# Intrinsic charm in the nucleon, forward production of charm meson and high-energy neutrino flux 

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## Motivation from the Neutrino Astronomy

## High-energy cosmic neutrinos $\Rightarrow$ excellent cosmic messenger particles

- Universe not transparent to extragalactic photons with $E_{\gamma}>10 \mathrm{TeV}$ (gamma rays) $\Rightarrow$ strongly absorbed by interactions with the cosmic microwave background (CMB).
- Neutrinos $\Rightarrow$ no absorption and no deflection by magnetic fields
- essentially no mass and no electric charge, weakly interacting
- can travel cosmic distances without distortion and can point back to their sources
- can escape dense astrophysical environments where they are produced


Low-energy extraterrestrial neutrinos

- MeV neutrinos from the Sun (the closet source)
- neutrinos from supernova 1987A

Pushed forward:

- elucidated neutrino properties, neutrino flavor changing puzzle
- fundamental physics, Sun's inner working, supernova physics
- Diffuse high-energy neutrinos $\Rightarrow$ information about the mechanism of cosmic ray production and cosmic ray sources
- e.g. probe of the high-energy neutrino-nucleon cross section
- many new physics phenomena (dark matter, leptoquarks, micro black holes, etc.)


## Cosmic Neutrino Detection

Unfortunately, their weak interactions also make cosmic neutrinos very difficult to detect...

- neutrino observatories require gigaton masses $\Rightarrow$ natural resources needed
- immense detectors to collect cosmic neutrinos in statistically significant numbers
- first efforts $\Rightarrow$ a large volume of deep natural water (DUMAND, ANTARES, KM3NeT, BGVD)
- next steps $\Rightarrow$ a large volume of transparent natural Antarctic ice (AMANDA and IceCube)



The IceCube Neutrino Observatory

- at the Amundsen-Scott South Pole Station in Antarctica
- an in-ice array (IceCube detector)
- a surface air shower array (IceTop)
- detector medium $\Rightarrow$ one cubic kilometre of the deep ultra-clear glacial ice


## Backgrounds



- conventional $\nu$-flux $\Rightarrow \Phi_{\nu} \sim E_{\nu}^{-3.7}$ $\triangleright$ decays of lighter mesons: $\pi^{ \pm}, K^{ \pm}$ $\triangleright$ long life-time: interactions occurs before decay $\triangleright$ mesons loose energy $\rightarrow$ steeply falling $\nu$-flux $\triangleright$ zenith-angle dependent, largest at horizon
- prompt $\nu$-flux (not yet identified) $\Rightarrow \Phi_{\nu} \sim E_{\nu}^{-2.7}$ $\triangleright$ decays of heavy mesons: $D$ and $B$ $\triangleright$ short life-time: decay before interactions $\triangleright$ more energy transferred to neutrino $\rightarrow$ flat $\nu$-flux $\triangleright$ isotropic


## From cosmic ray to prompt neutrino flux detection

Theoretical predictions of the prompt atmospheric neutrino flux at the detector level $\Rightarrow$


- a multi-stage problem with many sources of uncertainties
$\triangleright$ the initial cosmic ray flux: shape and composition
$\triangleright$ strong interaction cross section: charm production, framework, parton densities, nucelar effects, intrinsic charm
$\triangleright$ charm hadronization
$\triangleright$ semileptonic decay
$\triangleright$ neutrino interaction cross section
high-energy neutrinos $\left(E_{\nu}>10^{5} \mathrm{GeV}\right) \Rightarrow$ charmed meson production at very high energies and large forward rapidities
- QCD methods for the charmed meson production in the kinematics beyond the LHC
- validity of the collinear factorization in the forward kinematics
- the size of subleading fragmentation of light partons into heavy meson
- the presence (or not) of intrinsic heavy quarks in the hadronic wave function
- the presence (or not) of nonlinear (saturation) effects


## Initial cosmic ray (CR) flux

The energy spectra of cosmic rays on top of the Earth atmosphere



Parametrization by Gaisser
(Gaisser, Astropart. Phys.35, 801 (2012))
$\phi_{i}(E)=\sum_{j=1}^{3} a_{i, j} E^{-\gamma_{i, j}} \times \exp \left[-\frac{E}{Z_{i} R_{c, j}}\right]$

- 5 nuclei groups: $\mathrm{H}, \mathrm{He}, \mathrm{CNO}, \mathrm{Fe}, \mathrm{MgSi}$
- 3 populations characterised by different rigidities (1st: supernova remnants, 2nd: higher energy galactic, 3rd: extragalactic component)
- H3a and H3p (only protons in the 3rd pop.)

