Phenomenology of spin-3 tensor mesons

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16th International Workshop on Meson Physics

MESON2021, 17-21 May 2021

Motivation

- Mesons can be described as quark and anti-quark bound states $(q_i \bar{q}_j)$

$n^{2s+1}\ell_J$	J^{PC}	$I = 1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$I = \frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	$I = 0$ f'	$I = 0$ f	θ_{quad} [°]	θ_{lin} [°]
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$	−11.3	−24.5
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.2	36.5
1^1P_1	1^{+-}	$b_1(1235)$	$K_{1B}{}^{\dagger}$	$h_1(1415)$	$h_1(1170)$		
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
1^3P_1	1^{++}	$a_1(1260)$	$K_{1A}{}^{\dagger}$	$f_1(1420)$	$f_1(1285)$		
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)^{\dagger}$	$\eta_2(1870)$	$\eta_2(1645)$		
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)^{\ddagger}$		$\omega(1650)$		
1^3D_2	2^{--}	?	$K_2(1820)^{\dagger}$	?	?		
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8

Figure: [P.A.Zyla et al.(Particle Data Group), Prog.Theor.Exp.Phys.2020]

Study of the mesons

- quantum number 3^{--} : $\rho_3(1690)$, $K_3^*(1780)$, $\phi_3(1850)$ and $\omega_3(1670)$
- quantum number 2^{++} : $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$ and $f_2'(1525)$
- missing resonances with quantum number 2^{--}

One of the theoretical ways to investigate the mesons is the low energy effective model of QCD which imitates the symmetry of QCD

Symmetries of QCD

- QCD Lagrangian

$$\mathcal{L}_{QCD} = \mathrm{tr}\Big(\bar{q}_i(i\gamma_\mu D^\mu - m_i)q_i - \frac{1}{2}G_{\mu\nu}G^{\mu\nu}\Big),\ G_{\mu\nu:} = D_\mu A_\nu - D_\nu A_\mu - ig[A_\mu, A_\nu]$$

$$D_\mu := \partial_\mu - igA_\mu,\ A_\mu := A^a_\mu t^a,\ [t^a, t^b] = if^{abc}t^c$$

- Color symmetry: $SU(3)_c \to$ Confinement
- Chiral symmetry:
  $U(N_f)_R \times U(N_f)_L \equiv U(1)_{V=R+L} \times SU(N_f)_V \times SU(N_f)_A \times U(1)_{A=R-L}$:
  works in chiral limit ($m_i \to 0$)
- Can be broken: 1) explicitly by $m_i \neq 0$ and 2) spontaneously breaking to $SU(N_f = 3)_V \times U(1)_V$
- Spontaneous breaking is the essential property of hadronic world since they generate a mass
- Dilitation invariance: $x^\mu \to \lambda^{-1}x^\mu$ is satisfied in chiral limit and classically
- Quantum level $\to$ Trace anomaly
- $U(1)_A$: Classical symmetry, broken by quantum effects $\to$ Axial anomaly

Spin-3

- Phenomenology of $J^{PC} = 3^{--}$ tensor mesons [Sh.Jafarzade, A.Koenigstein, and F.Giacosa Phys.Rev.D (2021), (arXiv:2101.03195)]

Nonet transformations under the symmetries

- Mesons can be grouped to the nonets which transform under the adjoint transformation of the flavour symmetry $U_V(N_f = 3)$
- This symmetry leaves QCD lagrangian invariant under the exchange of light quarks $q_i = (u, d, s)$ for the same m_i
- Within the effective model we consider mesons as effective fields and $SU(N_f = 3)_V$ approximate symmetry as a guide symmetry

