Near-threshold hadron scattering using effective field theory

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Background

Exotic hadrons ⇨ $T_{cc}, X(3872), f_0(980), a_0, P_c, Z_c$

Internal structure ⇔ Scattering length a

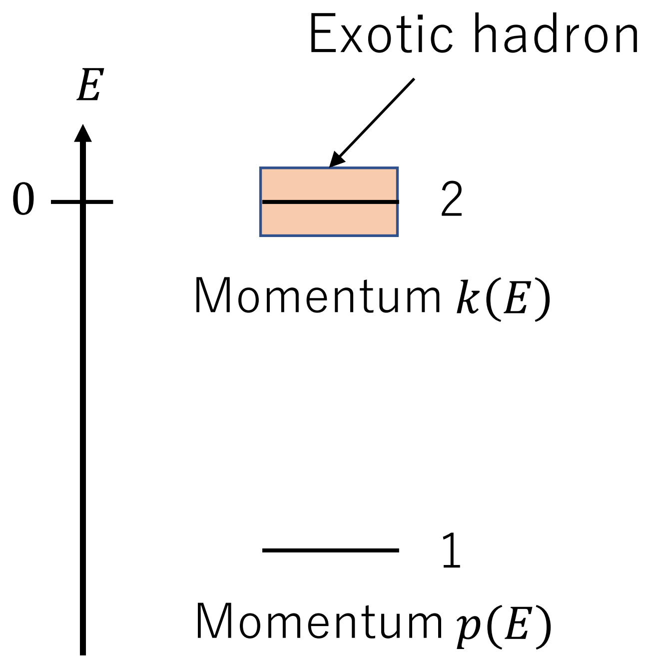
For near-threshold exotic hadrons,
channel couplings are important.

Unstable exotic hadron near the threshold of channel 2

⇨ Flatté amplitude has been used[1].

Scattering length a_F has been determined by the Flatté amplitude[2].

a in more general framework?



[1] R.Aaij et al. [LHCb], Phys. Rev. D 102, no.9, 092005 (2020)

[2] V. Baru et al. Eur. Phys. J. A, 23, 523-533 (2005)

Flatté amplitude

The Flatté amplitude

$$f^F = h(E)\begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix}$$

The Flatté parameters

g_1, g_2 : Real coupling constants

E_{BW} : Bare energy

The Flatté amplitude has the threshold effect.

$$h(E) = -\frac{1}{2}\frac{1}{E - E_{BW} + i\, g_1^2 p(E)/2 + i g_2^2\, k(E)/2}$$

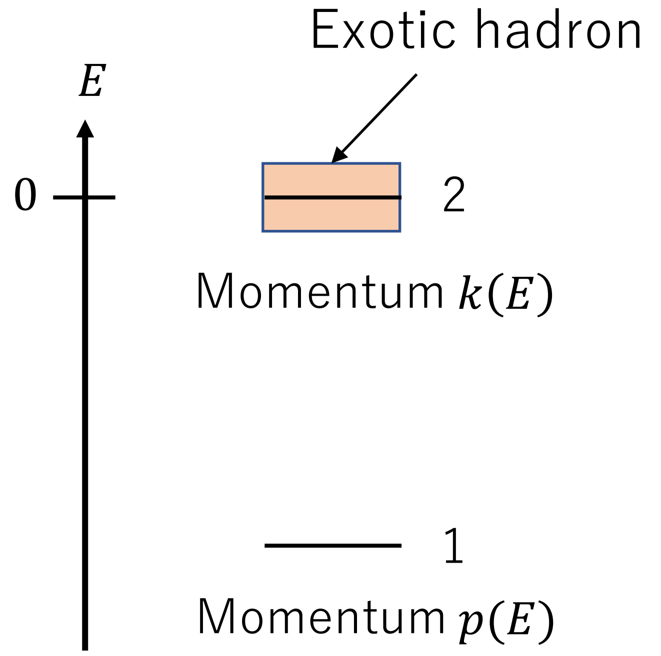
f_{11}^F, f_{22}^F can be written as the effective range expansion in k.

$$f_{11}^F, f_{22}^F \propto \left(-\frac{1}{a_F} + \frac{1}{2} r_F k^2 - ik + O(k^4)\right)^{-1}$$

a_F : Scattering length

r_F : Effective range

Problem of Flatté amplitude



$1/f_{11}^F$ up to order k^1 can be written by only two parameters R, α[2].

