

Measurement of $\bar{K}N$ scattering below the $\bar{K}N$ mass threshold

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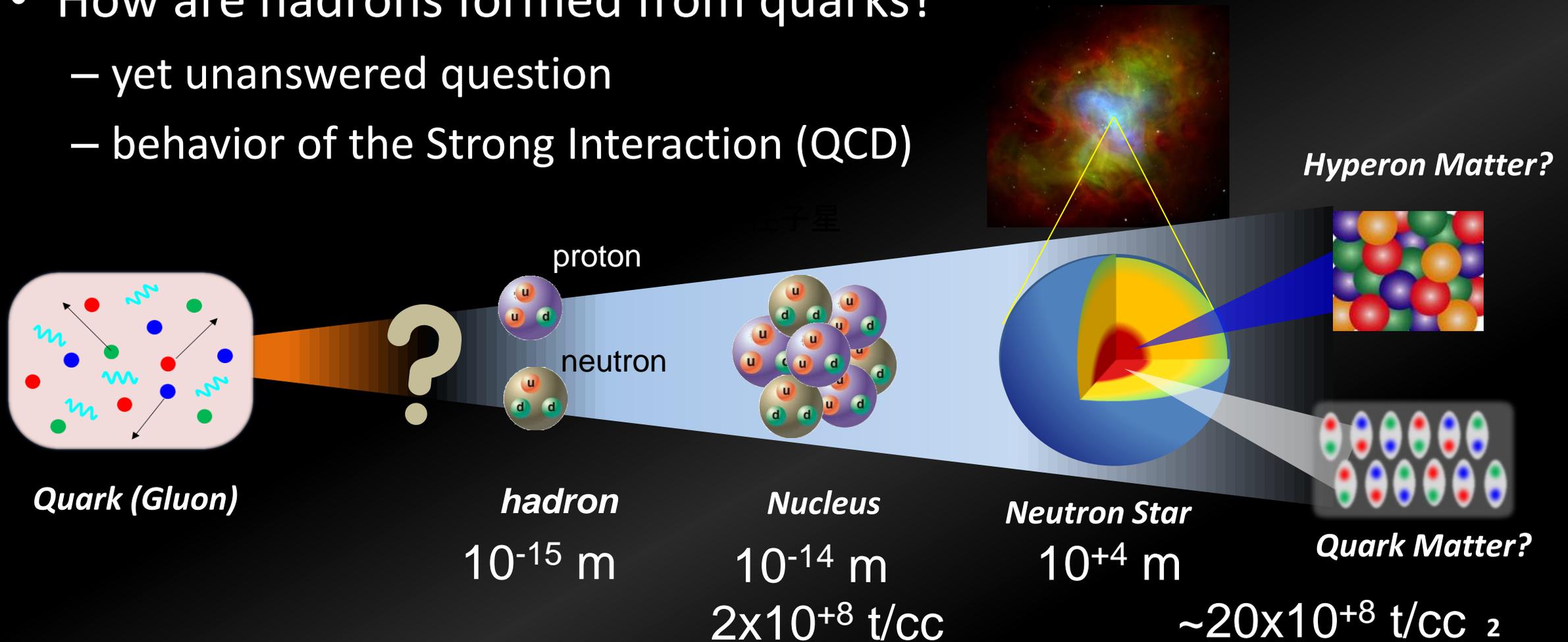
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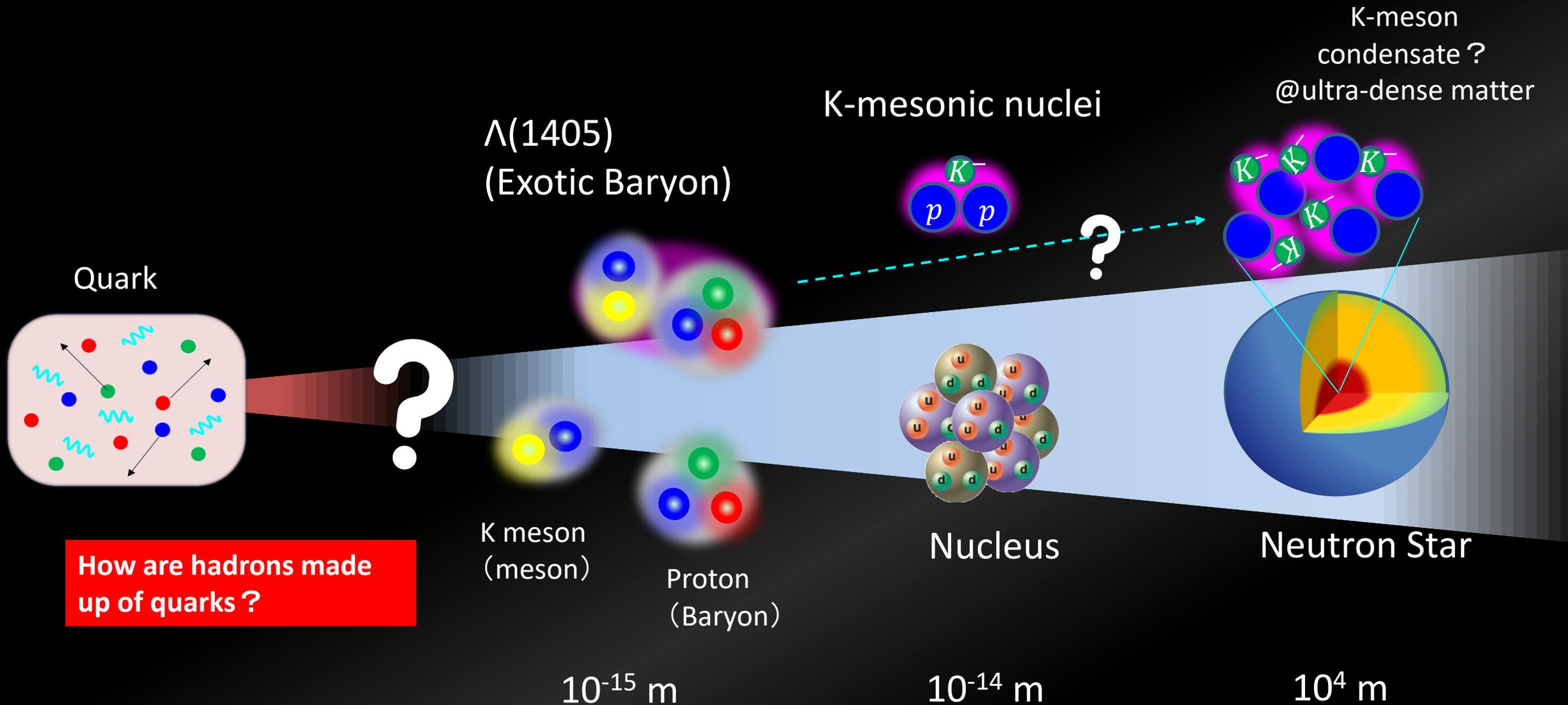
Based on the E31 collaboration, Phys. Lett. B837(2023)137637

Matter Formation and Evolution in the Universe

- Hadrons: complex system of quarks (and gluons)
- How are hadrons formed from quarks?
 - yet unanswered question
 - behavior of the Strong Interaction (QCD)



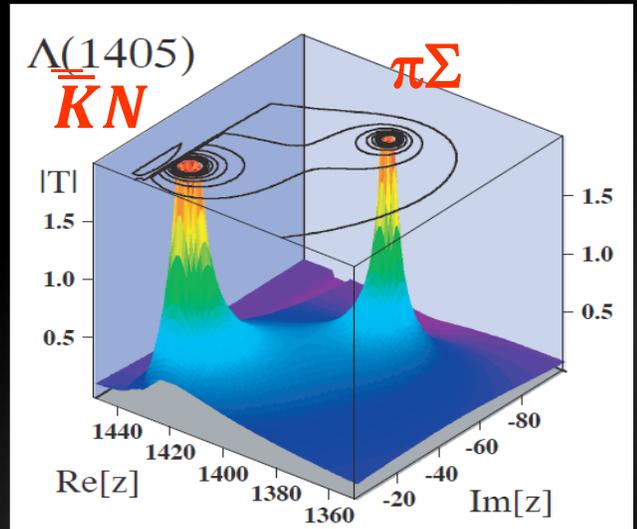
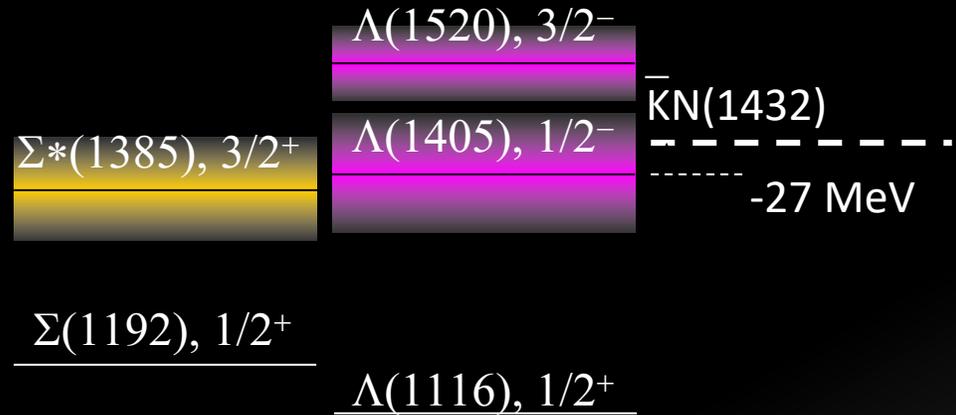
Matter Formation and Evolution in the Universe



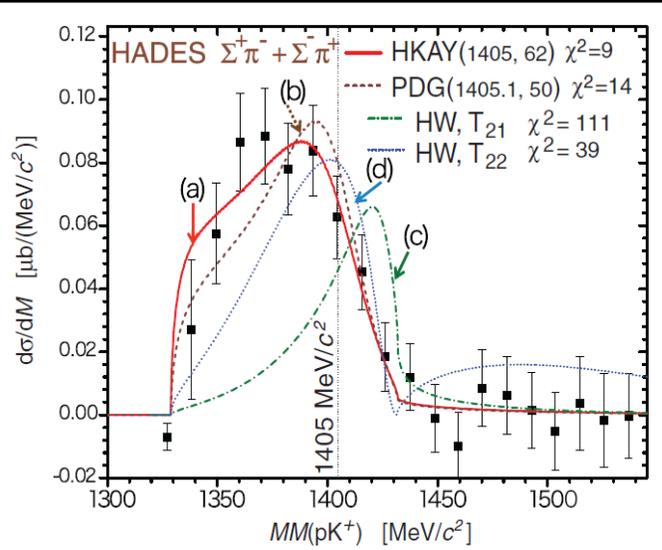
How are hadrons made up of quarks ?

$\Lambda(1405) : 1405.1^{+1.3}_{-0.9} \text{ MeV}$ (PDG in 2022) \leftrightarrow Double pole?

