Light-Front Hamiltonian Approach to Hadrons

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Dirac's forms of relativistic dynamics [Dirac, Rev. Mod. Phys. **21**, 392–1949] Instant form is the well-known form of dynamics starting with $x^0 = t = 0$ $K^i = M^{0i}$, $J^i = \frac{1}{2} \varepsilon^{ijk} M^{jk}$, $\varepsilon^{ijk} = (+1,-1,0)$ for (cyclic, anti-cyclic, repeated) indeces Front form defines relativistic dynamics on the light front (LF): $x^+ = x^0 + x^3 = t + z = 0$ $P^{\pm} \triangleq P^0 \pm P^3$, $\vec{P}^{\perp} \triangleq (P^1, P^2)$, $x^{\pm} \triangleq x^0 \pm x^3$, $\vec{x}^{\perp} \triangleq (x^1, x^2)$, $E^i = M^{+i}$.

E^+	$= I \pm I$ $= M^{+-},$	$F^i =$	M^{-i}	, 1), 1	— <i>x</i>	-(x,x), L	— <i>IVI</i>	,



Adapted from talk by Yang Li

Light Front (LF) Hamiltonian Defined by its Elementary Vertices in LF Gauge



Discretized Light Cone Quantization [H.C. Pauli & S.J. Brodsky, PRD32 (1985)] **Basis Light Front Quantization** [J.P. Vary, et al., PRC81 (2010)] $\phi\left(\vec{k}_{\perp},x\right) = \sum \left[f_{\alpha}\left(\vec{k}_{\perp},x\right)a_{\alpha} + f_{\alpha}^{*}\left(\vec{k}_{\perp},x\right)a_{\alpha}^{\dagger}\right]$ where $\{a_{\alpha}\}$ satisfy usual (anti-) commutation rules. Furthermore, $f_{\alpha}(\vec{x})$ are arbitrary except for conditions: Orthonormal: $\int f_{\alpha}\left(\vec{k}_{\perp},x\right)f_{\alpha'}^{*}\left(\vec{k}_{\perp},x\right)\frac{d^{2}k_{\perp}dx}{(2\pi)^{3}2x(1-x)} = \delta_{\alpha\alpha'}$ Complete: $\sum_{\alpha} f_{\alpha}\left(\vec{k}_{\perp}, x\right) f_{\alpha}^{*}\left(\vec{k}_{\perp}', x'\right) = 16\pi^{3}\sqrt{x(1-x)}\delta^{2}\left(\vec{k}_{\perp}-\vec{k}_{\perp}'\right)\delta\left(x-x'\right)$

For mesons we adopt (later extended to baryons): [Y. Li, et al., PLB758 (2016)]

$$f_{\alpha = \{nml\}}(\vec{k}_{\perp}, x) = \phi_{nm}(\vec{k}_{\perp}/\sqrt{x(1-x)})\chi_{l}(x)$$

$$\phi_{nm} \text{ 2D-HO functions as in AdS/QCD}$$

$$\chi_{l} \text{ Jacobi polynomials times } x^{a}(1-x)^{b}$$

BLFQ Symmetries & Constraints

Baryon number

Charge

Angular momentum projection (M-scheme)

Longitudinal momentum (Bjorken sum rule)

Longitudinal mode regulator (Jacobi)

Transverse mode regulator (2D HO)

"Internal coordinates"
$$\vec{k}_{i\perp} = \vec{p}_{i\perp} - x_i \vec{P}_{\perp} \implies \sum_{i\perp} \vec{P}_{i\perp}$$

 $H \to H + \lambda H_{CM} \longleftarrow$

Global Color Singlets (QCD)

Light Front Gauge

Optional Fock-Space Truncation

All $J \ge J_z$ states $\sum b_i = B$ in one calculation $\sum_{i} q_i = Q$ $\sum (m_i + s_i) = J_z$ Finite basis $\sum x_i = \sum$ regulators $\sum_{i} (2n_i + |m_i| + 1)$ $\hat{k}_{i+} = 0$ Preserve transverse i boost invariance

Light-Front Wavefunctions (LFWFs) $|\psi_h(P, j, \lambda)\rangle = \sum_n \int [d\mu_n] \psi_{n/h}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n) |\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\rangle$

LFWFs are *frame-independent* (boost invariant) and depend only on the relative variables: $x_i \equiv p_i^+/P^+$, $\vec{k}_{i\perp} \equiv \vec{p}_{i\perp} - x_i \vec{P}_{\perp}$

LFWFs provide intrinsic information of the structure of hadrons, and are indispensable for exclusive processes in DIS [Lepage '80]

- ► Overlap of LFWFs: structure functions (e.g. PDFs), form factors, ...
- Integrating out LFWFs: light-cone distributions (e.g. DAs)

"Hadron Physics without LFWFs is like Biology without DNA!"