Nonet	Parity (P)	Charge conjugation (C)	Flavour $(U_V(3))$
$0^{-+} = P$	$-P(t, -\vec{x})$	P^t	$UPU^\dagger$
$1^{--} = V^\mu$	$V_\mu(t, -\vec{x})$	$-(V^\mu)^t$	$UV^\mu U^\dagger$
$2^{++} = T_2^{\mu\nu}$	$T_{2\,\mu\nu}(t, -\vec{x})$	$(T_2^{\mu\nu})^t$	$UT_2^{\mu\nu} U^\dagger$
$3^{--} = W^{\mu\nu\rho}$	$W_{\mu\nu\rho}(t, -\vec{x})$	$-(W^{\mu\nu\rho})^t$	$UW^{\mu\nu\rho} U^\dagger$

$$P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}, \quad V^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega^\mu_{1,N}+\rho^{0\mu}_1}{\sqrt{2}} & \rho_1^{+\mu} & K_1^{*+\mu} \\ \rho^{-\mu} & \frac{\omega^\mu_N-\rho^{0\mu}}{\sqrt{2}} & K^{*0\mu} \\ K^{*-\mu} & \bar{K}^{*0\mu} & \omega^\mu_S \end{pmatrix}$$

Effective Lagrangians

- Interactions with minimal number of the derivative terms
- CPT-invariance, Poincaré and $U(3)_V$ symmetry

Decay Mode	Interaction Lagrangians
$3^{--} \longrightarrow 0^{-+} + 0^{-+}$	$\mathcal{L}_{WPP} = g_{WPP}\, \mathbf{tr}\Big[W^{\mu\nu\rho}\big[P\,,(\partial_\mu\partial_\nu\partial_\rho P)\big]_-\Big]$
$3^{--} \longrightarrow 0^{-+} + 1^{--}$	$\mathcal{L}_{WVP} = g_{WVP}\, \varepsilon^{\mu\nu\rho\sigma}\, \mathbf{tr}\Big[W_{\mu\alpha\beta}\big\{(V_{\nu\rho})\,,(\partial^\alpha\partial^\beta\partial_\sigma P)\big\}_+\Big]$
$3^{--} \longrightarrow 0^{-+} + 2^{++}$	$\mathcal{L}_{WT_2P} = g_{WT_2P}\, \varepsilon_{\mu\nu\rho\sigma}\, \mathbf{tr}\Big[W^{\mu}_{\alpha\beta}\big[(\partial^\nu T_2^{\rho\alpha})\,,(\partial^\sigma\partial^\beta P)\big]_-\Big]$

- Decay rate with momentum $|\vec{k}_{A,B}| = \frac{1}{2m_W}\sqrt{(m_W^2 - m_A^2 - m_B^2)^2 - 4m_A^2 m_B^2}$

$$\Gamma(W \to A + B) = \frac{|\vec{k}_{A,B}|}{8\pi m_W^2} \times |-i\mathcal{M}|^2 \times \kappa_i \times \Theta(m_W - m_A - m_B)$$

Decay Mode	$\frac{1}{7} \times |-i\mathcal{M}|^2$
$3^{--} \longrightarrow 0^{-+} + 0^{-+}$	$g_{WPP}^2 \times \frac{2|\vec{k}_{P_1,P_2}|^6}{35}$
$3^{--} \longrightarrow 0^{-+} + 1^{--}$	$g_{WVP}^2 \times \frac{8|\vec{k}_{V,P}|^6\, m_W^2}{105}$
$3^{--} \to 0^{-+} + 2^{++}$	$g_{WT_2P}^2 \times \frac{2|\vec{k}_{T_2,P}|^4\, m_W^2}{m_{T_2}^2\, 105}(2|\vec{k}_{T_2,P}|^2 + 7m_{T_2}^2)$

Polarization sums for vector and tensor-2 fields

- For massive vector fields $V_\mu \propto \int_{dk} \sum_{\lambda=1}^{3} \epsilon_\mu(\lambda,\, \vec{k})(ae^{-ikx} + a^\dagger e^{ikx})$
- $k_\mu \epsilon^\mu = 0$ and $\epsilon_\mu(\lambda)\epsilon^\mu(\lambda') = \delta_{\lambda,\lambda'}$

$$\sum_{\lambda=1}^{3} \epsilon_\mu(\lambda,\, \vec{k})\, \epsilon_\nu(\lambda,\, \vec{k}) = -G_{\mu\nu}$$