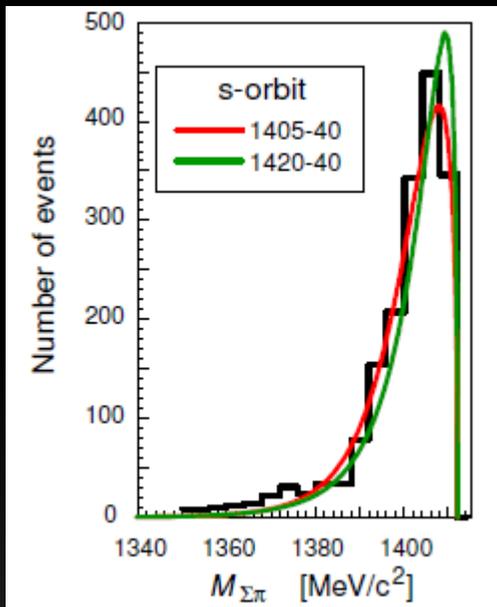
$J^P = \frac{1}{2}^-, I = 0, M_{\Lambda(1405)} < M_{K\bar{N}}$, lightest in neg. parity baryons



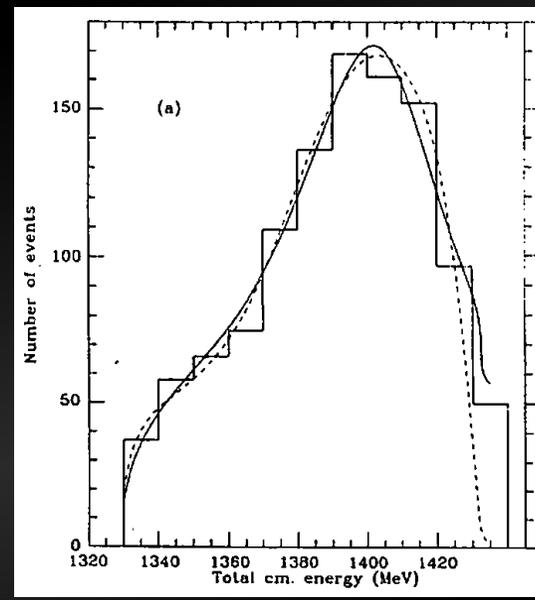
ChU model, T. Hyodo



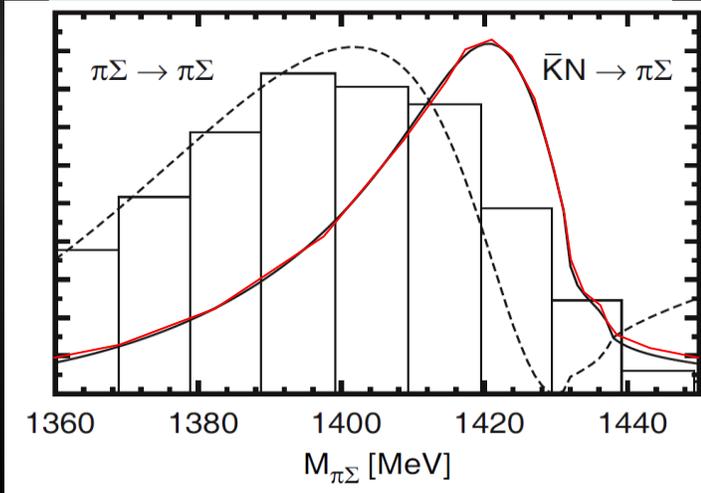
M. Hassanvand et al: $\pi\Sigma$ IM Spec. of $pp \rightarrow K^+\pi\Sigma$



J. Esmaili et al: $\pi\Sigma$ IM Spec. of Stopped K^- on ^4He



R.H. Dalitz et al: $\pi\Sigma$ IM Spec. in $K-p \rightarrow \pi\pi\Sigma$ w/ M-matrix



Chiral Unitary Model: D. Jido et al., NPA725(03)181

Pole Structure of the Lambda(1405) Region

PDG Reviews: Ulf-G. Meissner and T. Hyodo (since Nov. 2015)

Table 1: Comparison of the pole positions of $\Lambda(1405)$ in the complex energy plane from next-to-leading order chiral unitary coupled-channel approaches including the SIDDHARTA constraint.

approach	pole 1 [MeV]	pole 2 [MeV]
Refs. 11,12, NLO	$1424_{-23}^{+7} - i 26_{-14}^{+3}$	$1381_{-6}^{+18} - i 81_{-8}^{+19}$
Ref. 14, Fit II	$1421_{-2}^{+3} - i 19_{-5}^{+8}$	$1388_{-9}^{+9} - i 114_{-25}^{+24}$
Ref. 15, solution #2	$1434_{-2}^{+2} - i 10_{-1}^{+2}$	$1330_{-5}^{+4} - i 56_{-11}^{+17}$
Ref. 15, solution #4	$1429_{-7}^{+8} - i 12_{-3}^{+2}$	$1325_{-15}^{+15} - i 90_{-18}^{+12}$

$\Lambda(1405) : 1405.1_{-1.0}^{+1.3} \text{ MeV}$ (Part. Listing in '22)

$J^P = \frac{1}{2}^-, I = 0, M_{\Lambda(1405)} < M_{K\bar{K}N}$, lightest in neg. parity baryons

M. Hassanvand et al: $\pi\Sigma$ IM
Spec. of $pp \rightarrow K^+\pi\Sigma$

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in $K-p \rightarrow \pi\pi\Sigma$ w/ M-matrix

Pole Structure of the Lambda(1405) Region

PDG Reviews: Ulf-G. Meissner and T. Hyodo (since Nov. 2015)

Table 1: Comparison of $\Lambda(1405)$ in the complex plane to leading order chiral approaches including $N\bar{K}$ threshold constraint.

approach	pole 1
Refs. 11,12, NLO	1424_{-1}^{+1}
Ref. 14, Fit II	1421_{-1}^{+1}
Ref. 15, solution #2	1434_{-1}^{+1}
Ref. 15, solution #4	1429_{-1}^{+1}

Citation: R.L. Workman et al. (Particle Data Group), to be published (2022)

$\Lambda(1405) 1/2^-$ $I(J^P) = 0(\frac{1}{2}^-)$ Status: ****

In the 1998 Note on the $\Lambda(1405)$ in PDG 98, R.H. Dalitz discussed the S-shaped cusp behavior of the intensity at the $N\bar{K}$ threshold observed in THOMAS 73 and HEMINGWAY 85. He commented that this behavior "is characteristic of S-wave coupling; the other below threshold hyperon, the $\Sigma(1385)$, has no such threshold distortion because its $N\bar{K}$ coupling is P-wave. For $\Lambda(1405)$ this asymmetry is the sole direct evidence that $J^P = 1/2^-$."

A recent measurement by the CLAS collaboration, MORIYA 14, definitively established the long-assumed $J^P = 1/2^-$ spin-parity assignment of the $\Lambda(1405)$. The experiment produced the $\Lambda(1405)$ spin-polarized in the photoproduction process $\gamma p \rightarrow K^+ \Lambda(1405)$ and measured the decay of the $\Lambda(1405)$ (polarized) $\rightarrow \Sigma^+$ (polarized) π^- . The observed isotropic decay of $\Lambda(1405)$ is consistent with spin $J = 1/2$. The polarization transfer to the Σ^+ (polarized) direction revealed negative parity, and thus established $J^P = 1/2^-$.

See the related review(s):
[Pole Structure of the \$\Lambda\(1405\)\$ Region](#)

$\Lambda(1405) : 1405.1_{-1.0}^{+1.3} \text{ MeV (Part. Listing in '22)}$

$J^P = \frac{1}{2}^-, I = 0, M_{\Lambda(1405)} < M_{K\bar{N}}$, lightest in neg. parity baryons

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Pole Structure of the Lambda(1405) Region

PDG Reviews: Ulf-G. Meissner and T. Hyodo (since Nov. 2015)

Table 1: Comparison of the pole positions of $\Lambda(1405)$ in the complex energy plane from next-to-leading order chiral unitary coupled-channel

Citation: R.L. Workman et al. (Particle Data Group), to be published (2022)

IARTA con-

$\Lambda(1380) 1/2^-$

$$J^P = \frac{1}{2}^-$$

Status: **

OMITTED FROM SUMMARY TABLE

See the related review on "Pole Structure of the $\Lambda(1405)$ Region."

$\Lambda(1380)$ POLE POSITION

REAL PART

VALUE (MeV)

DOCUMENT ID

TECN

••• We do not use the following data for averages, fits, limits, etc. •••

1325 ± 15	¹ MAI	15	DPWA
1330^{+4}_{-5}	² MAI	15	DPWA
1388 ± 9	GUO	13	DPWA
1381^{+18}_{-6}	IKEDA	12	DPWA

pole 2 [MeV]

$1381^{+18}_{-6} - i 81^{+19}_{-8}$
$1388^{+9}_{-9} - i 114^{+24}_{-25}$
$1330^{+4}_{-5} - i 56^{+17}_{-11}$
$1325^{+15}_{-15} - i 90^{+12}_{-18}$

$\Lambda(1405) : 1405.1^{+1.3}_{-1.0}$ MeV (Part. Listing in '22)

$J^P = \frac{1}{2}^-, I = 0, M_{\Lambda(1405)} < M_{\bar{K}N}$, lightest in neg. parity baryons

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in $K-p \rightarrow \pi\pi\Sigma$ w/ M-matrix

Pole
PDG R

O. Morimatsu and K. Yamada, RPC100, 025201(2019)

Region
(2015)

TABLE II. Pole positions of the T -matrix in the $\bar{K}N$ and $\pi\Sigma$ single-channel scatterings and the $\bar{K}N$ - $\pi\Sigma$ coupled channels without on-shell factorization, A and B , and with on-shell factorization, C .