Basis Light-Front Quantization (BLFQ) Positronium in QED at Strong Coupling (α = 0.3) Systematic removal of regulators (b = HO momentum scale)



P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D 91, 105009 (2015)

Positronium in QED at Strong Coupling Covariant Basis Light-Front Quantization (BLFQ)



P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D 91, 105009 (2015)

Positronium with one dynamical photon: Light-front QED Hamiltonian

- QED Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^{\mu}D_{\mu} m_e)\Psi$
- Light-front QED Hamiltonian from standard Legendre transformation

$$P^{-} = \int d^{2}x^{\perp}dx^{-} F^{\mu+}\partial_{+}A_{\mu} + i\bar{\Psi}\gamma^{+}\partial_{+}\Psi - \mathcal{L} \qquad \text{Light-cone gauge: } (A^{+} = 0)$$

$$= \int d^{2}x^{\perp}dx^{-} \frac{1}{2}\bar{\Psi}\gamma^{+}\frac{m_{e}^{2} + (i\partial^{\perp})^{2}}{i\partial^{+}}\Psi + \frac{1}{2}A^{j}(i\partial^{\perp})^{2}A^{j} \qquad \text{kinetic energy terms}$$

$$+ ej^{\mu}A_{\mu} + \frac{e^{2}}{2}j^{+}\frac{1}{(i\partial^{+})^{2}}j^{+} \qquad \text{kinetic energy terms}$$

$$+ ej^{\mu}A_{\mu} + \frac{e^{2}}{2}j^{+}\frac{1}{(i\partial^{+})^{2}}j^{+} \qquad \text{instantaneous}$$

$$photon \qquad \text{interaction} \qquad photon \qquad \text{interaction}$$

Positronium with one dynamical photon: Interaction Part Of Hamiltonian



Kaiyu Fu, et al., in preparation

Mass Renormalization

- Mass counterterm $\Delta_m = m_{bare} - m_{phys}$ is needed for fermion self-energy correction



- Mass renormalization needs to be performed on single physical electron
 - Prediction power on positronium mass
- Mass counterterm is determined by fitting single electron mass
 - Complication: Δ_m depends on UV cutoff and thus is basis dependent.
 - An extension of sector-dependent renormalization is [Karmanov et al, 2008] needed: $\Delta_m(N_{max}, K)$



[Kaiyu Fu et al, in preparation]

vs.

• Mass counterterm is at higher order: $\Delta_m \propto \alpha m E_B \propto \alpha^2 m$

Here at $\alpha = 1/137$

Basis Scale and Rotational Symmetry

• Adjust the 2d harmonic oscillator basis scale parameter b to minimize the energy difference within the triplet 1^3S_1



• Maintaining rotational symmetry leads to a corresponding UV cutoff



[Kaiyu Fu et al, in preparation]

Wave Functions for S-Wave States



- Wave functions in $|e^+e^-\rangle$ Fock sector, dominant and non-dominant helicity component
- Nodal structure visible in non-dominant helicity component

[Kaiyu Fu et al, in preparation]

PDFs of the electron and photon



- $|e^+e^-\rangle$ Fock sector carries 99.1% probability.
- The peak of photon PDF is at small x region.

[Kaiyu Fu et al, in preparation]

Overview of BLFQ/tBLFQ applications to mesons and baryons

Common features

Transverse confinement from 2D HO (in common with LF Holography) Longitudinal confinement (Y. Li, et al, PLB 2016, PRD 2017) Basis states from exact solutions of a reference Hamiltonian Compare results with experiment, lattice, DSE/BSE, ...

Distinct features

For Veff

1) perturbative one-gluon exchange (Y. Li, et al, PLB 2016, PRD 2017)

2) NJL model for light meson applications (S. Jia, et al, PRC 2019)

For Fock space truncation

1) Valence sector

2) Valence sector plus dynamical gluon (plus sea quarks, plus ...)