Broken power-law
$\phi_{\boldsymbol{p}}^{0}(E)=1.7 E^{-2.7}$ for $E<5 \cdot 10^{6} \mathrm{GeV}$
$\phi_{\boldsymbol{p}}^{\mathbf{0}}(E)=174 E^{-3}$ for $E>5 \cdot 10^{6} \mathrm{GeV}$

- used in earlier works
- overestimates the highest energies


## Development of air-showers and lepton fluxes

- may be described by a set of CASCADE EQUATIONS

The flux $\phi_{j}\left(E_{j}, X\right)$ of a particle $j$ with energy $E_{j}$ that has traversed a slant depth $X$ is given by:

$$
\frac{\mathrm{d} \phi_{j}}{\mathrm{~d} X}=-\frac{\phi_{j}}{\lambda_{j}}-\frac{\phi_{j}}{\lambda_{j}^{\text {dec }}}+\sum_{k} S_{k j}\left(E_{j}, X\right),
$$

where $\lambda_{j}$ is the interaction length (the average amount of atmosphere (in $\mathrm{g} / \mathrm{cm}^{2}$ ) traversed between successive collisions with air nuclei), $\lambda_{j}^{\text {dec }}$ is the decay length, and $S_{k j}$ are '(re)generation functions' describing the production of particle $j$ from particle $k$. The (re)generation function is:

$$
S_{k j}\left(E_{j}, X\right)=\int_{E_{j}}^{\infty} \frac{\phi_{k}\left(E_{k}^{\prime}, X\right)}{\lambda_{k}\left(E_{k}^{\prime}\right)} \frac{\mathrm{d} n\left(k \rightarrow j ; E_{k}^{\prime}, E_{j}\right)}{\mathrm{d} E_{j}} \mathrm{~d} E_{k}^{\prime},
$$

where $\mathrm{d} n\left(k \rightarrow j ; E_{k}^{\prime}, E_{j}\right)$ is the differential transition rate between particle species $k$ and $j$ (the number of particles $j$ with energies between $E_{j}$ and $E_{j}+d E_{j}$ produced in the collision of the incoming particle $k$ with an air nucleus)

The equation says that as a particle ( $j$ ) traverses the atmosphere, its flux will decrease when the particle undergoes an interaction (thus losing energy) or decays, as well as increase from the decay or interaction of other particle species ( $k$ )

## Development of air-showers and lepton fluxes

Assuming that the particle flux factorises into components dependent respectively on the energy $E$ and the slant depth $X$, the (re)generation function can be rewritten more simply in terms of Z-moments as

$$
S_{k j}\left(E_{j}, X\right)=\frac{\phi_{k}\left(E_{j}, X\right)}{\lambda_{k}\left(E_{j}\right)} Z_{k j}\left(E_{j}\right)
$$

with the key property that the moment $Z_{k j}$,

$$
Z_{k j}\left(E_{j}\right)=\int_{E_{j}}^{\infty} \frac{\phi_{k}\left(E_{k}^{\prime}, X\right)}{\phi_{k}\left(E_{j}, X\right)} \frac{\lambda_{k}\left(E_{j}\right)}{\lambda_{k}\left(E_{k}^{\prime}\right)} \frac{\mathrm{d} n\left(k \rightarrow j ; E_{k}^{\prime}, E_{j}\right)}{\mathrm{d} E_{j}} d E_{k}^{\prime},
$$

is independent of the slant depth $X$ (which cancels in the ratio of fluxes)

- a set of coupled differential equations:

$$
\begin{aligned}
& \frac{\mathrm{d} \phi_{p}}{\mathrm{~d} X}=-\frac{\phi_{p}}{\lambda_{p}}+Z_{p p} \frac{\phi_{p}}{\lambda_{p}} \text { (protons) } \\
& \frac{\mathrm{d} \phi_{m}}{\mathrm{~d} X}=-\frac{\phi_{m}}{\rho \mathrm{~d}_{m}(E)}-\frac{\phi_{m}}{\lambda_{m}}+Z_{m m} \frac{\phi_{m}}{\lambda_{m}}+Z_{p m} \frac{\phi_{p}}{\lambda_{p}} \text { (mesons) } \\
& \frac{\mathrm{d} \phi_{l}}{\mathrm{~d} X}=\sum_{m} Z_{m \rightarrow I} \frac{\phi_{m}}{\rho d_{m}} \text { (leptons) }
\end{aligned}
$$

## Development of air-showers and lepton fluxes

The most crucial in our work is the nucleon to meson moment: $Z_{p m}$ which depends on the charm production cross-section in pp-collisions:

- the generic $Z$-moment can be written as:

$$
Z_{k j}\left(E_{j}\right)=\int_{E_{j}}^{\infty} \frac{\phi_{k}\left(E_{k}^{\prime}, X\right)}{\phi_{k}\left(E_{j}, X\right)} \frac{\lambda_{k}\left(E_{j}\right)}{\lambda_{k}\left(E_{k}^{\prime}\right)} \frac{\mathrm{d} n\left(k A \rightarrow j ; E_{k}^{\prime}, E_{j}\right)}{\mathrm{d} E_{j}} \mathrm{~d} E_{k}^{\prime}
$$

- $Z_{p D}$ : the number distribution can be related to the differential charm cross-section:

$$
\frac{\mathrm{d} n\left(p A \rightarrow D+X ; E^{\prime}, E\right)}{\mathrm{d} E}=\frac{1}{\sigma_{p A}\left(E^{\prime}\right)} \frac{\mathrm{d} \sigma\left(p A \rightarrow D+X ; E^{\prime}, E\right)}{\mathrm{d} E}
$$

- we assume: $\sigma(p A \rightarrow D+X) \simeq\langle A\rangle \sigma(p p \rightarrow D+X)$

The charmed hadron $Z$-moments are given by:

$$
\begin{aligned}
& Z_{p D}\left(E_{D}\right)=\int_{E_{D}}^{\infty} \frac{\phi_{p}\left(E_{p}^{\prime}\right)}{\phi_{p}\left(E_{D}\right)} \frac{\langle A\rangle}{\sigma_{p A}\left(E_{D}\right)} \frac{\mathrm{d} \sigma\left(p p \rightarrow D+X ; E_{p}^{\prime}, E_{D}\right)}{\mathrm{d} E_{D}} \mathrm{~d} E_{p}^{\prime} \\
& Z_{p D}\left(E_{D}\right)=\int_{0}^{1} \frac{d x_{F}}{x_{F}} \frac{\phi_{p}\left(E_{D} / x_{F}\right)}{\phi_{p}\left(E_{D}\right)} \frac{\langle A\rangle}{\sigma_{p A}\left(E_{D}\right)} \frac{d \sigma_{p p \rightarrow D}\left(E_{D} / x_{F}\right)}{d x_{F}}
\end{aligned}
$$

where $E$ is the energy of the $D$-meson, $x_{F}=E_{D} / E_{p}^{\prime}$ is the Feynman variable, $\sigma_{p A}$ is the inelastic p -Air cross section and $d \sigma / d x_{F}$ is the differential cross section for the charmed meson production $\Rightarrow$ INPUT

## Charm cross section in QCD

The basic ingredient for the prompt neutrino flux $\Rightarrow$ pQCD charm quark production

- the leading-order (LO) partonic processes for $Q \bar{Q}$ production $\Rightarrow$ $q \bar{q}$-annihilation and gluon-gluon fusion (dominant at high energies)

- main classes of the next-to-leading order (NLO) diagrams:

collinear approach:
- state of the art for single particle spectra at NLO (FONLL, GM-VFNS)
- MC@NLO+PS for correlations
- NNLO not available for charm/bottom



## the NLO and the NNLO corrections

of a special importance for charm $p_{T}$-differential cross section!
$\boldsymbol{k}_{T}$-factorizaton (high-energy factorization):

- exact kinematics from the very beginning
- correlation observables directly calculable
- some contributions even beyond the NLO available (also differentially)


## Forward charm production at the LHC

## Open charm LHCb data in $p p$-scattering at $\sqrt{s}=7,13 \mathrm{TeV}$ :



Detector acceptance: $2.0<y<4.5$ and $0<p_{T}<8 \mathrm{GeV}$

- inclusive $D$-meson spectra and $D \bar{D}$-pair correlation observables ( $M_{i n v}, \Delta \varphi, p_{T}$-pair)
- longitudinal momentum fractions probed:

$$
10^{-3}<x_{1}<10^{-1} \text { and } 10^{-5}<x_{2}<10^{-3}
$$

- $p_{T}$-differential cross section well described in different $y$-bins
- correct shapes of the correlation observables
(R.M., A. Szczurek, Phys.Rev.D 100 (2019) 5, 054001)


- $k_{T}$-factorizaton: $g^{*} g^{*} \rightarrow c \bar{c}+$ KMR uPDF $\Rightarrow$ works very well


## Kinematics probed with the IceCube prompt neutrino flux

Mapping the dominant regions of the phase space associated with $c \bar{c}$-pair production relevant for the prompt flux at IceCube
(V.P. Goncalves, R.M., R. Pasechnik, A. Szczurek, Phys.Rev.D 96 (2017) 9, 094026)


