Exotic meson spectroscopy from lattice QCD

"17th International Workshop on Meson Physics" – Krakow, Poland

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Introductory remarks

- In this talk only heavy exotic mesons:
  - tetraquarks $\bar{b}\bar{b}qq$,
  - tetraquarks $\bar{b}b\bar{q}q$,
  - hybrid mesons $\bar{b}b$ + gluons

  (light quarks $q \in \{u, d, s\}$).
  (possibly more about light exotic mesons in Gernot Eichmann's talk at 10:30)
- Lattice QCD = numerical QCD.
  - Lattice QCD is not a model, there are no approximations.
  - Results are in principle full and rigorous QCD results.
  - Lattice QCD simulations can be seen as computer experiments (based on the theory QCD).
  - However, the investigation of exotic mesons in lattice QCD is techically very difficult.
    $\rightarrow$ Even though we use lattice QCD, there are quite often assumtions and simplifying approximations (as you will see during the talk) ...

Two types of approaches

- Two types of approaches, when studying **heavy** exotic mesons with lattice QCD:
  - **Born-Oppenheimer approximation** (a 2-step procedure):
    (1) Compute the potential $V(r)$ of the two heavy quarks (approximated as static quarks) in the presence of two light quarks and/or gluons using lattice QCD.
    $\rightarrow$ full QCD results
    (2) Use standard techniques from quantum mechanics and $V(r)$ to study the dynamics of two heavy quarks (Schrödinger equation, scattering theory, etc.).
    $\rightarrow$ an approximation

    $\rightarrow$ **The main focus of this talk.**
  - **Full lattice QCD computations of eigenvalues of the QCD Hamiltonian**:
    * Masses of stable hadrons correspond to energy eigenvalues at infinite volume (comparatively easy).
    * Masses and decay widths of resonances can be calculated from the volume dependence of the energy eigenvalues (rather difficult).

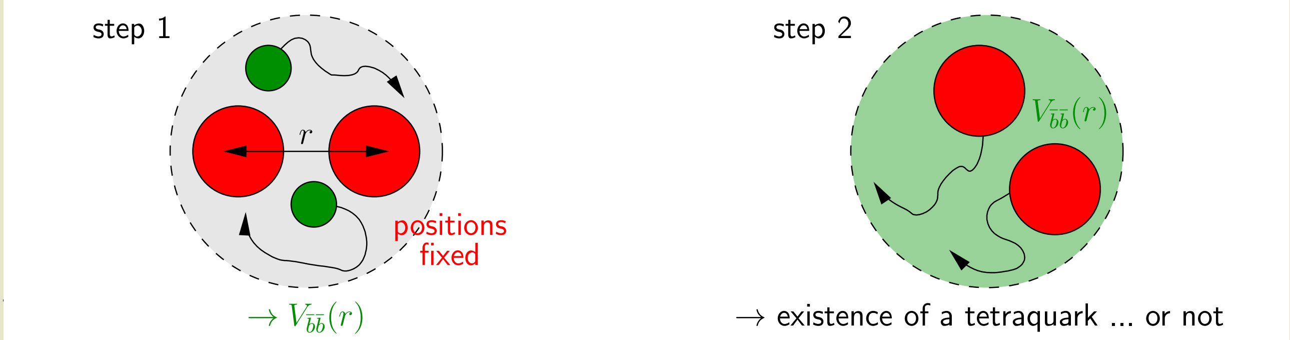
    $\rightarrow$ **Only a few short remarks at the end, if time is left.**
    (possibly more about this approach in Daniel Mohler's talk at 13:00)

Part 1:
Born-Oppenheimer approximation

Basic idea: lattice QCD + BO

- Start with $\bar{b}\bar{b}qq$.
- $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$ is the bottom counterpart of the experimentally observed T_{cc}.
  [R. Aaij *et al.* [LHCb], Nature Phys. **18**, 751-754 (2022) [arXiv:2109.01038]].
- Study such $\bar{b}\bar{b}qq$ tetraquarks in two steps:
  (1) **Compute potentials of the two static quarks $\bar{b}\bar{b}$ in the presence of two lighter quarks qq ($q \in \{u, d, s\}$) using lattice QCD.**
  (2) **Check, whether these potentials are sufficiently attractive to host bound states or resonances** ($\rightarrow$ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

(1) + (2) $\rightarrow$ Born-Oppenheimer approximation.



$\bar{b}\bar{b}qq$ / BB potentials

- To determine $\bar{b}\bar{b}$ potentials $V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)$, compute temporal correlation functions

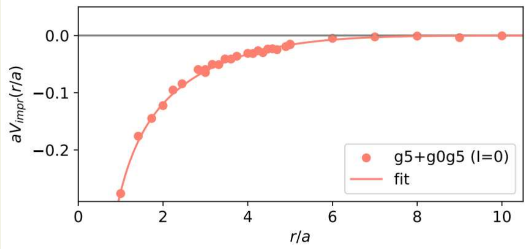
  $$\langle\Omega|\mathcal{O}^\dagger_{BB,\Gamma}(t)\mathcal{O}_{BB,\Gamma}(0)|\Omega\rangle \propto_{t\rightarrow\infty} e^{-V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)t}$$

  of operators

  $$\mathcal{O}_{BB,\Gamma} \quad = \quad 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)q^a_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)q^b_B(+\mathbf{r}/2)\Big).$$

- Many different channels: attractive as well as repulsive, different asymptotic values ...

- The most attractive potential of a $B^{(*)}B^{(*)}$ meson pair has $(I, |j_z|, P, P_x) = (0, 0, +, -)$:
  - $\psi^{(f)}\psi^{(f')} = ud - du$, $\Gamma \in \{(1+\gamma_0)\gamma_5\,,\,(1-\gamma_0)\gamma_5\}$.
  - $\bar{Q}\bar{Q} = \bar{b}\bar{b}$, $\tilde{\Gamma} \in \{(1+\gamma_0)\gamma_5\,,\,(1+\gamma_0)\gamma_j\}$ (irrelevant).



- Parameterize lattice results by

  $$V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r) \quad = \quad -\frac{\alpha}{r}\exp\bigg(-\Big(\frac{r}{d}\Big)^p\bigg) + V_0$$

  (1-gluon exchange at small r; color screening at large r).

