The interplay of axial mesons and short-distance constraints in $(g-2)_\mu$

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Based on arXiv:2005.11761, in Coll. with P. Masjuan & P. Roig

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Section 1

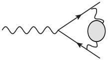
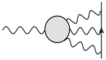
Motivation

Motivation

- The $a_\mu = (g-2)_\mu/2$ is a sensitive probe of new physics

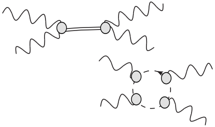
$$a_\mu^{\rm th} = 116591810(43) \times 10^{-11} \text{ vs. } a_\mu^{\rm exp} = 116592061(41) \times 10^{-11};$$
$$a_\mu^{\rm exp} - a_\mu^{\rm th} = 279(76) \times 10^{-11} \quad (4.2\sigma)$$

- New experiment at FNAL $\Delta a_\mu^{\rm exp} = 16 \times 10^{-11} \Rightarrow$ **Needs th error reduction!!**


$a_\mu^{\rm HVP} = 6845(40) \times 10^{-11}$

$a_\mu^{\rm HLbL} = 92(18) \times 10^{-11}$

- Among the dominant contributions to **HLbL** (10^{-11} units)[1]

$\pi, \eta, \eta^{(\prime)}$	$\pi\pi$+S-wave	....	Axials	SD
94(4)	$-[15.9(2) + 8(1)]$	...	6(6)	15(10)



- As we shall see, nice interrelation among axials and SD!

[1] Aoyama et al., Phys.Rept. 887 (2020) 1-166

Outline

- $A \to \gamma^* \gamma^*$ form factors
- Short-distance constraints: MV's limit
- Implications for axials contributions to $\langle VVA \rangle$ and HLbL

Section 2

$A \to \gamma^*\gamma^*$ form factors

$A \to \gamma^*\gamma^*$ form factors I

- The axial-vector form factors defined as[2] $(q_{12} \equiv q_1 + q_2, \quad \bar{q}_{12} \equiv q_1 - q_2)$

$$i \int d^4x e^{iq_1 \cdot x} \langle 0| T\{j^\mu(x) j^\nu(0)\} |A(q_{12})\rangle = \mathcal{M}_A^{\mu\nu\rho}(q_1, q_2)\varepsilon_{A\rho}$$

- Lorentz, C, P, T, and Bose symm., 8 form factors $[\bar{X}(q_1^2, q_2^2) = X(q_2^2, q_1^2)]$

$$\mathcal{M}_A^{\mu\nu\rho} = i\epsilon^{\mu\nu\rho}{}_\alpha(q_1^\alpha A_1 - q_2^\alpha \bar{A}_1) + i\epsilon^{q_1 q_2 \alpha\rho}[g_\alpha^\mu(q_1^\nu B_1 + q_2^\nu B_2) + g_\alpha^\mu(q_2^\mu \bar{B}_1 + q_1^\mu \bar{B}_2)] + i\epsilon^{\mu\nu q_1 q_2}[q_1^\rho C + q_2^\rho \bar{C}]$$

- Gauge invariance: 2 additional constraints

$$q_{1(2)}^{\mu(\nu)}\mathcal{M}_{A\mu\nu\rho} = 0 \Rightarrow A + (q_1 \cdot q_2)B_1 + q_2^2 B_2 = \bar{A} + (q_1 \cdot q_2)\bar{B}_1 + q_1^2 \bar{B}_2 = 0$$

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$$\epsilon^{\mu\nu q_1 q_2} q_1^\rho = \epsilon^{\rho\nu q_1 q_2} q_1^\mu + \epsilon^{\mu\rho q_1 q_2} q_1^\nu + \epsilon^{\mu\nu\rho q_2} q_1^2 + \epsilon^{\mu\nu q_1 \rho}(q_1 \cdot q_2) + \binom{\mu \leftrightarrow \nu}{q_1 \leftrightarrow q_2}$$

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[2] P. Roig, P. SP, Phys.Rev.D 101 (2020) 7, 074019

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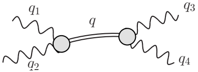
- However, when off-shell, dismissing C_S cumbersome



$$\mathcal{M}_A^{\mu\nu\alpha}\, i\frac{-g_{\alpha\beta}+\frac{q_\alpha q_\beta}{m_A^2}}{q^2-m_A^2}\mathcal{M}_A^{\rho\sigma\beta} \supset -i\epsilon^{\mu\nu q_1 q_2}\epsilon^{\rho\sigma q_3 q_4}\frac{q^2}{m_A^2}C_S \neq 0$$