- recent: up to $E_{\nu}=3 \cdot 10^{6} \mathrm{GeV} \Rightarrow$ the LHC energy range
- future: $E_{\nu}>10^{7} \mathrm{GeV} \Rightarrow$ energy range beyond that probed in the LHC Run2
- flux sensitive to the $p_{T}<5 \mathrm{GeV}$


## Kinematics probed with the IceCube prompt neutrino flux

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- projectile: $0.2<x_{1}<0.6$
- target: $10^{-6}<x_{2}<10^{-5}$ (IceCube recently) and even $10^{-8}<x_{2}<10^{-5}$ (future)
- far-forward production beyond the LHC range $\Rightarrow$ very asymmetric kinematics


## Hybrid high-energy factorization

How to treat theoretically the asymmetric configuration?
The hybrid approach for far-forward production $\Rightarrow$


- combined collinear- and $k_{T}$-factorization
- used in many phenomenological studies
- the differential cross section for $g g^{*} \rightarrow c \bar{c}$ mechanism:

$$
\begin{array}{r}
d \sigma_{p p \rightarrow c h a r m}\left(g g^{*} \rightarrow c \bar{c}\right)=\int d x_{1} \int \frac{d x_{2}}{x_{2}} \int d^{2} k_{t} \\
\times g\left(x_{1}, \mu^{2}\right) \cdot \mathcal{F}_{g}\left(x_{2}, k_{t}^{2}, \mu^{2}\right) \cdot d \hat{\sigma}_{g g^{*} \rightarrow c \bar{c}}
\end{array}
$$

- $g\left(x_{1}, \mu^{2}\right) \Rightarrow$ collinear large- $x$ gluon (the one from the incoming cosmic ray) we use the CT14nnlo PDF
- $\mathcal{F}_{g}\left(x_{2}, k_{t}^{2}, \mu^{2}\right) \Rightarrow$ off-shell small- $x$ gluon (the one from the target air nucleus) we use the KMR/MRW and the KS linear/nonlinear uPDFs
- $d \hat{\sigma}_{g g^{*} \rightarrow c \bar{c}}$ is the hard partonic cross section obtained from a gauge invariant off-shell tree-level amplitudes (available in KaTie)
- a derivation of the hybrid factorization from the dilute limit of the Color Glass Condensate approach can be found in the literature


## Charm production driven by the intrinsic charm

What if there is a non-perturbative charm content of the proton?
The charm quark in the initial state $\Rightarrow$

- perturbative: extrinsic charm (from gluon splitting)
- non-perturbative: intrinsic charm (IC)
- the differential cross section for $c g^{*} \rightarrow c g$ mechanism:

$$
\begin{array}{r}
d \sigma_{p p \rightarrow c h a r m}\left(c g^{*} \rightarrow c g\right)=\int d x_{1} \int \frac{d x_{2}}{x_{2}} \int d^{2} k_{t} \\
\times c\left(x_{1}, \mu^{2}\right) \cdot \mathcal{F}_{g}\left(x_{2}, k_{t}^{2}, \mu^{2}\right) \cdot d \hat{\sigma}_{c g^{*} \rightarrow c g}
\end{array}
$$

- $c\left(x_{1}, \mu^{2}\right) \Rightarrow$ collinear charm quark PDF (large- $x$ )
- $\mathcal{F}_{g}\left(x_{2}, k_{t}^{2}, \mu^{2}\right) \Rightarrow$ off-shell gluon uPDF (small-x)
- $d \hat{\sigma}_{c g^{*} \rightarrow c g} \Rightarrow$ only in the massless limit (also available in KaTie)
- regularization needed at $p_{T} \rightarrow 0 \Rightarrow$ we use PYTHIA prescription:
$F_{\text {sup }}\left(p_{T}\right)=\frac{p_{T}^{2}}{p_{T 0}^{2}+p_{T}^{2}}, \alpha_{S}\left(\mu_{R}^{2}+p_{T 0}^{2}\right)$, where $p_{T 0}=1.5 \mathrm{GeV}$ (free parameter)
- the charm quark PDF with IC content is taken at the initial scale: $c\left(x_{1}, \mu_{0}^{2}\right)$, where $\mu_{0}=1.3 \mathrm{GeV}$ so the perturbative charm contribution is intentionally not taken into account


## The concept of intrinsic charm in the nucleon

The intrinsic charm quarks $\Rightarrow$ multiple connections to the valence quarks of the proton