For observables

- 1) Single state properties and decays
- 2) Transitions between states
- 3) Non-perturbative probes (tBLFQ)

Next Methods

BLFQ on Quantum Computers

(Work by Meijian Li, et al)

(Work by Wenyang Qian, et al)

Heavy Quarkonia [Y.Li,PLB758,2016; PRD96,2017]

• Effective Hamiltonian in the $q\overline{q}$ sector



where
$$x = p_q^+ / P^+$$
, $\vec{k}_{\perp} = \vec{k}_{q\perp} = \vec{p}_{q\perp} - x\vec{P}_{\perp} = -\vec{k}_{\overline{q}\perp} = -(\vec{p}_{\overline{q}\perp} - (1-x)\vec{P}_{\perp})$, $\vec{r}_{\perp} = \vec{r}_{q\perp} - \vec{r}_{\overline{q}\perp}$

Confinement

transverse holographic confinement [S.J.Brodsky, PR584, 2015] longitudinal confinement [Y.Li, PLB758, 2016]

• One-gluon exchange with running coupling (Q^2)

$$V_g = -\frac{4}{3} \frac{4\pi\alpha_s(Q^2)}{Q^2} \bar{u}_{\sigma'} \gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$$

- Basis representation
 - valence Fock sector: $|q \bar{q}
 angle$
 - basis functions: eigenfunctions of H₀ (LF kinetic energy+ confinement)

Spectroscopy

[Y. Li, et al., Phys. Letts. B 758, 118 (2016); Phys. Rev. D 96, 016022 (2017)]



Yang Li, Meijian Li and James P. Vary, PRD 105, L071901 (2022)

and dilepton/diphoton widths combined

Diphoton width $\Gamma_{\gamma\gamma}$ of charmonia in BLFQ



NRQCD: Feng '15 & '17 NRQM: Babiarz '19 & '20 Transition form factor: η_c

$$\mathcal{M}^{\mu\nu} = 4\pi\alpha_{em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2, q_2^2)$$

- ✓ Diphoton width $\Gamma_{\gamma\gamma} = \frac{\pi}{4} \alpha_{em}^2 M_P^3 |F_{P\gamma\gamma}(0,0)|^2$
- ✓ Single-tag TFF $F_{P\gamma}(Q^2) = F_{P\gamma\gamma}(Q^2, 0) = F_{P\gamma\gamma}(0, Q^2)$

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_c} \int \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2k_{\perp}}{(2\pi)^3} \frac{\psi_{\uparrow\downarrow-\downarrow\uparrow}(x,k_{\perp})}{k_{\perp}^2 + m_f^2 + x(1-x)Q^2}$$

 q_2

 q_1



- BABAR data: described by monopole form with pole mass $\Lambda^2 = 8.5 \pm 0.6 \pm 0.7 \text{ GeV}^2$, and width 5.12 (53) keV
- BLFQ: using N_{max}=8 wave function corresponding to $\mu \approx 2m_c$. Basis sensitivity band is taken as the difference between the N_{max}=8,16 results.
- BLFQ/DA: prediction using the LCDA obtained from the LFWF
- Theoretical prediction in good agreement with both the width and the form factor.

Yang Li, Meijian Li and James P. Vary, PRD 105, L071901 (2022)

Lepage '80, Feldman '97, Babiarz, '19



Light Meson Mass Spectrum Including One Dynamical Gluon

[Lan, et al., (BLFQ Collaboration) PLB 825, 136890 (2022); arXiv 2106.04954]



BLFQ Basis States

BLFQ basis: expansion in Fock space

 $|\beta_{\text{meson}}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |gg\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}gg\rangle + |q\bar{q}q\bar{q}gg\rangle + |q\bar{q}q\bar{q}gg\rangle + \cdots$ $|\beta_{\text{barvon}}\rangle = |qqq\rangle + |qqqgg\rangle + |qqqq\bar{q}g\rangle + |qqqq\bar{q}gg\rangle + |qqqq\bar{q}gg\rangle + \cdots$

 $|\beta_{\text{deuterium}}\rangle = |qqq qqq\rangle + |qqq qqq g\rangle + |qqq qqq q\overline{q}\rangle + |qqq qqq g\overline{q}\rangle + \cdots$

Dimension of basis states increases with number of Fock sectors => motivation for quantum computing



Baryons with one dynamical gluon

$$|P_{baryon}\rangle = \Psi_{1}|qqq\rangle + \Psi_{2}|qqqg\rangle$$

$$P^{-} = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

$$H_{K.E.} = \sum_{i} \frac{p_{i}^{2} + m_{q}^{2}}{p_{i}^{+}}$$

 $H_{trans} \sim \kappa_T^4 r^2$ -- Brodsky, Teramond arXiv: 1203.4025

$$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$$
 ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$$H_{Interact} = H_{Vertex} + H_{inst} = g\overline{\psi} \gamma^{\mu} T^{a} \psi A^{a}_{\mu} + \frac{g^{2}C_{F}}{2} j^{+} \frac{1}{(i\partial^{+})^{2}} j^{+}$$



Unpolarized Parton Distribution Functions



The data are extracted from MARATHON data

Including the One Dynamical Gluon Fock Sector, the gluon distribution is closer to the global fit.

