In medium properties and effects of vector mesons from effective field theories: recent advances

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MESON 2021, $17^{\rm th} - 20^{\rm th}$ May 2021

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Supported by the UNKP-20-5 New National Excellence Program of the Ministry for

Innovation and Technology.



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Overview

- 1. Introduction/motivation
- 2. ePQM model Lagrange parameters
- 3. (Axial)vector curvature masses T-dependence of the masses
- 4. N_c scaling Phase boundary
- 5. Hybrid star M R curves Hadron-quark crossover
- 6. Conclusion

Envisaged phase diagram of QCD



Important details of the phase diagram is still unknown (mainly at large baryon density)

Properties of the phase diagram especially at finite baryon densities/baryochemical potential can be well investigated with the help of effective field theories of QCD \rightarrow e.g. details of the phase boundary like existence and location of the CEP, in medium dependence of meson masses, or properties of compact stars etc.

The ePQM model

Lagrangian of the ePQM

 $\mathcal L$ constructed based on linearly realized global $U(3)_L\times U(3)_R$ symmetry and its explicit breaking

$$\begin{split} \mathcal{L} &= \mathrm{Tr}[(D_{\mu}\Phi)^{\dagger}(D_{\mu}\Phi)] - m_{0}^{2}\mathrm{Tr}(\Phi^{\dagger}\Phi) - \lambda_{1}[\mathrm{Tr}(\Phi^{\dagger}\Phi)]^{2} - \lambda_{2}\mathrm{Tr}(\Phi^{\dagger}\Phi)^{2} \\ &+ c_{1}(\det\Phi + \det\Phi^{\dagger}) + \mathrm{Tr}[H(\Phi + \Phi^{\dagger})] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^{2} + R_{\mu\nu}^{2}) \\ &+ \mathrm{Tr}\left[\left(\frac{m_{1}^{2}}{2}\mathbbm{1} + \Delta\right)(L_{\mu}^{2} + R_{\mu}^{2})\right] + i\frac{g_{2}}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^{\mu}, L^{\nu}]\} + \mathrm{Tr}\{R_{\mu\nu}[R^{\mu}, R^{\nu}]\}) \\ &+ \frac{h_{1}}{2}\mathrm{Tr}(\Phi^{\dagger}\Phi)\mathrm{Tr}(L_{\mu}^{2} + R_{\mu}^{2}) + h_{2}\mathrm{Tr}[(L_{\mu}\Phi)^{2} + (\Phi R_{\mu})^{2}] + 2h_{3}\mathrm{Tr}(L_{\mu}\Phi R^{\mu}\Phi^{\dagger}) \\ &+ \bar{\Psi}\left(i\gamma^{\mu}D_{\mu} - g_{F}(S - i\gamma_{5}P)\right))\Psi - g_{V}\bar{\Psi}\left(\gamma^{\mu}(V_{\mu} + \gamma_{5}A_{\mu})\right)\Psi, \end{split}$$

$$\begin{split} \Phi &= S + iP \equiv \sum_{a=0}^{8} (S_{a}\lambda_{a} + iP_{a}\lambda_{a}) \\ D^{\mu}\Phi &= \partial^{\mu}\Phi - ig_{1}(L^{\mu}\Phi - \Phi R^{\mu}) - ieA_{e}^{\mu}[T_{3}, \Phi], \\ L^{\mu\nu} &= \partial^{\mu}L^{\nu} - ieA_{e}^{\mu}[T_{3}, L^{\nu}] - \left\{\partial^{\nu}L^{\mu} - ieA_{e}^{\nu}[T_{3}, L^{\mu}]\right\}, \\ R^{\mu\nu} &= \partial^{\mu}R^{\nu} - ieA_{e}^{\mu}[T_{3}, R^{\nu}] - \left\{\partial^{\nu}R^{\mu} - ieA_{e}^{\nu}[T_{3}, R^{\mu}]\right\}, \\ D^{\mu}\Psi &= \partial^{\mu}\Psi - iG^{\mu}\Psi, \text{ with } G^{\mu} = g_{5}G_{a}^{\mu}T_{a}. \end{split}$$

+ Polyakov loop potential (for T > 0)