- strong evidence for internal strangeness and somewhat smaller for internal charm
$\mathrm{xc}(\mathrm{x}, \mathrm{Q})$, comparison
- global experimental data put only loose constraints on the $P_{i c}$ probability
- dfferent pictures of non-perturbative $c \bar{c}$ content:
- sea-like models
- valence-like models
- we use the IC distributions from the Brodsky-Hoyer-Peterson-Sakai (BHPS) model as adopted in the CT14nnloIC PDF


- the presence of an intrinsic component implies a large enhancement of the charm distribution at large $x(>0.1)$ in comparison to the extrinsic charm prediction
- the models do not allow to predict precisely the absolute probability $P_{\text {ic }}$


## Prompt neutrino fluxes




- when intrinsic charm is included the behaviour of the $x_{F}$-distribution is strongly modified in the $0.03 \leq x_{F} \leq 0.6$ range
- the Feynman $x_{F}$-distribution for large $x_{F}$ is dominated by the $c g^{*} \rightarrow c g$ mechanism
- extrinsic charm negligible
- the inclusion of the $c g^{*} \rightarrow c g$ mechanism driven by the intrinsic charm (IC) has a strong effect on the prompt neutrino flux
- the flux is enhanced by one order of magnitude when intrinsic charm is present ( $P_{i c}=1 \%$ here)


## Predictions and IceCube limits for intrinsic charm



- the impact of the prompt flux is small in the current kinematical range probed by IceCube as long as only the gluon-gluon fusion mechanism is taken into account
- the intrinsic charm mechanism implies a large enhancement of the prompt flux at large $E_{\nu}$, with the associated magnitude being strongly dependent on the value of $P_{\text {ic }}$
- linear QCD dynamics $\Rightarrow P_{i c} \leq 0.5 \%$


## Predictions and IceCube limits including saturation



- within the saturation scenario the impact of the prompt flux driven by the gluon-gluon fusion mechanism is even smaller and becomes negligible
- nonlinear QCD dynamics $\Rightarrow P_{\text {ic }} \leq 1.0 \%$
- consistent with the central CT14nnloIC PDF set


## Conclusions

Currently we have two acceptable solutions when the intrinsic charm mechanism is included in the analysis of the IceCube prompt neutrino flux:

- the QCD dynamics is described by a linear evolution equation and the amount of IC in the proton wave function has the upper limit $P_{\text {ic }} \leq 0.5 \%$ that is smaller than the value predicted by the central CT14nnloIC parameterization
- the amount of IC at the level of about $1.0 \%$ is correctly described by the central CT14nnloIC parameterization and the saturation effects are needed to describe the IceCube prompt neutrino flux at the highest energies rapidities

One has that if the amount of IC is constrained in hadronic colliders, the IceCube data for the atmospheric neutrino flux can be considered as a probe of the QCD dynamics at high energies. Inversely, if the saturation effects are probed in hadronic colliders, the IceCube data can be used to constrain the amount of the IC. Such results demonstrate synergy between IceCube and the LHC, and strongly motivate new experimental and theoretical analyses in the future.

- one of such alternatives is the analysis of the $D$-meson and $\nu_{\mu}$ neutrino at FASER taking into account both effects, which we intend to study in a forthcoming publication


## Thank You!

## BackUp slides (just in case)

## IceCube Detector

## The detector volume is instrumented with:




- 5160 Digital Optical Modules (DOMs)
- distributed on 86 read out and support cables ("strings")
- deployed between 1.5 and 2.5 km below the surface
- neutrino energy threshold about 10 GeV

Cherenkov angle: $\cos \theta_{\mathrm{c}}=1 /(\beta n), \theta_{c}=42^{\circ}$ water


The DOMs register the Cherenkov light emitted by relativistic charged particles passing through the detector

- Cherenkov light is emitted when particle velocity exceeds the speed of light in the given medium
- it is emitted by a charged particle: either prompt (like atm. muons) or resulting from neutrino interaction with ice or bedrock


## Experimental signatures

There are two principle classes of Cherenkov events (red early in time, blue late in time):


Fig. 2 Two examples of events observed with IceCube. The left plot shows a muon track from a $v_{\mu}$ interaction crossing the detector. Each coloured dot represents a hit DOM. The size of the dot is proportional to the amount of light detected and the colour code is related to the
relative timing of light detection: read denotes earlier hits, blue, later hits. The right plot shows a $v_{e}$ or $v_{\tau}$ charged-current (or any flavour neutral-current) interaction inside the detector

- TRACKS: through-going track-like pattern (left panel)
- CASCADES: spherical light distribution (right panel)
- starting tracks (cascade + track)