  [P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]
  [L. Müller, unpublished ongoing work]

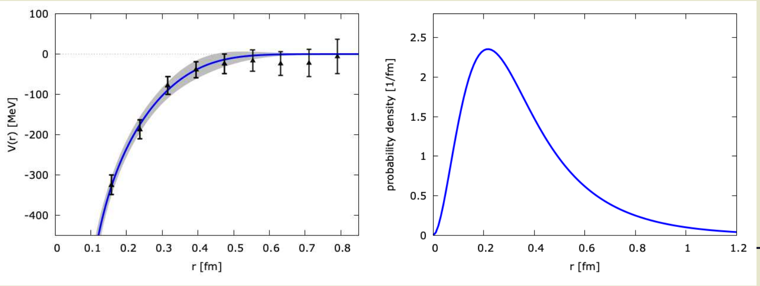
Stable $\bar{b}\bar{b}qq$ tetraquarks

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$ using the previously computed $\bar{b}\bar{b}qq$ / BB potentials,

$$\left(\frac{1}{m_b}\left(-\frac{d^2}{dr^2}+\frac{L(L+1)}{r^2}\right)+V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)-2m_B\right)R(r) \quad = \quad ER(r).$$

- Possibly existing bound states, i.e. $E < 0$, indicate QCD-stable $\bar{b}\bar{b}qq$ tetraquarks.
- There is a bound state for orbital angular momentum $L = 0$ of $\bar{b}\bar{b}$:
  - Binding energy $E = -90^{+43}_{-36}$ MeV with respect to the BB^* threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$.

[P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]

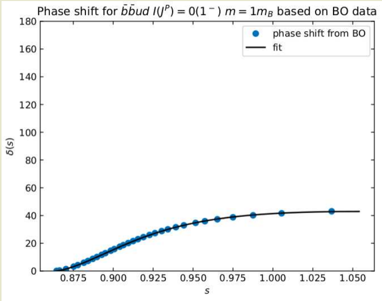


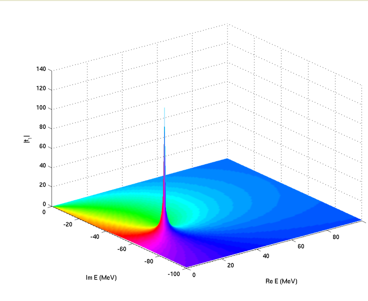
Further $\bar{b}\bar{b}qq$ results (1)

- **Are there further QCD-stable $\bar{b}\bar{b}qq$ tetraquarks with other $I(J^P)$ and light flavor quantum numbers?**
  - $\rightarrow$ No, not for $qq = ud$ (both $I = 0, 1$), not for $qq = ss$.
    [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]
  - $\rightarrow$ $\bar{b}\bar{b}us$ was not investigated.
  - – Strong evidence from full QCD computations that a QCD-stable $\bar{b}\bar{b}us$ tetraquark exists (see part 2 of this talk).
- **Effect of heavy quark spins:**
  - – Expected to be $\mathcal{O}(m_{B^*} - m_B) = \mathcal{O}(45\,\text{MeV})$.
  - – Previously ignored (potentials of static quarks are independent of the heavy spins).
  - – In [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]] included in a crude phenomenological way via a BB^* and a B^*B^* coupled channel Schrödinger equation with the experimental mass difference $m_{B^*} - m_B$ as input.
  - $\rightarrow$ Binding energy reduced from around $90\,\text{MeV}$ to $59\,\text{MeV}$.
  - $\rightarrow$ Physical reason: the previously discussed attractive potential does not only correspond to a lighter BB^* pair, but has also a heavier B^*B^* contribution.

Further $\bar{b}\bar{b}qq$ results (2)

- **Are there $\bar{b}\bar{b}qq$ tetraquark resonances?**





  – In
    [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]]
    resonances studied via standard scattering theory from quantum mechanics textbooks.

  $\rightarrow$ Heavy quark spins ignored.

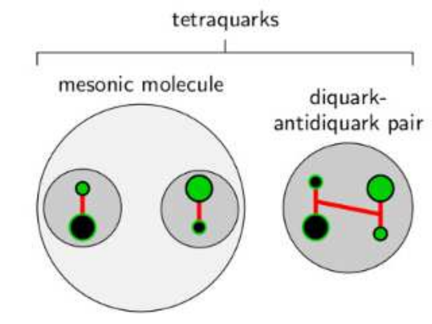
  $\rightarrow$ Indication for $\bar{b}\bar{b}ud$ tetraquark resonance with $I(J^P) = 0(1^-)$ found, $E = 17^{+4}_{-4}$ MeV above the BB threshold, decay width $\Gamma = 112^{+90}_{-103}$ MeV.

  – In
    [J. Hoffmann, A. Zimermmane-Santos and M.W., PoS **LATTICE2022**, 262 (2023) [arXiv:2211.15765]]
    heavy quark spins included.

  $\rightarrow$ $\bar{b}\bar{b}ud$ resonance not anymore existent.

  $\rightarrow$ Physical reason: the relevant attractive potential does not only correspond to a lighter BB pair, but has also a heavier B^*B^* contribution.

Further $\bar{b}\bar{b}qq$ results (3)



- Structure of the QCD-stable $\bar{b}\bar{b}ud$ tetraquark with $I(J^P) = 0(1^+)$: meson-meson (BB) versus diquark-antidiquark (Dd).
  - Use not just one but two operators,

$$\mathcal{O}_{BB,\Gamma} = 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)\psi^{(f)a}_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)\psi^{(f')b}_B(+\mathbf{r}/2)\Big)$$

