Theoretical study of the $D^+ \to \pi^+\eta\eta$ and $D^+ \to \pi^+\pi^0\eta$ reactions

M. Bayar

Kocaeli University

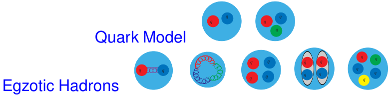
MESON 2021
19 May 2021
Collaborators: Natsumi Ikeno, Eulogio Oset

Outline

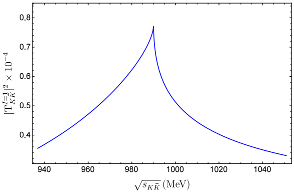
- Introduction
- Formalism :
  - weak decay
  - hadronization
  - the final state interactions
- Results for $D^+ \to \pi^+\pi^0\eta$
- Results for $D^+ \to \pi^+\eta\eta$
- Summary and Conclusion

Introduction

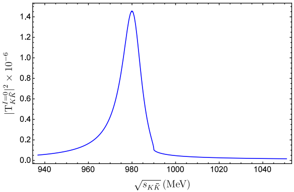
- D weak decays into three light mesons
  $\Rightarrow$ crucial to explore the strong and weak interaction effects J.R. Ellis, M.K. Gaillard, D.V. Nanopoulos, Nucl. Phys. B 100, 313 (1975); M. Matsuda, M. Nakagawa, K. Odaka, S. Ogawa, M. Shin-Mura, Prog. Theor. Phys. 59, 1396 (1978); M. Nakagawa, Prog. Theor. Phys. 60, 1595 (1978)
  $\Rightarrow$ provide information the meson-meson interaction E.M. Aitala et al. [E791 Collaboration], Phys. Rev. Lett. 86, 765 (2001); J.M. Link et al. [FOCUS], Phys. Lett. B 585, 200 (2004); E. Klempt, M. Matveev, A.V. Sarantsev, Eur. Phys. J. C 55, 39 (2008); J.A. Oller, Phys. Rev. D 71, 054030 (2005);F. Niecknig, B. Kubis, Phys. Lett. B 780, 471 (2018); B. Aubert et al. [BaBar], Phys. Rev. Lett. 99, 251801 (2007)
- The nature of the low-lying light scalar resonances are still problematic
- Their nature is still discussed, either as $q\bar{q}$, hybrids, tetraquarks and meson-meson molecules.



- Both $a_0(980)$ and $f_0(980)$ have a mass around the $K\bar{K}$ threshold, and couple to $K\bar{K}$



(a) $K\bar{K}$ amplitude in $I = 1$; couples to $a_0(980)$,



(b) $K\bar{K}$ amplitude in $I = 0$; couples to $f_0(980)$

Introduction

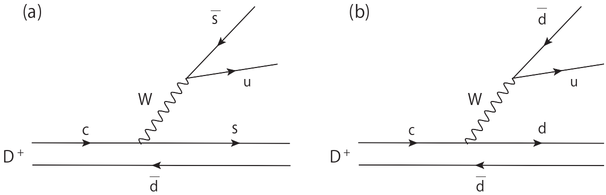
- CLEO Collaboration measured the branching ratio of $D^+ \to \pi^+\pi^0\eta$ (M. Artuso et al. [CLEO Collaboration], Phys. Rev. D 77, 092003(2008))
- BESIII Collaboration : amplitude analysis of the decay $D_s^+ \to \pi^+\pi^0\eta$ and the W-annihilation dominant decays $D_s^+ \to a_0(980)^+\pi^0$, $D_s^+ \to a_0(980)^0\pi^+$ (M. Ablikim *et al.* PRL 123, 112001 (2019))
- BESIII Collaboration measured the absolute branching fractions of the $D^+ \to \pi^+\eta\eta$ and $D^{0(+)} \to \pi^+\pi^{-(0)}\eta$ reactions (M. Ablikim *et al.* PRD 101, 052009 (2020))
- The $D_s^+ \to \pi^+\pi^0\eta$ decay and the nature of a0(980) (R. Molina, JJ. Xie, WH. Liang, LS. Geng, E. Oset Phys. Lett. B803(2020)135279) (R. Molina, Monday,P.S. A1, 17:45)
- Role of scalar a0(980) in the single Cabibbo Suppressed process $D^+ \to \pi^+\pi^0\eta$ (MY. Duan, JY. Wang, GY. Wang, E. Wang, DM. Li, Eur. Phys.J. C 80, 1041 (2020))

Formalism

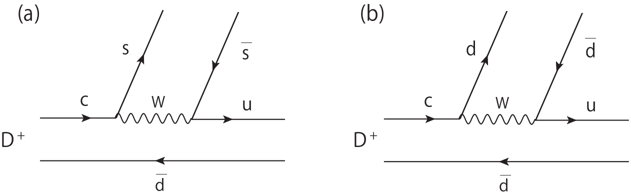
- Weak decay
- Hadronization
- The final state interactions

Weak Decay

- **External emission:** (a) Cabibbo suppressed $Wu\bar{s}$ vertex, (b) Cabibbo suppressed Wcd vertex.