	Single channel		Coupled channels	
	$\bar{K}N$	$\pi\Sigma$	$\bar{K}N$	$\bar{K}N$ - $\pi\Sigma$
A	1432 MeV	1388-179 <i>i</i> MeV	1434-7 <i>i</i> MeV	1418-160 <i>i</i> MeV
B	1425 MeV	1382-169 <i>i</i> MeV	1419-19 <i>i</i> MeV	1424-146 <i>i</i> MeV
C	1427 MeV	1388-96 <i>i</i> MeV	1432-17 <i>i</i> MeV	1398-73 <i>i</i> MeV

Refs. 11,12, NLO	1424 ⁺⁷ ₋₂₃	- <i>i</i> 26 ⁺³ ₋₁₄	1381 ⁺¹⁸ ₋₆	- <i>i</i> 81 ⁺¹⁹ ₋₈
Ref. 14, Fit II	1421 ⁺³ ₋₂	- <i>i</i> 19 ⁺⁸ ₋₅	1388 ⁺⁹ ₋₉	- <i>i</i> 114 ⁺²⁴ ₋₂₅
Ref. 15, solution #2	1434 ⁺² ₋₂	- <i>i</i> 10 ⁺² ₋₁	1330 ⁺⁴ ₋₅	- <i>i</i> 56 ⁺¹⁷ ₋₁₁
Ref. 15, solution #4	1429 ⁺⁸ ₋₇	- <i>i</i> 12 ⁺² ₋₃	1325 ⁺¹⁵ ₋₁₅	- <i>i</i> 90 ⁺¹² ₋₁₈

$\Lambda(1405) : 1405.1^{+1.3}_{-1.0}$ MeV (Part. Listing in '22)

$J^P = \frac{1}{2}^-$, $I = 0$, $M_{\Lambda(1405)} < M_{\bar{K}N}$, lightest in neg. parity baryons

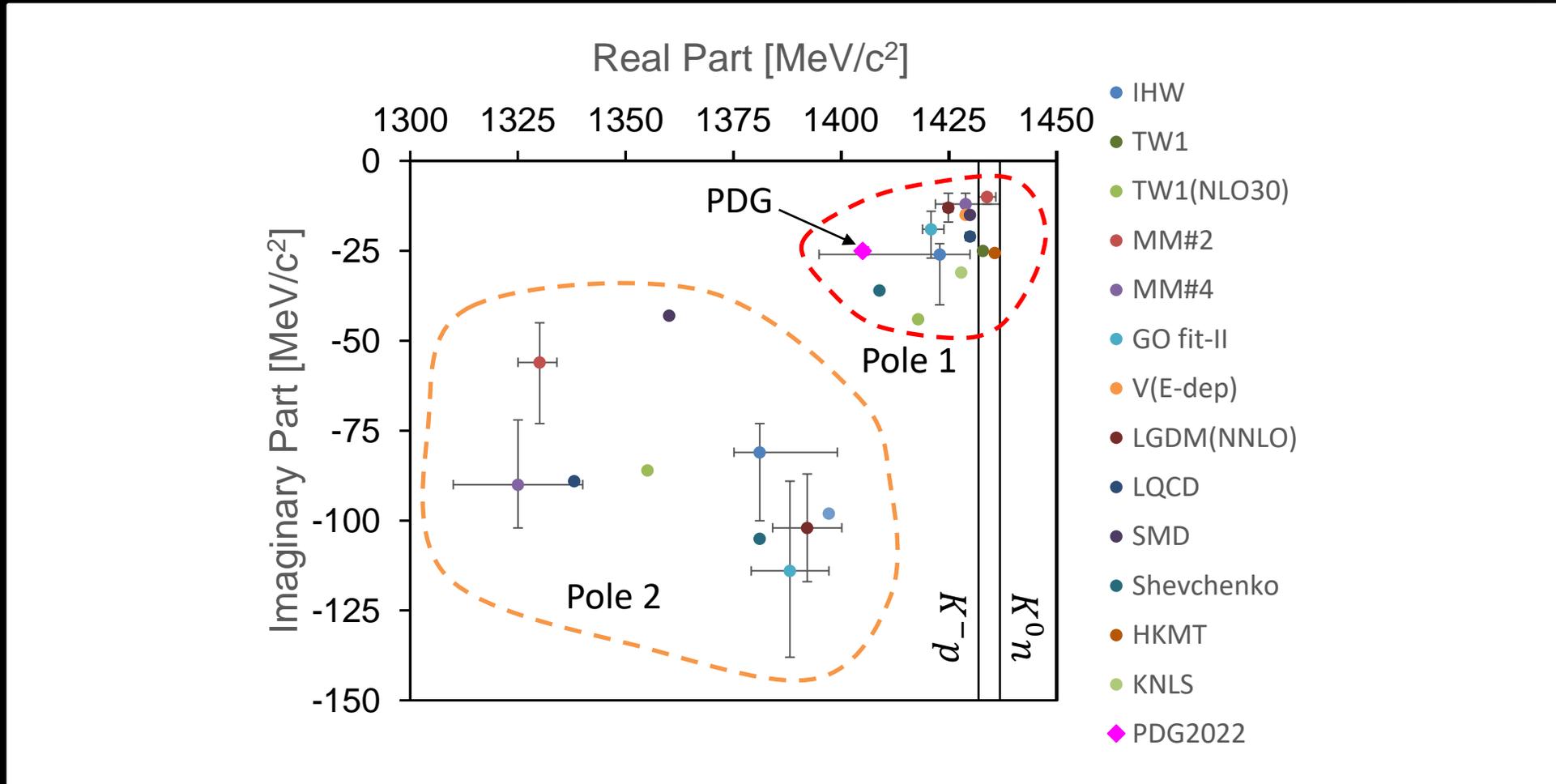
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Two-pole structure of $\Lambda(1405)$ in Meson-Baryon dynamics (theoretical analyses constraint by $\bar{K}N$ scat., Kaonic X-ray data, etc)

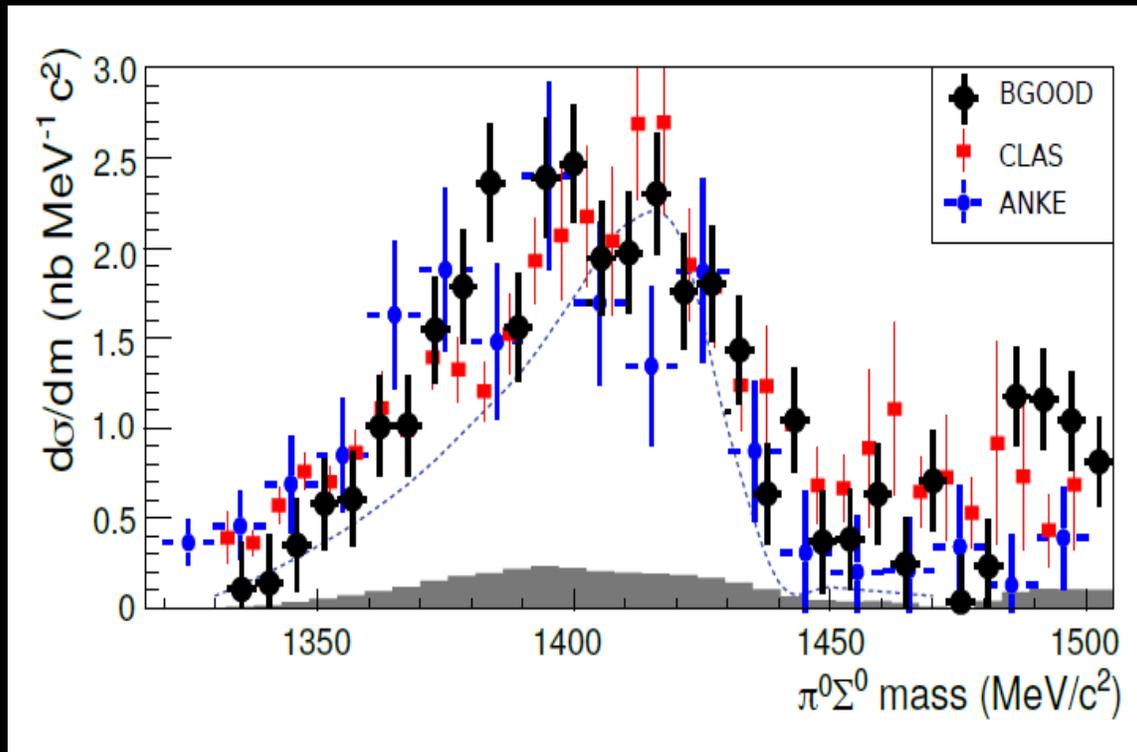
Recent Reviews: M.Mai, , *Eur. Phys. J. S T.* (2021) 230:15931607, T.Hyodo, M.Niiyama, *Prog. Part. Nucl. Phys.* 120(2021)103868



Need direct access to the $\bar{K}N$ Scat. Amp. and pole position.

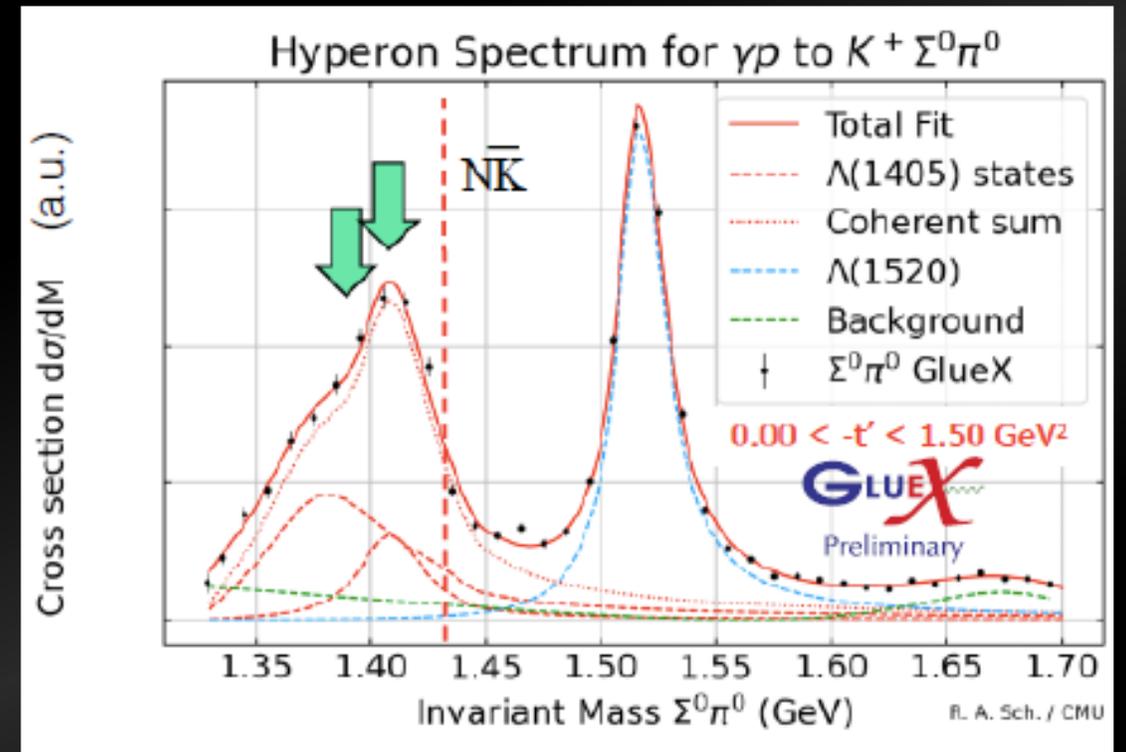
Recent two results on gamma-induced $\pi^0\Sigma^0$ spectra