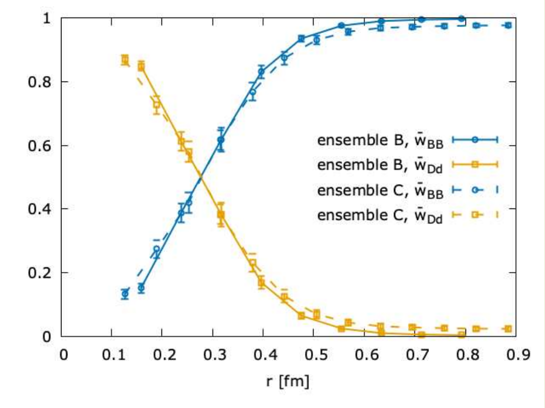
$$\mathcal{O}_{Dd,\Gamma} = -N_{Dd}\epsilon^{abc}\Big(\psi^{(f)b}_A(\mathbf{z})(\mathcal{C}\Gamma)_{AB}\psi^{(f')c}_B(\mathbf{z})\Big)$$
$$\epsilon^{ade}\Big(\bar{Q}^f_C(-\mathbf{r}/2)U^{fd}(-\mathbf{r}/2;\mathbf{z})(\mathcal{C}\tilde{\Gamma})_{CD}\bar{Q}^g_D(+\mathbf{r}/2)U^{ge}(+\mathbf{r}/2;\mathbf{z})\Big),$$

    compare the contribution of each operator to the $\bar{b}\bar{b}$ potential $V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)$.
    [P. Bicudo, A. Peters, S. Velten, M.W., Phys. Rev. D **103**, 114506 (2021) [arXiv:2101.00723]]

$\rightarrow$ $r \lesssim 0.2\,\text{fm}$: Clear diquark-antidiquark dominance.

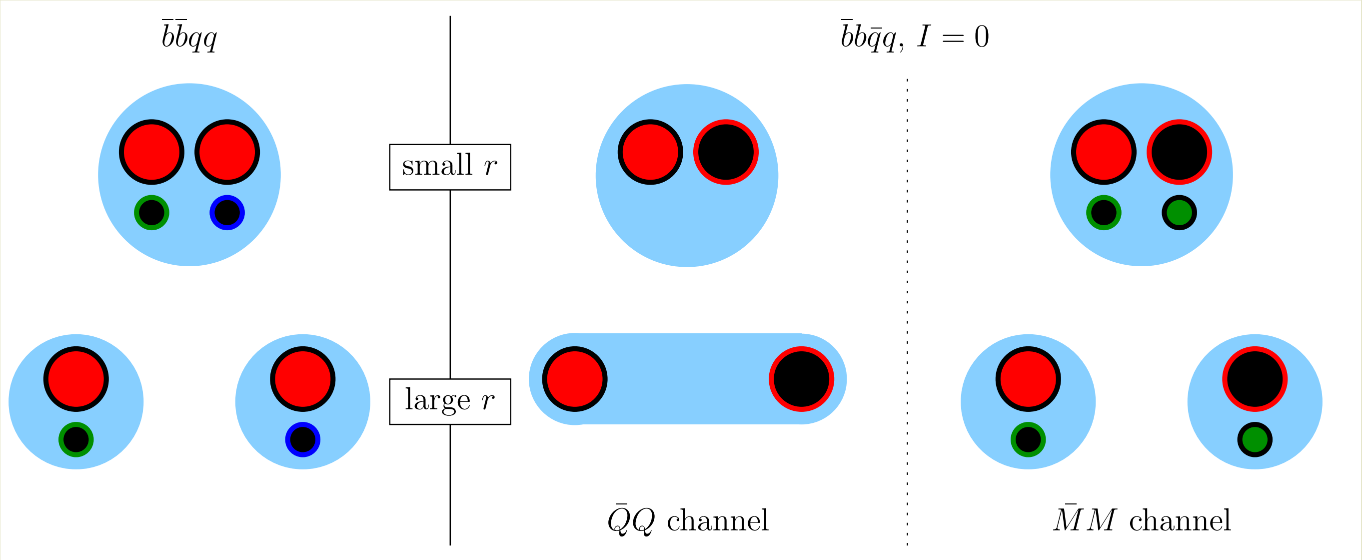
$\rightarrow$ $0.5\,\text{fm} \lesssim r$: Essentially a meson-meson system.

$\rightarrow$ Integrate over t to estimate the composition of the tetraquark: $\%BB \approx 60\%$, $\%Dd \approx 40\%$.

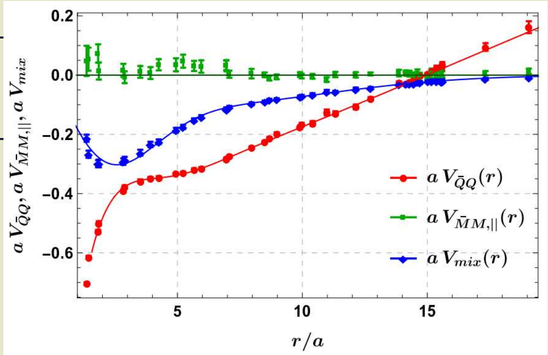


Bottomonium, $I = 0$: difference to $\bar{b}\bar{b}qq$

- Now bottomonium with $I = 0$, i.e. $\bar{b}b$ and/or $\bar{b}b\bar{q}q$ (with $\bar{q}q = (\bar{u}u + \bar{d}d)/\sqrt{2}, \bar{s}s$).
- Technically more complicated than $\bar{b}\bar{b}qq$, because there are two channels:
  - Quarkonium channel, $\bar{Q}Q$ (with $Q \equiv b$).
  - Heavy-light meson-meson channel, $\bar{M}M$ (with $M = \bar{Q}q$), "string breaking".



Bottomonium, $I = 0$: ...



- Lattice computation of potentials for both channels ($\bar{Q}Q$ and $\bar{M}M$) needed, additionally also a mixing potential:
  - Pioneering work:
    [G. S. Bali *et al.* [SESAM Collaboration], Phys. Rev. D **71**, 114513 (2005) [hep-lat/0505012]]
    Rather heavy u/d quark masses ($m_\pi \approx 650$ MeV), only 2 flavors, not $2+1$.
  - More recent work:
    [J. Bulava, B. Hörz, F. Knechtli, V. Koch, G. Moir, C. Morningstar and M. Peardon, Phys. Lett. B **793**, 493-498 (2019) [arXiv:1902.04006]]
    Unfortunately, mixing potential not computed.
  - Several assumptions needed to adapt the "Bali results" to $2+1$ flavors and physical quark masses.