S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

[EPJC 77 (2017) 663]

Nucleon Spin with BLFQ



Helicity Parton Distribution Functions



• $\Delta G = \int_0^1 dx \Delta g(x) = 0.131 \pm 0.003$, is sizeable to the proton spin.

• PHENIX Collaboration: $\Delta G^{[0.02,0.3]} = 0.2 \pm 0.1$ [PRL 103 (2009) 012003

The sea quarks' contributions come from the DGLAP evolution

S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

N. Sato et al. [JAM], PRD93 (2016); E. R. Nocera et al. [NNPDF], NPB 887 (2014).

3-Dimension Structure of Nucleon



Orbital angular momentum distributions



Canonical: $\ell_d = -0.0114 \pm 0.0004$ $\ell_u = 0.0327 \pm 0.0013$ $\ell_g = -0.0065 \pm 0.0005$

At the LC gauge : $\frac{1}{2}\Delta\Sigma = 0.359 \pm 0.002$ $\Delta G = 0.131 \pm 0.003$

[In preparation, Siqi Xu, C. Mondal et.al]

Light-Front QCD Hamiltonian

 $|P_{baryon}\rangle = \Psi_1 |qqq\rangle + \Psi_2 |qqqg\rangle + \Psi_{31} |qqq u\bar{u}\rangle + \Psi_{32} |qqq d\bar{d}\rangle + \Psi_{33} |qqq s\bar{s}\rangle$



Parton Distribution Function



Now return to applications to baryons solved in the qqq + qqqg sectors

Transverse-momentum dependent distribution



 Within the Basis Light-front Quantization (BLFQ) we expand proton to |qqq> + |qqqg> Fock sector, obtain the corresponding LFWFs and calculate T-even TMDs of gluon and quark

Zhi Hu, et al., in preparation



- Define $\langle |p^{\perp}|^n \rangle_{f_1}^q = \int d^2 p^{\perp} |p^{\perp}|^n \times f_1^q$ then we know that $\frac{\langle |p^{\perp}|^2 \rangle_{f_1}^q}{\langle |p^{\perp}|^0 \rangle_{f_1}^q}$ would be the average transverse momentum of flavor q
- Average transverse momentum of d quark is slightly larger than that of u, the same as our $|qqq\rangle$ Fock sector conclusion.
- With |qqqg> Fock sector we now also know that transverse momentum of gluon is larger than that of quark
 Zhi Hu, et al., in preparation

All-charm tetraquark using BLFQ

• New issues compared with mesons and baryons

- Cluster decomposition principle for interactions
- Identical particles issue
- More than one color singlet
- Hamiltonian
 - Transverse confining potential like in AdS/QCD
 - Longitudinal confining potential (Głazek et al. PLB773, 172-178 (2017), different than BLFQ₀)
 - One-gluon-exchange spin-dependent potential (Wiecki et al.)
- Problem with negative M^2 solved by ad hoc modification of the Hamiltonian (which breaks cluster decomposition principle)



Kamil Serafin, et al., Phys Rev. D 105, 094028 (2022)

Forward quark jet-nucleus scattering in a light-front Hamiltonian approach

Time-dependent Basis Light-Front Quantization (tBLFQ)

First-principles:

In the light-front Hamiltonian formalism, the state obeys the time-evolution equation, and the Hamiltonian is derived from the QCD Lagrangian

$$\frac{1}{2}P^{-}(x^{+})|\psi(x^{+})\rangle = i\frac{\partial}{\partial x^{+}}|\psi(x^{+})\rangle$$

Nonperturbative treatment:

The time evolution operator is divided into many small timesteps, each timestep is evaluated numerically and intermediate states are accessible,

$$\begin{split} |\psi(x^{+})\rangle &= \mathcal{T}_{+} \exp\left[-\frac{i}{2} \int_{0}^{x^{+}} dz^{+} P^{-}(z^{+})\right] |\psi(0)\rangle \\ &= \lim_{n \to \infty} \prod_{k=1}^{n} \mathcal{T}_{+} \exp\left[-\frac{i}{2} \int_{x^{+}_{k-1}}^{x^{+}_{k}} dz^{+} P^{-}(z^{+})\right] |\psi(0)\rangle \end{split}$$

Basis representation:

Optimal basis has the same symmetries of the system, and it is the key to numerical efficiency We consider scattering of a high-energy quark moving in the positive z direction, on a high-energy nucleus moving in the negative z direction.

