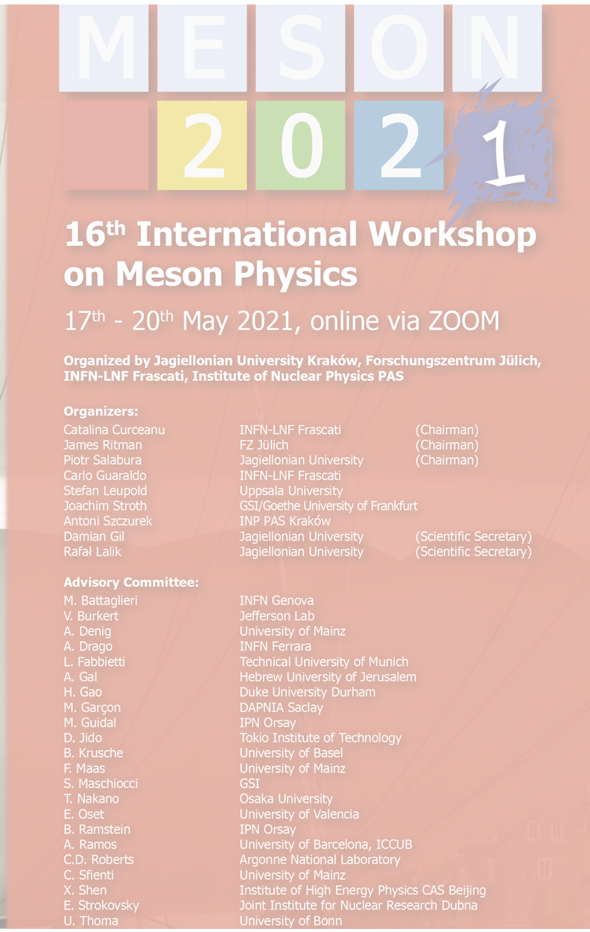


In-medium spectral functions of vector and axial-vector mesons from aFRG flow equations

Lorenz von Smekal

Meson 2021 — 16th International Workshop on Meson Physics

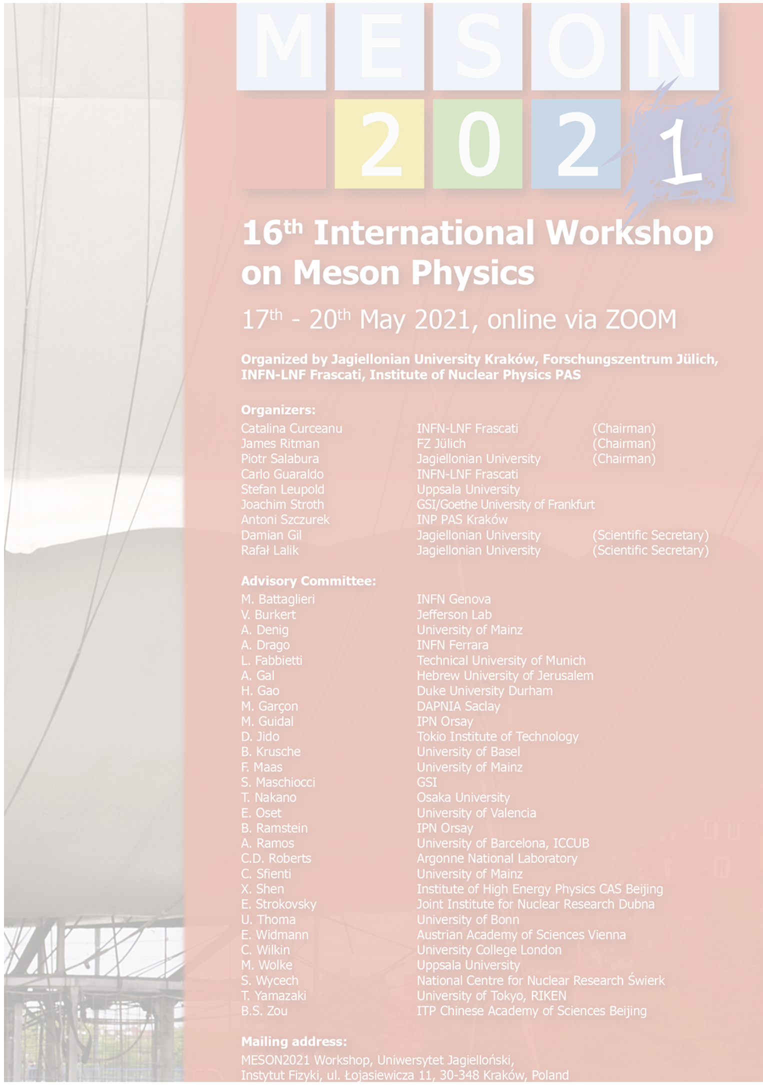
18 May 2021

Bundesministerium für Bildung und Forschung

HFHF Helmholtz Forschungsakademie Hessen für FAIR

CRC-TR 211 Strong-interaction matter under extreme conditions

Outline


MESON 2021

16th International Workshop on Meson Physics

17th - 20th May 2021, online via ZOOM

Organized by Jagiellonian University Kraków, Forschungszentrum Jülich, INFN-LNF Frascati, Institute of Nuclear Physics PAS

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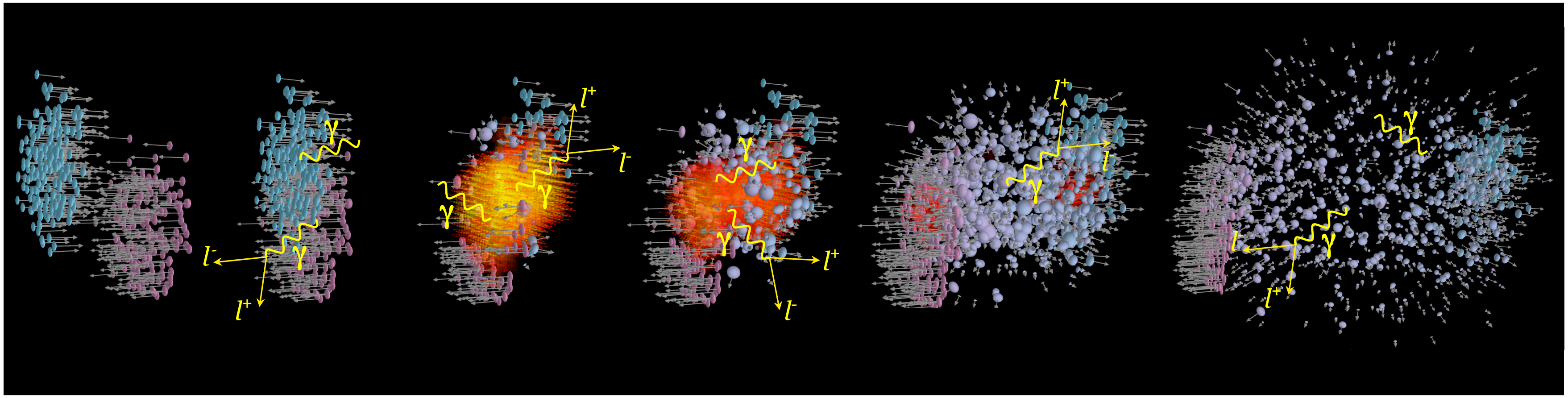
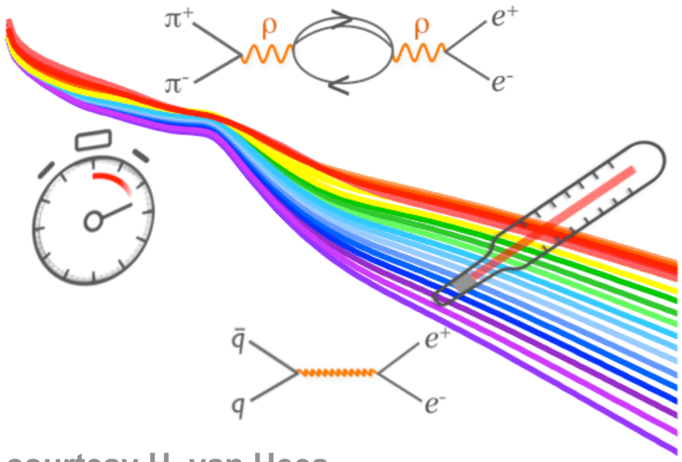
- **Dileptons in HIC**
- **GiBUU transport simulations for SIS18 energies**
- **Spectral functions from aFRG flows**
- **Vector and axial-vector spectral functions**
- **Effective hadronic theory for dense nuclear matter**
- **Summary and Outlook**

Photons and Dileptons in HIC



T. Galatyuk et al., Physik Journal 17 (2018) no. 10



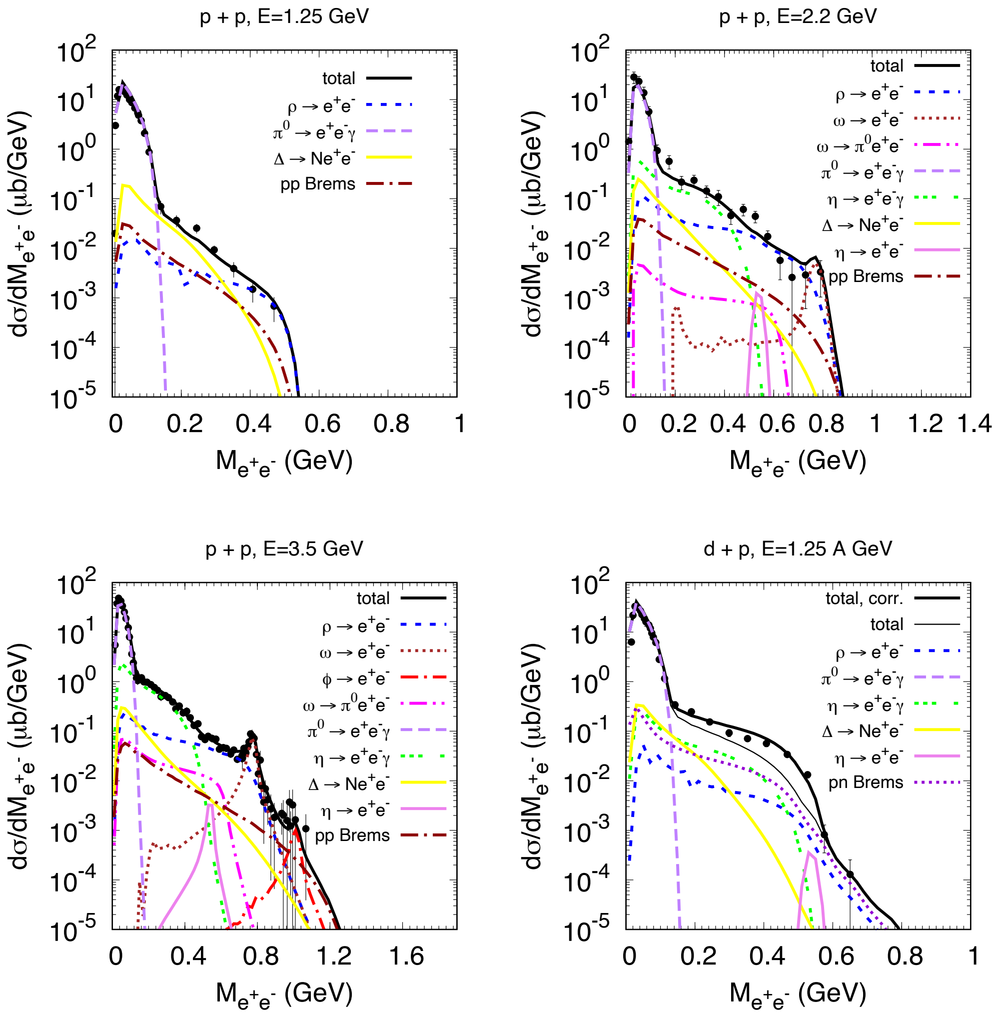
courtesy H. van Hees

- from all stages of the collision
- measure temperature in QGP, lifetime of fireball…