$\rightarrow$ Potential for a coupled channel Schrödiger equation (see next slide):

$$V(\mathbf{r}) = \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\text{mix}}(r)(1 \otimes \mathbf{e}_r) & (1/\sqrt{2})V_{\text{mix}}(r)(1 \otimes \mathbf{e}_r) \\ V_{\text{mix}}(r)(1 \otimes \mathbf{e}_r) & V_{\bar{M}M}(r) & 0 \\ (1/\sqrt{2})V_{\text{mix}}(r)(1 \otimes \mathbf{e}_r) & 0 & V_{\bar{M}M}(r) \end{pmatrix}.$$

Bottomonium, $I = 0$: SE

- Schrödinger equation non-trivial:
  - 3 coupled channels, $\bar{b}b$, BB (3 components), $B_s B_s$ (3 components).
  - Static potentials used as input have other symmetries and quantum numbers than bottomonium states (Λ_η^ϵ versus J^{PC}).

$$\left(-\frac{1}{2}\mu^{-1}\left(\partial_r^2 + \frac{2}{r}\partial_r - \frac{\mathbf{L}^2}{r^2}\right) + V(\mathbf{r}) + \begin{pmatrix} E_{\text{threshold}} & 0 & 0 \\ 0 & 2m_M & 0 \\ 0 & 0 & 2m_{M_s} \end{pmatrix} - E\right)\psi(\mathbf{r}) \quad = \quad 0.$$

  - Project to definite total angular momentum,
    - 7 coupled PDEs $\rightarrow$ 3 coupled ODEs for $\tilde{J} = 0$,
    - 7 coupled PDEs $\rightarrow$ 5 coupled ODEs for $\tilde{J} \geq 1$

    ($\tilde{J}$: total angular momentum excluding the heavy quark spins).
  - Add scattering boundary conditions.
- Determine scattering amplitudes and T matrices from the Schrödinger equation, find poles of $\mathrm{T}_{\tilde{J}}$ in the complex energy plane to identify bound states and resonances.
- The components of the resulting wave functions provide the compositions of the states, i.e. the quarkonium and meson-meson percentages $\%\bar{Q}Q$ and $\%\bar{M}M$.

		theory		experiment			
$\tilde{J}^{PC}$	n	m[GeV]	Γ[MeV]	name	m[GeV]	Γ[MeV]	$I^G(J^{PC})$
0^{++}	1	9.618^{+10}_{-15}	-	$\eta_b(1S)$	9.399(2)	10(5)	$0^+(0^{+-})$
				$\Upsilon_b(1S)$	9.460(0)	≈ 0	$0^-(1^{--})$
	2	10.114^{+7}_{-11}	-	$\eta_b(2S)_{\text{BELLE}}$	9.999(6)	-	$0^+(0^{+-})$
				$\Upsilon(2S)$	10.023(0)	≈ 0	$0^-(1^{--})$
	3	10.442^{+7}_{-9}	-	$\Upsilon(3S)$	10.355(1)	≈ 0	$0^-(1^{--})$
	4	10.629^{+1}_{-1}	$49.3^{+5.4}_{-3.9}$	$\Upsilon(4S)$	10.579(1)	21(3)	$0^-(1^{--})$
	5	10.773^{+1}_{-2}	$15.9^{+2.9}_{-4.4}$	$\Upsilon(10750)_{\text{BELLE II}}$	10.753(7)	36(22)	$0^-(1^{--})$
	6	10.938^{+2}_{-2}	$61.8^{+7.6}_{-8.0}$	$\Upsilon(10860)$	10.890(3)	51(7)	$0^-(1^{--})$
	7	11.041^{+5}_{-7}	$45.5^{+13.5}_{-8.2}$	$\Upsilon(11020)$	10.993(1)	49(15)	$0^-(1^{--})$
1^{--}	1	9.930^{+43}_{-52}	-	$\chi_{b0}(1P)$	9.859(1)	-	$0^+(0^{++})$
				$h_b(1P)$	9.890(1)	-	$?^?(1^{+-})$
				$\chi_{b1}(1P)$	9.893(1)	-	$0^+(1^{++})$
				$\chi_{b2}(1P)$	9.912(1)	-	$0^+(2^{++})$
	2	10.315^{+29}_{-40}	-	$\chi_{b0}(2P)$	10.233(1)	-	$0^+(0^{++})$
				$\chi_{b1}(2P)$	10.255(1)	-	$0^+(1^{++})$
				$h_b(2P)_{\text{BELLE}}$	10.260(2)	-	$?^?(1^{+-})$
				$\chi_{b2}(2P)$	10.267(1)	-	$0^+(2^{++})$
	3	10.594^{+32}_{-28}	-	$\chi_{b1}(3P)$	10.512(2)	-	$0^+(0^{++})$
	4	10.865^{+37}_{-21}	$67.5^{+5.1}_{-4.9}$				
	5	10.932^{+33}_{-54}	$101.8^{+7.3}_{-5.1}$				
	6	11.144^{+52}_{-75}	$25.0^{+1.1}_{-1.3}$				
2^{++}	1	10.181^{+35}_{-46}	-	$\Upsilon(1D)$	10.164(2)	-	$0^-(2^{--})$
	2	10.486^{+32}_{-36}	-				
	3	10.799^{+2}_{-2}	$13.0^{+2.1}_{-2.0}$				
	4	11.038^{+30}_{-44}	$40.8^{+2.0}_{-2.8}$				
3^{--}	1	10.390^{+28}_{-39}	-				
	2	10.639^{+31}_{-25}	$2.4^{+1.5}_{-0.9}$				
	3	10.944^{+20}_{-29}	$46.8^{-4.6}_{+6.2}$				
	4	11.174^{+51}_{-69}	$1.9^{+2.1}_{-1.4}$				