$$f_{11}^F = \frac{g_1^2}{2E_{BW} - ig_1^2 p_0 - ig_2^2 k} = \frac{1/R}{\alpha p_0/R - ip_0/R - ik}$$

$$\alpha = \frac{2E_{BW}}{g_1^2 p_0} \qquad R = \frac{g_2^2}{g_1^2}$$

We find $1/f_{22}^F$ up to k^1 can also be written by only two parameters R, α.

$$f_{22}^F = \frac{g_2^2}{2E_{BW} - ig_1^2 p_0 - ig_2^2 k} = \frac{1}{\alpha p_0/R - ip_0/R - ik}$$

p_0 : channel 1 momentum at $E = 0$

$f^F(g_1^2, g_2^2, E_{BW})$ three parameters ⇨ $f^F(R, \alpha)$ two parameters(near the threshold)

⇨ **The Flatté amplitude can be written by only two parameters near the threshold.**

[2] V. Baru et al. Eur. Phys. J. A, 23, 523-533 (2005)

General form : EFT amplitude

In general, a scattering amplitude satisfies the optical theorem.

$$f_{ij} - f_{ji}^* = \sum_n 2ip_n f_{in} f_{nj}^*$$

The Flatté amplitude satisfies the optical theorem.

One of the general solutions of the above equation derived from EFT.

EFT amplitude[3] up to first order of k.

$$f^{EFT} = \left\{ \frac{1}{a_{12}^2} - \left(\frac{1}{a_{22}} + ik \right) \left(\frac{1}{a_{11}} + ip_0 \right) \right\}^{-1} \begin{pmatrix} \left(\frac{1}{a_{22}} + ik \right) & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \left(\frac{1}{a_{11}} + ip_0 \right) \end{pmatrix}$$

a_{11}, a_{12}, a_{22} : Three EFT parameters

[3]T.D.Cohen et al. , Phys. Lett. B 588 (2004) 57-66

EFT parameters near-threshold

$1/f_{11}^{EFT}$ up to order k^1

$$f_{11}^{EFT} = \frac{a_{12}^2/a_{22}^2}{\frac{1}{a_{22}} - \frac{a_{12}^2}{a_{11}a_{22}^2} - i\frac{a_{12}^2}{a_{22}^2}p_0 - ik}$$

$1/f_{22}^{EFT}$ up to order k^1

$$f_{22}^{EFT} = \frac{1}{\frac{1}{a_{12}^2\left(\frac{1}{a_{11}} + ip_0\right)} - \frac{1}{a_{22}} - ik}$$

$\boldsymbol{f^{EFT}(a_{11}, a_{12}, a_{22})}$ three parameters(near the threshold)

Are there any relations between the EFT amplitude and the Flatté amplitude?

Comparison

What is the difference between the EFT and Flatté ?

⇨ **Invers amplitude**

EFT

$$(f^{EFT})^{-1} = \begin{pmatrix} -\frac{1}{a_{11}} - ip_0 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - ik \end{pmatrix}$$

Flatté

$$f^F = h(E)\begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix}$$

⇨ $(f^F)^{-1}$ = does not exist

unitarity relation

$\boldsymbol{f^{EFT}(a_{11}, a_{12}, a_{22})}$		$\boldsymbol{f^F(\alpha, R)}$
・**Invers matrix exist**	⇏	・**Invers matrix does not exist**

EFT amplitude does not reduce to Flatté amplitude directly

General amplitude

We construct the new representation including EFT and Flatté.

⇨ **General amplitude $\boldsymbol{f^G(A_{22},\gamma,\epsilon)}$**

$$(f^{EFT})^{-1} = \begin{pmatrix} -\frac{1}{a_{11}} - ip_0 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - ik \end{pmatrix} \quad \Rightarrow \quad (f^G)^{-1} = \begin{pmatrix} -\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0 & \frac{1}{A_{22}}\frac{\sqrt{\epsilon-\gamma}}{\gamma} \\ \frac{1}{A_{22}}\frac{\sqrt{\epsilon-\gamma}}{\gamma} & -\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik \end{pmatrix}$$

$$f^G(A_{11},\gamma,\epsilon) = \frac{1}{-\frac{1}{A_{22}^2} - i\frac{1}{A_{22}}\epsilon p_0 - i\frac{1}{A_{22}}k + \gamma p_0 k}\begin{pmatrix} \frac{1}{A_{22}}\epsilon + i\gamma k & \frac{1}{A_{22}}\sqrt{\epsilon-\gamma} \\ \frac{1}{A_{22}}\sqrt{\epsilon-\gamma} & \frac{1}{A_{22}} + i\gamma p_0 \end{pmatrix}$$