- For massive spin-2 tensors $X_{\mu\nu} \propto \int_{dk} \sum_{\lambda=1}^{5} \epsilon_{\mu\nu}(\lambda,\, \vec{k})(ae^{-ikx} + a^\dagger e^{ikx})$
- Fierz-Pauli constraints: $X^{\mu\nu} - X^{\nu\mu} = 0, \quad g_{\mu\nu} X^{\mu\nu} = 0$ and $\partial_\mu X^{\mu\nu} = 0$
- Orthonormality condition $\epsilon^{\mu\nu}(\lambda)\epsilon_{\mu\nu}(\lambda') = -\delta_{\lambda\lambda'}$

$$\sum_{\lambda=1}^{5} \epsilon_{\mu\nu}(\lambda,\, \vec{k})\, \epsilon_{\alpha\beta}(\lambda,\, \vec{k}) = -\frac{G_{\mu\nu} G_{\alpha\beta}}{3} + \frac{G_{\mu\alpha} G_{\nu\beta} + G_{\mu\beta} G_{\nu\alpha}}{2}$$

where

$$G_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}\,, \quad \eta_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Polarization sum for spin-3 fields

- Fierz-Pauli conditions for spin-3 fields lead:

$$\epsilon^{(\mu\nu\rho)} = 0, \quad \epsilon^{\mu\nu}_{\mu} = 0, \quad k_\mu \epsilon^{\mu\nu\rho} = 0$$

- Orthonormality condition

$$\epsilon^{\mu\nu\rho}(\lambda)\epsilon_{\mu\nu\rho}(\lambda') = -\delta_{\lambda\lambda'}$$

$$\sum_{\lambda=1}^{7} \epsilon_{\mu\nu\rho}(\lambda)\,\epsilon_{\alpha\beta\gamma}(\lambda) = \frac{1}{15}\Big\{ G_{\mu\nu}\left(G_{\rho\alpha}G_{\beta\gamma} + G_{\rho\beta}G_{\alpha\gamma} + G_{\rho\gamma}G_{\alpha\beta}\right)$$

$$G_{\mu\rho}\left(G_{\nu\alpha}G_{\beta\gamma} + G_{\nu\beta}G_{\alpha\gamma} + G_{\nu\gamma}G_{\alpha\beta}\right) + G_{\nu\rho}\left(G_{\mu\alpha}G_{\beta\gamma} + G_{\mu\beta}G_{\alpha\gamma} + G_{\mu\gamma}G_{\alpha\beta}\right)$$

$$-\frac{5}{2}\Big[G_{\mu\alpha}\left(G_{\nu\beta}G_{\rho\gamma} + G_{\nu\gamma}G_{\rho\beta}\right) + G_{\mu\beta}\left(G_{\nu\alpha}G_{\rho\gamma} + G_{\nu\gamma}G_{\rho\alpha}\right)$$

$$+G_{\mu\gamma}\left(G_{\nu\alpha}G_{\rho\beta} + G_{\nu\beta}G_{\rho\alpha}\right)\Big]\Big\}$$

Results for $W \to P + P$

- Two pseudoscalars decay $g_{WPP}\,\mathbf{tr}\left[W^{\mu\nu\rho}\left[P\,,(\partial_\mu\partial_\nu\partial_\rho P)\right]_-\right]$
- For the coupling constant g^2_{WPP}, experimental results $\tilde{g}_i^2$ and errors on them Δg^2, $\Delta\tilde{g}_i^2$ we define $\chi^2 \equiv \sum_{i=1}^{N}\frac{(\tilde{g}-\tilde{g}_i)^2}{\Delta\tilde{g}_i^2}$
- Minimizing χ^2 with respect to coupling $\frac{d\chi^2}{d\tilde{g}} = 0$ leads to