Dileptons at SIS18 Energies

- simulate with GiBUU transport model

O. Buss, T. Gaitanos, K. Gallmeister et al. (A. Larionov), Phys. Rept. 512 (2012) 1



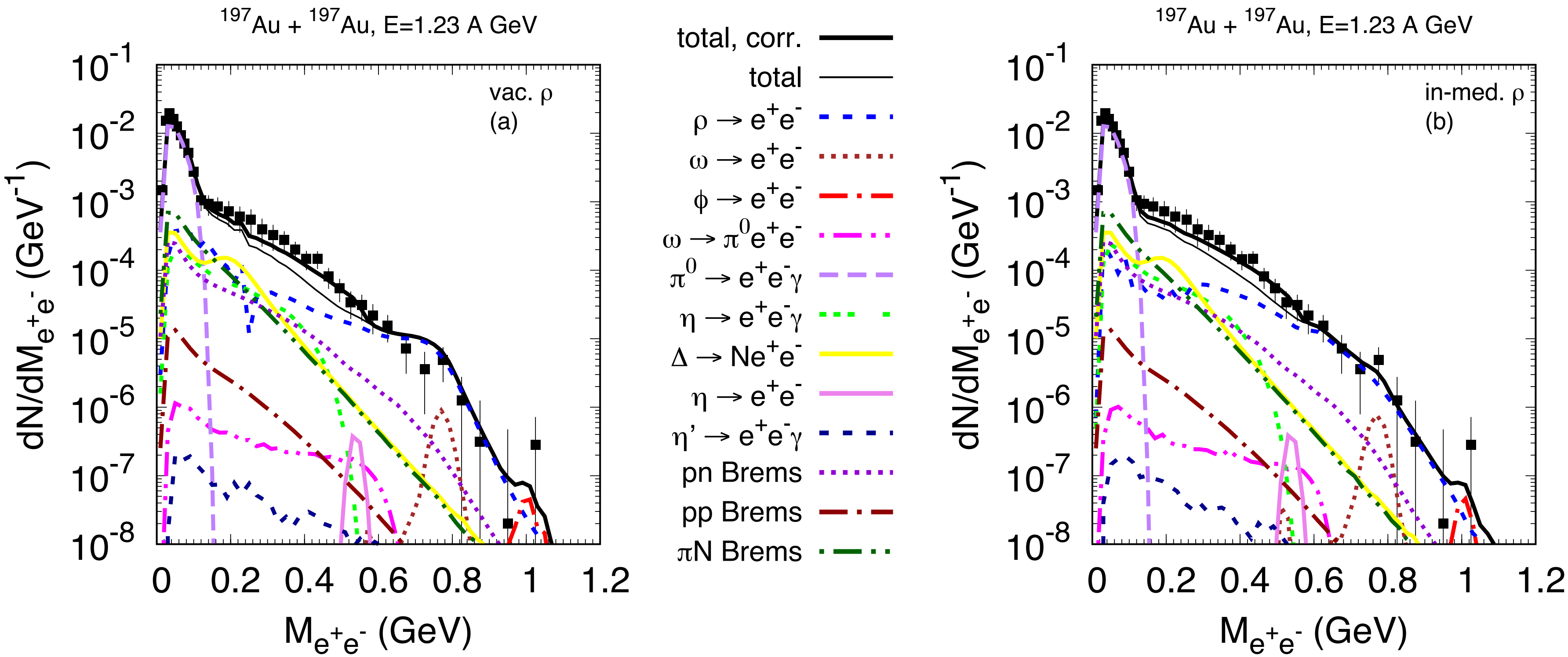
Data: HADES collaboration,
PLB 690 (2010) 118
PRC 85 (2012) 054005
EPJA 48 (2012) 64
PRC 95 (2017) 065205

A. Larionov, U. Mosel & L.v.S., Phys. Rev. C 102 (2020) 064913

Transport Simulations

- **Dileptons from GiBUU**

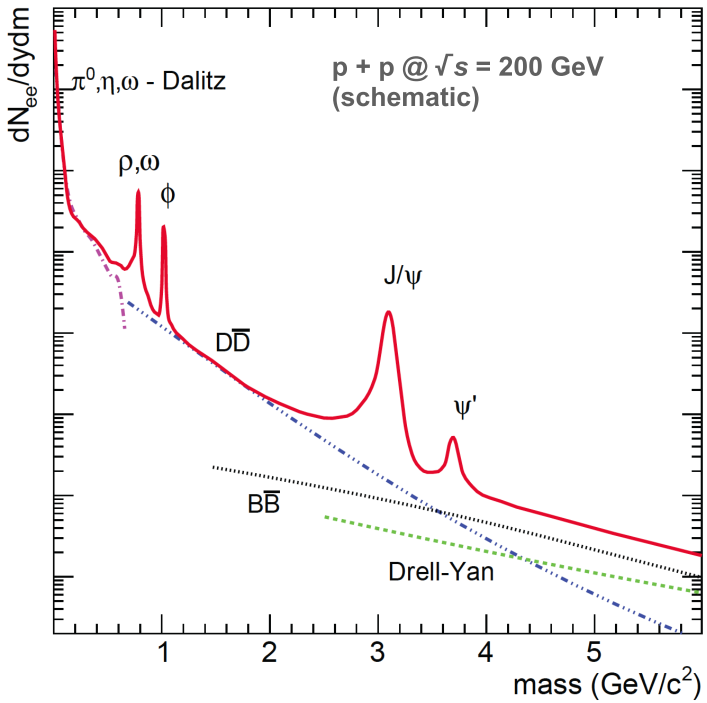
Data: HADES Collaboration, Nature Physics 15 (2019) 1040



A. Larionov, U. Mosel & L.v.S., Phys. Rev. C 102 (2020) 064913

Dilepton Spectra

A. Drees, NPA 830 (2009) 435



dilepton rate (local thermal equilibrium):

$$\frac{dN_{ll}}{d^4x d^4q} = -\frac{\alpha_{\rm em}^2}{\pi^3 M^2}\,\frac{1}{3} g_{\mu\nu}\,\mathrm{Im}\,\Pi^{\mu\nu}_{\rm em}\big(M, |\vec{q}|; \mu, T\big)$$

electromagnetic correlator:

$$\Pi^{\mu\nu}_{\rm em}\big(q;\mu,T\big) = -i\int d^4x\; e^{iqx}\theta(x^0)\,\big\langle\big[j^\mu_{\rm em}(x), j^\nu_{\rm em}(0)\big]\big\rangle$$

vector meson dominance & quark counting:

$$\mathrm{Im}\,\Pi^{\mu\nu}_{\rm em}\big(M \le 1\mathrm{GeV}\big) \sim \mathrm{Im}\,D^{\mu\nu}_\rho + \frac{1}{9}\mathrm{Im}\,D^{\mu\nu}_\omega + \frac{2}{9}\mathrm{Im}\,D^{\mu\nu}_\phi$$

Spectral Functions

- some theory basics

[courtesy L. Holicki]

Spectral Functions

commutator of interacting fields:

free fields

$$\langle[\phi(x),\phi(0)]\rangle = \int_0^\infty dm^2\, \rho(m^2)\, i\Delta(x;m^2)$$

Fourier transform:

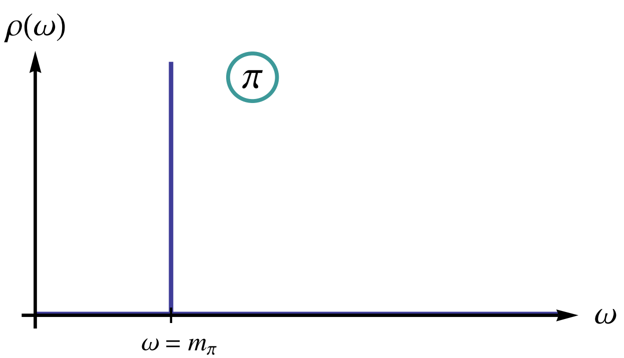
$$\rho(\omega,\vec{p}) = \int d^4x\, e^{ipx}\, i\langle[\phi(x),\phi(0)]\rangle$$

spectral function:

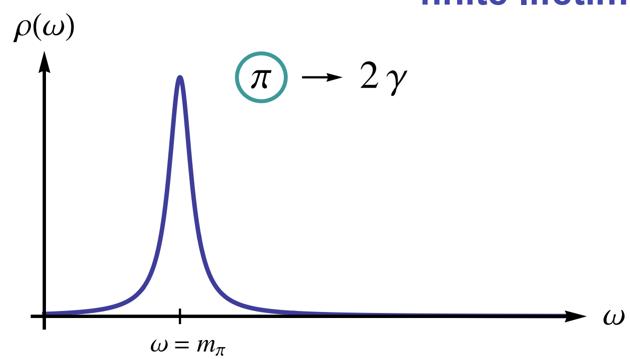
$$\Rightarrow \qquad \rho(\omega,\vec{p}) = 2\pi i\, \epsilon(\omega)\theta(p^2)\, \rho(p^2)$$

$$\rho(p^2) = (2\pi)^3 \sum_\psi \delta^4(p-q_\psi)\, \left|\langle\Omega|\phi(0)|\psi\rangle\right|^2 \;, \quad p_0 > 0$$

free fields (stable pion):

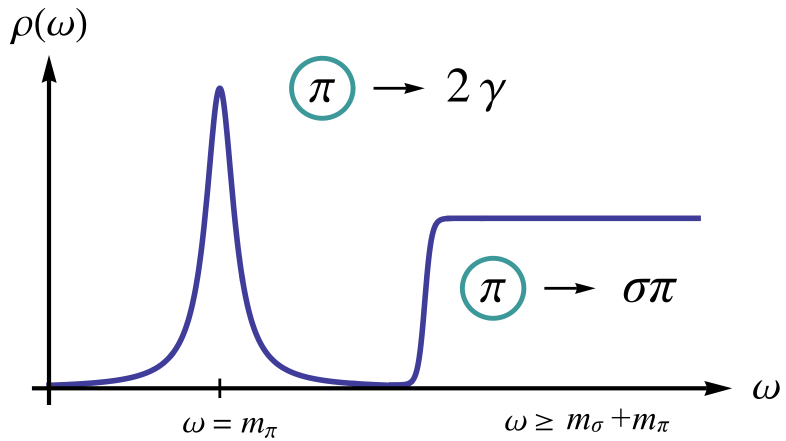


finite lifetime/width



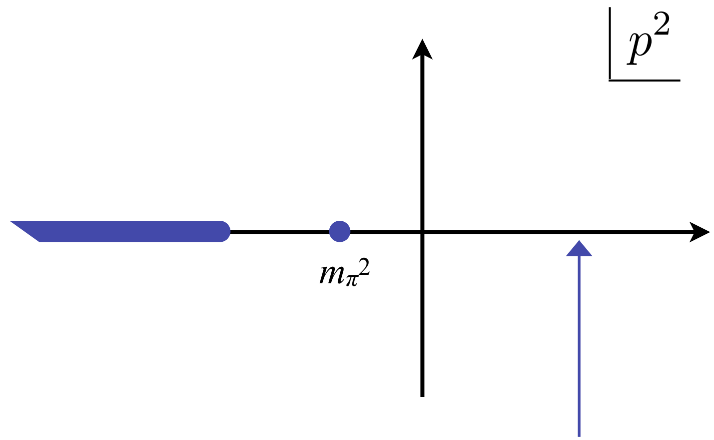
Spectral Functions

two-particle thresholds:



discontinuity at cut of propagator:

$$D(p) = \int_0^\infty dm^2 \, \rho(m^2) \, \frac{1}{p^2 + m^2}$$



retarded, imaginary part: $\rho(p^2) = -\frac{1}{\pi} \, \mathrm{Im} \, D_R(p)$

Euclidean space:

$p^2 > 0$

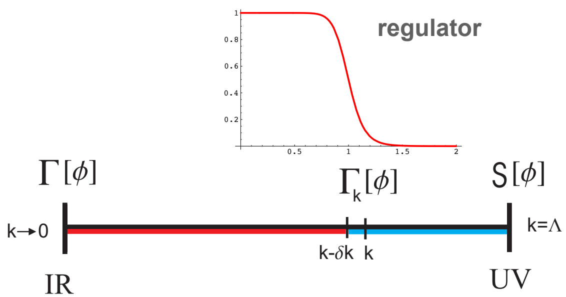
Euclidean data: $D(t, \vec{p} = 0) = \int_0^\infty dm \, \rho(m^2) \, \exp\{-mt\}$