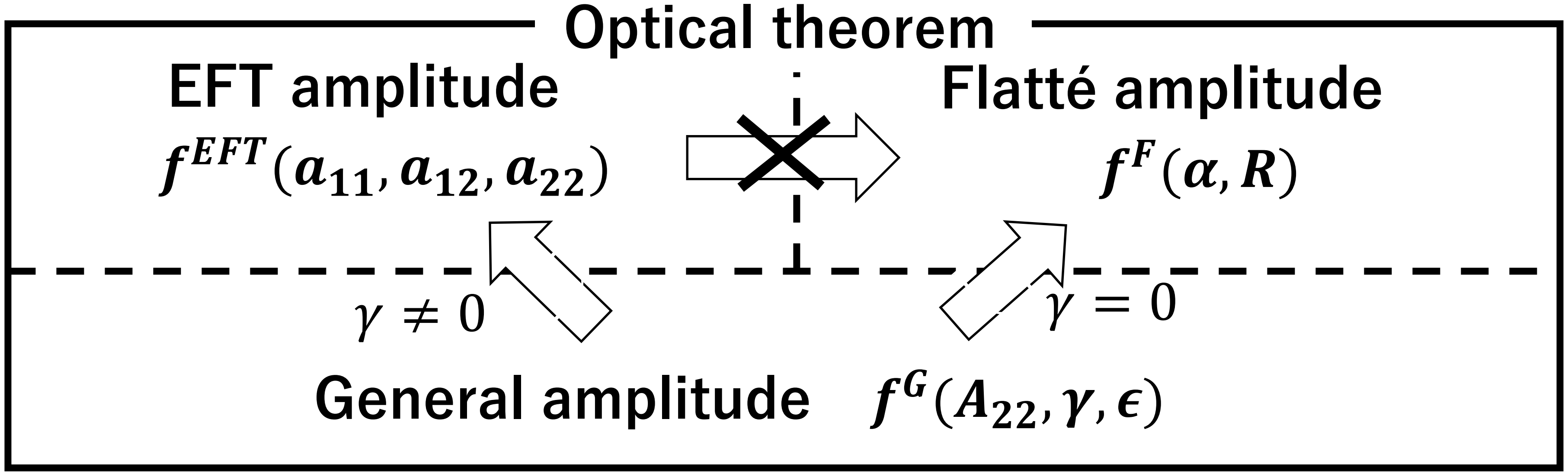
A_{22} : scattering length of channel two in the absence of channel couplings

property

Flatté form

$$f^G(A_{22},\gamma,\epsilon) \xrightarrow{\gamma=0} f^G(A_{22},0,\epsilon) = \frac{1}{-\frac{1}{A_{22}} - i\epsilon p_0 - ik}\begin{pmatrix} \epsilon & \sqrt{\epsilon} \\ \sqrt{\epsilon} & 1 \end{pmatrix}$$

$$(f^G)^{-1}(A_{22},\gamma,\epsilon) = \begin{pmatrix} -\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0 & \frac{1}{A_{22}}\frac{\sqrt{\epsilon-\gamma}}{\gamma} \\ \frac{1}{A_{22}}\frac{\sqrt{\epsilon-\gamma}}{\gamma} & -\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik \end{pmatrix} \xrightarrow{\gamma=0} (f^G)^{-1} = \text{does not exist}$$



Determination of a_F

We consider the region near the threshold 2(region Ⅱ and Ⅲ).

2→2 scattering does not occur in region Ⅱ.