$E_\gamma = 1.55 \sim 2.3$ GeV



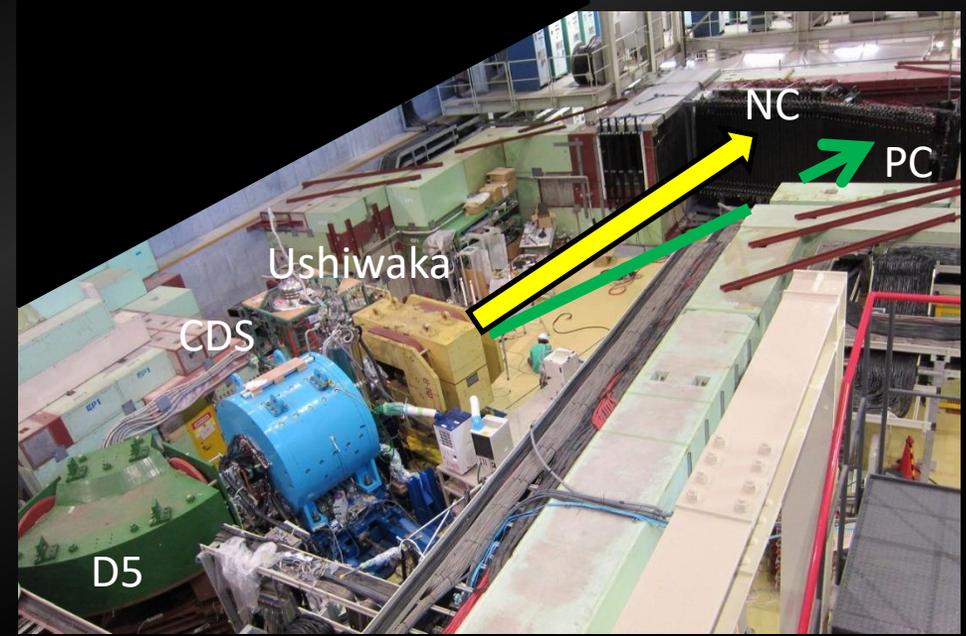
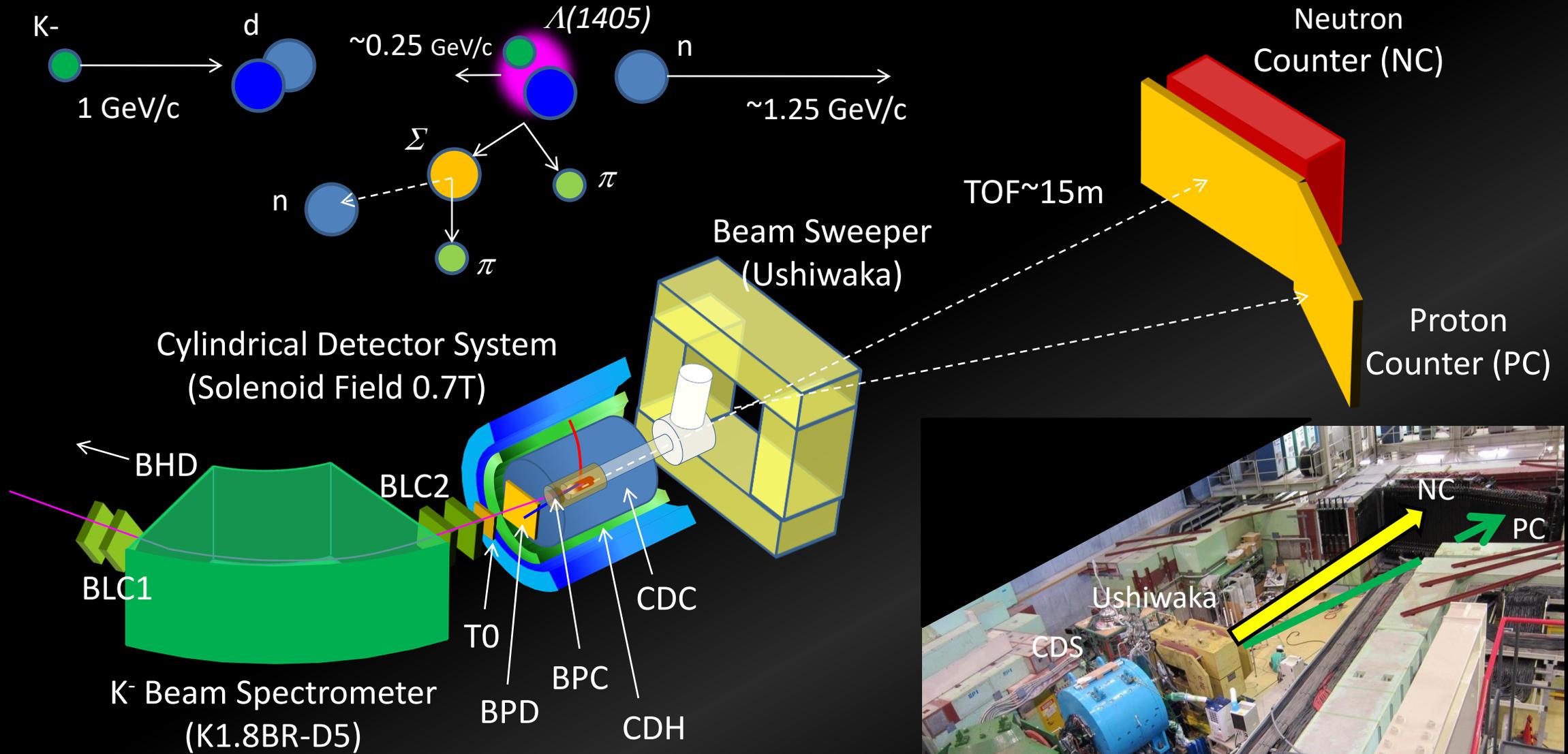
G. Scheluchin *et al.* [BGOOD collab.]
 Phys. Lett. B833 (2022) 137375

$E_\gamma = 6.5 \sim 11.6$ GeV

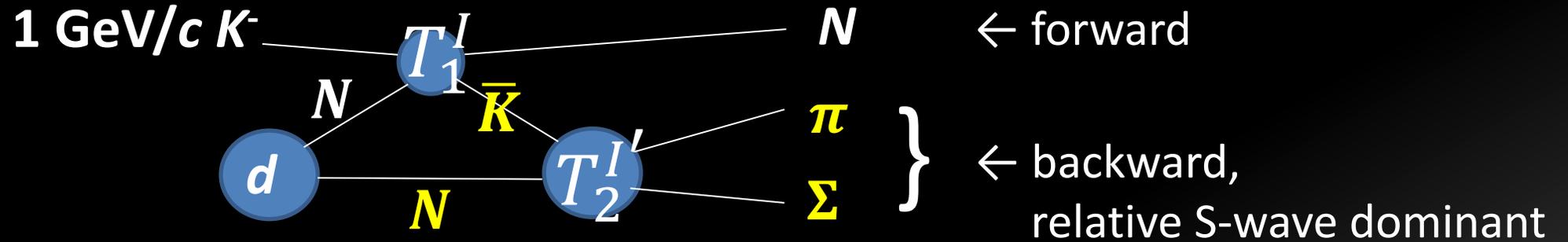


N. Wickramaarachchi for GlueX
 arXiv:2209.06230

Experimental Setup for E31 at J-PARC K1.8BR



$\bar{K}N$ scattering below the $\bar{K}N$ mass threshold (J-PARC E31)

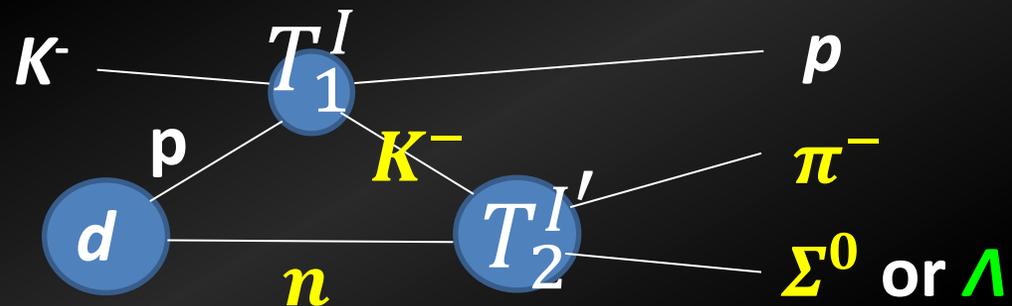
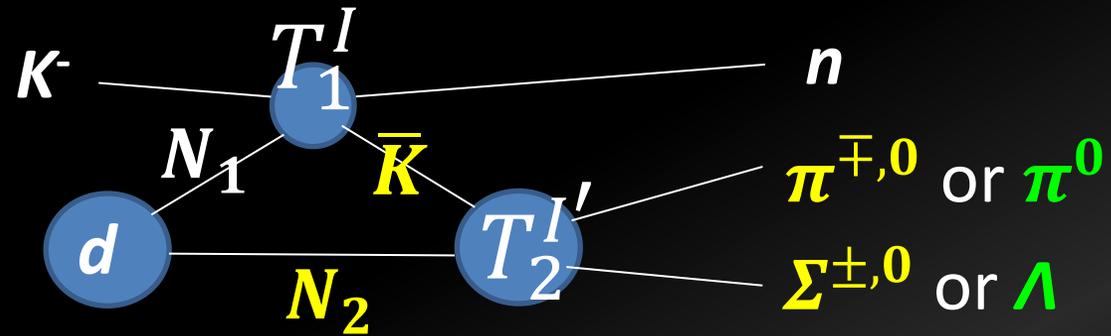


- measuring an **S-wave** $\bar{K}N \rightarrow \pi\Sigma$ scattering below the $\bar{K}N$ threshold in the $d(K^-, n)\pi\Sigma$ reactions at a forward angle of N .
- ID's all the final states to decompose the $l=0$ and 1 ampl's.

Fwd N	$\pi\Sigma$ mode	Isospin	Expected resonance
n	$\pi^\pm \Sigma^\mp$	0, 1	$\Lambda(1405)$ interference btw $l=0$ and 1 ampl's.
p	$\pi^- \Sigma^0$	1	P-wave $\Sigma^*(1385)$ to be suppressed
n	$\pi^0 \Sigma^0$	0	$\Lambda(1405)$

What we measured: missing $\pi\Sigma/\pi\Lambda$ mass spectra

- $d(K^-, n)X_{\pi^\pm\Sigma^\mp}$
- $d(K^-, n)X_{\pi^0\Sigma^0}$
- $d(K^-, p)X_{\pi^-\Sigma^0}$
- $d(K^-, p)X_{\pi^-\Lambda}$
- $d(K^-, n)X_{\pi^0\Lambda}$