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Particle content

• Vector and Axial-vector meson nonets

$$V^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_{N} + \rho^{0}}{\sqrt{2}} & \rho^{+} & K^{*+} \\ \rho^{-} & \frac{\omega_{N} - \rho^{0}}{\sqrt{2}} & K^{*0} \\ K^{*-} & \overline{K^{*0}} & \omega_{5} \end{pmatrix}^{\mu} A^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_{1}^{0}}{\sqrt{2}} & a_{1}^{+} & K_{1}^{+} \\ a_{1}^{-} & \frac{f_{1N} - a_{1}^{0}}{\sqrt{2}} & K_{1}^{0} \\ K_{1}^{-} & \overline{K_{1}^{0}} & f_{15} \end{pmatrix}^{\mu}$$

$$\rho \to \rho(770), K^{*} \to K^{*}(894) \qquad \qquad a_{1} \to a_{1}(1230), K_{1} \to K_{1}(1270) \\ \omega_{N} \to \omega(782), \omega_{5} \to \phi(1020) \qquad \qquad f_{1N} \to f_{1}(1280), f_{1S} \to f_{1}(1426)$$

• Scalar (~ $\bar{q}_i q_j$) and pseudoscalar (~ $\bar{q}_i \gamma_5 q_j$) meson nonets

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_0^{\star +} \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_0^{\star 0} \\ K_0^{\star -} & K_0^{\star 0} & \sigma_S \end{pmatrix} P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & K^0 & \eta_S \end{pmatrix}$$
multiple possible assignments $\pi \to \pi(138) \ K \to K(495)$

multiple possible assignments $\pi \to \pi(138), K \to K(495)$ mixing in the $\sigma_N - \sigma_S$ sectormixing: $\eta_N, \eta_S \to \eta(548), \eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$ fields shifted by their expectation value: $\sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S}$

In case of compact stars, also vector condensates (see later)

Determination of the parameters

14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_5, \Phi_N, \Phi_5, g_F) \longrightarrow$ determined by the min. of χ^2 :

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^M \left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\exp}}{\delta Q_i} \right]^2,$$

 $(x_1,\ldots,x_N) = (m_0,\lambda_1,\lambda_2,\ldots), Q_i(x_1,\ldots,x_N) \longrightarrow \text{from the model}, Q_i^{\exp} \longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$ multiparametric minimalization \longrightarrow MINUIT

▶ PCAC \rightarrow 2 physical quantities: f_{π}, f_{K}

• Curvature masses \rightarrow 16 physical quantities: $m_{u/d}, m_s, m_{\pi}, m_{\eta}, m_{\eta'}, m_K, m_{\rho}, m_{\Phi}, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}, m_{H_1^H}, m_{H_1^H$

• Decay widths \rightarrow 12 physical quantities: $\Gamma_{\rho \to \pi\pi}, \Gamma_{\Phi \to KK}, \Gamma_{K^{\star} \to K\pi}, \Gamma_{a_1 \to \pi\gamma}, \Gamma_{a_1 \to \rho\pi}, \Gamma_{f_1 \to KK^{\star}}, \Gamma_{a_0}, \Gamma_{K_S \to K\pi},$ $\Gamma_{f_{L}^{L} \to \pi\pi}, \Gamma_{f_{L}^{L} \to KK}, \Gamma_{f_{L}^{H} \to \pi\pi}, \Gamma_{f_{L}^{H} \to KK}$