Bottomonium, $I = 0$: results

- Results for masses of bound states and resonances consistent with experimentally observed states within expected errors.
- Errors might be large:
  - Lattice QCD results for the potentials computed with unphysically heavy u/d quarks.
  - Heavy quark spin effects and corrections due to the finite b quark mass not included.
- Several bound states in the sectors $\tilde{J} = 0, 1, 2$ with clear experimental counterparts.
- Two resonance candidates for $\Upsilon(10753)$ recently found by Belle:
  - S wave state, $\tilde{J} = 0$, $n = 5$ ($\%\bar{Q}Q \approx 24$, $\%\bar{M}M \approx 76$).
  - D wave state, $\tilde{J} = 2$, $n = 3$ ($\%\bar{Q}Q \approx 21$, $\%\bar{M}M \approx 79$).
- $\Upsilon(10860)$ confirmed as an S wave state, $\tilde{J} = 0$, $n = 6$ ($\%\bar{Q}Q \approx 35$, $\%\bar{M}M \approx 65$).
  [P. Bicudo, M. Cardoso, N. Cardoso, M.W., Phys. Rev. D **101**, 034503 (2020) [arXiv:1910.04827]]
  [P. Bicudo, N. Cardoso, L. Müller, M.W., Phys. Rev. D **103**, 074507 (2021) [arXiv:2008.05605]]
  [P. Bicudo, N. Cardoso, L. Müller, M.W., Phys. Rev. D **107**, 094515 (2023) [arXiv:2205.11475]]

Bottomonium, $I = 0$: $1/m_Q$ corrections

- Potentials of static quarks are independent of the heavy spins.
  $\rightarrow$ Systematic errors are possibly large, $\mathcal{O}(m_{B^*} - m_B) = \mathcal{O}(45\,\mathrm{MeV})$.

- Such spin effects and further corrections due to the finite b quark mass can be expressed order by order in $1/m_b$.
  [E. Eichten and F. Feinberg, Phys. Rev. D **23**, 2724 (1981)]
  [N. Brambilla, A. Pineda, J. Soto and A. Vairo, Phys. Rev. D **63**, 014023 (2001) [arXiv:hep-ph/0002250]]

- The corresponding correlation functions are Wilson loops with field strength insertions.

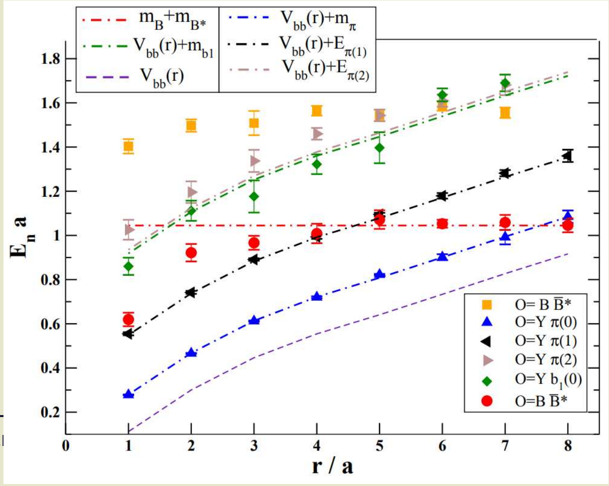
- Computations in pure SU(3) lattice gauge theory (no light quarks) up to order $1/m_Q^2$ in
  [Y. Koma and M. Koma, Nucl. Phys. B **769**, 79-107 (2007) [arXiv:hep-lat/0609078]]

- $1/m_Q$ and $1/m_Q^2$ corrections used to predict low lying (stable) bottomonium states with 1st order stationary perturbation theory.
  [Y. Koma and M. Koma, PoS **LATTICE2012**, 140 (2012) [arXiv:1211.6795 [hep-lat]]
  $\rightarrow$ Improvements, but still no satisfactory agreement with experimental results.

- Onging efforts
  - to compute these $1/m_Q$ and $1/m_Q^2$ corrections more precisely using gradient flow,
  - to replace perturbation theory by a non-perturbative coupled channel SE.

Bottomonium, $I = 1$: potentials

- Now bottomonium with $I = 1$, $\bar{b}b\bar{q}q$.
- Bottomonium with $I = 1$ includes the experimentally observed Z_b tetraquarks.
- Technically even more complicated than bottomonium with $I = 0$, because the relevant $\bar{B}^{(*)}B^{(*)}$ channel does not correspond to the ground state, but to an excited state.
  - Ordinary bottomonium $\Upsilon \equiv \bar{b}b$ and a pion (possibly with non-vanishing momentum) have the same quantum numbers, but lower energies.
  - In lattice QCD you can compute the energy of an excited state, but only if you also compute all energy levels below.
- The relevant low-lying potentials were recently computed for the first time.
  [S. Prelovsek, H. Bahtiyar, J. Petkovic, Phys. Lett. B **805**, 135467 (2020) [arXiv:1912.02656]]
- The relevant $\bar{B}^{(*)}B^{(*)}$ potential is represented by the red points.
- For small separations it corresponds to the 2nd excited state ($\Upsilon + \pi$ at rest [blue] and with 1 quantum of momentum [black] are below).



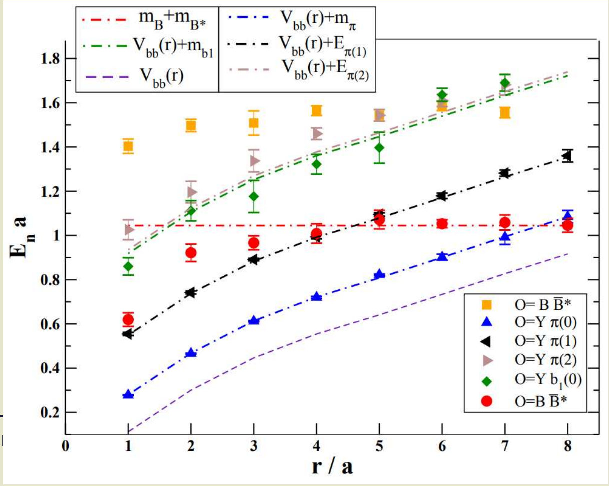
Bottomonium, $I = 1$: BO results

- Single-channel Schrödinger equation with the computed $\bar{B}^{(*)}B^{(*)}$ potential:
  - $\rightarrow$ There seems to be a bound state close to the $\bar{B}^{(*)}B^{(*)}$ threshold, binding energy $E = -48^{+41}_{-108}$ MeV.
  - $\rightarrow$ Probably related to $Z_b(10610)$ and $Z_b(10650)$.

  $\rightarrow$ A very interesting and impressive result.
  [S. Prelovsek, H. Bahtiyar, J. Petkovic, Phys. Lett. B **805**, 135467 (2020) [arXiv:1912.02656]]

- However, possibly large systematic errors:
  - Heavy spin effects and corrections due to the finite b quark mass neglected.
  - No coupling of the $\bar{B}^{(*)}B^{(*)}$ channel to the other channels, in particular $\Upsilon + \pi$.