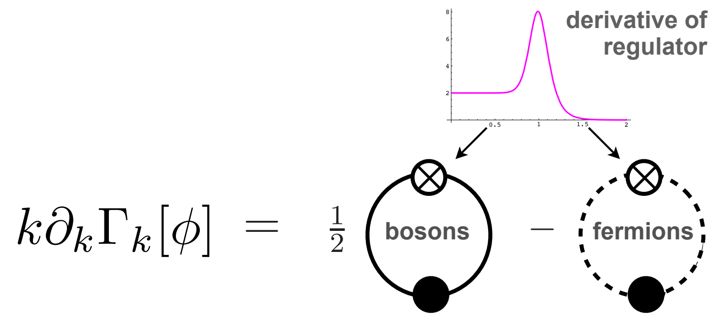
(inverse Laplace, try e.g. MEM, but ill-posed numerical problem)

Functional RG (Flow) Equations

- **compute effective (average) action:**

  Ch. Wetterich, PLB 301 (1993) 90



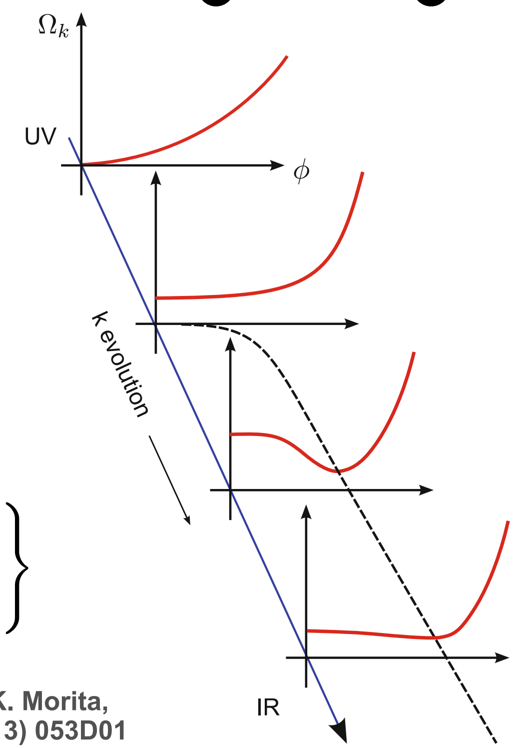


- **flow of Landau free energy density:**

  (quark-meson model, leading order derivative expansion)

$$\partial_k \Omega_k(T,\mu;\phi^2) =$$

$$\frac{k^4}{12\pi^2}\left\{ \frac{1}{E_k^\sigma}\,\coth\left(\frac{E_k^\sigma}{2T}\right) + \frac{3}{E_k^\pi}\,\coth\left(\frac{E_k^\pi}{2T}\right) - \frac{2N_cN_f}{E_k^q}\left[\tanh\left(\frac{E_k^q-\mu}{2T}\right) + \tanh\left(\frac{E_k^q+\mu}{2T}\right)\right]\right\}$$



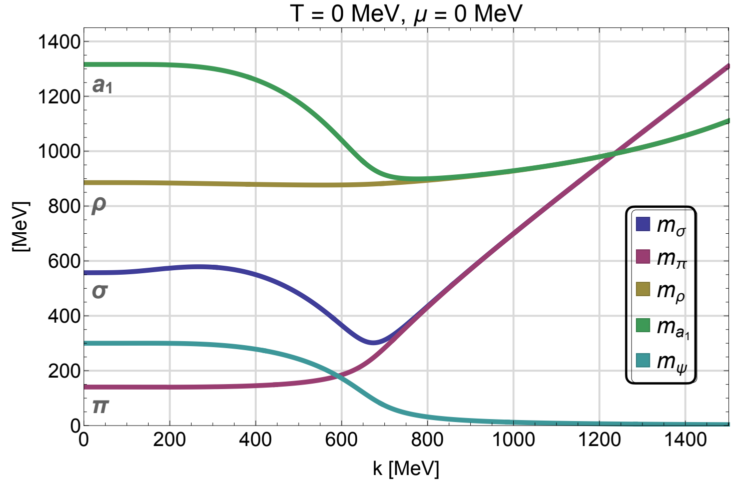
- **include mixing with density fluctuations**

  K. Kamikado, T. Kunihiro, K. Morita, A. Ohnishi, PTEP 2013 (2013) 053D01

Euclidean Mass Parameters

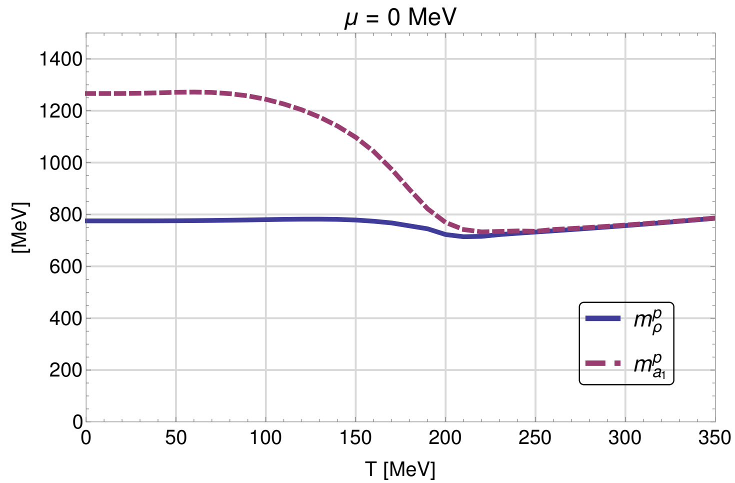
Flow of Euclidean (curvature) masses

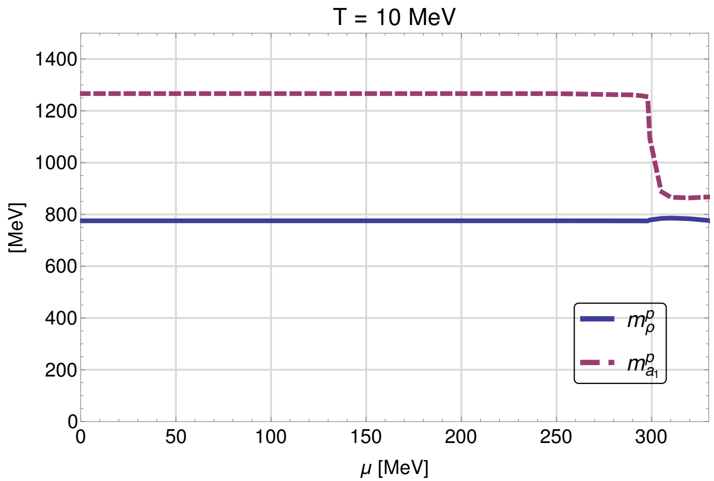
including (axial-)vector mesons



- **physical pole masses:**

chiral restoration at finite T and μ





Ch. Jung & L.v.S., PRD 100 (2019) 116009

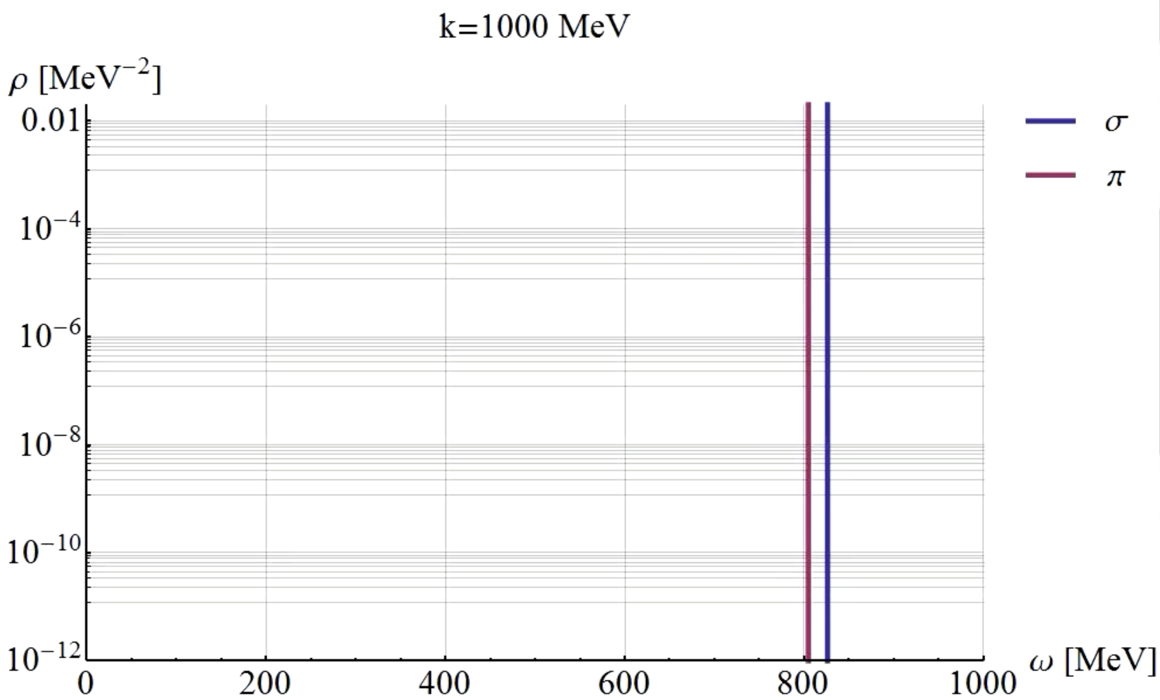
Analytically Continued aFRG Flows

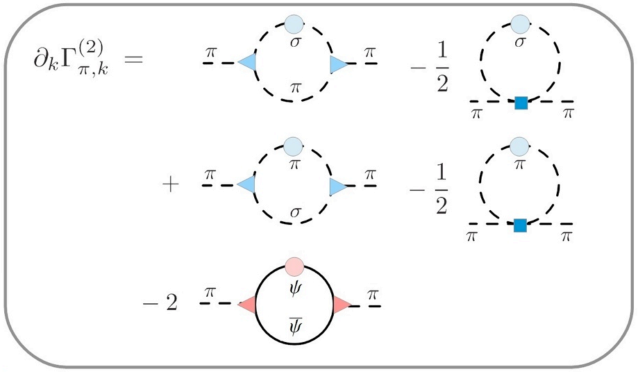
- **quark-meson model, $T = \mu = 0$:**

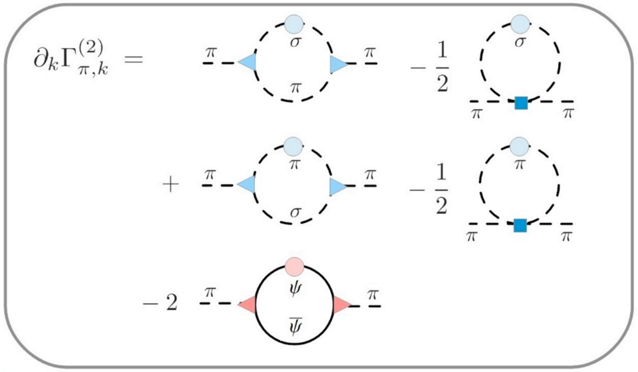
$$p_0 = -i(\omega + i\varepsilon) \quad \textbf{(retarded)}$$

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi} \mathrm{Im}\, D^R(\omega, \vec{p})$$

- **for $\varepsilon \to 0$:**







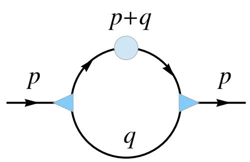
$$p_0 = -i(\omega + i\varepsilon)$$

K. Kamikado, N. Strodthoff, L.v.S. & J. Wambach, EPJC 74 (2014) 2806

aFRG Flow at Finite Temperature

- **analytic continuation not unique:**

exploit one-loop structure, 3-dim. regulators

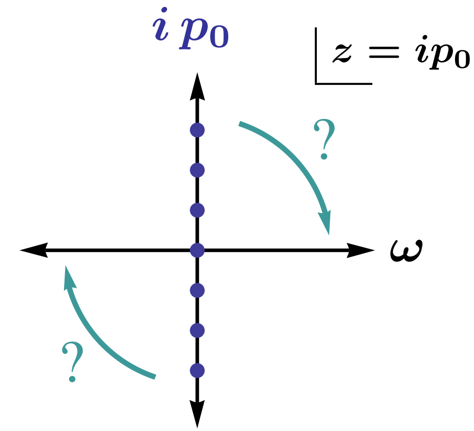


1) Use periodicity in external energy $p_0 = n 2\pi T$:

$$n_{B,F}(E + ip_0) \to n_{B,F}(E)$$

2) Substitute p_0 by continuous real frequency:

$$\Gamma^{(2),R}(\omega) = -\lim_{\epsilon \to 0} \Gamma^{(2),E}(p_0 = i\omega - \epsilon)$$