- CC: $\nu_{\tau}+N \rightarrow \tau+$ hadrons (double cascades)

- CC: $\nu_{e}+N \rightarrow e+$ hadrons (cascades)

- NC: $\nu_{\alpha}+N \rightarrow \nu_{\alpha}+$ hadrons (cascades)



## How to reduce the atmospheric background?

Two complementary strategies:

- $\nu_{\mu}$ tracks from Northern Sky $\triangleright$ using the Earth as a filter by selecting up-going track events $\triangleright$ atm. muons sufficiently reduced $\triangleright$ vertex outside the detector
- Starting Events (HESE, $E_{\nu} \gtrsim 30 \mathrm{TeV}$ ) $\triangleright$ high-energy $\nu$ interacting inside the detector $\triangleright$ all directions in the sky
$\triangleright$ a virtual veto region
$\triangleright$ rejects atmospheric muons and neutrinos


- HESE veto data $\Rightarrow$ the first observation of high-energy astrophysical neutrinos by IceCube (no atmospheric background)
- study of the $\nu_{\mu}$ tracks data from Northern Sky (before HESE veto) $\Rightarrow$ could be use to constrain the prompt atmospheric neutrino flux and physics behind in the kinematical limits beyond the LHC

(IceCube Collaboration, Astrophys. J. 833 (2016))


## Development of air-showers and lepton fluxes

Next step: Simulation of the propagation of high energy particles and their decay products through the atmosphere

- The aim is to solve a series of coupled differential equations dependent on the slant depth $X(I, \theta)$ measuring the amount of atmosphere traversed by a particle:

$$
X(I, \theta) \equiv \int_{I}^{\infty} \rho\left[h\left(I^{\prime}, \theta\right)\right] \mathrm{d} I^{\prime}
$$

where $\rho$ is the density of the atmosphere dependent on the distance from the ground $I$ (along the particle trajectory) as well as on the zenith angle $\theta$

- an isothermal model of the atmosphere $\Rightarrow$ appropriate for atmospheric depths 10-40 km within which the bulk of particle production occurs:

$$
\rho(h)=\rho_{0} \exp \left(-h / h_{0}\right), \quad \rho_{0}=2.03 \times 10^{-3} \mathrm{gm} \mathrm{~cm}^{-3}, \quad h_{0}=6.4 \mathrm{~km}
$$

- the horizontal depth of the atmosphere is $X \simeq 3.6 \times 10^{4} \mathrm{gm} \mathrm{cm}^{-2}$ while its vertical depth is $\simeq 1.3 \times 10^{3} \mathrm{gm} \mathrm{cm}^{-2}$ (values which adequately describe the density of the stratosphere)
- concerning the atmospheric composition, a good approximation, valid up to a height of 100 km , is $78.4 \%$ nitrogen, $21.1 \%$ oxygen and $0.5 \%$ argon. This leads to an average atomic number of $\langle A\rangle=14.5$
for a detailed discussion of the cascade formalism see e.g. R. Gauld et al., JHEP (2016) 130,


## Development of air-showers and lepton fluxes

The solution of these equations is in general quite involved (Monte Carlo methods needed)

- but there are simple (approximate) asymptotic analytic solutions $\Rightarrow$

The equation for the proton flux can be trivially integrated to give

$$
\phi_{p}(E, X)=\phi_{p}^{(0)}(E) \exp \left(-X / \Lambda_{p}(E)\right)
$$

where $\Lambda_{p}(E) \equiv \lambda_{p}(E) /\left(1-Z_{p p}(E)\right)$ is the nucleon attenuation length that depends in general on the nucleon's interaction length in the atmosphere: $\lambda_{p}(E)=\langle A\rangle / N_{0} \sigma_{p A}(E)$, where $\langle A\rangle=14.5$ is the average atomic number of air molecules, $N_{0}$ is Avogadro's number, and the total inelastic proton-air cross-section is denoted by $\sigma_{P A}$.
Then, the meson flux in the two asymptotic regions reads:

- at low energies the interaction and regeneration terms neglected

$$
\phi_{m}^{\text {low }}(E)=\phi_{p}^{(0)}(E) \frac{Z_{p m}(E)}{\Lambda_{p}\left(1-Z_{p p}\right)} \rho d_{m} e^{-X / \Lambda_{p}}
$$

- at high energies the decay terms neglected

$$
\phi_{m}^{\mathrm{high}}(E)=\phi_{p}^{(0)}(E) \frac{Z_{p m}(E)}{\left(1-Z_{p p}\right)} \frac{\left(e^{-X / \Lambda_{m}}-e^{-X / \Lambda_{p}}\right)}{1-\Lambda_{p} / \Lambda_{m}}
$$