$$g^2_{WPP} = \frac{\sum_{i=1}^{N}\frac{\tilde{g}_i^2}{\Delta\tilde{g}_i^2}}{\sum_{j=1}^{N}\frac{1}{\Delta\tilde{g}_j^2}},\ \Delta g^2_{WPP} = \sqrt{\frac{1}{\sum_{j=1}^{N}\frac{1}{\Delta g_j^2}}} \to g^2_{WPP} = (1.5 \pm 0.1)\cdot 10^{-10}(\mathrm{MeV})^{-4}$$

Decay process (in model)	Theory (MeV)	PDG (MeV)
$\rho_3(1690) \longrightarrow \pi\,\pi$	32.7 ± 2.3	$38.0 \pm 3.2 \leftrightarrow (23.6 \pm 1.3)\%$
$\rho_3(1690) \longrightarrow \bar{K}\,K$	4.0 ± 0.3	$2.54 \pm 0.45 \leftrightarrow (1.58 \pm 0.26)\%$
$K_3^*(1780) \longrightarrow \pi\,\bar{K}$	18.5 ± 1.3	$29.9 \pm 4.3 \leftrightarrow (18.8 \pm 1.0)\%$
$K_3^*(1780) \longrightarrow \bar{K}\,\eta$	7.4 ± 0.6	$47.7 \pm 21.6 \leftrightarrow (30 \pm 13)\%$
$\omega_3(1670) \longrightarrow \bar{K}\,K$	3.0 ± 0.2	
$\phi_3(1850) \longrightarrow \bar{K}\,K$	18.8 ± 1.4	seen

Results for $W \to V + P$

- Vector and pseudoscalar decay $g_{WVP}\,\varepsilon^{\mu\nu\rho\sigma}\,\mathbf{tr}\big[W_{\mu\alpha\beta}\{(V_{\nu\rho}),(\partial^{\alpha}\partial^{\beta}\partial_{\sigma}P)\}_{+}\big]$

$\rho_3(1690) \to \rho(770)\,\eta$	3.8 ± 0.8	seen
$\rho_3(1690) \to \bar{K}^*(892)\,K$	3.4 ± 0.7	
$\rho_3(1690) \to \omega(782)\,\pi$	35.8 ± 7.4	25.8 ± 9.8
$\rho_3(1690) \to \phi(1020)\,\pi$	0.036 ± 0.007	
$K_3^*(1780) \to \rho(770)\,K$	16.8 ± 3.5	49.3 ± 15.7
$K_3^*(1780) \to \bar{K}^*(892)\,\pi$	27.2 ± 5.6	31.8 ± 9.0
$K_3^*(1780) \to \bar{K}^*(892)\,\eta$	0.09 ± 0.02	
$K_3^*(1780) \to \omega(782)\,\bar{K}$	4.3 ± 0.9	
$K_3^*(1780) \to \phi(1020)\,\bar{K}$	1.2 ± 0.3	

$\omega_3(1670) \to \rho(770)\,\pi$	97 ± 20	seen
$\omega_3(1670) \to \bar{K}^*(892)\,K$	2.9 ± 0.6	
$\omega_3(1670) \to \omega(782)\,\eta$	2.8 ± 0.6	
$\omega_3(1670) \to \phi(1020)\,\eta$	$(7.6 \pm 1.6)\cdot 10^{-6}$	
$\phi_3(1850) \to \rho(770)\,\pi$	1.1 ± 0.2	
$\phi_3(1850) \to \bar{K}^*(892)\,K$	35.5 ± 7.3	seen
$\phi_3(1850) \to \omega(782)\,\eta$	0.015 ± 0.003	
$\phi_3(1850) \to \omega(782)\,\eta'(958)$	0.003 ± 0.001	
$\phi_3(1850) \to \phi(1020)\,\eta$	3.8 ± 0.8	