- **for $\varepsilon \to 0$:**

$$p_0 = -i(\omega + i\varepsilon)$$

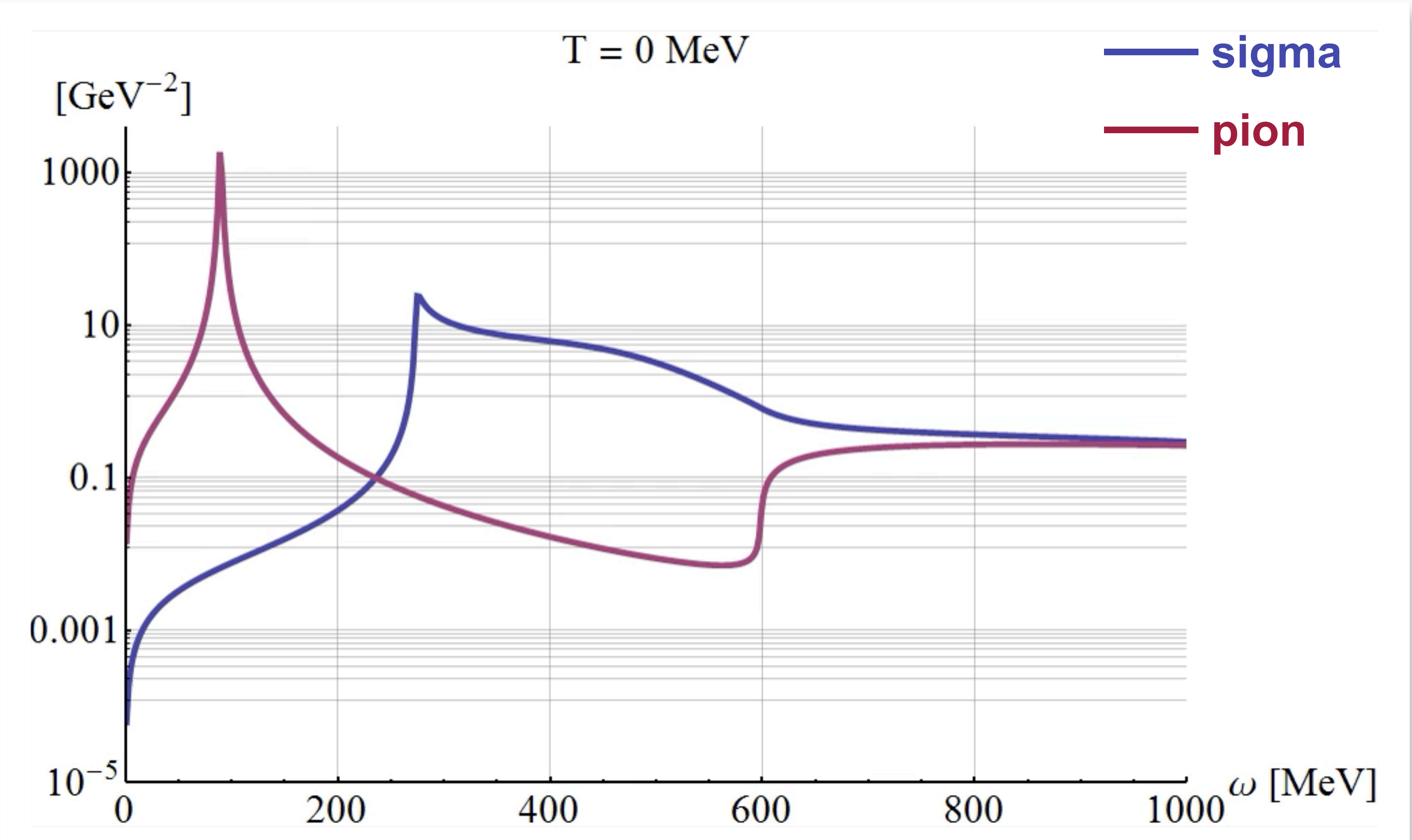
$\rightsquigarrow$ **Baym-Mermin physical b.c.'s satisfied**

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi} \mathrm{Im}\, D^R(\omega, \vec{p}) = \frac{1}{\pi} \frac{\mathrm{Im}\,\Gamma^{(2),R}(\omega, \vec{p})}{\left(\mathrm{Re}\,\Gamma^{(2),R}(\omega, \vec{p})\right)^2 + \left(\mathrm{Im}\,\Gamma^{(2),R}(\omega, \vec{p})\right)^2}$$

In-Medium Spectral Functions

- **quark-meson model:**

$\mu = 0$

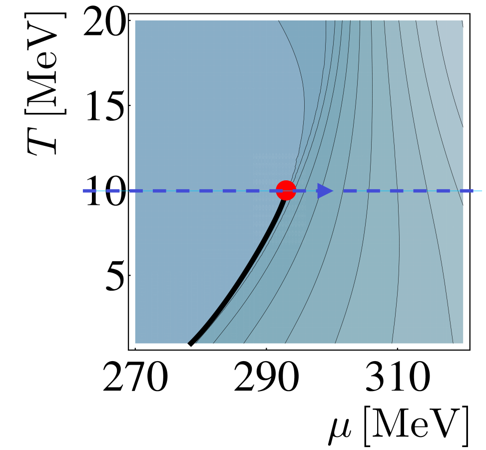


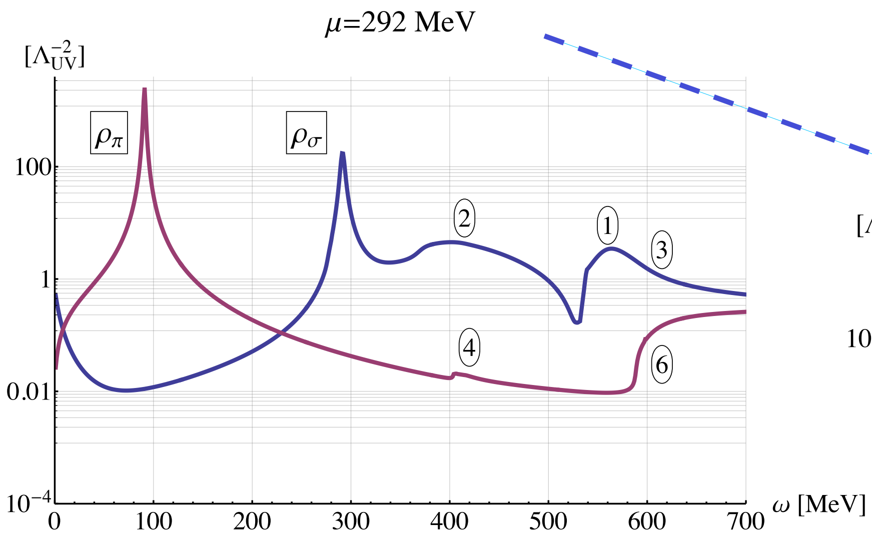
A. Tripolt, N. Strodthoff, L.v.S. & J. Wambach, PRD 89 (2014) 34010

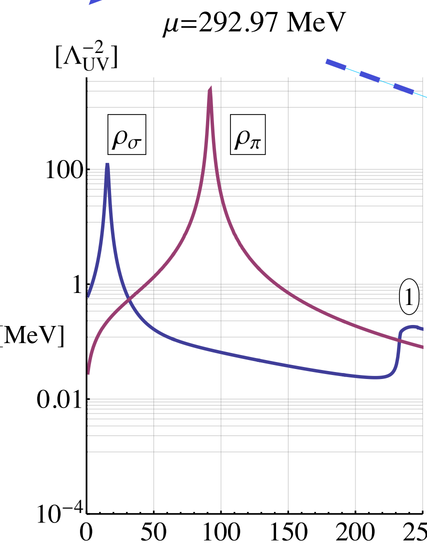
In-Medium Spectral Functions

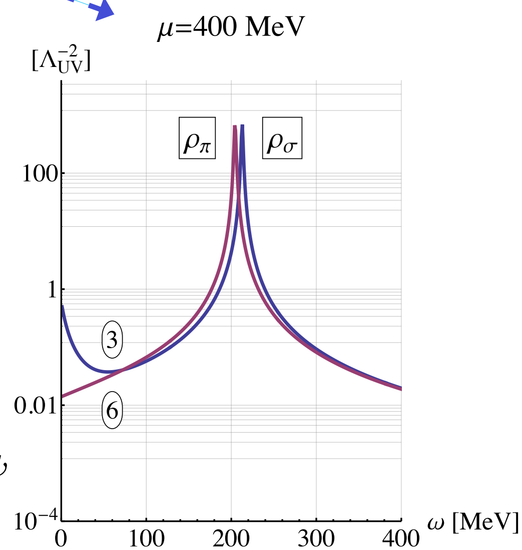
- **quark-meson model:**

  $T = 10\,\mathrm{MeV}$







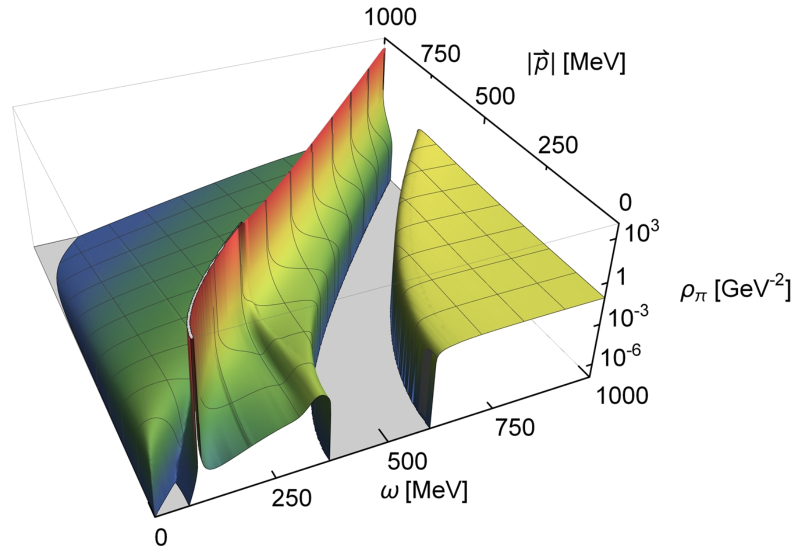


1: $\sigma^* \to \sigma\sigma$, 2: $\sigma^* \to \pi\pi$, 3: $\sigma^* \to \bar\psi\psi$, 4: $\pi^* \to \sigma\pi$, 5: $\pi^*\pi \to \sigma$, 6: $\pi^* \to \bar\psi\psi$

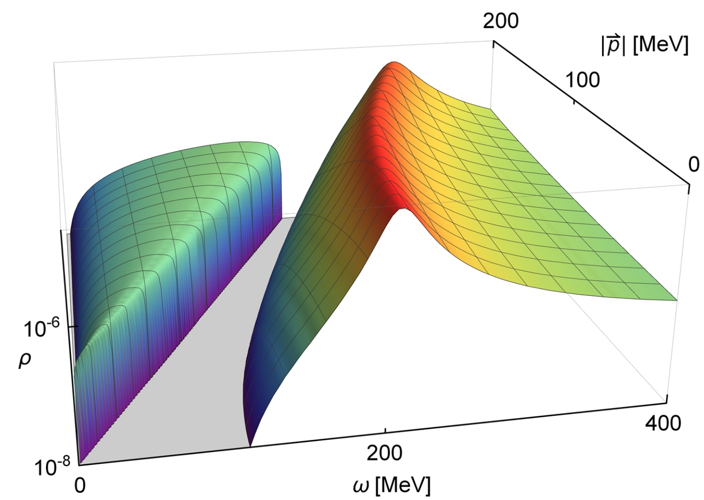
A. Tripolt, N. Strodthoff, L.v.S. & J. Wambach, PRD 89 (2014) 34010