▶ Pseudocritical temperature T_c at $\mu_B = 0$

Field equation

Four coupled field equations are obtained by extremizing the grand potential $\Omega(T,\mu_q) = U_{\mathrm{meson}}^{\mathrm{tree}}(\langle M \rangle) + \Omega_{\bar{a}a}^{(\mathbf{0})\mathrm{vac}} + \Omega_{\bar{a}a}^{(\mathbf{0})T}(T,\mu_q) + \mathcal{U}_{\mathrm{log}}(\Phi,\bar{\Phi})$ using $\frac{\partial \Omega}{\partial \phi_{\mu}} = \frac{\partial \Omega}{\partial \phi_{\mu}} = \frac{\partial \Omega}{\partial \phi} = \frac{\partial \Omega}{\partial \phi} = 0$ $E_{f}^{\pm}(p) = E_{f}(p) \mp \mu_{q}, \quad E_{f}^{2}(p) = p^{2} + m_{f}^{2}$ 1) $-\frac{1}{T^4} \frac{d U(\Phi, \bar{\Phi})}{d\Phi} + \frac{6}{T^3} \sum_{f=u,d,c} \int \frac{d^3 p}{(2\pi)^3} \left(\frac{e^{-\beta E_f^-(p)}}{g_f^-(p)} + \frac{e^{-2\beta E_f^+(p)}}{g_f^+(p)} \right) = 0$ 2) $-\frac{1}{T^4} \frac{dU(\Phi,\bar{\Phi})}{d\bar{\Phi}} + \frac{6}{T^3} \sum_{e,\dots,d_r} \int \frac{d^3p}{(2\pi)^3} \left(\frac{e^{-\beta E_f^+(p)}}{g_f^+(p)} + \frac{e^{-2\beta E_f^-(p)}}{g_e^-(p)} \right) = 0$ 3) $m_0^2 \phi_N + \left(\lambda_1 + \frac{\lambda_2}{2}\right) \phi_N^3 + \lambda_1 \phi_N \phi_S^2 - \frac{c_1}{\sqrt{2}} \phi_N \phi_S - h_{0N} + \frac{3}{2} g_F \left(\langle \bar{q}_u q_u \rangle_T + \langle \bar{q}_d q_d \rangle_T\right) = 0$ 4) $m_0^2 \phi_S + (\lambda_1 + \lambda_2) \phi_S^3 + \lambda_1 \phi_N^2 \phi_S - \frac{\sqrt{2}}{4} c_1 \phi_N^2 - h_{0S} + \frac{3}{\sqrt{2}} g_F \langle \bar{q}_S q_S \rangle_T = 0$ renormalized fermion tadpole: $m_{u,d} = \frac{g_F}{2} \phi_N$ and $m_s = \frac{g_F}{\sqrt{2}} \phi_S$

 $\langle \bar{q}_f q_f \rangle_T = 4m_f \left[-\frac{m_f^2}{16\pi^2} \left(\frac{1}{2} + \ln \frac{m_f^2}{M_0^2} \right) + \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_f(p)} (f_f^-(p) + f_f^+(p)) \right]$

Axial(vector) curvature masses

Curvature masses

$$\Delta \hat{m}_{ab}^{2,(X)} \equiv \frac{d^2 U_f(\phi,\xi)}{dX_a dX_b} \bigg|_{\xi=0}, \quad X \in \{S, P\}$$

$$\Delta \hat{M}_{\mu\nu,ab}^{2,(Y)} \equiv -\frac{d^2 U_f(\phi,\xi)}{dY_a^{\mu} dY_b^{\nu}} \bigg|_{\xi=0}, \quad Y \in \{V, A\}, \quad \xi \in \{X_a, Y_a\}, \quad \phi \in \{\phi_N, \phi_S, \Phi, \bar{\Phi} \dots\}$$

where U_f is the fermionic contribution to the effective potential,

$$U_f(\phi, \xi) = i \operatorname{Tr}_D \int_{\mathcal{K}} \log(i \mathcal{S}^{-1}(\mathcal{K};\xi) \big|_{\xi=0} - \frac{i}{2} \operatorname{Tr} \int_{\mathcal{K}} \log\left(i \mathcal{D}_{(\mu\nu),ab}^{-1}(\mathcal{K}) - \Pi_{(\mu\nu),ab}(\mathcal{K})\right)$$

On the other hand: The curvature masses are the one-loop self-energies at vanishing momentum:

$$\Pi_{ab}^{(V/A)\mu\nu}(Q) = i2N_c g_V^2 \int_K \frac{g^{\mu\nu}(\pm m_a m_b - K^2 + K \cdot Q) + 2K^{\mu}K^{\nu} - K^{\mu}Q^{\nu} - Q^{\mu}K^{\nu}}{(K^2 - m_a^2)((K - Q)^2 - m_b^2)}$$
(1)

- At $T = 0 \rightarrow$ vacuum self-energy \rightarrow renormalization \rightarrow dimensional regularization
- At $T \neq 0$ \longrightarrow matter part (with statistical function) \rightarrow Wick rotation, Matsubara sum, $\int_{K} \rightarrow iT \sum_{-} \int \frac{d^{3}k}{(2\pi)^{3}}$