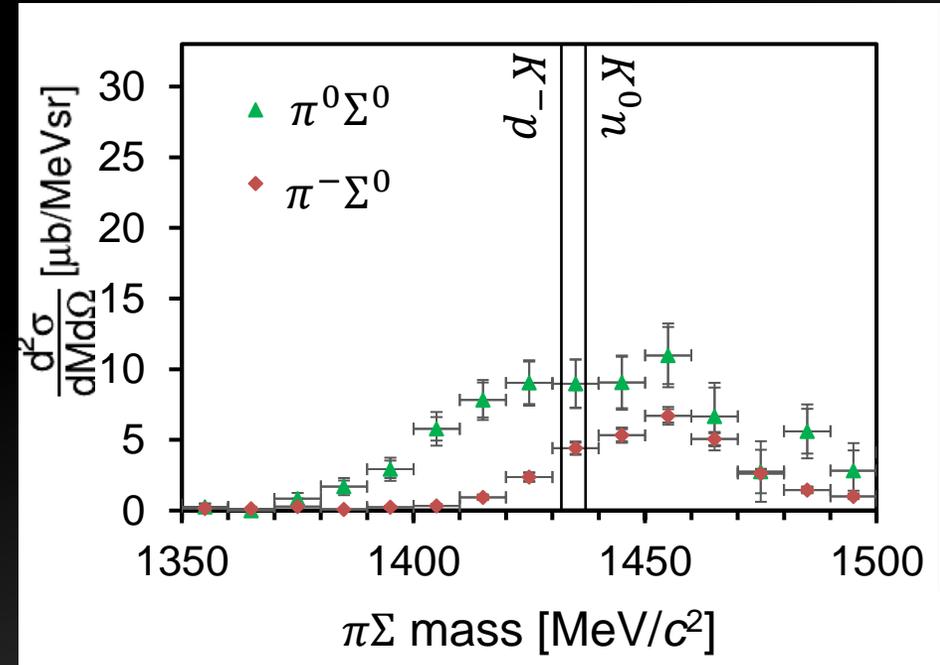
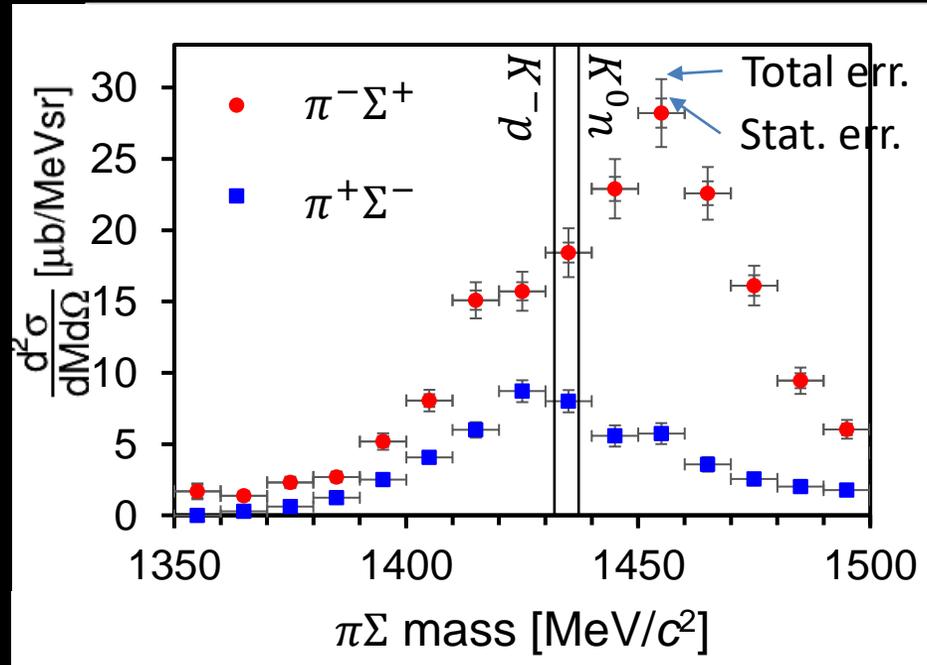


$$\pi^+\Sigma^- / \pi^-\Sigma^+$$

$$(I' = 0, 1)$$

$$\pi^0\Sigma^0 (I' = 0) \leftarrow \text{predominant}$$

$$\pi^-\Sigma^0 (I' = 1)$$



$$\frac{d\sigma}{d\Omega} (\pi^-\Sigma^+ / \pi^+\Sigma^-)$$

$$\propto \left| \frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} T_2^{I'=0} \pm \frac{T_1^{I=0} + T_1^{I=1}}{4\sqrt{2}} T_2^{I'=1} \right|^2$$

$$\frac{d\sigma}{d\Omega} (\pi^0\Sigma^0) \propto \left| -\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} T_2^{I'=0} \right|^2$$

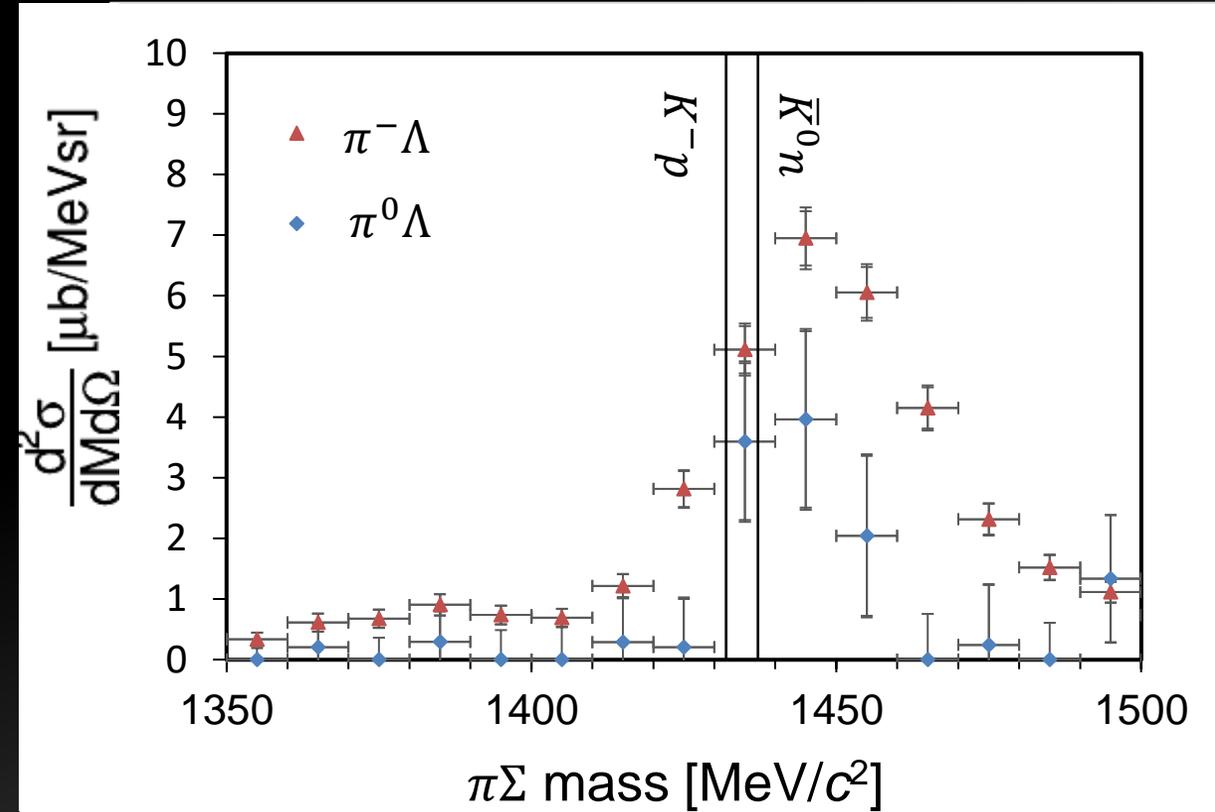
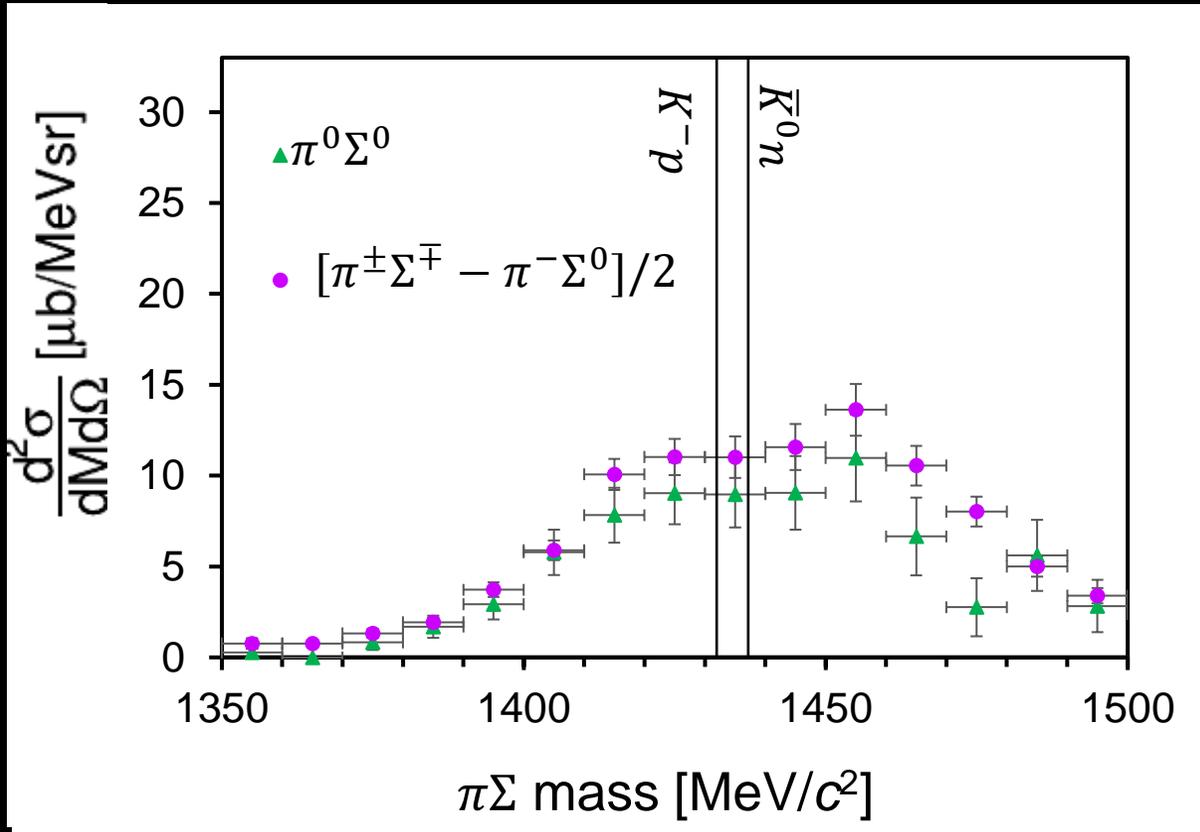
$$\frac{d\sigma}{d\Omega} (\pi^-\Sigma^0) \propto \left| -\frac{T_1^{I=0} + T_1^{I=1}}{4} T_2^{I'=1} \right|^2$$

Interference btw $I' = 0$ and 1 is clearly observed.

Isospin relations seem to be satisfied well.