- 3 related four-quark sectors with quantum numbers differing in parity and charge conjugation do not show any sign of a bound state.
  [M. Sadl, S. Prelovsek, Phys. Rev. D **104**, 114503 (2021) [arXiv:2109.08560]]



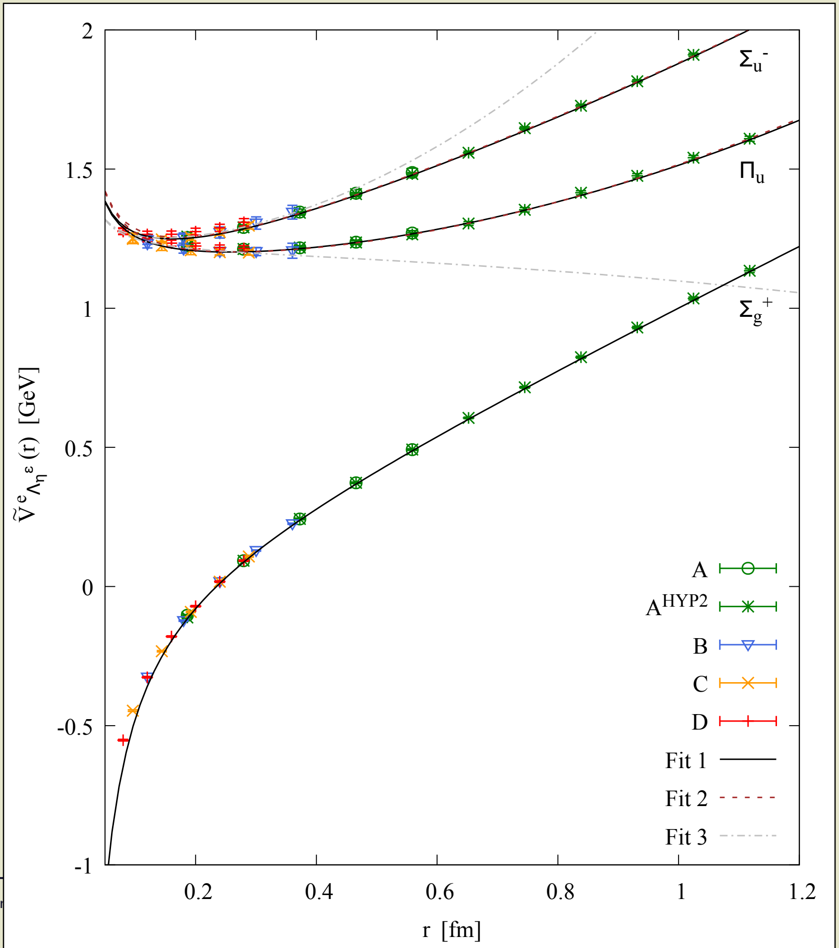
Heavy hybrid mesons: potentials (1)

- Now heavy hybrid mesons, i.e. $\bar{b}b +$ gluons.
- (Hybrid) static potentials can be characterized by the following quantum numbers:
  - Absolute total angular momentum with respect to the $\bar{Q}Q$ separation axis (z axis): $\Lambda = 0, 1, 2, \ldots \equiv \Sigma, \Pi, \Delta, \ldots$
  - Parity combined with charge conjugation: $\eta = +, - = g, u$.
  - Relection along an axis perpendicular to the $\bar{Q}Q$ separation axis (x axis): $\epsilon = +, -$.
- The ordinary static potential has quantum numbers $\Lambda_\eta^\epsilon = \Sigma_g^+$.
- Particularly interesting: the two lowest hybrid static potentials with $\Lambda_\eta^\epsilon = \Pi_u, \Sigma_u^-$.
- References:

[K. J. Juge, J. Kuti, C. J. Morningstar, Nucl. Phys. Proc. Suppl. **63**, 326 (1998) [hep-lat/9709131]
[C. Michael, Nucl. Phys. A **655**, 12 (1999) [hep-ph/9810415]
[G. S. Bali *et al.* [SESAM and TχL Collaborations], Phys. Rev. D **62**, 054503 (2000) [hep-lat/0003012]
[K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. **90**, 161601 (2003) [hep-lat/0207004]
[C. Michael, Int. Rev. Nucl. Phys. **9**, 103 (2004) [hep-lat/0302001]
[G. S. Bali, A. Pineda, Phys. Rev. D **69**, 094001 (2004) [hep-ph/0310130]
[P. Bicudo, N. Cardoso, M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815 [hep-lat]]]
[S. Capitani, O. Philipsen, C. Reisinger, C. Riehl. M.W., Phys. Rev. D **99**, 034502 (2019) [arXiv:1811.11046 [hep-lat]]]

Heavy hybrid mesons: potentials (2)

- [C. Schlosser, M.W., Phys. Rev. D **105**, 054503 (2022) [arXiv:2111.00741]]



Heavy hybrid mesons: SE

- Solve Schrödinger equations for the relative coordinate of $\bar{b}b$ using hybrid static potentials,

$$\bigg(-\frac{1}{2\mu}\frac{d^2}{dr^2} + \frac{L(L+1) - 2\Lambda^2 + J_{\Lambda_\eta^\epsilon}(J_{\Lambda_\eta^\epsilon}+1)}{2\mu r^2} + V_{\Lambda_\eta^\epsilon}(r)\bigg) u_{\Lambda_\eta^\epsilon;L,n}(r) \quad = \quad E_{\Lambda_\eta^\epsilon;L,n} u_{\Lambda_\eta^\epsilon;L,n}(r).$$

  Energy eigenvalues $E_{\Lambda_\eta^\epsilon;L,n}$ correspond to masses of $\bar{b}b$ hybrid mesons.
  [E. Braaten, C. Langmack, D. H. Smith, Phys. Rev. D **90**, 014044 (2014) [arXiv:1402.0438]]
  [M. Berwein, N. Brambilla, J. Tarrus Castella, A. Vairo, Phys. Rev. D **92**, 114019 (2015) [arXiv:1510.04299]]
  [R. Oncala, J. Soto, Phys. Rev. D **96**, 014004 (2017) [arXiv:1702.03900]]