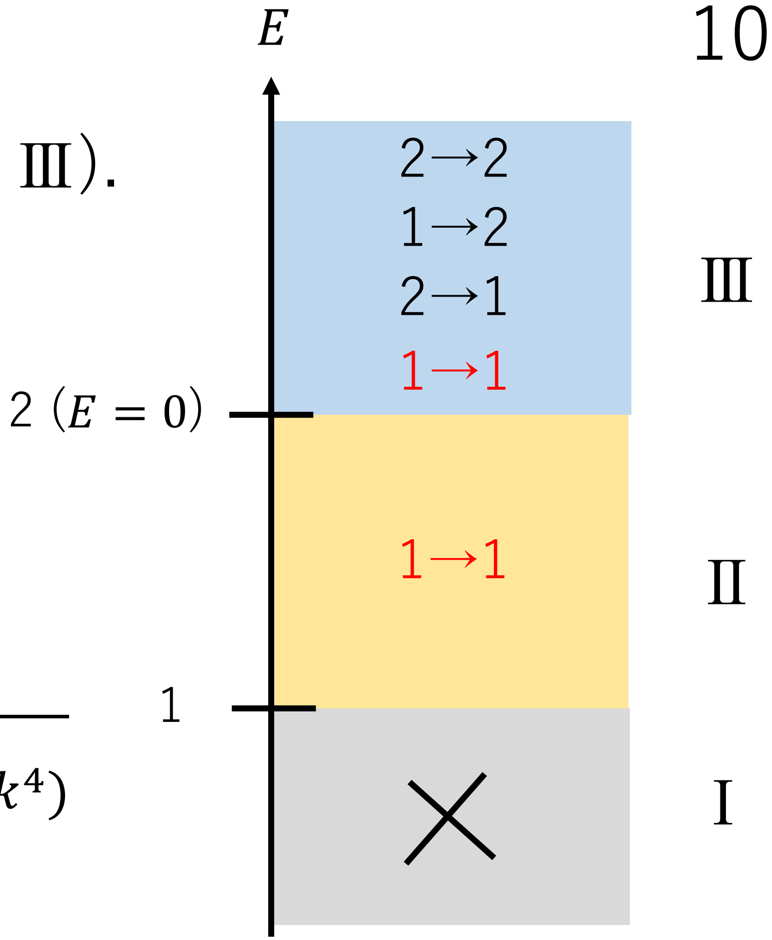
1→1 scattering occurs in both region Ⅱ and Ⅲ.

⇨ a_F is determined from f_{11}^F (ex. [1][4] for $X(3872)$).

$$f_{11}^F = \frac{\frac{g_1^2}{g_2^2}}{\left(\frac{2E_{BW}}{g_2^2} - i\frac{g_1^2}{g_2^2}p_0\right) - \left(\frac{2}{m_k g_2^2} + i\frac{g_1^2}{2p_0 g_2^2}\right)k^2 - ik + O(k^4)}$$

$$a_F = -\frac{g_2^2}{2E_{BW} - ig_1^2 p_0}$$

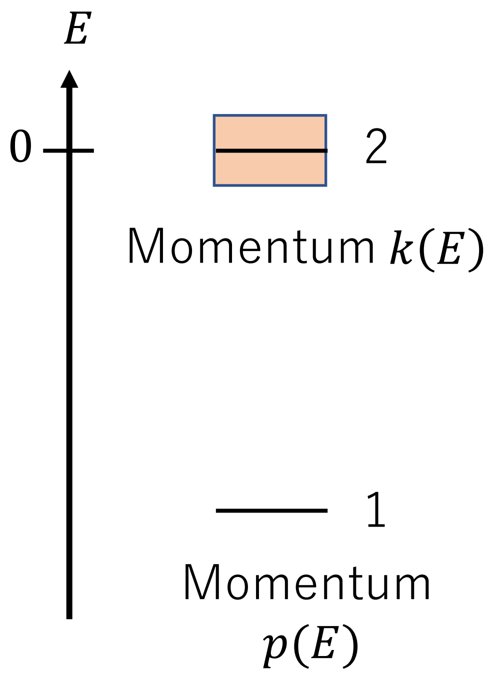
Scattering length



[1] R. Aaij et al. [LHCb], Phys. Rev. D102, no.9, 092005 (2020)