- Radiative decays $V_{\mu\nu} \to V_{\mu\nu} + \frac{e}{g_\rho}F_{\mu\nu}Q$ where $Q = \mathrm{diag}\{2/3, -1/3, -1/3\}$

	Γ/keV
$\rho_3^{\pm/0}(1690) \to \gamma\,\pi^{\pm/0}$	69 ± 14
$\rho_3^0(1690) \to \gamma\,\eta$	157 ± 32
$\rho_3^0(1690) \to \gamma\,\eta'(958)$	20 ± 4
$K_3^{\pm}(1780) \to \gamma\,K^{\pm}$	58 ± 12
$K_3^0(1780) \to \gamma\,K^0$	231 ± 48

$\omega_3(1670) \to \gamma\,\pi^0$	560 ± 120
$\omega_3(1670) \to \gamma\,\eta$	19 ± 4
$\omega_3(1670) \to \gamma\,\eta'(958)$	1.4 ± 0.3
$\phi_3(1850) \to \gamma\,\pi^0$	4 ± 1
$\phi_3(1850) \to \gamma\,\eta$	129 ± 26
$\phi_3(1850) \to \gamma\,\eta'(958)$	35 ± 7

- Comparison to Lattice QCD data [C.Johnson and J.Dudek (arXiv:2012.00518)]

Decay process (in model)	Theory (MeV)	LQCD (MeV)
$\rho_3(1690) \longrightarrow \bar{K}^*(892)\, K + \mathbf{c}.\mathrm{c}.$	3	2
$\rho_3(1690) \longrightarrow \omega(782)\, \pi$	36	22
$\omega_3(1670) \longrightarrow \rho(770)\, \pi$	97	62
$\omega_3(1670) \longrightarrow \bar{K}^*(892)\, K + \mathbf{c}.\mathrm{c}.$	2.9	2
$\omega_3(1670) \longrightarrow \omega(782)\, \eta$	2.8	1
$\phi_3(1850) \longrightarrow \bar{K}^*(892)\, K + \mathbf{c}.\mathrm{c}.$	36	20
$\phi_3(1850) \longrightarrow \phi(1020)\, \eta$	4	3

- Results of tensor-pseudoscalar $g_{WT_2P}\, \varepsilon_{\mu\nu\rho\sigma}\, \mathbf{tr}\left[W^{\mu}_{\alpha\beta}\left[(\partial^{\nu} T_2^{\rho\alpha}), (\partial^{\sigma}\partial^{\beta} P)\right]_-\right]$

decay process	theory Γ/MeV	experiment Γ/MeV
$\rho_3(1690) \to a_2(1320)\, \pi$	20.9 ± 8.7	seen
$K_3^*(1780) \to \bar{K}_2^*(1430)\, \pi$	5.8 ± 2.4	$< 25.4 \pm 3.4$
$K_3^*(1780) \to f_2(1270)\, \bar{K}$	$(5.4 \pm 2.2) \cdot 10^{-5}$	

Spin-2

- "Tensor mesons within extended Linear Sigma Model" [Sh.Jafarzade, A.Vereijken, M.Piotrowska and F.Giacosa, (arXiv:21xx.xxxxx)]

Extended Linear Sigma Model (eLSM)

- Fermionic (zero quark mass) part of the QCD lagrangian is invariant under global $U(N_f)_L \times U(N_f)_R \longrightarrow \quad \mathrm{Tr}\Big(\bar{q}_{i,L}(iD)q_{i,L} + \bar{q}_{i,R}(iD)q_{i,R}\Big)$
- Chiral invariant lagrangian for $\Phi := \sum_a \phi_a T^a$ with T^a generators of $U(N_f)$

$$\mathcal{L} = \mathrm{Tr}\Big\{(\partial_\mu \Phi^\dagger)(\partial^\mu \Phi)\Big\} - m^2 \mathrm{Tr}\Big\{\Phi^\dagger \Phi\Big\} - \lambda_1 \mathrm{Tr}\Big\{(\Phi^\dagger \Phi)^2\Big\} - \lambda_2 \Big(\mathrm{Tr}\Big\{\Phi^\dagger \Phi\Big\}\Big)^2$$