Finite Momenta

pion SF $\rho(\omega, \vec{p})$ below T_c



sigma meson SF $\rho(\omega, \vec{p})$ above T_c



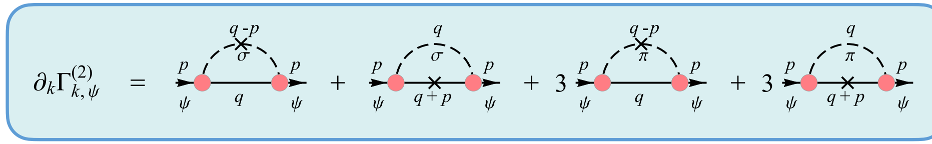
$\rightsquigarrow$ **transport coefficients**

A. Tripolt, L.v.S. & J. Wambach, PRD 90 (2014) 074031

Fermionic SFs

- **aFRG for fermionic two-point functions**

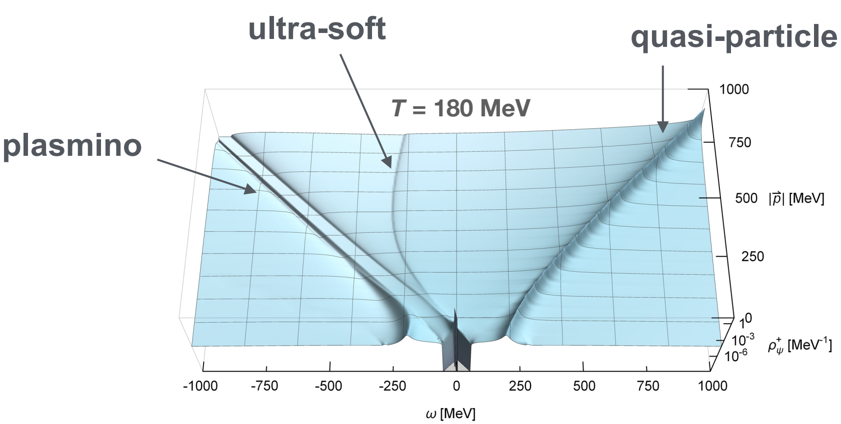
$$\Gamma^{(2)}_{k,\psi}(\omega,\vec{p}) = \gamma_0 C_k(\omega,\vec{p}) + i\vec{\gamma}\cdot\hat{p}A_k(\omega,\vec{p}) - B_k(\omega,\vec{p})$$

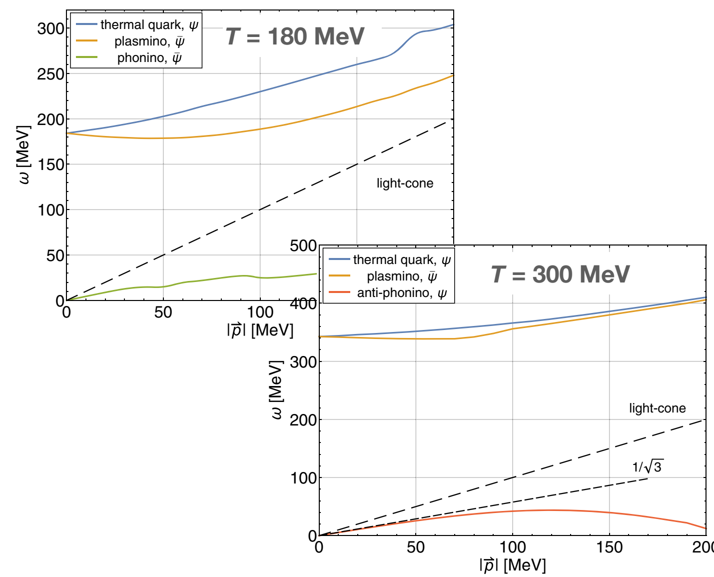


- **spectral functions**

$$\rho_{k,\psi}(\omega,\vec{p}) = \gamma_0\rho^{(C)}_{k,\psi}(\omega,\vec{p}) + i\vec{\gamma}\cdot\hat{p}\rho^{(A)}_{k,\psi}(\omega,\vec{p}) + \rho^{(B)}_{k,\psi}(\omega,\vec{p})$$

- **describe fermionic excitations at finite T**





A. Tripolt, J. Weyrich, L.v.S. & J. Wambach, PRD 98 (2018) 094002
A. Tripolt, D. Rischke, L.v.S. & J. Wambach, PRD 101 (2020) 094010

Vector and Axial-Vector SFs

- extended linear-sigma model with quarks:

  Ch. Jung, F. Rennecke, A. Tripolt, L.v.S. & J. Wambach, PRD95 (2017) 036020

- electromagnetic SF from gauging and mixing:

  A. Tripolt, Ch. Jung, N. Tanji, L.v.S. & J. Wambach, NPA 982 (2019) 775

- include fluctuating (axial-)vectors in aFRG flows for SFs:

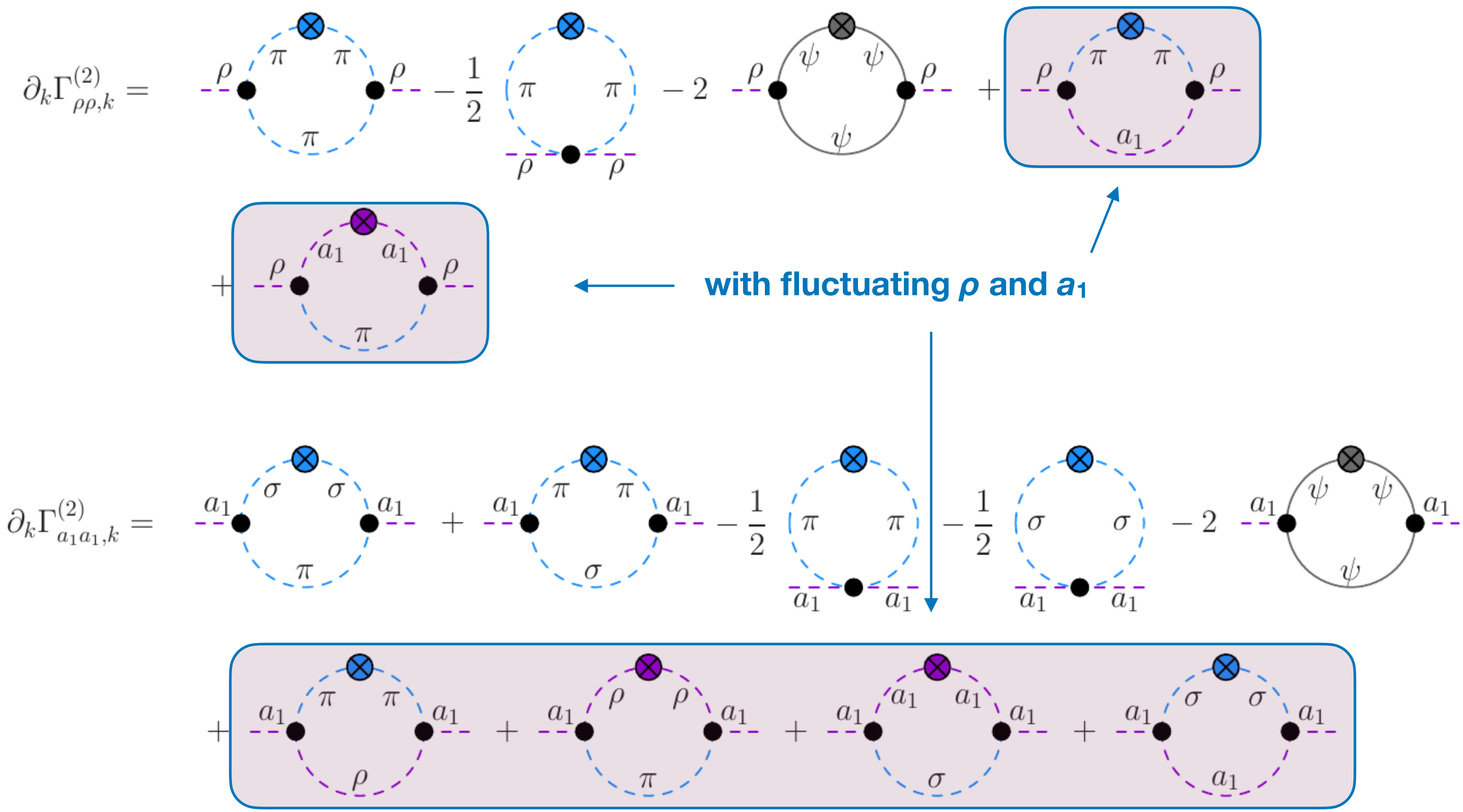
  Ch. Jung & L.v.S., PRD 100 (2019) 116009

- (axial-)vector SFs in hadronic effective theory for dense nuclear matter:

  A. Tripolt, Ch. Jung, L.v.S. & J. Wambach, arXiv:2105.00861 [hep-ph]

Fluctuating (Axial-) Vectors

- aFRG flows for ρ and a_1 at finite T and μ:



Ch. Jung & L.v.S., PRD 100 (2019) 116009

Fluctuating (Axial-)Vectors

- **spectral representation of conserved current:**

$$\langle T_{\rm cov}\, j_\mu(x) j_\nu(0)\rangle = -\,\mathrm{i}\int_0^\infty ds\, \frac{\rho(s)}{s} \int \frac{d^4p}{(2\pi)^4}\, \mathrm{e}^{-\mathrm{i}px}\, \frac{p^2 g_{\mu\nu} - p_\mu p_\nu}{p^2 - s + \mathrm{i}\epsilon}$$

- **current-field identity, transverse vector propagator:**

$$D^{T,V}_{\mu\nu}(p) \;= -\mathrm{i}\frac{Z}{m_v^2}\, \frac{p^2 g_{\mu\nu} - p_\mu p_\nu}{p^2 - m_v^2 + \mathrm{i}\epsilon} \;+\; \dots$$

- **Euclidean two-point function, single-particle contribution:**

$$\Gamma^{(2)T}_{\mu\nu}(p) = -\frac{m_0^2}{p^4}(p^2 + m_v^2)\left(p^2\delta_{\mu\nu} - p_\mu p_\nu\right) \qquad m_{0,k}^2 \;=\; m_{v,k}^2/Z_k$$

- **(axial-)vectors from (anti-)selfdual field strengths:**

$$\mathcal{L}_0^\rho = -\frac{1}{4}\,\mathrm{tr}\,(\partial_\mu \rho_{\mu\nu})\partial_\sigma \rho_{\sigma\nu} + \frac{m_v^2}{8}\,\mathrm{tr}\,\rho_{\mu\nu}\rho_{\mu\nu}$$

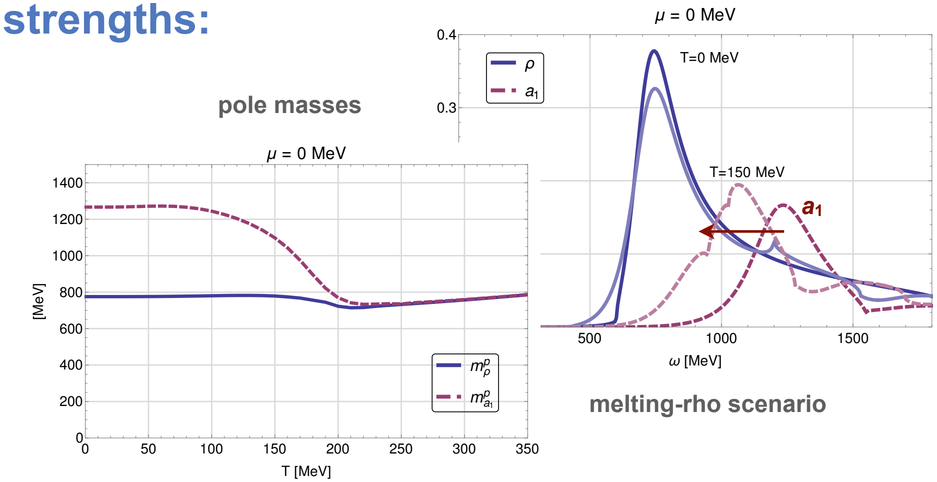
Fluctuating (Axial-)Vectors

- **(axial-)vectors from (anti-)selfdual field strengths:**

$$\rho_{\mu\nu} = \vec{\rho}^{\,+}_{\mu\nu} \cdot \vec{T}_R + \vec{\rho}^{\,-}_{\mu\nu} \cdot \vec{T}_L$$