Mode decomposition

Vacuum contribution: $\Pi_{\text{vac}}^{\mu\nu}(Q) = \Pi_{\text{vac},L}(Q)P_L^{\mu\nu} + \Pi_{\text{vac},T}(Q)P_T^{\mu\nu}$ **Thermal** contribution: $\Pi^{\mu\nu}(Q) = \sum_{x=l,t,L} \Pi_x(Q)P_x^{\mu\nu} + \Pi_C(Q)C^{\mu\nu}$ Where the 4-long./transv., 3-long./transv. projectors and C are ([1,2])

$$\begin{split} P_L^{\mu\nu} &= \frac{Q^{\nu}Q^{\mu}}{Q^2}, \ P_T^{\mu\nu} = g^{\mu\nu} - P_L^{\mu\nu} \\ P_l^{\mu\nu} &= \frac{u_T^{\mu}u_T^{\nu}}{u_T^2}, \ P_t^{\mu\nu} = g^{\mu\nu} - P_L^{\mu\nu} - P_l^{\mu\nu}, \ C^{\mu\nu} = \frac{Q^{\mu}u_T^{\nu} + Q^{\nu}u_T^{\mu}}{\sqrt{(Q \cdot u)^2 - Q^2}}, \ u_T^{\mu} = u^{\mu} - (Q \cdot u)Q^{\mu}/Q^2 \end{split}$$

(Pseudo)scalar curvature masses
 Tree-level

$$\begin{array}{ccc} \text{Tree-level} & T = 0 & T \neq 0 \\ \hat{m}^2 & \longrightarrow & \hat{M}^2 = \hat{m}^2 + \Pi_{\text{vac}}(0) & + & \Pi_{\text{mat}}(0) \end{array}$$

(Axial) vector curvature masses

 $\begin{array}{ll} \text{Tree-level} & \text{Fermion correction} \\ \hat{m}^2 = \hat{m}_L^2 = \hat{m}_T^2 & \xrightarrow[T \neq 0]{T \neq 0} & \hat{M}_{\text{vac}}^2 = \hat{M}_{\text{vac},L/T}^2 = \hat{m}_{L/T}^2 + \Pi_{\text{vac},L/T}(0) \\ & \xrightarrow[T \neq 0]{T \neq 0} & \hat{M}_{L/I/t}^2 = \hat{m}_{L/I/t}^2 + \Pi_{L/I/t}(0) \end{array}$

[1] M. Le Bellac, Thermal Field Theory, (1996) Cambridge University Press

[2] Buchmuller et al Nucl. Phys. B 407, 387-411 (1993)

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T dependence of (pseudo)scalar masses



From the χ^2 fit the vector coupling: $g_v\approx 5$

${\cal T}$ dependence of (axial)vector masses



From the χ^2 fit the vector coupling: $g_v\approx 5$

Large N_c thermodynamics

$N_{\boldsymbol{C}}$ scaling

Some properties of mesons and baryons for Large N_c

• $g_{QCD} \sim \frac{1}{\sqrt{N_c}}$

- ▶ quark loops are $1/N_c$ suppressed
- leading diagrams are palanar diagrams with minimum number of quark loops
- mesons are free, stable, and non-interacting
- ▶ mesons are pure $q\bar{q}$ for Large N_c
- ▶ meson masses ~ N_c^0
- ▶ meson decay amplitudes $\sim 1/\sqrt{N_c}$
- ▶ for one meson creation: $< 0|J|m > \sim \sqrt{N_c}$
- ▶ k meson vertex ~ $N_c^{1-k/2}$. Specifically, the three- and four-meson vertices are ~ $1/\sqrt{N_c}$ and ~ $1/N_c$, respectively
- ▶ baryon masses ~ N_c . Consequently constituent quark masses ~ N_c^0

G. 't Hooft. (1974), Nucl. Phys. B 72:461

G. 't Hooft. (1974), Nucl. Phys. B 75:461-470

E. Witten. (1979), Nucl. Phys. B 160:57-115

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N_c scaling of the Lagrange parameters

The parameters are: $m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_5, \Phi_N, \Phi_5, g_F, h_N, h_S$

- m_0^2 , m_1^2 , $\delta_s \sim N_c^0$, because terms of tree level meson masses
- $g_1, g_2 \sim \frac{1}{\sqrt{N_c}}$, three couplings
- λ_2 , h_2 , $h_3 \sim \frac{1}{N_c}$, four couplings
- $\lambda_1, h_1 \sim \frac{1}{N_2^2}$, four couplings with different trace structure
- $c_1 \sim \frac{1}{N_c^{3/2}}~U_A(1)$ anomaly term has extra $1/N_c$ suppression

•
$$\Phi_{N/S} \sim \sqrt{N_c}, \ \Phi_N = Z_\pi f_\pi, \ f_\pi \sim \sqrt{N_c}$$