- Extension of the Linear Sigma Model to (axial)-vector mesons in [D. Parganlija, F. Giacosa, et al. PRD 87 (2013) 014011]

$$L^\mu := \sum_{i=0}^{8} (V_i^\mu + A_i^\mu) T_i, \quad R^\mu := \sum_{i=0}^{8} (V_i^\mu - A_i^\mu) T_i, \quad D_\mu \Phi = \partial_\mu \Phi - i g_1 [L_\mu \Phi - \Phi R_\mu]$$

- Mesons within chiral nonets

$$\Phi = \sum_{i=0}^{8} (S_i + iP_i) T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

$$L^\mu = \sum_{i=0}^{8} (V_i^\mu + A_i^\mu) T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{\star +} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} + K_1^0 \\ K^{\star -} + K_1^- & \bar{K}^{\star 0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}^\mu$$

$$R^\mu = \sum_{i=0}^{8} (V_i^\mu - A_i^\mu) T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{\star +} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} - K_1^0 \\ K^{\star -} - K_1^- & \bar{K}^{\star 0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}^\mu$$

Spin-2 mesons within (eLSM)

- Extension of the Linear Sigma Model to (axial)-tensor mesons in

$$\mathbf{L}^{\mu\nu} := \sum_{i=0}^{8}(T_i^{\mu\nu} + A_i^{\mu\nu})T_i, \quad \mathbf{R}^{\mu\nu} := \sum_{i=0}^{8}(T_i^{\mu\nu} - A_i^{\mu\nu})T_i$$

Nonet	Parity (P)	Charge conjugation (C)	Chiral symmetry
$\Phi(t,\vec{x})$	$\Phi^\dagger(t,-\vec{x})$	$\Phi^t(t,\vec{x})$	$U_L \Phi U_R^\dagger$
$R^\mu(t,\vec{x})$	$L_\mu(t,-\vec{x})$	$-(L^\mu(t,\vec{x}))^t$	$U_R R^\mu U_R^\dagger$
$L^\mu(t,\vec{x})$	$R_\mu(t,-\vec{x})$	$-(R^\mu(t,\vec{x}))^t$	$U_L L^\mu U_L^\dagger$
$\mathbf{R}^{\mu\nu}(t,\vec{x})$	$\mathbf{L}_{\mu\nu}(t,-\vec{x})$	$(\mathbf{L}^{\mu\nu}(t,\vec{x}))^t$	$U_R \mathbf{R}^{\mu\nu} U_R^\dagger$
$\mathbf{L}^{\mu\nu}(t,\vec{x})$	$\mathbf{R}_{\mu\nu}(t,-\vec{x})$	$(\mathbf{R}^{\mu\nu}(t,\vec{x}))^t$	$U_L \mathbf{L}^{\mu\nu} U_L^\dagger$

Table: Transformations of the fields

- Chiral and dilatation invariant with $\Delta := \mathrm{diag}\{\delta_N, \delta_N, \delta_S\}$

$$\mathcal{L} = \mathrm{Tr}\Big\{\big(\frac{m^2 G^2}{2G_0^2} + \Delta\big)(\mathrm{L}_{\mu\nu}^2 + \mathrm{R}_{\mu\nu}^2)\Big\} + \frac{h_1^{\mathrm{ten}}}{2}\mathrm{Tr}\{\Phi^\dagger\Phi\}\mathrm{Tr}\{\mathbf{L}^{\mu\nu}\mathbf{L}_{\mu\nu} + \mathbf{R}^{\mu\nu}\mathbf{R}_{\mu\nu}\} +$$
$$+ h_2^{\mathrm{ten}}\mathrm{Tr}\{\Phi^\dagger\mathbf{L}^{\mu\nu}\mathbf{L}_{\mu\nu}\Phi + \Phi\mathbf{R}^{\mu\nu}\mathbf{R}_{\mu\nu}\Phi^\dagger\} + 2h_3^{\mathrm{ten}}\mathrm{Tr}\{\Phi\mathbf{R}^{\mu\nu}\Phi^\dagger\mathbf{L}_{\mu\nu}\},$$