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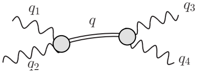

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- Noteworthy if comparing different (uses of Schouten id. to get a) basis (PPVdH PRD 2012 & M. Knecht, JHEP 2020)

$$B_2 \sim [\bar{F}_A^1 + \frac{q_1^2}{q_1\cdot q_2}F_A^1], \quad \bar{B}_2 \sim [F_A^1 + \frac{q_2^2}{q_1\cdot q_2}\bar{F}_A^1], \quad C_A = \frac{(q_1\cdot q_2)(q_1^2-q_2^2)}{(q_1\cdot q_2)^2-q_1^2q_2^2}F_A^{(0)}, \quad C_S = \frac{(q_1\cdot q_2)\bar{q}_{12}^2}{(q_1\cdot q_2)^2-q_1^2q_2^2}F_A^{(0)}$$

$$B_{2S} = \frac{B_2+\bar{B}_2}{2} = -W_2, \quad B_{2A} = \frac{B_2-\bar{B}_2}{2} = W_3, \quad C_A = W_1, \quad C_S = W_2$$

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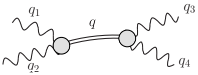

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- Results cannot depend on each theorists' taste!

Is there a sensible way to fix this?? SDCs!

Section 3

Short-distance constraints: MV's limit

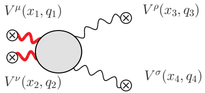
Short-Distance Constraints (I)

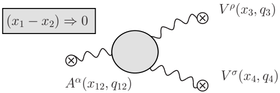
- Melnikhov-Vainstein's OPE[1]

In the limit $q_{1,2}^2\ (q_1-q_2)^2 \gg q_{3,4,12}^2$

$$\int ... e^{i(q_1\cdot x + q_2\cdot x + q_3 \cdot x)} \langle 0 | T\{V^\mu(x_1) V^\nu(x_2) V^\rho(x_3) V^\sigma(0)\} | 0 \rangle$$
$$\to \frac{4i\epsilon^{\mu\nu\alpha(q_1-q_2)}}{(q_1-q_2)^2} \int ... e^{i(q_{12}\cdot x_{12} + q_3\cdot z)} \langle 0 | T\{A^\alpha(x_{12}) V^\rho(z) V^\sigma(0)\} | 0 \rangle$$

Relates HLbL to the $\langle VVA \rangle$ Green's function!





[1] K. Melnikov, A. Vainshtein, Phys.Rev.D 70 (2004)

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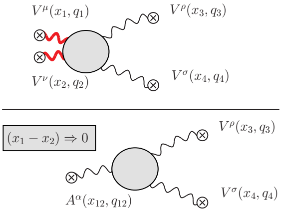

- For $m_q \to 0$, anomaly $\langle VV\partial A \rangle$ an exact result $\Rightarrow$ Confront it for π^0 models!!

$$q_{12\rho}\left(q_{12}^\rho F_\pi \frac{1}{q_{12}^2} \frac{\epsilon^{\mu\nu q_1 q_2}}{4\pi^2 F_\pi} \tilde{F}_{P\gamma^*\gamma^*}(q_1^2,q_2^2) \right) = \epsilon^{\mu\nu q_1 q_2} \frac{1}{4\pi^2} \tilde{F}_{P\gamma^*\gamma^*}(q_1^2,q_2^2)$$

$$q_{12\rho}\langle V^\mu(q_1)V^\nu(q_2)A^{\rho,3}(q_{12})\rangle = \epsilon^{\mu\nu q_1 q_2}\frac{1}{4\pi^2}$$

- $\tilde{F}_{P\gamma^*\gamma^*}(q_1^2,q_2^2)$ spoils for virtual photons (motivating many models), but ...