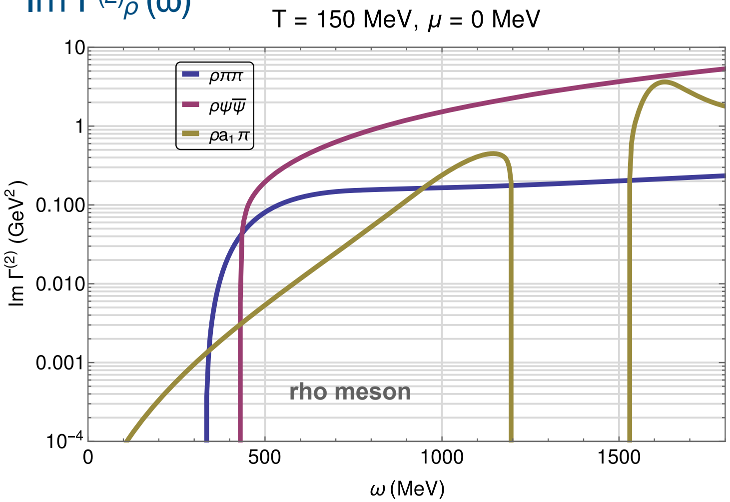
$$\vec{\rho}_{\mu} = \frac{1}{2m_v}\,\mathrm{tr}\big(\partial_\sigma \rho_{\sigma\mu} \vec{T}_V\big)$$

$$\vec{a}_{1\mu} = \frac{1}{2m_v}\,\mathrm{tr}\big(\partial_\sigma \rho_{\sigma\mu} \vec{T}_A\big)$$

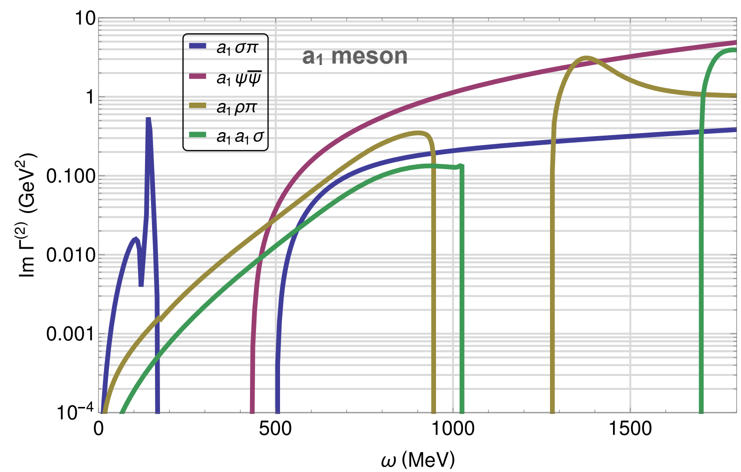


- **new processes / imaginary parts:**

$\mathrm{Im}\,\Gamma^{(2)}_\rho(\omega)$



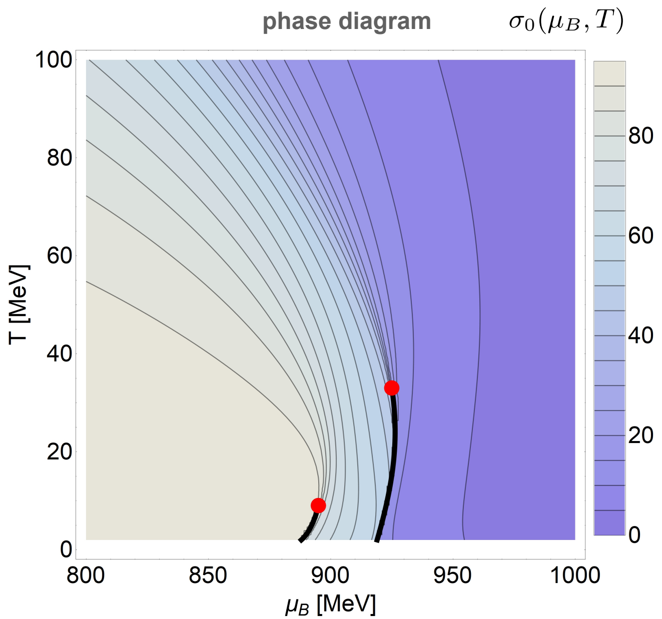
$\mathrm{Im}\,\Gamma^{(2)}_a(\omega)$

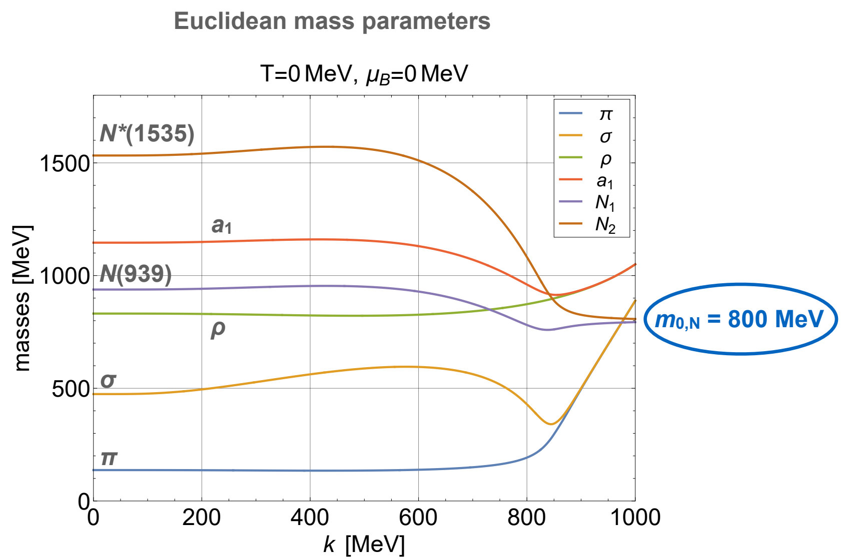


Parity-Doublet Model

- gauged linear-sigma model with *N*(939) and *N**(1535) iso-doublets:

$$\Gamma_k = \int d^4x \, \Big\{ \bar{N}_1 \left(\not{\partial} - \mu_B \gamma_0 + h_{s,1}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma^5) + h_{v,1}(\gamma_\mu \vec{\tau}\cdot\vec{\rho}_\mu + \gamma_\mu\gamma^5\vec{\tau}\cdot\vec{a}_{1,\mu}) \right) N_1$$
$$+ \bar{N}_2 \left(\not{\partial} - \mu_B \gamma_0 + h_{s,2}(\sigma - i\vec{\tau}\cdot\vec{\pi}\gamma^5) + h_{v,2}(\gamma_\mu \vec{\tau}\cdot\vec{\rho}_\mu - \gamma_\mu\gamma^5\vec{\tau}\cdot\vec{a}_{1,\mu}) \right) N_2 + m_{0,N}(\bar{N}_1\gamma^5 N_2 - \bar{N}_2\gamma^5 N_1)$$
$$+ U_k(\phi^2) - c\sigma + \frac{1}{2}(D_\mu\phi)^\dagger D_\mu\phi - \frac{1}{4}\,\mathrm{tr}\,\partial_\mu\rho_{\mu\nu}\partial_\sigma\rho_{\sigma\nu} + \frac{m_v^2}{8}\,\mathrm{tr}\,\rho_{\mu\nu}\rho_{\mu\nu} \Big\}$$

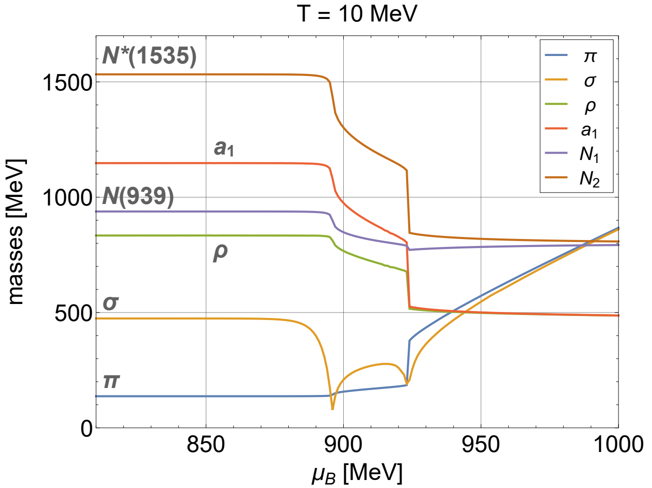


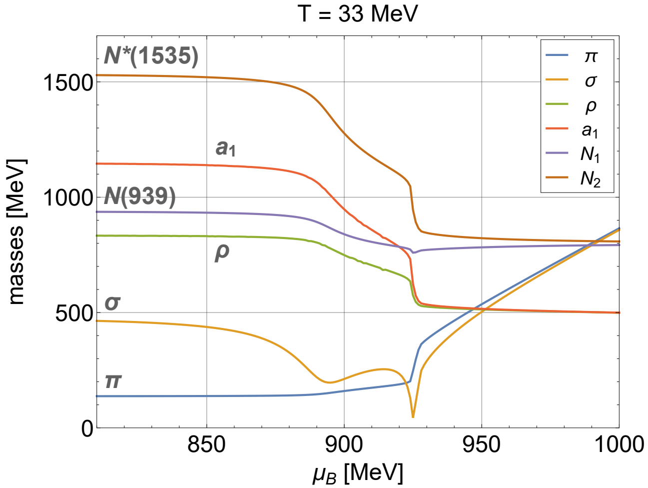


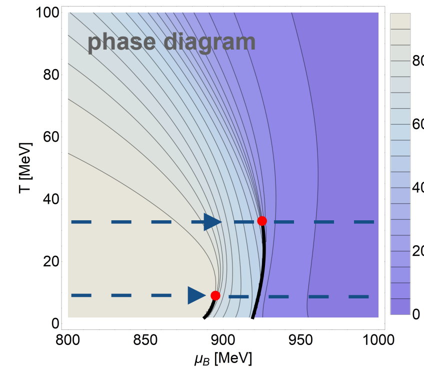
J. Weyrich, N. Strodthoff & L.v.S., PRC 92 (2015) 015214
A. Tripolt, Ch. Jung, L.v.S. & J. Wambach, arXiv:2105.00861 [hep-ph]