Time evolution of a quark state in the $|q\rangle + |qg\rangle$ Fock space observed from the transverse momentum space

nucleus A^µ



M. Li, T. Lappi and X. Zhao, Phys. Rev. D 104, 056104 (2021)

Wenyang Qian, et al

Quantum Simulation of QFT in the Front Form

2002.04016, 2105.10941, 2011.13443, 2009.07885

NIS	Q	Resour	rce requireme	ents Fault-tolera	Fault-tolerant,				
benchma	rking	Low		High ab initio					
VE	E	Adi	abetic	Scattering					
spectroscopy									
-	O(TT)	' 0		LF QFT features	Advantages for QC				
$\Omega_{\text{Direct}} =$	$= O(K \operatorname{loc})$	$\log K$)	Resources	No ghost fields	Low qubit count				
ຳ –	$-O(\sqrt{K})$	$\log K$		Linear EoM					
$\mathcal{L}_{\mathrm{Compact}} =$	$= O(\sqrt{N})$	$\log K$		LF momentum > 0	Efficient encoding				
	Trotter	Oracle	Evolution	Sparse Hamiltonians	Using sparsity-based methods				
Direct	\checkmark	✓		LF wavefunction \rightarrow					
Compact	Compact X			\rightarrow static quantities;	Simple form of				
·			Measurement	Simple form of operators	measurement				
		`		in the second-quantized	operators				
$O_F x,i\rangle$	= x, x	$\left. y_{i} ight angle ,$		formalism					
$O_{II} r u$	$0 \rangle - r $	H	Othor	Trivial vacuum, fewer cut-offs, no fermion doubling,					
$\circ_{H x}, g,$	$\cup / - x $	$y, \mathbf{r}_{xy/}$	Other	form invariance of H					

Light front approach to hadrons on quantum computers

- Quantum computers: New tool to simulate many-body quantum system.
 (quantum mechanical nature and high scalability)
- In the Noisy Intermediate-Scale Quantum (NISQ) era, the Variational Quantum Eigensolver (VQE) and Subspace-search VQE (SSVQE) approaches are promising tools to solve nuclear physics problems.
- Advantages of light front Hamiltonian formalism are directly applicable
- We first formulate the problem on the light front and then map the Hamiltonian to qubits (quantum bits)



Wenyang Qian, et al. Phys. Rev. Research 4, 043193 (2022)

Formulating the problem on qubits



- Basis representation (BLFQ) is key to represent the Hamiltonian on qubits.
- Small-size Hamiltonians (4-by-4 and 16-by-16) are used.
- Direct encoding and compact encoding are compared.

[Seeley, 1208.5986] [Kreshchuk, 2002.04016]

	$N_{\rm f}$	$\alpha_{ m s}(0)$	$\kappa \ ({\rm MeV})$	$m_q \; ({\rm MeV})$	N_{\max}	L_{\max}	Matrix dimension
$H_{ m eff}^{(1,1)}$	3	0.89	560 ± 10	300 ± 10	1	1	4 by 4
$H_{\mathrm{eff}}^{(4,1)}$					4	1	16 by 16

$$\begin{split} H_{\rm direct}^{(1,1)} &= 2269462\,{\rm IIII} - 284243\,({\rm ZIII} + {\rm IIZI}) \\ &- 850488\,({\rm IZII} + {\rm IIIZ}) + 12714\,({\rm XZXI} + {\rm YZYI}) \\ &- 7883\,({\rm IXZX} + {\rm IYZY}), \end{split} \qquad \begin{aligned} H_{\rm compact}^{(1,1)} &= 1134731\,{\rm II} - 566245\,{\rm IZ} \\ &+ 4831\,{\rm XI} + 20598\,{\rm XZ} \\ &+ 7883\,({\rm IXZX} + {\rm IYZY}), \end{aligned}$$

Wenyang Qian, et al. Phys. Rev. Research 4, 043193 (2022)

Summary and Outlook

Basis Light Front Quantization approach to mesons and baryons yields competitive descriptions and predictions

Positronium test applications found successful
Bound states and transitions of hadrons are described
Time-dependent scattering applications are advancing
Plan: continue to expand the Fock spaces (e.g. more gluons)
Plan: continue to develop renormalization & counterterms
Efficient utilization of supercomputing resources
Well-positioned to exploit advances in quantum computing

Thank you for your attention