[1] K. Melnikov, A. Vainshtein, Phys.Rev.D 70 (2004)

Short-Distance Constraints (II)

- ... let's have a closer look to $\langle VVA\rangle$ (only $w_L = \frac{2N_c}{q_{\mathbf{12}}^2}$ contributes to anomaly)

$$\langle V^\mu(q_1)V^\nu(q_2)A^\rho(q_{12})\rangle = \frac{-1}{8\pi^2}\left[-\epsilon_{\mu\nu\mathbf{q_1}\mathbf{q_2}}q_{12\rho}w_L + t^{(+)}_{\mu\nu\rho}w_T^{(+)} + t^{(-)}_{\mu\nu\rho}w_T^{(-)} + \tilde{t}^{(-)}_{\mu\nu\rho}\tilde{w}_T^{(-)}\right]$$

$$t^{(+)}_{\mu\nu\rho} = \epsilon^{\mathbf{q_1}\mathbf{q_2}\mu\rho}q_1^\nu + \epsilon^{\mathbf{q_2}\mathbf{q_1}\nu\rho}q_2^\mu - (q_1\cdot q_2)\epsilon^{\mu\nu\rho\bar{\mathbf{q}}_{\mathbf{12}}} + \frac{q_1^2+q_2^2-q_{12}^2}{q_{12}^2}\epsilon^{\mu\nu\mathbf{q_1}\mathbf{q_2}}q_{12}^\rho,$$

$$t^{(-)}_{\mu\nu\rho} = \epsilon^{\mu\nu\mathbf{q_1}\mathbf{q_2}}\left[\bar{q}_{12}^\rho - \frac{q_1^2-q_2^2}{q_{12}^2}q_{12}^\rho\right], \tilde{t}^{(-)}_{\mu\nu\rho} = \epsilon^{\mathbf{q_1}\mathbf{q_2}\mu\rho}q_1^\nu - \epsilon^{\mathbf{q_2}\mathbf{q_1}\nu\rho}q_2^\mu - (q_1\cdot q_2)\epsilon^{\mu\nu\rho\mathbf{q_{12}}},$$

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$$\langle V^\mu(q_1) V^\nu(q_2) A^\rho(q_{12}) \rangle = \frac{-1}{8\pi^2} \left[-\epsilon_{\mu\nu q_1 q_2} q_{12\rho} w_L + t^{(+)}_{\mu\nu\rho} w_T^{(+)} + t^{(-)}_{\mu\nu\rho} w_T^{(-)} + \tilde{t}^{(-)}_{\mu\nu\rho} \tilde{w}_T^{(-)} \right]$$

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- But $\langle VVA \rangle$ singularities at $q_{12}^2 = 0$ should only correspond to pGBs!

$$\lim_{q_{12}^2 \to 0} \langle V^\mu V^\nu A^\rho \rangle = \lim_{q_{12}^2 \to 0} \frac{\epsilon^{\mu\nu q_1 q_2} q_{12}^\rho}{8\pi^2}\left[\frac{\mathrm{Res}(w_L)|_{q_{12}^2=0}}{q_{12}^2} - \frac{q_1^2 + q_2^2}{q_{12}^2} w_T^{(+)} + \frac{q_1^2 - q_2^2}{q_{12}^2} w_T^{(-)} \right] = w_L|_{pGB}$$

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$$w_L = w_L|_{\mathrm{pGB}} + \frac{q_1^2+q_2^2}{q_{12}^2} w_{T0}^{(+)} - \frac{q_1^2-q_2^2}{q_{12}^2} w_{T0}^{(-)}, \quad \lim_{q_{12}^2 \to 0} w_T^{(\pm)}(q_1^2, q_2^2, q_{12}^2) \equiv w_{T0}^{(\pm)}(q_1^2, q_2^2)$$

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- Implying a "sum rule" for transverse contributions

$$(q_1^2 + q_2^2) w_{T0}^{(+)}(q_1^2, q_2^2) - (q_1^2 - q_2^2) w_{T0}^{(-)}(q_1^2, q_2^2) = 2N_c[1 - \tilde{F}_{P\gamma\gamma}(q_1^2, q_2^2)]$$

Section 4

Implications for axials contributions to $\langle VVA \rangle$ and HLbL

Implications for axials contributions (I): $\langle VVA\rangle$

- The axial-vector form factor:

$$\mathcal{M}_A^{\mu\nu\rho} = i\epsilon^{\mu\alpha\tau q_1}(q_{2\alpha}q_2^\nu - g_\alpha^\nu q_2^2)B_2 + i\epsilon^{\nu\alpha\tau q_2}(q_{1\alpha}q_1^\mu - g_\alpha^\mu q_1^2)\bar{B}_2 + i\epsilon^{\mu\nu q_1 q_2}[\bar{q}_{12}^\tau C_A + q_{12}^\tau \not{C}_S]$$