[4] A. Esposito et al., Phys. Rev. D 105 (2022) 3, L031503

f_{22} component



Effective range expansion for f_{22}^G

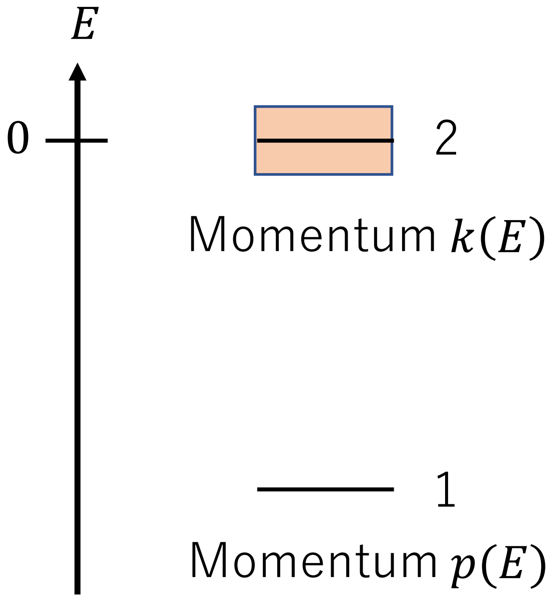
$$f_{22}^G = \frac{-\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0}{\left(-\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0\right)\left(-\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik\right) - \frac{\epsilon - \gamma}{A_{22}^2\gamma^2}}$$

$$= \frac{1}{-\frac{1}{A_{22}}\left(\frac{\frac{1}{A_{22}} + i\epsilon p_0}{\frac{1}{A_{22}} + i\gamma p_0}\right) - \frac{i(\epsilon - \gamma)}{2(1 + iA_{22}\gamma p_0)^2 p_0}k^2 + O(k^4)}$$

$$a_G = A_{22}\left(\frac{\frac{1}{A_{22}} + i\gamma p_0}{\frac{1}{A_{22}} + i\epsilon p_0}\right)$$: scattering length

f_{22}^G can be written as the effective range expansion in k.

f_{11} component



Effective range expansion for f_{11}^G

$$f_{11}^G = \frac{-\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik}{\left(-\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0\right)\left(-\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik\right) - \frac{\epsilon-\gamma}{A_{22}^2\gamma^2}}$$

$$= \frac{\frac{\epsilon^2}{\epsilon-\gamma}}{-\frac{1}{A_{22}}\frac{\epsilon}{\epsilon-\gamma} - i\frac{\epsilon^2}{\epsilon-\gamma}p_0 - \left(A_{22}\frac{\gamma}{\epsilon} + i\frac{\epsilon^2}{2(\epsilon-\gamma)p_0}\right)k^2 - ik + O(k^3)}$$

f_{11}^G cannot be written as the effective range expansion in k.

⇨ a_G should not be defined in f_{11}^G.

The correct scattering length must be defined by f_{22}.

Scattering length

The constant term of the denominator of f_{22}^{G}

⇨ general scattering length a_G

$$a_G = A_{22}\left(\frac{\frac{1}{A_{22}} + i\gamma p_0}{\frac{1}{A_{22}} + i\epsilon p_0}\right)$$

$\gamma = 0$ ⇨

Flatté scattering length a_F

$$a_F = \frac{1}{\frac{1}{A_{22}} + i\epsilon p_0}$$

The constant term of the denominator of f_{11}^{G}

$$\frac{1}{\frac{1}{A_{22}}\frac{\epsilon}{\epsilon-\gamma} + i\frac{\epsilon^2}{\epsilon-\gamma}p_0}$$

$\gamma = 0$ ⇨

$$a_F = \frac{1}{\frac{1}{A_{22}} + i\epsilon p_0}$$

Except for the case with gamma being zero we should not use the Flatté amplitude

Application

We study the effect of γ on the scattering length a_G.

Analysis of the $\pi\pi$-$K\bar{K}$ system with $f_0(980)$ [5].