$[\pi^\pm \Sigma^\mp - \pi^- \Sigma^0]/2$ vs $\pi^0 \Sigma^0$ ($I' = 0$)

$\pi^- \Lambda$ vs $\pi^0 \Lambda$ ($I' = 1$)



$$\frac{d\sigma}{d\Omega}([\pi^\pm \Sigma^\mp - \pi^- \Sigma^0]/2) \propto \left| -\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} T_2^{I'=0} \right|^2$$

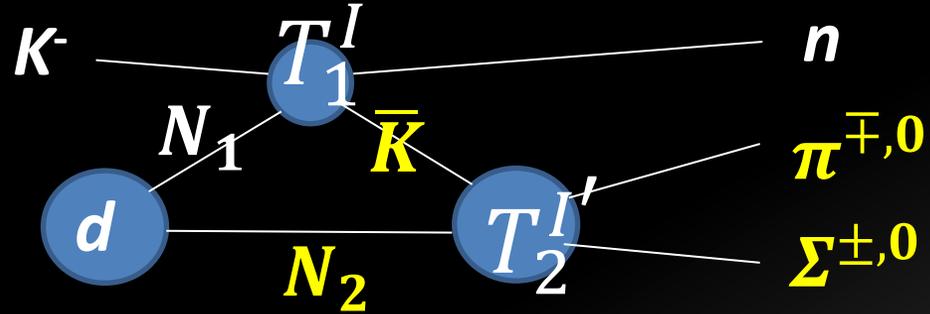
$$\frac{d\sigma}{d\Omega}(\pi^- \Lambda) \propto \left| \frac{T_1^{I=0} + T_1^{I=1}}{2\sqrt{2}} T_2^{I'=1} \right|^2$$

$$\approx \frac{d\sigma}{d\Omega}(\pi^0 \Sigma^0) \propto \left| -\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} T_2^{I'=0} \right|^2$$

$$\approx 2 \times \frac{d\sigma}{d\Omega}(\pi^0 \Lambda) \propto \left| -\frac{T_1^{I=0} + T_1^{I=1}}{4} T_2^{I'=1} \right|^2$$

Extracting Scattering Amplitude

- 2-step process



$$\frac{d\sigma}{dM_{\pi\Sigma}} \Big|_{\theta_n=3^\circ} \sim \left| \left\langle n\pi\Sigma \left| T_2^{I'} (\bar{K}N_2 \rightarrow \pi\Sigma) G_0 T_1^I (K^- N_1 \rightarrow \bar{K}n) \right| K^- \Phi_d \right\rangle \right|^2$$

$$\sim \left| T_2^{I'} (\bar{K}N \rightarrow \pi\Sigma) \right|^2 F_{\text{res}}(M_{\pi\Sigma})$$

Factorization Approximation

$$F_{\text{res}}(M_{\pi\Sigma}) \sim \left| \int_0^\infty dq_{N_2}^3 T_1^I \frac{1}{E_{\bar{K}} - E_{\bar{K}}(q_{\bar{K}}) + i\epsilon} \Phi_d(q_{N_2}) \right|^2, q_{\bar{K}} + q_{N_2} = q_{\pi\Sigma}$$

E31: Response Function, $F_{\text{res}}(M_{\pi\Sigma})$

- $F_{\text{res}}(M_{\pi\Sigma}) = \left| \int G_0(q_2, q_1) T_1 \Phi_d(q_2) d^3 q_2 \right|^2$
- $G_0(q_2, q_1) = \frac{1}{q_0^2 - q'^2 + i\varepsilon} f(q_0, q') \frac{\left(\sqrt{P_{\pi\Sigma}^2 + M_{\pi\Sigma}^2} + \sqrt{P_{\pi\Sigma}^2 + W(q')^2} \right)}{M_{\pi\Sigma} + W(q')}$,
- $f(q_0, q')^{-1} = [E_1(q_0) + E_1(q')]^{-1} + [E_2(q_0) + E_2(q')]^{-1}$

Miyagawa and Haidenbauer, PRC85, 065201(2012)

- $T_1: K^- n \rightarrow K^- n (I = 1), K^- p \rightarrow \bar{K}^0 n (I = 0, 1)$ amplitude,
Gopal et al., NPB119, 362(1977)

- $T_1(K^- n \rightarrow K^- n) = f(I = 1)$
- $T_1(K^- p \rightarrow \bar{K}^0 n) = [f(I = 1) - f(I = 0)]/2$

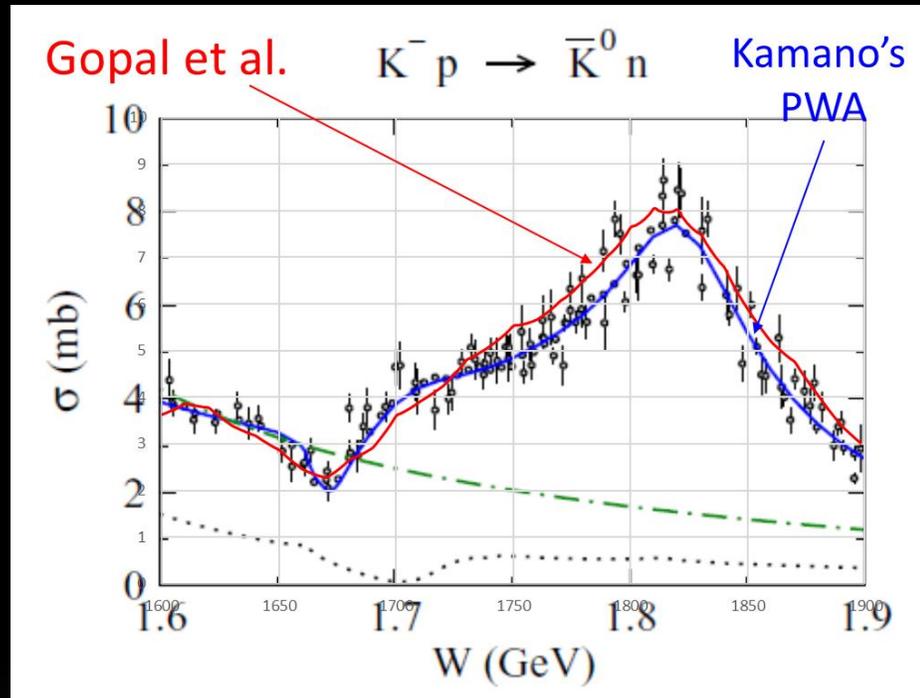
Off-shell treatment :See eq.(17) in PRC94, 065205

- $\Phi_d(q_2)$: deuteron wave function, PRC63, 024001(2001)

E31: Response Function, $F_{\text{res}}(M_{\pi\Sigma})$

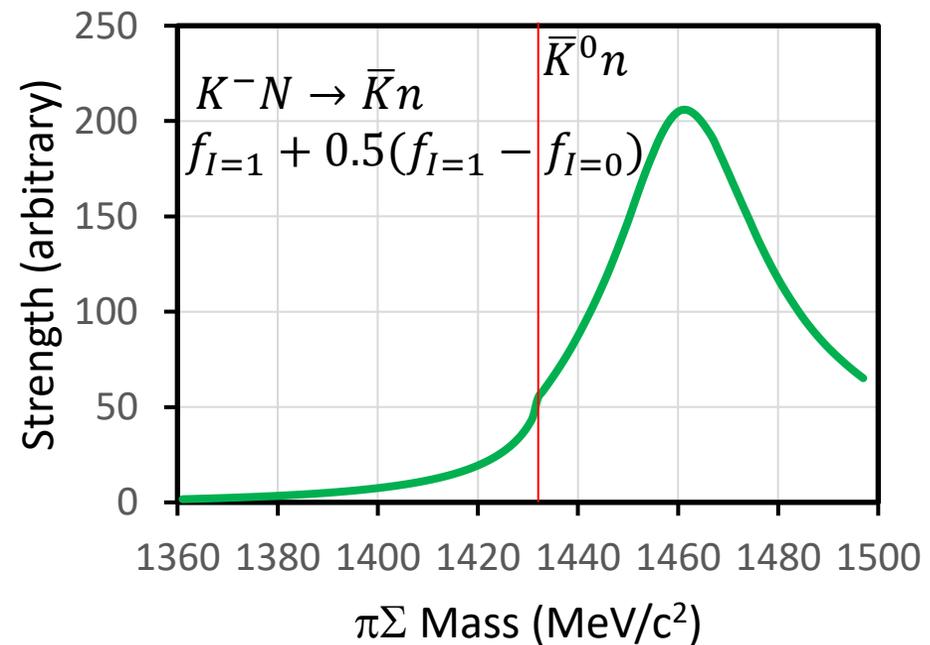
$$F_{\text{res}}(M_{\pi\Sigma}) \sim p_{\pi}^{cm} p_n^2 / |(E_{K^-} + m_d)\beta_n - p_{K^-} \cos \theta| \times \left| \int d\Omega_{\pi}^{cm} E_{\pi} E_{\Sigma} \left| \int q_2 T_1^I(p_{K^-}, q_N, p_n, q_{\bar{K}}, \cos \theta_{n\bar{K}}; M_{\pi\Sigma}) G_0(q_2, q_1) \Phi_d(q_2) d^3 q_2 \right|^2 \right.$$

Elementary Cross Section for T_1^I



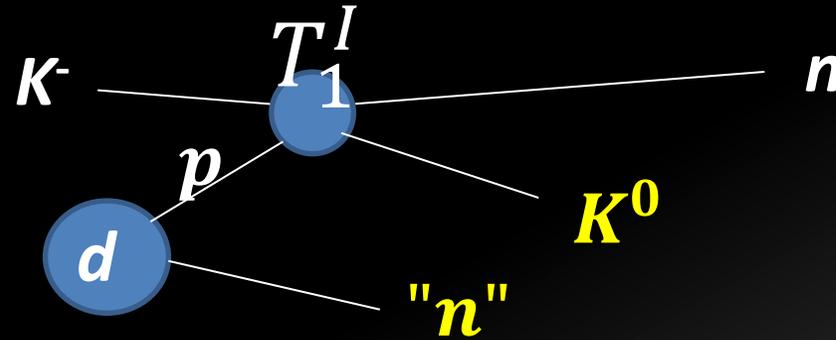
Gopal et al., NPB119, 362(1977)

$F_{\text{res}}(M_{\pi\Sigma})$



Demonstration of the T_1^I amplitude

- 1-step process



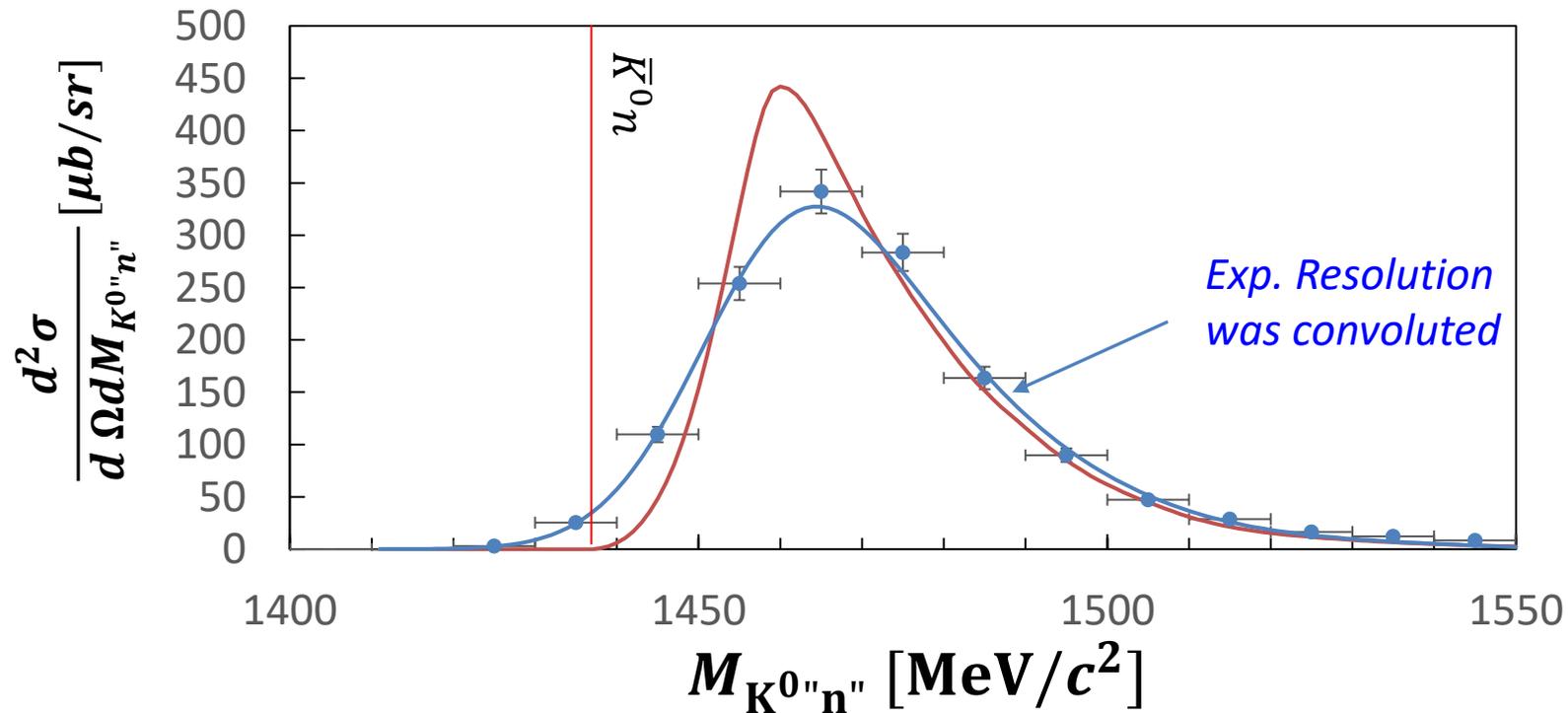
$$\frac{d\sigma}{dM_{\pi\Sigma}} \Big|_{\theta_n=3^\circ} \sim |\langle nK^0 n | T_1^I (K^- p \rightarrow \overline{K^0} n) | K^- \Phi_d \rangle|^2$$