Parity-Doublet Model

- **mass parameters at finite T and μ:**

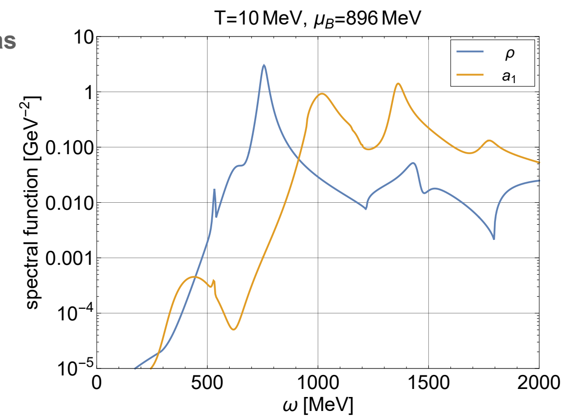


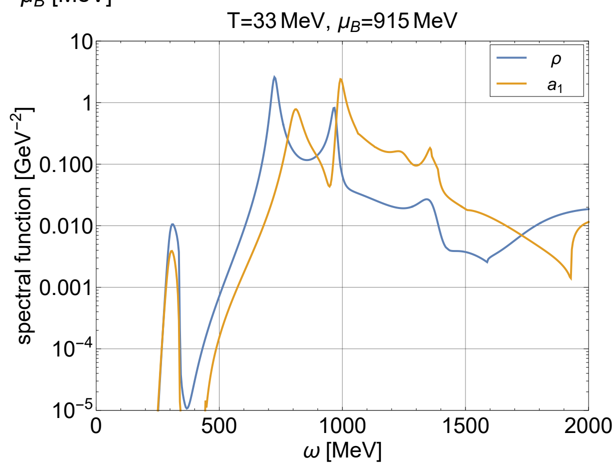




- **(axial-)vector SFs inside nuclear matter:**

nuclear liquid-gas CEP



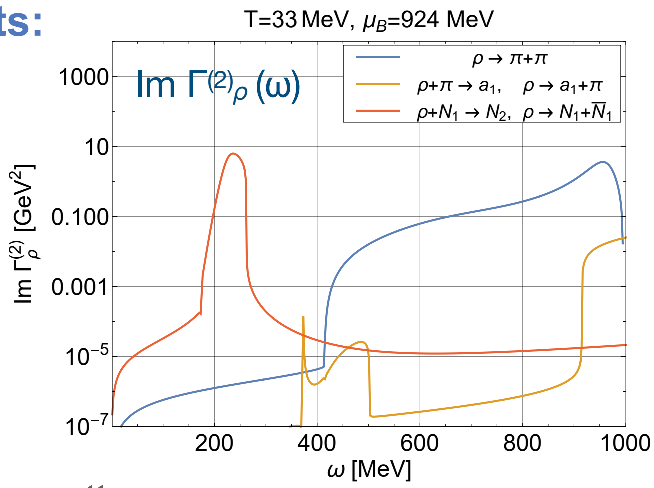


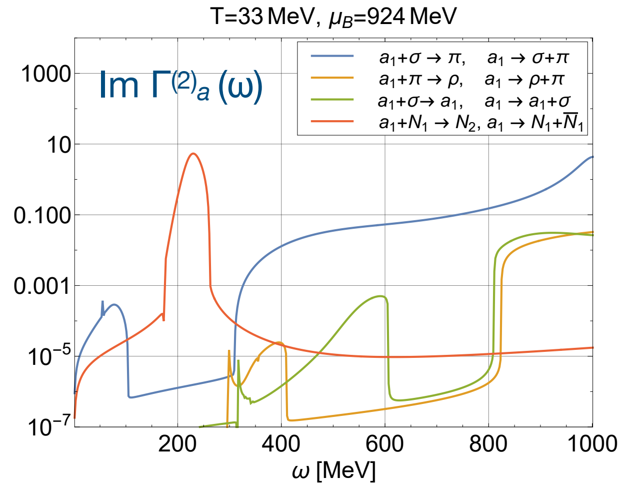
approaching chiral CEP inside dense nuclear matter

A. Tripolt, Ch. Jung, L.v.S. & J. Wambach, arXiv:2105.00861 [hep-ph]

(Axial-)Vector SFs in Dense NM

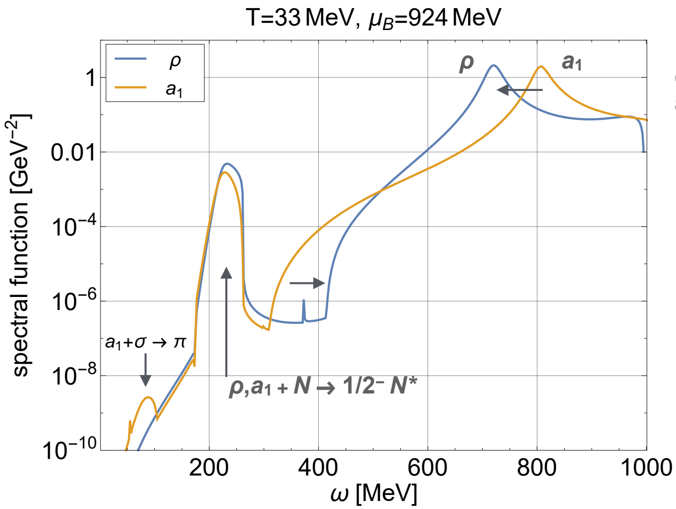
- **imaginary parts:**





near chiral CEP
inside dense nuclear matter

- **(axial-)vector SFs:**

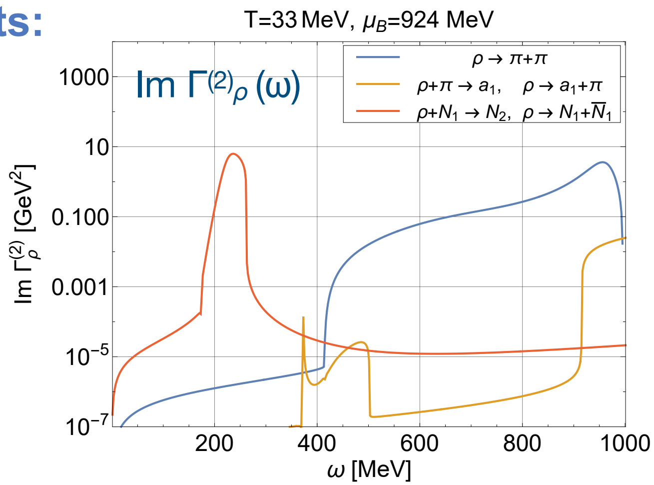


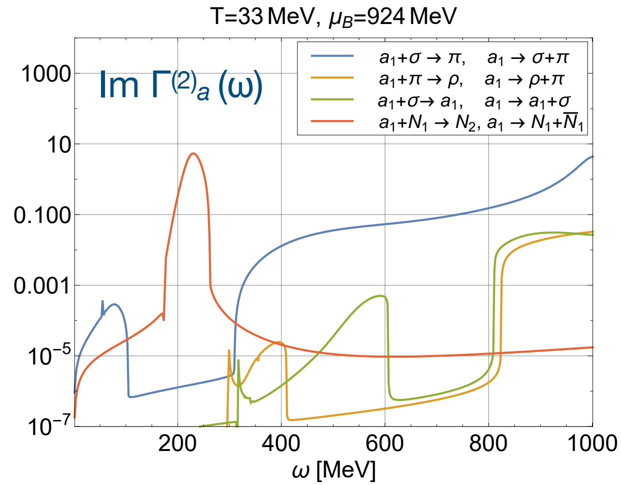
degeneration at high density

Bundesministerium für Bildung und Forschung

(Axial-)Vector SFs in Dense NM

- **imaginary parts:**





near chiral CEP
inside dense nuclear matter

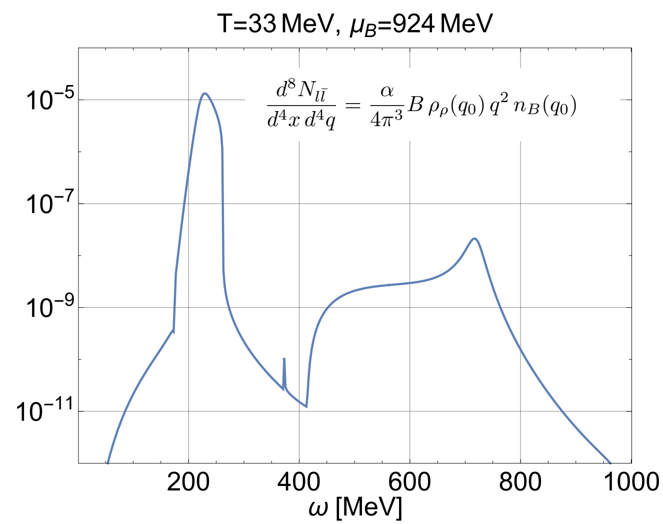
- **electromagnetic SF:**

convert to thermal dilepton rate

$$\frac{d^8 N_{l\bar{l}}}{d^4x\, d^4q} =$$

$$\frac{\alpha}{12\pi^3}\left(1+\frac{2m^2}{q^2}\right)\left(1-\frac{4m^2}{q^2}\right)^{1/2} q^2(2\rho_T+\rho_L)\, n_B(q_0)$$

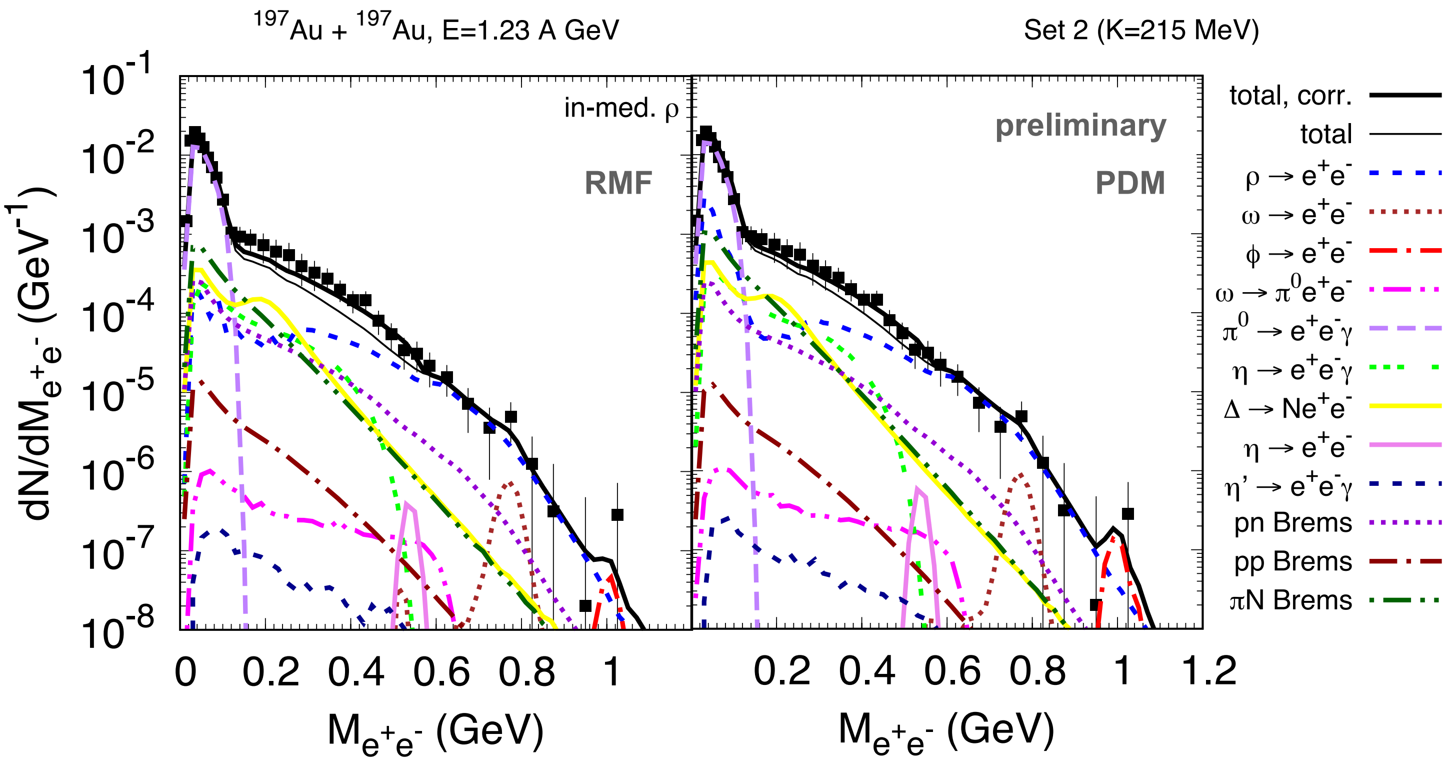
Weldon, PRD 42 (1990) 2385



Parity-Doublet MFT in GiBUU

- **RMF mode: replace mean fields by those of PDM**

PDM parameters: M. Kim, S. Jeon, Y.-M. Kim, Y. Kim, & C.-H. Lee, PRC 101 (2020) 064614