Results and Conclusion

- Mass splitting relations for spin-2 mesons

$$m_{\rho_2}^2 - m_{a_2}^2 = -h_3^{\text{ten}}\phi_N^2, \quad m_{K_{2A}}^2 - m_{K_2}^2 = -\sqrt{2}h_3^{\text{ten}}\phi_N\phi_S, \quad m_{f_{2s}}^2 - m_{\omega_{2,S}}^2 = 2h_3^{\text{ten}}\phi_S^2$$

$$m_{\rho_2}^2 = m_{\omega_{2,N}}^2\ , \quad m_{a_2}^2 = m_{f_{2n}}^2$$

Resonance	Mass (in MeV)	Resonance	Mass (in MeV)
$a_2(1320)$	**1317**	$\rho_2(?)$	1661
$K_2^*(1430)$	**1427**	$K_2^*(1820)$	**1819**
$f_2(1270)$	1315	$\omega_{2,N}(?)$	1661
$f_2'(1525)$	1522	$\omega_{2,S}(?)$	1966

- $T_2 \longrightarrow P + P\ , \quad A_2 \longrightarrow V + P$

$$\mathcal{L} = g_2\Big(\text{Tr}\big\{\mathbf{L}_{\mu\nu}L^\mu L^\nu\big\} + \text{Tr}\big\{\mathbf{R}_{\mu\nu}R^\mu R^\nu\big\}\Big)$$

- $T_2 \longrightarrow V + P$

$$\mathcal{L}_2 = c_1\,\text{tr}\Big[\partial^\mu \mathbf{L}^{\nu\alpha}\tilde{L}_{\mu\nu}L_\alpha + \partial^\mu \mathbf{R}^{\nu\alpha}R_\alpha\tilde{R}_{\mu\nu}\Big] - c_1\,\text{tr}\Big[\partial^\mu \mathbf{R}^{\nu\alpha}\tilde{R}_{\mu\nu}R_\alpha + \partial^\mu \mathbf{L}^{\nu\alpha}L_\alpha\tilde{L}_{\mu\nu}\Big]$$

Two pseudoscalars decay

$$\Gamma^{tl}_{T_2 \longrightarrow P^{(1)}+P^{(2)}}(m_{T_2}, m_{P^{(1)}}, m_{P^{(2)}}) = \frac{g_2^2 |\vec{k}_{P^{(1)},P^{(2)}}|^5}{60\,\pi\, m_{T_2}^2} \times \kappa_i \times \Theta(m_{T_2} - m_{P^{(1)}} - m_{P^{(2)}})$$