$$\frac{d\sigma}{dM_{\pi\Sigma}} \sim \left| \int_0^\infty dq_{N_2}^3 T_1^I \delta(p_{K^-} + p_p - p_n - p_{K^0}) \Phi_d(q_{N_2}) \right|^2$$

Demonstration for fitting data with the 1-step

$K^- d \rightarrow n \bar{K}^0 n$ reaction calculation

- Data: $d(K^-, n) \bar{K}^0 n$ Ks/KL, BR(Ks- \rightarrow pi \pm) corrected (K. Inoue)



$\bar{K}N$ Scattering Amplitude

L. Lensniak, arXiv:0804.3479v1(2008)

- $T_2^{I'}(\bar{K}N \rightarrow \bar{K}N) = \frac{A}{1 - iAk_2 + \frac{1}{2}ARk_2^2}$

- $T_2^{I'}(\bar{K}N \rightarrow \pi\Sigma) = \frac{1}{\sqrt{k_1}} e^{i\delta_0} \frac{\sqrt{\text{Im}A - \frac{1}{2}|A|^2 \text{Im}Rk_2^2}}{1 - iAk_2 + \frac{1}{2}ARk_2^2}$

- $T_2^{I'}(\pi\Sigma \rightarrow \pi\Sigma)$

$$= \frac{e^{i\delta_0} \left(\sin \delta_0 + i \text{Im} \left(e^{-i\delta_0} A \right) k_2 - \frac{1}{2} \text{Im} \left(e^{-i\delta_0} AR \right) k_2^2 \right)}{k_1 \left(1 - iAk_2 + \frac{1}{2}ARk_2^2 \right)}$$

- 5 real number parameters (effective range expansion)

- A : scattering length, R : effective range, δ_0 : phase

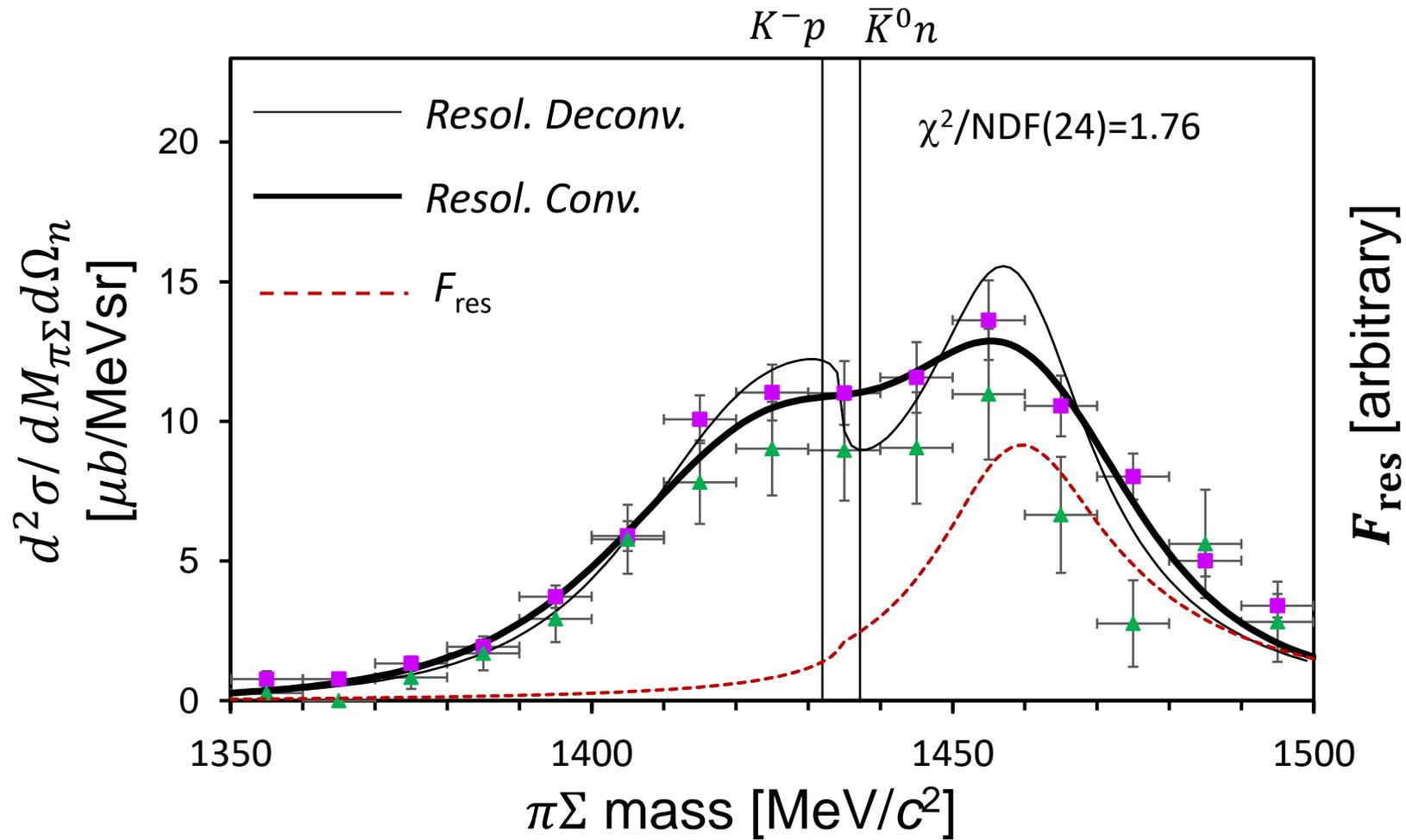
$$T_{11} = k_2 T_2^{I'}(\bar{K}N \rightarrow \bar{K}N),$$

$$T_{12} = \sqrt{k_1 k_2} T_2^{I'}(\bar{K}N \rightarrow \pi\Sigma),$$

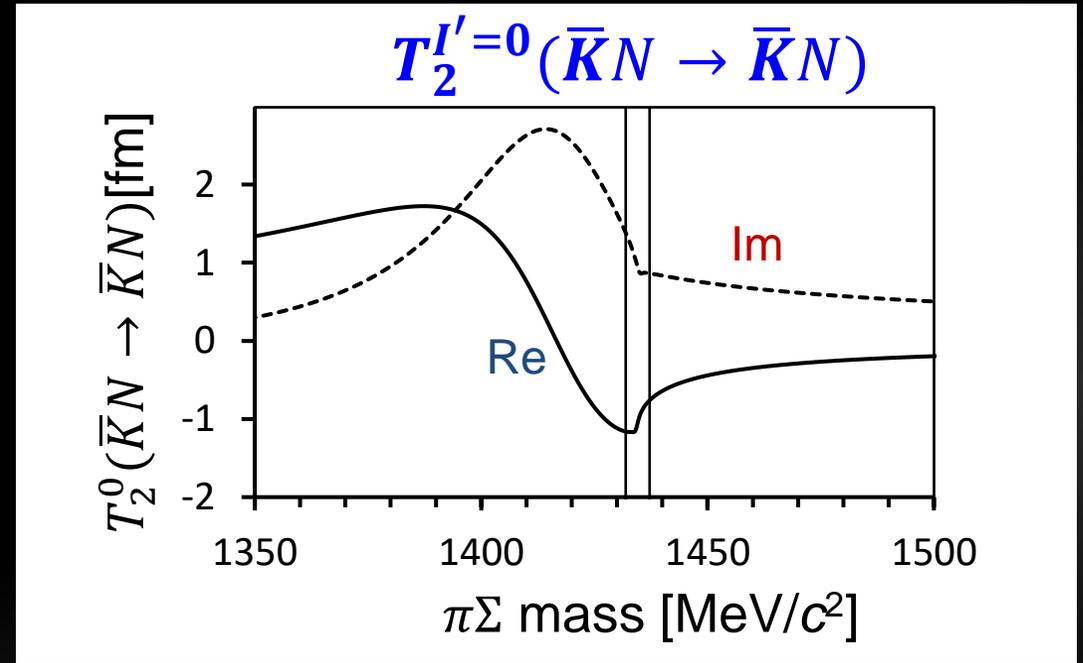
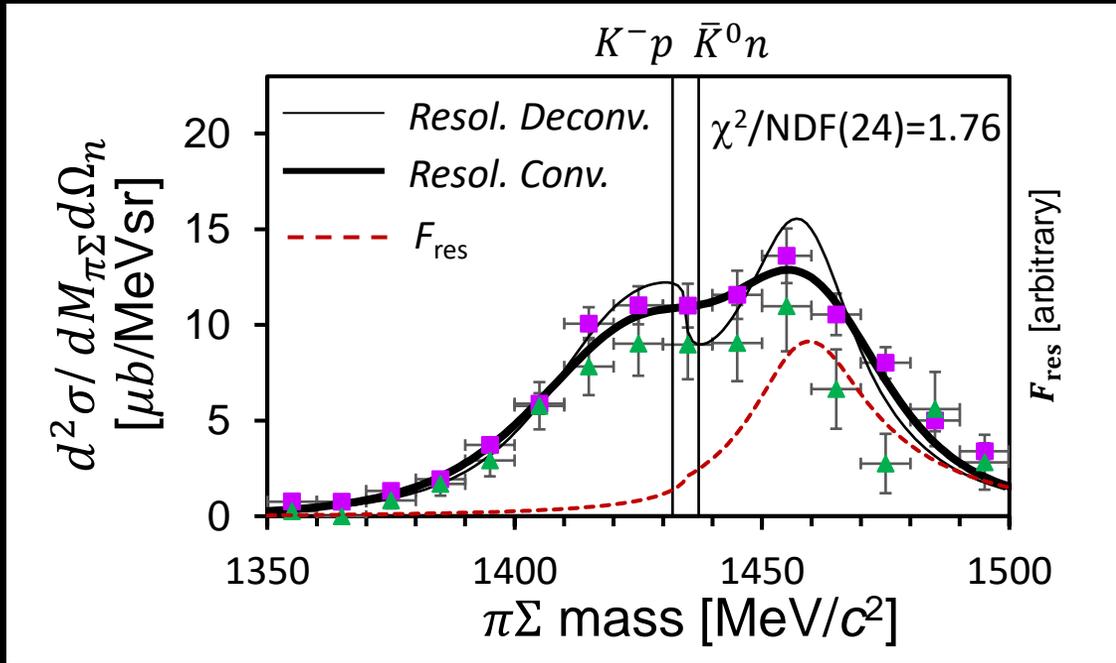
$$|T_{11}|^2 + |T_{12}|^2 = \text{Im}T_{11},$$

$$S = I + 2iT,$$

Fit the spectra to deduce $\bar{K}N$ scattering amplitude



Best fit $\bar{K}N$ scattering amplitude



A pole at $(1417.7_{-7.4}^{+6.0} + 1.1_{-1.0}^{+1.7}) + (-26.1_{-7.9}^{+6.0} - 2.0_{-2.0}^{+1.7})i$ MeV/ c^2