- **Internal emission:** (a) Cabibbo suppressed $W\bar{s}u$ vertex, (b) Cabibbo suppressed Wcd vertex.



Hadronization

- Hadronization ⇒ a $\bar{q}q$ pair SU(3) singlet $\bar{u}u + \bar{d}d + \bar{s}s$
- The hadronization of the $u\bar{s}$ pair as:

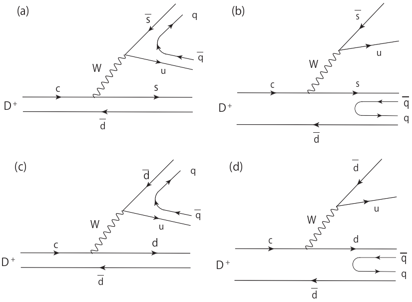
$$u\bar{s} \rightarrow \sum_i u\bar{q}_i q_i \bar{s} = \sum_i M_{1i} M_{i3} = (M^2)_{13}, \qquad (1)$$

M is the $q\bar{q}$ matrix ⇒ in terms of physical mesons

$$M \rightarrow P \equiv \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} \end{pmatrix} \qquad (2)$$

Hadronization: external emission

$$(a):(M^2)_{13}\bar{K}^0=\left(\frac{\pi^0K^+}{\sqrt{2}}+\pi^+K^0\right)\bar{K}^0,\quad (b):(M^2)_{32}K^+=\left(K^-\pi^+-\frac{\pi^0\bar{K}^0}{\sqrt{2}}\right)K^+, \qquad (3)$$



$$(c):\quad (M^2)_{12}\left(-\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{3}}\right)=\left(\frac{2}{\sqrt{3}}\eta\pi^++K^+\bar{K}^0\right)\left(-\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{3}}\right), \qquad (4)$$

$$(d):\quad (M^2)_{22}\pi^+=\left(\pi^-\pi^++\frac{\pi^0\pi^0}{2}+\frac{\eta\eta}{3}-\frac{2}{\sqrt{6}}\pi^0\eta+K^0\bar{K}^0\right)\pi^+, \qquad (5)$$

Hadronization: external emission

- (a)and (c) [(b)and (d)] $\Rightarrow$ the same topology and the same Cabibbo suppressing factor

$$H_1 = \pi^+ K^0 \bar{K}^0 - \frac{2}{\sqrt{6}} \eta \pi^+ \pi^0 + \frac{2}{3} \eta\eta\pi^+ + \frac{1}{\sqrt{2}} \pi^0 K^+ \bar{K}^0, \qquad (6)$$

$$H_2 = K^+ K^- \pi^+ + K^0 \bar{K}^0 \pi^+ - \frac{1}{\sqrt{2}} \pi^0 \bar{K}^0 K^+ \qquad (7)$$

$$+\pi^+\pi^-\pi^+ + \frac{1}{2}\pi^0\pi^0\pi^+ + \frac{1}{3}\eta\eta\pi^+ - \frac{2}{\sqrt{6}}\pi^0\eta\pi^+.$$

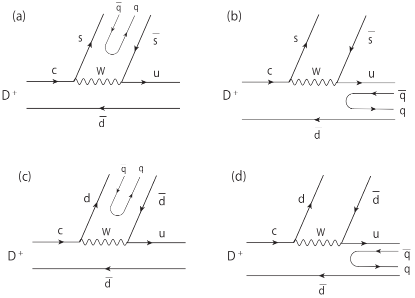
$$H_2 \equiv -\frac{1}{\sqrt{2}}\pi^0 \bar{K}^0 K^+ + \frac{1}{3}\eta\eta\pi^- - \frac{2}{\sqrt{6}}\pi^0\eta\pi^+.$$

- $K^- K^+ + K^0 \bar{K}^0 \Rightarrow I = 0$ and $\pi^0 \eta \Rightarrow I = 1 \Rightarrow (K^- K^+ + K^0 \bar{K}^0)\pi^+$ cannot give rise to $\pi^0 \eta \pi^+$
- $\pi^+\pi^- + \frac{\pi^0\pi^0}{2} \Rightarrow I = 0$ and hence cannot give $\pi^0\eta$
- Conctibute to $\eta\eta\pi^+$ production but $\eta\eta$ is far away from the narrow $f_0(980)$ resonance