## Development of air-showers and lepton fluxes

The final vertical flux of leptons expected at the detector:

$$
\begin{array}{ll}
\phi_{l, m}^{\text {low }}(E)=\phi_{p}^{(0)}(E) \frac{Z_{p m}(E)}{1-Z_{p p}} Z_{m l}^{\text {low }}(E) & E<\epsilon_{m} \\
\phi_{l, m}^{\mathrm{high}}(E)=\phi_{p}^{(0)}(E) \frac{\epsilon_{m}}{E} \frac{Z_{p m}(E)}{1-Z_{p p}} \frac{\ln \left(\Lambda_{m} / \Lambda_{p}\right)}{1-\Lambda_{p} / \Lambda_{m}} Z_{m l}^{\mathrm{high}}(E) & E>\epsilon_{m}
\end{array}
$$

where $\epsilon_{m}$ is a critical energy below which the probability of a meson to decay is greater than it is to interact:

$$
\epsilon_{m}=\frac{m_{m} c^{2} h_{0}}{c \tau_{m} \cos \theta} \simeq 3.7-9.5 \times 10^{7} \mathrm{GeV}
$$

- the smaller the critical energy, the longer the decay length, hence the more energy the particle will lose by interactions in the atmosphere before it actually decays.

The final step in solving the cascade equations in the $Z$-moment approach is the geometrical interpolation of the low- and high-energy asymptotic solutions:

$$
\phi_{l}(E)=\sum_{m} \frac{\phi_{l, m}^{\mathrm{low}}(E) \times \phi_{l, m}^{\mathrm{high}}(E)}{\phi_{l, m}^{\mathrm{low}}(E)+\phi_{l, m}^{\mathrm{high}}(E)} .
$$

- sum over mesons contributing to the prompt flux (the leptonic decays of): $D^{0}, \bar{D}^{0}, D^{ \pm}, D_{s}^{ \pm}$and $\Lambda_{c}^{ \pm}$


## $k_{T}$-factorization (high-energy factorization) approach



## off-shell initial state partons $\Rightarrow$

initial transverse momenta explicitly included $\boldsymbol{k}_{1, t}, \boldsymbol{k}_{2, t} \neq \mathbf{0}$

- additional hard dynamics coming from transverse momenta of incident partons (virtualities taken into account)
- very efficient for less inclusive studies of kinematical correlations
- more exclusive observables, e.g. pair transverse momentum or azimuthal angle very sensitive to the incident transverse momenta
multi-differential cross section:

$$
\begin{aligned}
& \frac{d \sigma}{d y_{1} d y_{2} d^{2} p_{1, t} d^{2} p_{2, t}}=\int \frac{d^{2} k_{1, t}}{\pi} \frac{d^{2} k_{2, t}}{\pi} \frac{1}{16 \pi^{2}\left(x_{1} x_{2} s\right)^{2}} \\
& \times \delta^{2}\left(\vec{k}_{1, t}+\vec{k}_{2, t}-\vec{p}_{1, t}-\vec{p}_{2, t}\right) \mathcal{F}_{g}\left(x_{1}, k_{1, t}^{2}, \mu\right) \mathcal{F}_{g}\left(x_{2}, k_{2, t}^{2}, \mu\right)
\end{aligned}
$$



- the $2 \rightarrow 3$ and $2 \rightarrow 4$ processes (higher-order) only at tree-level (KaTie Monte Carlo)
- $\mathcal{F}_{g}\left(x, k_{t}^{2}, \mu\right)$ - transverse momentum dependent - unintegrated PDFs (uPDFs)
pair creation with gluon emission

flavour excitation

gluon splitting

- part of higher-order (real) corrections might be effectively included in uPDF


## Unintegrated parton distribution functions (uPDFs)

Transverse momentum dependet PDFs: $\mathcal{F}_{g}\left(x, k_{t}^{2}, \mu\right)$

- CCFM evolution: Jung-Hautmann (JH2013)
- Parton Branching + DGLAP: Bermudez Martinez-Connor-Jung-Lelek-Zlebcik
- linear/nonlinear BK (saturation): Kutak-Sapeta (KS)
- modified DGLAP-BFKL: Kimber-Martin-Ryskin-Watt (KMR, MRW)
- modified BFKL-DGLAP: Kwieciński-Martin-Staśto (KMS)



## The quark to meson transition

Heavy quark to open heavy meson fragmentation: $c \rightarrow D$ and $\bar{c} \rightarrow \bar{D}$
The independent parton fragmentation picture:

- the charmed meson $x_{F}$-distributions at large $x_{F}$ can be obtained from the charm quark/antiquark $x_{F}^{c}$-distributions as:

$$
\frac{d \sigma_{p p \rightarrow D}\left(x_{F}\right)}{d x_{F}}=\int_{x_{F}}^{1} \frac{d z}{z} \frac{d \sigma_{p p \rightarrow c h a r m}\left(x_{F}^{c}\right)}{d x_{F}^{c}} D_{c \rightarrow D}(z),
$$

- where $x_{F}^{c}=x_{F} / z$ and $D_{c \rightarrow D}(z)$ is the relevant fragmentation function (FF)
- the fragmentation procedure leads to a decrease of the $x_{F}$ range for meson with respect to $x_{F}^{c}$ of the parent quark

- $c \rightarrow D: \operatorname{Peterson}(z), \varepsilon=0.05$ (well known from $e^{+} e^{-}$data)
- $\eta_{D}=\eta_{c}, x_{F}=z \cdot x_{F}^{c}, z \in(0,1)$
- fragmentation fractions well known (Particle Data Group)



## Intrinsic charm at the LHC and beyond

A possible impact of the intrinsic charm component on the forward charm particle production in already existing or future experiments at different energies: (R.M, A. Szczurek, JHEP 10 (2020) 135)

- Future Circular Collider (FCC) (D-meson production)


- the intrinsic charm important at $|y|>7$
- transverse momentum distribution visibly enhanced


## Intrinsic charm at the LHC and beyond

A possible impact of the intrinsic charm component on the forward charm particle production in already existing or future experiments at different energies: (R.M, A. Szczurek, JHEP 10 (2020) 135)

- FASER at the LHC (dedicated to a measurement of forward neutrinos originating from semileptonic decays of $D$ mesons)


- the intrinsic charm important at $|y|>6$
- transverse momentum distribution visibly enhanced


## Intrinsic charm at the LHC and beyond

A possible impact of the intrinsic charm component on the forward charm particle production in already existing or future experiments at different energies:
(R.M, A. Szczurek, JHEP 10 (2020) 135)

- SHIP at the SPS CERN at $\sqrt{s}=27.4 \mathrm{GeV}$ (dedicated to a measurement of forward $\nu_{\tau}$ neutrinos originating from semileptonic decays of $D_{s}$ mesons)


- at the lower energy $\Rightarrow$ the intrinsic charm important in the whole rapidity spectrum
- transverse momentum distribution visibly enhanced


## Intrinsic charm at the LHC and beyond

A possible impact of the intrinsic charm component on the forward charm particle production in already existing or future experiments at different energies: (R.M, A. Szczurek, JHEP 10 (2020) 135)

- Fixed-target LHCb mode at $\sqrt{s}=86.6 \mathrm{GeV}$ ( $D$-meson production)


- at the lower energy $\Rightarrow$ the intrinsic charm important already at $|y|>1$


## Intrinsic charm at the LHC and beyond

The fixed-target data on forward open charm meson production already exists:

- Fixed-target LHCb mode at $\sqrt{s}=86.6 \mathrm{GeV}$ ( $D$-meson production)

- some problems with understanding the LHCb fixed-target open charm data identified (R.M., Phys.Rev.D 102 (2020) 1, 014028)
- a new scenario proposed with the intrinsic charm contribution needed to describe the data points in the backward direction and at larger $p_{T}$ 's
- R.M, A. Szczurek, a paper in preparation


## Outlook

a work in progress...
The subleading fragmentation of light quarks $(u, d, s)$ into heavy meson $(D, B) \Rightarrow$ a possible source of enhancement of charm production cross section at large values of $x_{F}$ (R.M, A. Szczurek, Phys.Rev.D 97 (2018) 7, 074001)



- already discussed for the $\nu_{\tau}$ neutrino flux at IceCube (V.P. Goncalves, R.M, A. Szczurek, Phys.Lett.B 794 (2019) 29-35)
- might be also important for understanding the IceCube prompt $\nu_{\mu}$ neutrino flux (stay tuned)


## Prompt neutrino fluxes and saturation effects




- sum of both production mechanisms: $g g^{*}$-fusion and the $c g^{*}$ with IC BHPS $1 \%$
- the KMR and KS linear predictions are similar $\Rightarrow$ BFKL effects not important for IceCube (which probes $0.2<x_{F}<0.5$ )
- the KS nonlinear is a factor $\approx 3$ smaller for $x_{F}=0.2$
$\Rightarrow$ saturation effects strongly modifies the magnitude of the distribution