• $h_{N/S} \sim \sqrt{N_c}$, Goldstone-theorem: $m_\pi^2 \Phi_N = Z_\pi^2 h_N$

•
$$g_F \sim rac{1}{\sqrt{N_c}}, \ m_{u/d} = g_F \Phi_N$$

practically: $g_1 \longrightarrow g_1 \sqrt{\frac{3}{N_c}}, \Phi_{N/S} \longrightarrow \Phi_{N/S} \sqrt{\frac{N_c}{3}} \dots etc.$

Parameter sets

For lower $m_{\sigma} = 600$ MeV

Φ _N	0.092
Φ _S	0.095
m_0^2	-0.036
m_1^2	0.395
λ_1	-17.01
λ_2	82.47
h1	-9.0
h ₂	11.659
h ₃	4.703
δ_{S}	0.153
<i>C</i> 1	0.0
g1	-5.894
g ₂	-2.996
gг	6.494

For higher $m_{\sigma} = 1300$ MeV

Φ_N	0.162
Φ ₅	0.124
m_0^2	-0.754
m_{1}^{2}	0.395
λ_1	0.0
λ_2	65.322
h_1	0.0
h ₂	11.659
h ₃	4.703
δ_S	0.153
<i>C</i> 1	1.121
g 1	-5.894
g ₂	-2.996
<i>g</i> F	4.943

Field equations, masses in Large N_c

Field equations:

$$\begin{split} m_0^2 \Phi_N \sqrt{\frac{N_c}{3}} + \left(\lambda_1 \frac{3}{N_c} + \frac{\lambda_2}{2}\right) \Phi_N^3 \sqrt{\frac{N_c}{3}} + \lambda_1 \Phi_N \Phi_S^2 \sqrt{\frac{3}{N_c}} - \frac{1}{\sqrt{2}} c_1 \sqrt{\frac{3}{N_c}} \Phi_N \Phi_S \\ - h_N \sqrt{\frac{N_c}{3}} + \frac{3}{2} g_F (\operatorname{Tad}_u + \operatorname{Tad}_d) = 0 \end{split}$$

$$\begin{split} m_0^2 \Phi_5 \sqrt{\frac{N_c}{3}} + \left(\lambda_1 \frac{3}{N_c} + \lambda_2\right) \Phi_5^3 \sqrt{\frac{N_c}{3}} + \lambda_1 \Phi_N^2 \Phi_5 \sqrt{\frac{3}{N_c}} - \frac{1}{2\sqrt{2}} c_1 \sqrt{\frac{3}{N_c}} \Phi_N^2 \\ - h_5 \sqrt{\frac{N_c}{3}} + \frac{3}{\sqrt{2}} g_F \operatorname{Tad}_s = 0 \end{split}$$

Pion three-level mass:

$$m_{\pi}^{2} = Z_{\pi}^{2} \left(m_{0}^{2} + \left(\lambda_{1} \frac{3}{N_{c}} + \frac{\lambda_{2}}{2} \right) \Phi_{N}^{2} + \lambda_{1} \frac{3}{N_{c}} \Phi_{S}^{2} - c_{1} \frac{3}{N_{c}} \frac{\Phi_{S}}{\sqrt{2}} \right)$$

$\phi_N(T)$ at diff. N_c values ($\mu_B = 0, m_\sigma = 1300$ MeV)



N_c scaling of the pseudocrit. T_c ($m_\sigma = 1300$ MeV)



From $N_c=3$ to $N_c=100{\rm :}~T_c$ changes $\approx 4\%$

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N_c scaling of the pseudocrit. T_c ($m_\sigma = 600$ MeV)



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N_c scaling of $h_{N/S}$ ($m_\sigma = 1300$ MeV)



The expected N_c scaling $(h_{N/S}\sim \sqrt{N_c}$ are calculated from the field equations at $T=\mu_B=0)$

N_c scaling of meson masses ($m_\sigma = 1300$ MeV)



N_c scaling of meson masses ($m_\sigma = 600$ MeV)



Phase boundary at diff. N_c 's ($m_\sigma = 600$ MeV)



T_c scaling at different μ_B 's



For both low and high μ_B : T_c scales as $\sim N_c^0$. What happens if there is a CEP at large N_c ?