⇨ We determine the constant term of the denominator of $f_{11}^G(f_{\pi\pi})$

$$-\frac{1}{A_{22}}\frac{\epsilon}{\epsilon-\gamma}-i\frac{\epsilon^2}{\epsilon-\gamma}p_0=-1.0-1.0\mathrm{i}\ [\mathrm{GeV}]$$

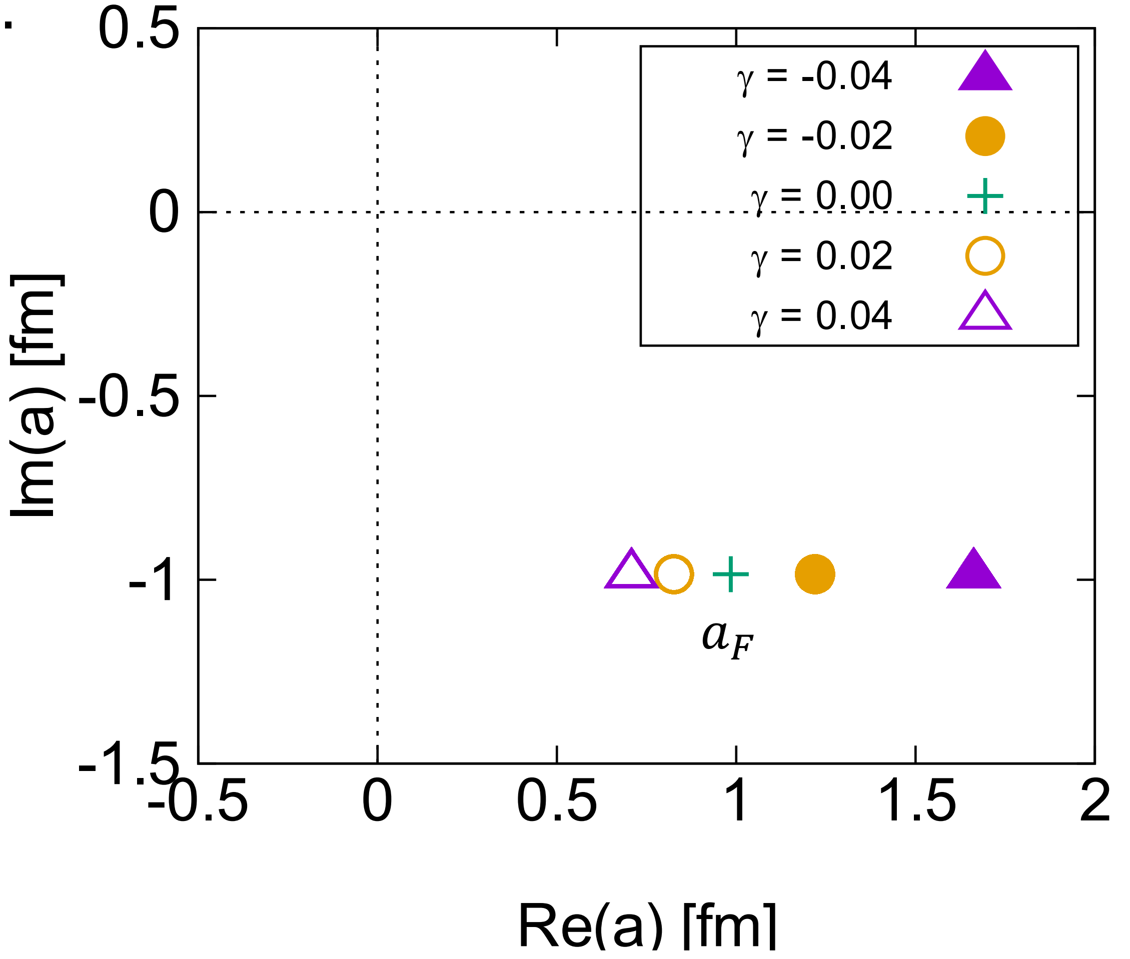
⇩

Two conditions

$A_{22}(\gamma), \epsilon(\gamma)$

⇩

We can determine $a_G(\gamma)$



The imaginary part is stable against of the variation of the γ.

[5] R.R. Akhametshin et al., Phys. Lett B 462, 380 (1999)

Summary

- We study the number of parameters of the scattering amplitude near the threshold.

⇨ The Flatté amplitude is written by only two parameters.

The EFT amplitude is written by three parameters.

⇩

The EFT amplitude is more general than the Flatté amplitude.

However, the EFT amplitude does not reduce to the Flatté amplitude directly.

- We propose a new parametrization of the EFT amplitude .

⇨ The general amplitude $f^G(A_{22}, \gamma, \epsilon)$ reduces to the Flatté amplitude when $\gamma = 0$.

- We study the effect of γ on the scattering length numerically.

⇨ The correct scattering length a_G is deviated from a_F by Flatté for nonzero γ.