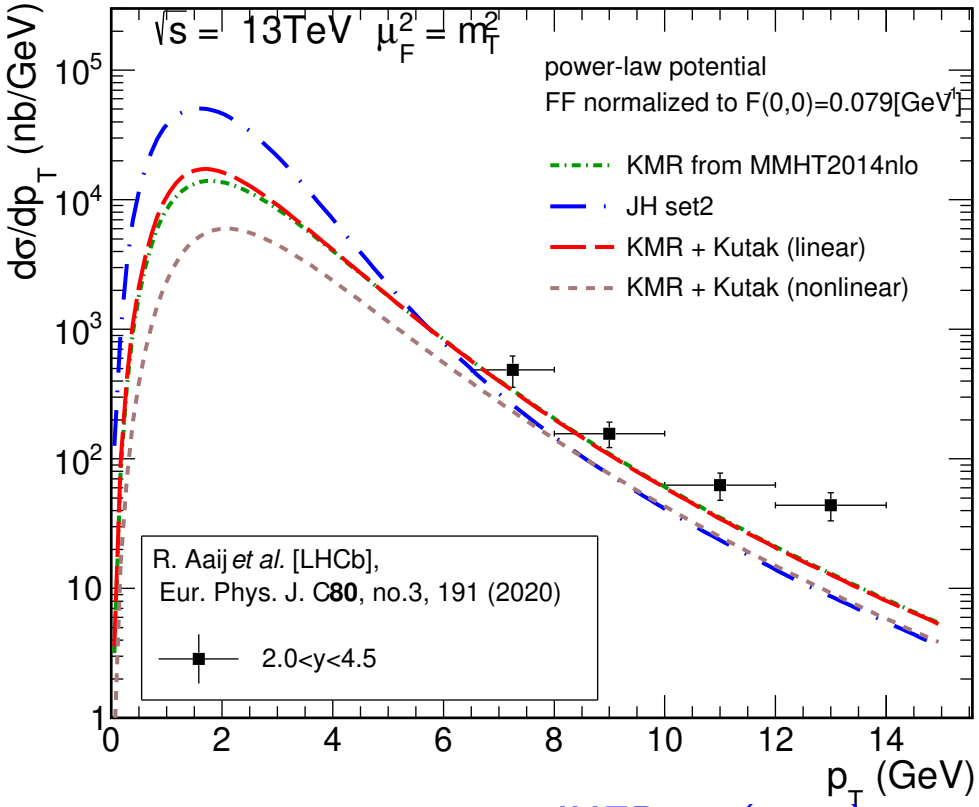



$Q^2 > 0.5\,\mathrm{GeV}^2$

e-Print: 2306.00754 [hep-ph]

# Hadroproduction of $\eta_c(1S, 2S)$ via gluon-gluon fusion

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$$\mathcal{M}^{ab}_{\mu\nu}(g^* g^* \to \eta_c) = (-i) 4\pi\alpha_s \, \varepsilon_{\mu\nu\alpha\beta} \, q_1^\alpha q_2^\beta \times \frac{\mathrm{Tr}[t^a t^b]}{\sqrt{N_c}} \, \frac{F(Q_1^2, Q_2^2)}{e_c^2 \sqrt{N_c}}$$

$$\frac{d\sigma}{dy d^2\vec{p}_\perp} = \int \frac{d^2\vec{q}_{1\perp}}{\pi \vec{q}_{1\perp}^{\,2}} \mathcal{F}(x_1, \vec{q}_{1\perp}{}^2, \mu_F) \int \frac{d^2\vec{q}_{2\perp}}{\pi \vec{q}_{2\perp}{}^2} \mathcal{F}(x_2, \vec{q}_{2\perp}{}^2, \mu_F) \, \delta^{(2)}(\vec{q}_{1\perp} + \vec{q}_{2\perp} - \vec{p}_\perp) \times \frac{\pi}{(x_1 x_2 s)^2} \overline{|\mathcal{M}(g^* g^* \to \eta_c)|^2}$$

# Summarize

- We derived the transition form factor for two off-shell photons $F(Q_1^2, Q_2^2)$.
- We estimate the rapidity, transverse momentum, and $Q^2$ distributions considering the energy configurations expected in the future electron-ion colliders at the BNL(USA), CERN and in China.
- Our results indicate that the electron-ion colliders can be considered as an alternative and provide supplementary data to those obtained in $e^+e^-$ colliders.
- The results derived in this paper indicate that the cross sections for the future electron-ion colliders are of the order of 0.1-60 nb