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M - R curves of hybrid stars

Structure of compact stars



Fig. from F. Weber, J. Phys. G 27, 465 (2001)

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Various M - R curves for different compact star EoS's

- QCD directly unsolvable at finite density
- One can use effective models in the zero temperature finite density region
- Neutron star observations restrict such models [1,2]



- [1] P. Demorest et al. (2010), Nature 467, 1081
- [2] J. Antoniadis et al. (2013), Science 340, 6131

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Elements of the quark EOS

For large density e(P)QM model is used, but with additional vector condensates

- ▶ non-zero vector condensates: $\langle (\rho^0)^0 \rangle = \phi_{\rho}, \langle (\omega)^0 \rangle = \phi_{\omega}, \langle (\Phi)^0 \rangle = \phi_{\Phi}$
- ▶ free electron gas + β -equilibrium
- ▶ modified chemical potentials:

$$\mu_u = \mu_q - \frac{2}{3}\mu_e - \frac{1}{2}g_V(\phi_\omega + \phi_\rho)$$
$$\mu_d = \mu_q + \frac{1}{3}\mu_e - \frac{1}{2}g_V(\phi_\omega - \phi_\rho)$$
$$\mu_s = \mu_q + \frac{1}{3}\mu_e - \frac{1}{\sqrt{2}}g_V\phi_\Phi$$

charge neutrality: ²/₃n_u - ¹/₃n_d - ¹/₃n_s - n_e = 0
 field equations:

$$\frac{\partial\Omega_{tot}}{\partial\phi_{N}} = \frac{\partial\Omega_{tot}}{\partial\phi_{S}} = \frac{\partial\Omega_{tot}}{\partial\phi_{\rho}} = \frac{\partial\Omega_{tot}}{\partial\phi_{\omega}} = \frac{\partial\Omega_{tot}}{\partial\phi_{\Phi}} = 0 \quad \rightarrow p(\varepsilon) \text{curve}$$

Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter \longrightarrow TOV eqs.:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{\left[p(r) + \varepsilon(r)\right]\left[M(r) + 4\pi r^3 p(r)\right]}{r[r - 2M(r)]} \tag{2}$$

with

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \varepsilon(r)$$

These are integrated numerically for a specific $p(\varepsilon)$

- ▶ For a fixed ε_c central energy density Eq. (2) is integrated until p = 0
- ▶ Varying ε_c a series of compact stars is obtained (with given *M* and *R*)
- ▶ Once the maximal mass is reached, the stable series of compact stars ends

Schematic picture of pressure (H-Q crossover)



In the crossover region hadrons starts to overlap \longrightarrow both low and high ρ_B models loose their validity. Gibbs condition (extrapolation from the dashed lines) can be misleading.

Hadron-quark crossover with P-interpolation

Features of the model:

- H-EOS: Relativistic Mean Field (RMF) models: Steiner-Fischer-Hempel (SFHo) [1], density-dependent RMF model (DD2) [2]
- \blacktriangleright Q-EOS: e(P)QM model with u, d, s quarks and vector interaction
- ▶ mean-field approximation
- *P*-interpolation ($\rho = \rho_B$):

$$P(\rho) = P_{H}(\rho)f_{-}(\rho) + P_{Q}(\rho)f_{+}(\rho), \qquad (3)$$

$$\overline{f}_{\pm}(\rho) = \frac{1}{2} \left(1 \pm \tanh\left(\frac{\rho - \overline{\rho}}{\Gamma}\right) \right)$$
(4)

$$\varepsilon(\rho) = \varepsilon_{H}(\rho)f_{-}(\rho) + \varepsilon_{Q}(\rho)f_{+}(\rho) + \Delta\varepsilon$$
(5)

$$\Delta \varepsilon = \rho \int_{\bar{\rho}}^{\rho} (\varepsilon_{\mathcal{H}}(\rho') - \varepsilon_{\mathcal{Q}}(\rho')) \frac{g(\rho')}{\rho'} d\rho'$$
(6)

[1]A. W. Steiner et al., Astrophys.J.774, 17 (2013)

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 [2]S. Typel et al., Phys. Rev. C81, 015803 (2010)
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M - R relations of BPS+SFHo+e(P)QM



 $\bar{\rho} = 3\rho_0, \Gamma = \rho_0$, From (axial)vector curvature masses: $g_V \approx 5$

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Conclusion

Conclusion

- ePQM model can be used for various in medium investigations
- (Axial)vector meson curvature masses were calculated at one-loop level
- ▶ Parameterization determines g_V
- ▶ Large N_c scaling was also investigated
- ▶ Hybrid star M R curves were calculated and curves compatible with current observations

Plans

- Solve the model in the Gaussian approximation
- Determine the phase boundary and the CEP
- ▶ Investigate the Large N_c scaling of the CEP if it exists
- ▶ Beside the M R curves also calculate tidal deformabilities
- Consequences of the g_V determination on the compact star properties

Thank you for your attention!