- Important recent and ongoing work to include heavy spin and $1/m_b$ corrections.
  [N. Brambilla, G. Krein, J. Tarrus Castella, A. Vairo, Phys. Rev. D **97**, 016016 (2018) [arXiv:1707.09647]]
  [N. Brambilla, W. K. Lai, J. Segovia, J. Tarrus Castella, A. Vairo, Phys. Rev. D **99**, 014017 (2019) [arXiv:1805.07713]]

Hybrid flux tubes (1)

- We are interested in

$$\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x}) \quad = \quad \langle 0_{\Lambda^\epsilon_\eta}(r)|F^2_{\mu\nu}(\mathbf{x})|0_{\Lambda^\epsilon_\eta}(r)\rangle - \langle\Omega|F^2_{\mu\nu}|\Omega\rangle.$$

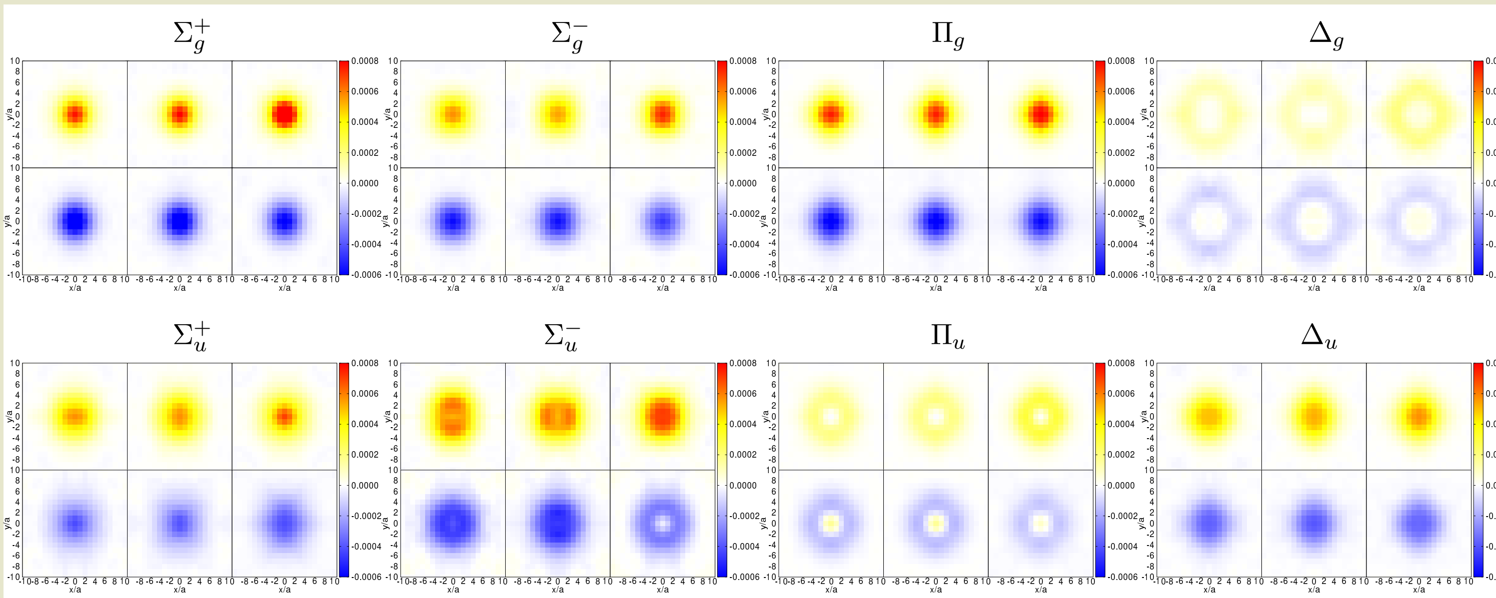
  - $F^2_{\mu\nu}(\mathbf{x})$, $F^2_{\mu\nu}$: squared chromoelectric/chromomagnetic field strength.
  - $|0_{\Lambda^\epsilon_\eta}(r)\rangle$: "hybrid static potential (ground) state" (r denotes the $\bar{Q}Q$ separation).
  - $|\Omega\rangle$: vacuum state.

- The sum over the six independent $\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x})$ is proportional to the chromoelectric and -magnetic energy density of hybrid flux tubes.

Hybrid flux tubes (2)

- $\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x})$, SU(2), mediator plane (x-y plane with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,\mathrm{fm}$.
  [L. Müller, O. Philipsen, C. Reisinger, M.W., Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]
- For results for $\Lambda^\epsilon_\eta = \Sigma^+_g, \Sigma^+_u, \Pi_u$ see also
  [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815]]