- Contribution to $\langle VVA\rangle$

$$\frac{\{w_T^{(+)}, w_T^{(-)}, \tilde{w}_T^{(-)}\}}{8\pi^2} = \frac{\{B_{2S}, B_{2A}-C_A, -B_{2A}\}}{q_{12}^2 - m_A^2} m_A F_A^a, \quad \frac{w_L}{8\pi^2} = -[C_S + \frac{q_1^2+q_2^2}{q_{12}^2}B_{2S} - \frac{q_1^2-q_2^2}{q_{12}^2}(B_{2A}-C_A)]\frac{F_A^a}{m_A},$$

- In a model with pGB and axials (heavy π's decouple in chiral limit)

$$\frac{w_L}{8\pi^2} = \frac{N_c\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)}{4\pi^2}\frac{\tilde{F}_{P\gamma\gamma}(q_1^2,q_2^2)}{q_{12}^2 - m_{\mathrm{pGB}}^2} - \sum_A \frac{F_A^a}{m_A}\left[C_S + \frac{q_1^2+q_2^2}{q_{12}^2}B_{2S} - \frac{q_1^2-q_2^2}{q_{12}^2}(B_{2A}-C_A)\right] = \frac{N_c\,\mathrm{tr}(\mathcal{Q}^2\lambda^a)}{4\pi^2 q_{12}^2},$$

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$$\frac{\{w_T^{(+)}, w_T^{(-)}, \tilde{w}_T^{(-)}\}}{8\pi^2} = \frac{\{B_{2S}, B_{2A}-C_A, -B_{2A}\}}{q_{12}^2 - m_A^2} m_A F_A^a, \quad \frac{w_L}{8\pi^2} = -[C_S + \frac{q_1^2+q_2^2}{q_{12}^2}B_{2S} - \frac{q_1^2-q_2^2}{q_{12}^2}(B_{2A}-C_A)]\frac{F_A^a}{m_A},$$

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- To fulfill the anomaly $C_S = 0$! Other basis require additional form factors!

$$B_{2S} = \frac{B_2+\bar{B}_2}{2} = -W_2, \quad B_{2A} = \frac{B_2-\bar{B}_2}{2} = W_3, \quad C_A = W_1, \quad C_S = W_0 + W_2 = W_0 - B_{2S}$$

Anomaly fixes ambiguities! In our choice $C_S = 0$, but above $W_0 = -W_2 \neq 0$

Implications for axials contributions (II): HLbL

- Apply OPE for $\int d^4x e^{iq\cdot x} T\{j^\mu(x) j^\nu(0)\} = \frac{-4}{\hat{q}^2}\epsilon^{\mu\nu\alpha(q_1-q_2)} \int d^4z e^{iq_{12}\cdot z} j_{5\alpha}(z)$

$$i\int d^4x e^{iq_1\cdot x} \langle 0|\, T\{j^\mu(x) j^\nu(0)\}\,|A(q_{12})\rangle = \mathcal{M}_A^{\mu\nu\rho}(q_1,q_2)\varepsilon_{A\rho}$$

$$\Rightarrow \frac{1}{\hat{q}^2}\langle 0|\, j_5^\rho \,|A\rangle \equiv \sum_a \frac{m_A F_A^a}{\hat{q}^2}\varepsilon_A^\rho \,\mathrm{tr}\, \mathcal{Q}^2\lambda^a = \lim_{\hat{q}^2\to\infty} \hat{q}^2 B_{2S}(\hat{q}^2,\hat{q}^2)\varepsilon_A^\rho$$

[3] J. Leutgeb, A. Rebhan, Phys.Rev.D 101 (2020) 11; L. Capiello, et al., Phys.Rev.D 102 (2020) 1
[4] Aoyama et al., Phys.Rept. 887 (2020) 1-166

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- This implies $B_{2S}(Q^2,Q^2) \sim Q^{-4}$, while it is known that $B_{2S}(Q^2,0) \sim Q^{-4}$
This was missing in existing estimates considering B_{2S}!

$$B_{2S}^{\mathrm{Fact}}(q_1^2,q_2^2) = \frac{B_{2S}(0,0)\Lambda^8}{(q_1^2-\Lambda^2)^2(q_2^2-\Lambda^2)^2} \Rightarrow B_{2S}^{\mathrm{OPE}}(q_1^2,q_2^2) = \frac{B_{2S}(0,0)\Lambda^4}{(q_1^2+q_2^2-\Lambda^2)^2}$$