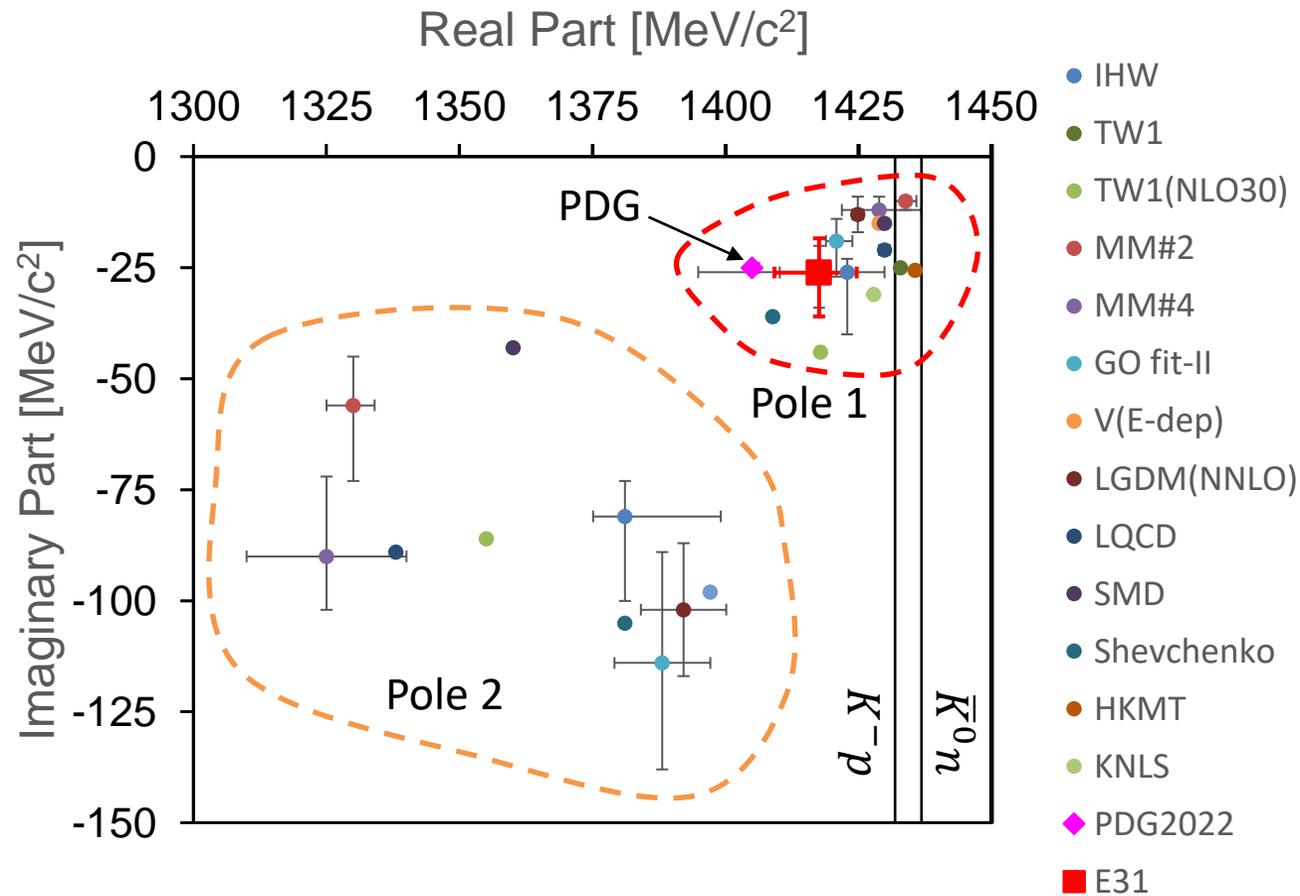
$$\left| T_2^{I'=0}(\bar{K}N \rightarrow \bar{K}N) \right|^2 / \left| T_2^{I'=0}(\bar{K}N \rightarrow \pi\Sigma) \right|^2 = 2.2_{-0.6}^{+1.0} + 0.3_{-0.3}^{+0.3}$$

$$\mathbf{A}^{I'=0} = (-1.12 \pm 0.11_{-0.07}^{+0.10}) + i(0.84 \pm 0.12_{-0.07}^{+0.08}) \text{ fm}$$

$$\mathbf{R}^{I'=0} = (-0.18 \pm 0.31_{-0.06}^{+0.08}) + i(0.41 \pm 0.13_{-0.09}^{+0.09}) \text{ fm}$$

*best fit value \pm fitting error \pm systematic error
systematic errors assuming the K^-p/K^0n mass threshold

Two-pole structure of $\Lambda(1405)$ in Meson-Baryon dynamics (theoretical analyses constraint by $\bar{K}N$ scat., Kaonic X-ray data, etc.)



E31 result → *Phys. Lett. B837(2023)137637*

Conclusion

- We measured the $\pi\Sigma$ mass spectra in the $K^-d \rightarrow N\pi\Sigma$ reactions, knocked-out N measured at ~ 0 degree.
 - well described with the **two-step reaction process**, $K^-N_1 \rightarrow N\bar{K}$, $\bar{K}N_2 \rightarrow \pi\Sigma$
 - **S-wave $\bar{K}N_2 \rightarrow \pi\Sigma$ scattering** is dominant.
 - No $\Sigma^*(1385)$ nor $\Lambda(1520)$ is observed: P/D-wave contributions seem to be negligible.
 - **Isospin relations** among the cross sections are well satisfied:
$$\frac{d\sigma}{d\Omega}([\pi^\pm\Sigma^\mp - \pi^-\Sigma^0]/2) = \frac{d\sigma}{d\Omega}(\pi^0\Sigma^0), l=0$$
$$\frac{d\sigma}{d\Omega}(\pi^-\Lambda) = 2 \times \frac{d\sigma}{d\Omega}(\pi^0\Lambda), l=1$$
- S-wave $\bar{K}N$ scattering amplitude ($l=0$) was deduced.
- We found a resonance pole at **1417.7 – 26.1i [MeV]**, which seems consistent to that of the so-called higher pole of $\Lambda(1405)$ suggested by the ChUM based calculations.
- The pole is likely to couple to the $K^{\text{bar}}N$ state.

Backup

Comparison w/ Recent Theoretical Works

	$A^{I=0}$ [fm]	Pole 1 [MeV]	Pole2 [MeV]	reference
This work	$-1.12 \pm 0.11_{-0.07}^{+0.10}$ $+i0.84 \pm 0.12_{-0.07}^{+0.08}$	$1417.7_{-7.4-1.0}^{+6.0+1.1}$ $-i26.1_{-6.0-1.7}^{+7.9+2.0}i$		
IHW	$-1.97 + i1.05$ $^{\$)}$	$1424_{-23}^{+7} - i26_{-14}^{+3}$	$1381_{-6}^{+18} - i81_{-8}^{+19}$	ChPT full NLO, $\bar{K}N$ scatt., SIDDHARTA data constraint
TW1 (NLO30)	$-1.61 + i1.02$ $^{\$)}$	$1433 - i25$ (1418 - i44)	$1371 - i54$ (1355 - i86)	ChPT LO(NLO), $\bar{K}N$ scatt., SIDDHARTA data constraint
MM#4 (#2)	$-1.81_{-0.28}^{+0.30}$ $+i0.92_{-0.23}^{+0.29}$ $^{\#)}$	$1429_{-7}^{+8} - i12_{-3}^{+2}$ (1434 $_{-2}^{+2} - i10_{-1}^{+2}$)	$1325_{-15}^{+15} - i90_{-18}^{+12}$ (1330 $_{-5}^{+4} - i56_{-11}^{+17}$)	ChPT NLO, $\bar{K}N$, CLAS and SIDDHARTA data constraint
GO Fit-II	$-1.79_{-0.14}^{+0.13}$ $+i1.36_{-0.19}^{+0.18}$	$1421_{-2}^{+3} - i19_{-5}^{+8}$	$1388_{-9}^{+9} - i114_{-25}^{+24}$	ChPT, $\bar{K}N$ scatt., SIDDHARTA data constraint
$V^{E\text{-dep}}$	$-1.89 + i1.11$ $^{\$)}$	$1429 - i15$	$1344 - i49$	ChPT LO, $\bar{K}N$ scatt., SIDDHARTA data constraint
LGDM	$-1.2 + 1.1i$ $^{\%)}$	$1425 \pm 1 - i(13 \pm 4)$	$1392 \pm 8 - i(102 \pm 15)$	ChPT NNLO, πN , $\bar{K}N$, SIDDHARTA, baryon resonances
LQCD	$-1.77 + i1.08$	$1430 - i21$	$1338 - i89$	Lattice QCD
Anisovich		$1421 \pm 3 - i(23 \pm 3)$		BnGa PWA fitted w/ CLAS, CB, $\bar{K}N$ scatt. Data, ...

IHW: Ikeda, Hyodo, Weise, NPA **881**(2012)98, $^{\$)}$ value found in PRC **97**(2019)055209

TW1: Cieply, Smejkal, NPA **881**(2012)115, $^{\$)}$ value found in PRC **97**(2019)055209

GO Fit-II: Guo, Oller, PRC **87**(2013)035202

MM#4: Mai, Meissner, EPJA **51**(2015)30, $^{\#)}$ NPA900(2013)51

$V^{E\text{-dep}}$: Ohnishi, Ikeda, Hyodo, Weise, PRC **93**(2016)025207, $^{\$)}$ value found in PRC **97**(2019)055209

LGDM: Lu, Geng, Doering, Mai, Phys. Rev. Lett. **130**(2023)071902, $^{\%)}$ value in private communication

LQCD: Liu, Wu, Leinweber, Thomas, PLB **808**(2020)135652, Liu, Hall, Leinweber, Thomas, Wu, PRD **95**(2017)014506

Anisovich et al., EPJA56(2020)139