Decay process (in model)	eLSM	PDG-2020
$a_2(1320) \longrightarrow \bar{K}\,K$	7.62 ± 0.17	$5.145 \pm 0.849 \leftrightarrow (4.9 \pm 0.8)\%$
$a_2(1320) \longrightarrow \pi\,\eta$	19.82 ± 0.44	$15.23 \pm 1.31 \leftrightarrow (14.5 \pm 1.2)\%$
$a_2(1320) \longrightarrow \pi\,\eta'(958)$	0.79 ± 0.02	$0.58 \pm 0.10 \leftrightarrow (0.55 \pm 0.09)\%$
$K_2^*(1430) \longrightarrow \pi\,\bar{K}$	38.39 ± 0.85	$49.9 \pm 1.9 \leftrightarrow (49.9 \pm 0.6)\%$
$f_2(1270) \longrightarrow \bar{K}\,K$	12.85 ± 0.28	$8.51^{+0.94}_{-0.75} \leftrightarrow (4.6^{+0.5}_{-0.4})\%$
$f_2(1270) \longrightarrow \pi\,\pi$	129.08 ± 2.86	$156.5^{+5.88}_{-2.43} \leftrightarrow (84.2^{+2.9}_{-0.9})\%$
$f_2(1270) \longrightarrow \eta\,\eta$	0.53 ± 0.01	$0.74 \pm 0.15 \leftrightarrow (0.4 \pm 0.08)\%$
$f_2'(1525) \longrightarrow \bar{K}\,K$	87.02 ± 1.93	$76.12 \pm 3.32 \leftrightarrow (87.6 \pm 2.2)\%$
$f_2'(1525) \longrightarrow \pi\,\pi$	0.74 ± 0.02	$0.72 \pm 0.14 \leftrightarrow (0.83 \pm 0.16)\%$
$f_2'(1525) \longrightarrow \eta\,\eta$	3.74 ± 0.08	$10.08 \pm 1.95 \leftrightarrow (11.6 \pm 2.2)\%$

$$\beta_T = \arctan\left(\sqrt{\frac{4m_{K_2}^2 - m_{a_2}^2 - 3m_{f_2'}^2}{-4m_{K_2}^2 + m_{a_2}^2 + 3m_{f_2}^2}}\right)$$

PDG: $= 28.0^\circ$ Th: $= 32.4^\circ$

Vector+ Pseudoscalar Decay

- Decays of 2^{++} mesons

Decay process	eLSM	PDG-2020
$a_2(1320) \longrightarrow \rho(770)\,\pi$	75.26 ± 2.79	$73.61 \pm 3.35 \leftrightarrow (70.1 \pm 2.7)\%$
$K_2^*(1430) \longrightarrow \bar{K}^*(892)\,\pi$	29.31 ± 1.09	$26.92 \pm 2.14 \leftrightarrow (24.7 \pm 1.6)\%$
$K_2^*(1430) \longrightarrow \rho(770)\,K$	5.44 ± 0.20	$9.48 \pm 0.97 \leftrightarrow (8.7 \pm 0.8)\%$
$K_2^*(1430) \longrightarrow \omega(782)\,\bar{K}$	1.90 ± 0.07	$3.16 \pm 0.88 \leftrightarrow (2.9 \pm 0.8)\%$
$f_2'(1525) \longrightarrow \bar{K}^*(892)\,K + \mathrm{c.c.}$	10.14 ± 0.38	
$K_2^{\pm}(1430) \longrightarrow \gamma\,K^{\pm}$	0.68 ± 0.03	0.24 ± 0.05
$a_2^{\pm}(1320) \longrightarrow \gamma\,\pi^{\pm}$	1.08 ± 0.04	0.31 ± 0.03

- Some predictions for missing 2^{--} resonances

Decay process (in model)	Theory (MeV)
$\rho_2(?) \longrightarrow \rho(770)\,\eta$	86
$\rho_2(?) \longrightarrow \bar{K}^*(892)\,K + \mathrm{c.c.}$	75
$\rho_2(?) \longrightarrow \omega(782)\,\pi$	306
$\rho_2(?) \longrightarrow \phi(1020)\,\pi$	0.7

Summary and conclusion

- Phenomenology of the spin-3 and spin-2 mesons is studied
- Tree level decay rates are overall agreement with the PDG data
- $SU(N_f = 3)_V$ approximate symmetry is considered as a main symmetry for effective model for studying spin-3
- Simple model results for spin-3 are qualitatively similar to advanced lattice QCD
- We predict some values for decay rates which can be tested in future *GlueX* and *CLAS12* experiments at Jefferson Lab
- For spin-2 we use the zero-quark mass symmetry of QCD - $U(3)_L \times U(3)_R$
- Mass prediction for missing $\rho_2(?)$ is near to [S. Godfrey and N. Isgur PRD (1985) 32, 189]

Thank you for the attention!