ΔE_x^2	ΔE_y^2	ΔE_z^2
ΔB_x^2	ΔB_y^2	ΔB_z^2

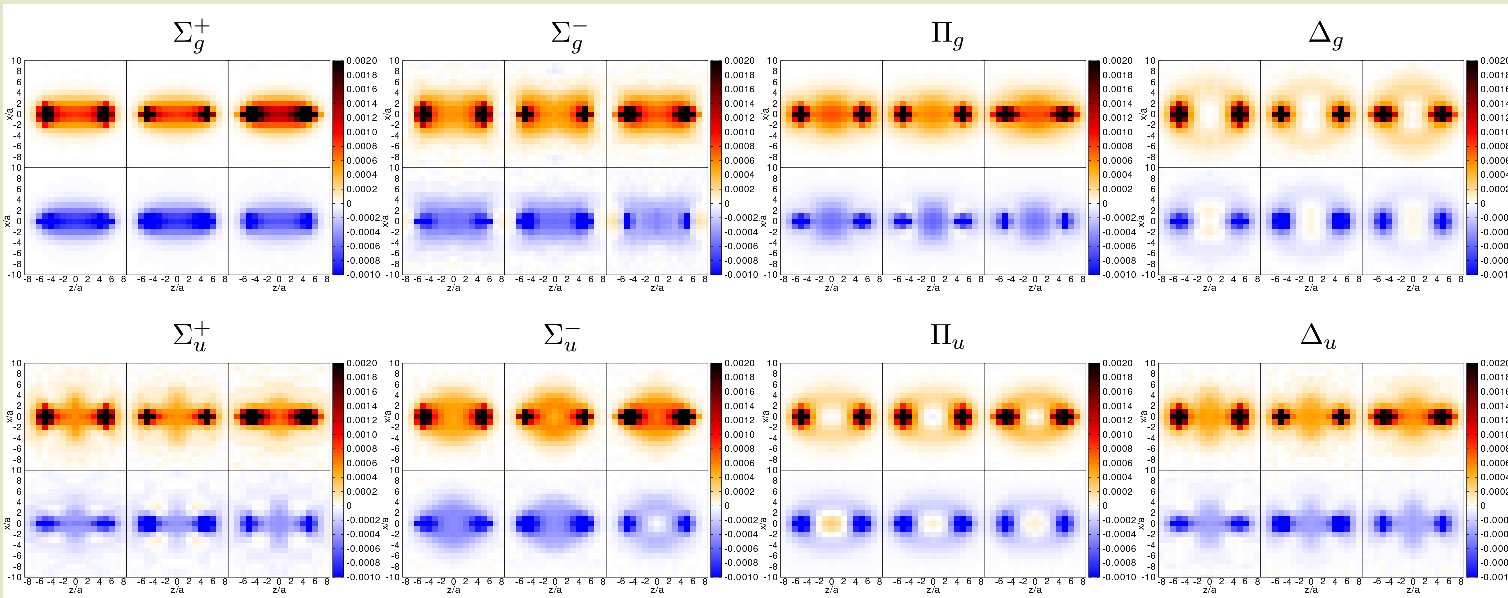


Hybrid flux tubes (3)

- $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$, SU(2), separation plane (x-z plane with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,\mathrm{fm}$.
  [L. Müller, O. Philipsen, C. Reisinger, M.W., Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]

- For results for $\Lambda_\eta^\epsilon = \Sigma_g^+, \Sigma_u^+, \Pi_u$ see also
  [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815]]

ΔE_x^2	ΔE_y^2	ΔE_z^2
ΔB_x^2	ΔB_y^2	ΔB_z^2



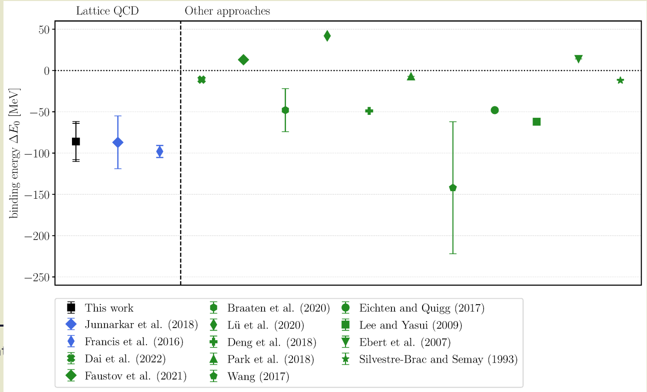
Part 2:
Full lattice QCD computations of eigenvalues of the QCD Hamiltonian

Full lattice QCD computations

- Do not treat the heavy b or c quarks as static.
- Do not separate the computations for heavy and for light quarks, i.e. no potentials.
- Compute eigenvalues of the QCD Hamiltonian at finite spatial volume.
- For QCD-stable states that might already be sufficient.
- For resonances:
  - Relate finite volume energy levels to infinite volume scattering phases (or equivalently scattering amplitudes).
  - Fit an ansatz for the scattering amplitude to the few data points from the previous step.
  - Find poles in the complex energy plane.

$\bar{b}\bar{b}ud$, $I(J^P) = 0(1^+)$ and $\bar{b}\bar{b}us$, $J^P = 1^+$

- QCD-stable $\bar{b}\bar{b}ud$ tetraquark, $I(J^P) = 0(1^+)$, $\approx 130\,\text{MeV}$ below the BB^* threshold.
- QCD-stable $\bar{b}\bar{b}us$ tetraquark, $J^P = 1^+$, $\approx 90\,\text{MeV}$ below the BB_s^* threshold.
- Lattice QCD results from independent groups consistent within statistical errors.
  [A. Francis, R. J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. Lett. **118**, 142001 (2017) [arXiv:1607.05214]] ($\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$)
  [P. Junnarkar, N. Mathur, M. Padmanath, Phys. Rev. D **99**, 034507 (2019) [arXiv:1810.12285]] ($\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$)
  [L. Leskovec, S. Meinel, M. Pflaumer, M.W., Phys. Rev. D **100**, 014503 (2019) [arXiv:1904.04197]] ($\bar{b}\bar{b}ud$)
  [P. Mohanta, S. Basak, Phys. Rev. D **102**, 094516 (2020) [arXiv:2008.11146]] ($\bar{b}\bar{b}ud$)
  [S. Meinel, M. Pflaumer, M.W., Phys. Rev. D **106**, 034507 (2022) [arXiv:2205.13982]] ($\bar{b}\bar{b}us$)
  [R. J. Hudspith, D. Mohler, Phys. Rev. D **107**, 114510 (2023) [arXiv:2303.17295]] ($\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$)
  [T. Aoki, S. Aoki, T. Inoue, [arXiv:2306.03565]] ($\bar{b}\bar{b}ud$)
- Strong discrepancies between non-lattice QCD results.



Conclusions

- Significant progress and interesting lattice QCD results in the past ≈ 10 years on heavy exotic mesons ... but still a lot to do and several problems to solve.

- This talk: focus on heavy exotics with two bottom (anti)quarks in the Born-Oppenheimer approximation.

  - Lattice QCD used to compute bb and $\bar{b}b$ potentials in QCD.
  - Majority of presented results obtained with static b quarks.
    $\rightarrow$ Crude, errors of order $\mathcal{O}(m_{B^*} - m_B) = \mathcal{O}(45\,\text{MeV})$ expected.
  - The computation of potentials provides interesting insights, e.g. composition of exotic mesons or hybrid flux tubes.
  - For solid quantitative results heavy spin and finite b quark mass corrections needed (ongoing work, challenge for the near future).

- Full lattice QCD computations, i.e. not Born-Oppenheimer: mostly studies of $\bar{Q}\bar{Q}qq$.

- At the moment quantitatively reliable results only for two systems, the QCD-stable tetraquarks $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$ and $\bar{b}\bar{b}us$ with $J^P = 1^+$.