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$$\Rightarrow \frac{1}{\hat{q}^2}\langle 0| j_5^\rho |A\rangle \equiv \sum_a \frac{m_A F_A^a}{\hat{q}^2}\varepsilon_A^\rho \operatorname{tr} \mathcal{Q}^2\lambda^a = \lim_{\hat{q}^2\to\infty} \hat{q}^2 B_{2S}(\hat{q}^2,\hat{q}^2)\varepsilon_A^\rho$$

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	f_1	f_1'	a_1	Total
Fact	$4.3(^{+1.8}_{-1.5})$	$1.2(^{+0.6}_{-0.5})$	$2.8(^{+1.9}_{-1.7})$	$8.3(^{+2.7}_{-3.4})$
OPE	$8.3(^{+3.4}_{-2.9})$	$2.3(^{+1.1}_{-0.9})$	$5.4(^{+3.7}_{-3.3})$	$16.0(^{+5.1}_{-4.5})$

- In line with holographic models;[3] compare to WP[4] $a_\mu^{\mathrm{HLbL};A} = 6(6)\times 10^{-11}$

[3] J. Leutgeb, A. Rebhan, Phys.Rev.D 101 (2020) 11; L. Capiello, et al., Phys.Rev.D 102 (2020) 1
[4] Aoyama et al., Phys.Rept. 887 (2020) 1-166

Implications for axials contributions (III): HLbL

- Relevance of basis independence: basis as in M. Knecht JHEP 2020

$$\epsilon^{\mu\nu q_1 q_2} q_{12}^{\rho} \cancel{W_0} + \epsilon^{\mu\nu q_1 q_2} \bar{q}_{12}^{\rho} W_1 + \left[q_1^{\nu}\epsilon^{\mu\rho q_1 q_2} - q_2^{\mu}\epsilon^{\nu\rho q_1 q_2} - (q_1\cdot q_2)\epsilon^{\mu\nu\rho\bar{q}_{12}}\right] W_2 + \left[q_1^{\mu}\epsilon^{\nu\rho q_1 q_2} - q_2^{\nu}\epsilon^{\mu\rho q_1 q_2} - q_1^2\epsilon^{\mu\nu\rho q_2} - q_2^2\epsilon^{\mu\nu\rho q_1}\right] W_3$$

$$B_{2S} = \frac{B_2+\bar{B}_2}{2} = -W_2, \quad B_{2A} = \frac{B_2-\bar{B}_2}{2} = W_3, \quad C_A = W_1, \quad C_S = W_0 + W_2$$

- Take $W_2 = -B_{2S}$ (needed for OPE) and $W_0 = 0$ (equivalent on-shell!)

	f_1	f_1'	a_1	Total
$C_S = 0$	8.3	2.3	5.4	16.0
$W_0 = 0$	3.5	1.0	2.2	6.7

- In order to recover $C_S = 0$ result, needs $W_0 = -W_2 \neq 0$!
- The effect is not small!

Further implications

- MV's SDC (HLbL$\sim\langle VVA\rangle$): interplay of longitudinal/transverse dof
- Models with constant/modified π^0 TFF or extended π_H sector not consistent
- Decomposing $D^{\alpha\beta}(q^2) = -\frac{q^2 g^{\alpha\beta} - q^\alpha q^\beta}{m_A^2(q^2-m_A^2)} + \frac{g^{\alpha\beta}}{m_A^2} \equiv \bar{D}^{\alpha\beta}(q^2) + \frac{g^{\alpha\beta}}{m_A^2}$
  - $\bar{D}^{\alpha\beta}$ contributes to $w_T(q_1^2,q_2^2,q_{12}^2) - w_{T0}$ "subtractedly" $\sim$ RχT
  - Contact term modelled (numerically dominates)
- Sum rule to constraint higher axial-vector mesons: ok with holographic

$$(q_1^2+q_2^2)w_{T0}^{(+)}(q_1^2,q_2^2) - (q_1^2-q_2^2)w_{T0}^{(-)}(q_1^2,q_2^2) = 2N_c[1-\tilde{F}_{P\gamma\gamma}(q_1^2,q_2^2)]$$

Conclusions & Outlook

- Including axial-vector mesons off-shell in Green's functions $\to$ complications
- SDCs HLbL$\sim$ $\langle VVA \rangle$ solves ambiguities
- Complying with the OPE for Axials TFF doubles their contribution to a_μ
- Allows to derive a sum-rule in chiral limit $\to$ build a model (future work)
- Models with constant/modified π^0 TFF or extended π_H sector not consistent
- $m_q \neq 0$ corrections via $\langle VV\partial A \rangle \sim \mathrm{ano} + \langle VVP \rangle$ (future work)