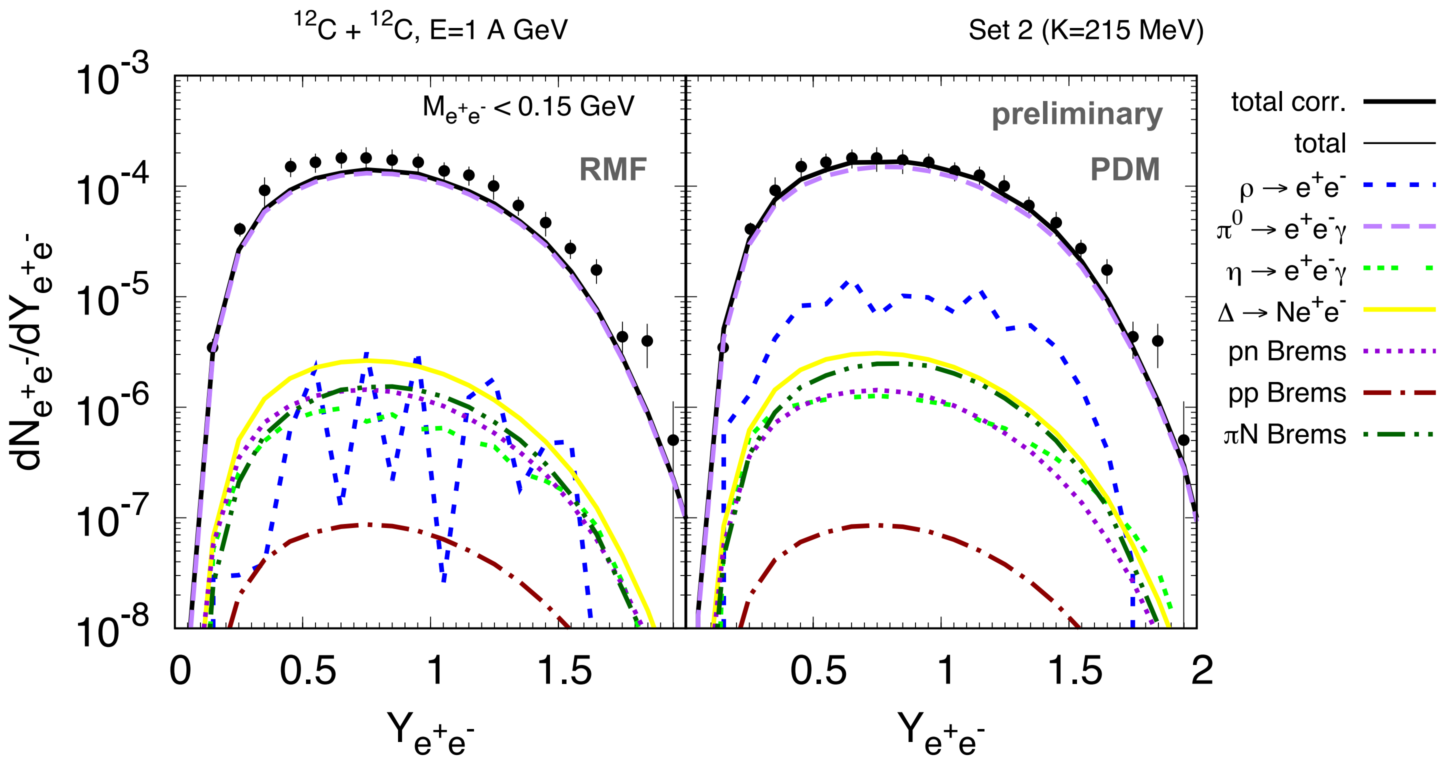
A. Larionov, U. Mosel & L.v.S., Phys. Rev. C 102 (2020) 064913

A. Larionov & L.v.S., in preparation

Parity-Doublet MFT in GiBUU

- RMF mode: replace mean fields by those of PDM

Data: Y. Pachmayer, PhD thesis, GU Frankfurt, 2008



A. Larionov, U. Mosel & L.v.S., Phys. Rev. C 102 (2020) 064913

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Parity-Doublet MFT in GiBUU

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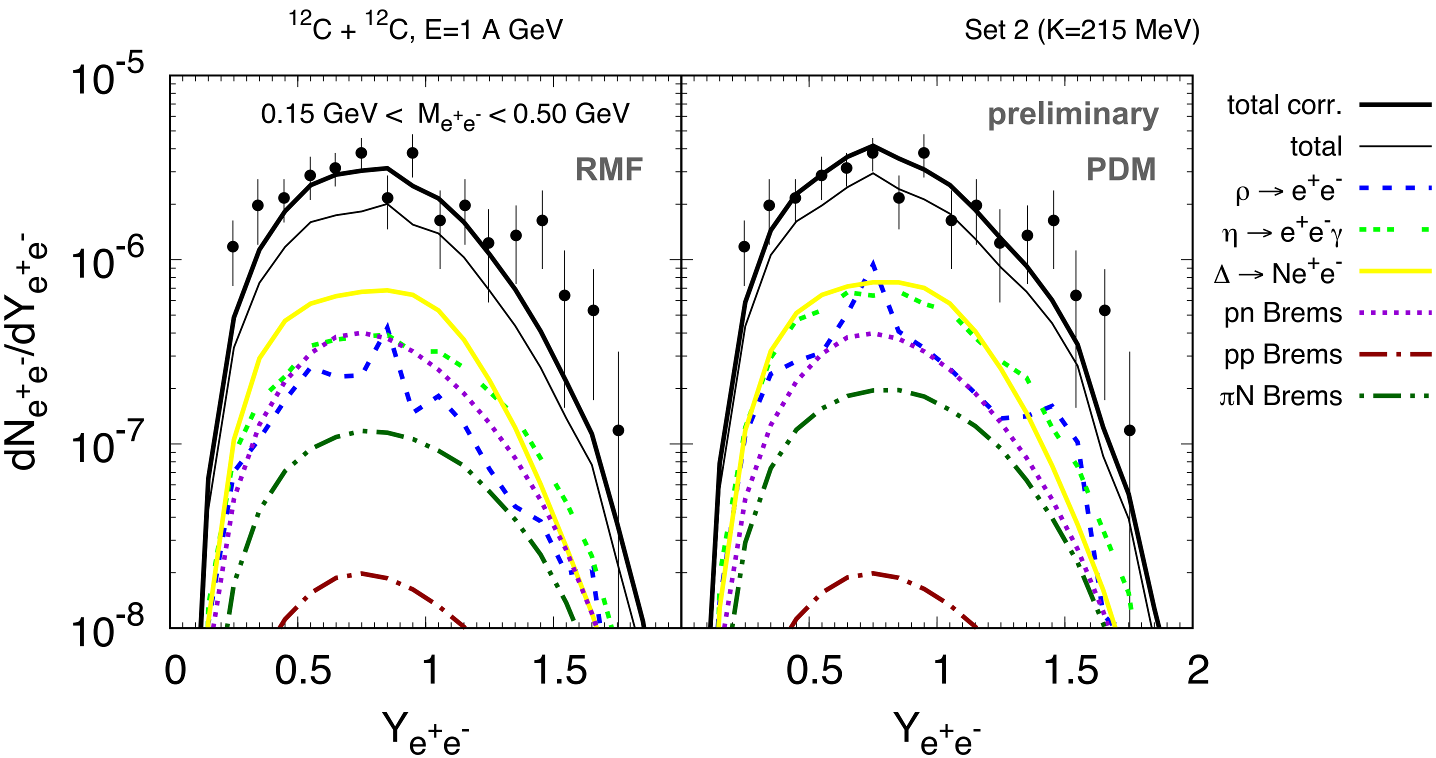


A. Larionov, U. Mosel & L.v.S., Phys. Rev. C 102 (2020) 064913

A. Larionov & L.v.S., in preparation

Summary

- **Spectral functions from analytically contd. aFRG flows**

  effective theories (chiral, linear)

- **Vector and axial-vector SFs at finite T and μ**

  melting-rho scenario

- **Electromagnetic spectral function**

  U(1) gauging, mixing

- **Fermionic spectral functions**

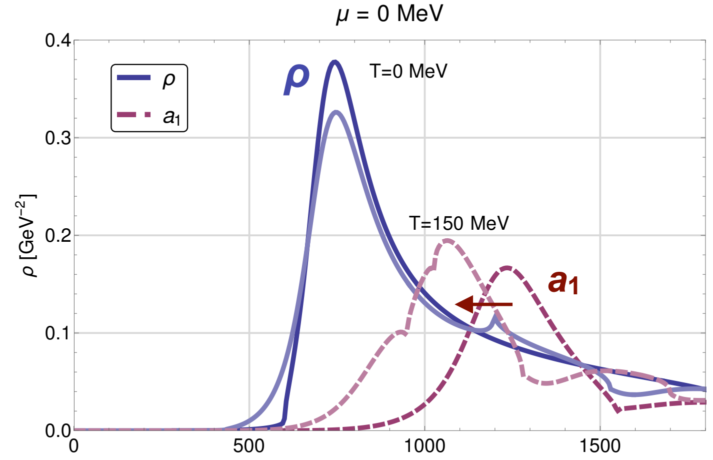
  use for baryonic SFs in dense matter

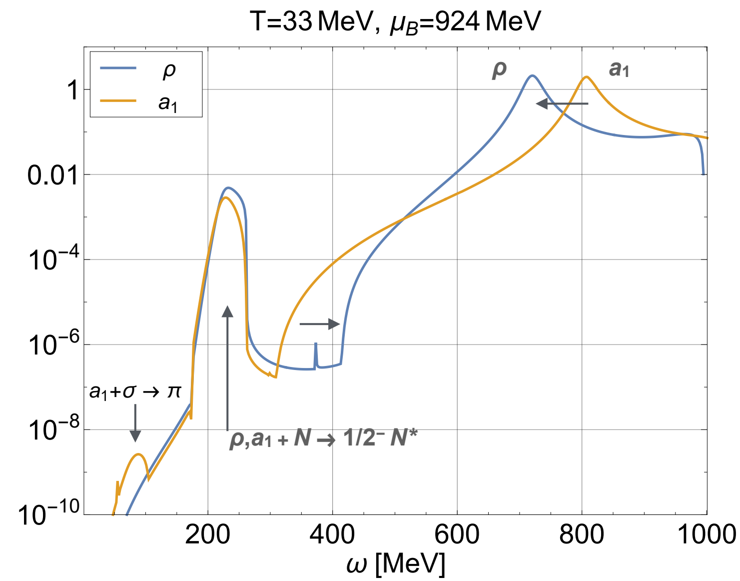
- **(Axial-)Vector SFs in nuclear matter, parity doubling**

  effective hadronic theory with chiral PT

- **Parity-doublet chiral MFT in GiBUU**

  enhanced low energy ρ and η signals





Outlook

- **parity-doublet model with fluctuating ω and ρ**

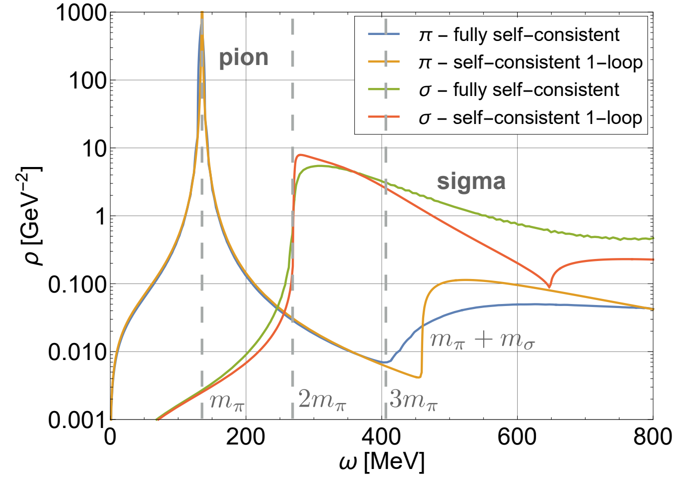
  symmetric nuclear and neutron matter

- **ρ-a_1 mixing and signatures of CEP in HIC**

  electromagnetic → dilepton rates
  weak → neutron star mergers…

- **self-consistent spectral functions**

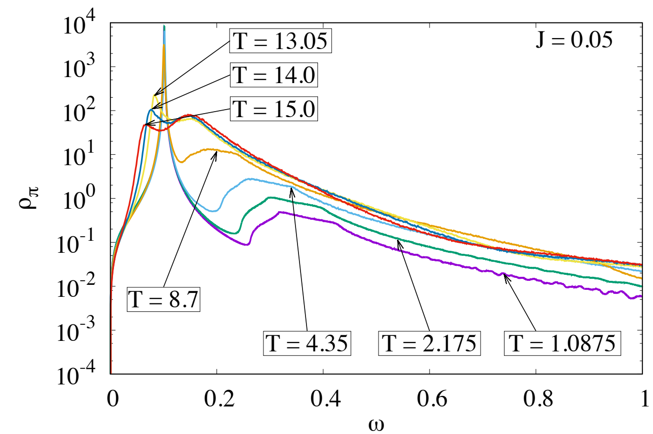
  O(4)-model, in preparation



- **universal critical SFs from classical-statistical simulations**

  O(4)-model, S. Schlichting, D. Smith & Lv.S,
  NPB 950 (2020) 114868

  universal dynamic scaling functions,
  S. Schlichting, D. Schweitzer & Lv.S, NPB 960 (2020) 115165



- **Gaussian-state approximation, real-time FRG…**

Thank you for your attention!