Hadronization: internal emission

$$(a): \quad (M^2)_{33}\pi^+ = \left(K^-K^+ + \bar{K}^0K^0 + \frac{\eta\eta}{3}\right)\pi^+, \tag{8}$$

$$(b): \quad (M^2)_{12}\left(-\frac{\eta}{\sqrt{3}}\right) = \left(\frac{2}{\sqrt{3}}\eta\pi^+ + K^+\bar{K}^0\right)\left(-\frac{\eta}{\sqrt{3}}\right), \tag{9}$$



$$(c): \quad (M^2)_{22}\pi^+ = \left(\pi^-\pi^+ + \frac{\pi^0\pi^0}{2} + \frac{\eta\eta}{3} - \frac{2}{\sqrt{6}}\pi^0\eta + K^0\bar{K}^0\right)\pi^+, \tag{10}$$

$$(d): \quad (M^2)_{12}\left(-\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}}\right) = \left(\frac{2}{\sqrt{3}}\eta\pi^+ + K^+\bar{K}^0\right)\left(-\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}}\right). \tag{11}$$

Hadronization: internal emission

- (a)and (c) [(b)and (d)] the same topology and the same Cabibbo suppressing factor

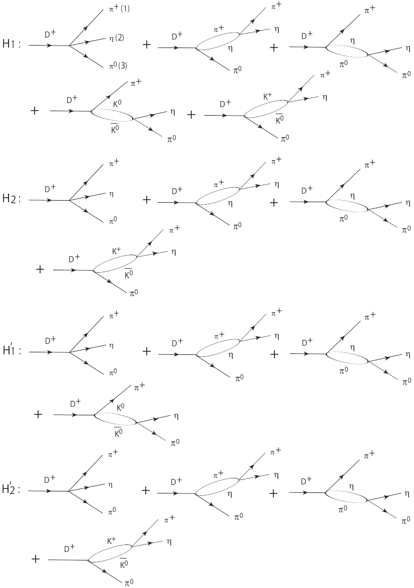
$$H_1' = K^- K^+ \pi^+ + 2K^0 \bar{K}^0 \pi^+ + \frac{2}{3}\eta\eta\pi^+ + \pi^-\pi^+\pi^+ \tag{12}$$

$$+\frac{1}{2}\pi^0\pi^0\pi^+ - \frac{2}{\sqrt{6}}\pi^0\eta\pi^+,$$

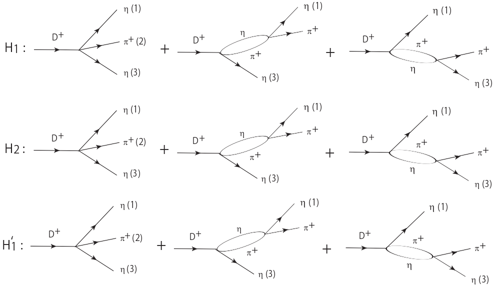
$$H_1' \equiv -\frac{2}{\sqrt{6}}\pi^0\eta\pi^+ + K^0\bar{K}^0\pi^+ + \frac{2}{3}\eta\eta\pi^+,$$

$$H_2' = -\frac{2}{\sqrt{6}}\eta\pi^0\pi^+ - \frac{1}{\sqrt{2}}\pi^0 K^+ \bar{K}^0. \tag{13}$$

Final state interaction to produce $\pi^+\pi^0\eta$



Final state interaction to produce $\pi^+\eta\eta$



$H'_2 :\quad 0$

Formalism: the amplitudes:

we give weights to the different terms;

$$H_1 \to A\beta, \quad H_2 \to A, \quad H_1' \to B, \quad H_2' \to B\gamma. \tag{14}$$

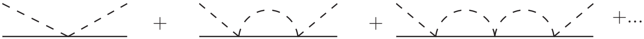
$$\begin{aligned} t_{D^+\to\pi^+\pi^0\eta} &= \left(h_{\pi^+\pi^0\eta} A\beta + \bar{h}_{\pi^+\pi^0\eta} A + h'_{\pi^+\pi^0\eta} B + \bar{h}'_{\pi^+\pi^0\eta} B\gamma\right) \\ &\cdot \left(1 + G_{\pi\eta}(M_{\rm inv}(\pi^+\eta))\, t_{\pi^+\eta,\pi^+\eta}(M_{\rm inv}(\pi^+\eta)) + G_{\pi\eta}(M_{\rm inv}(\pi^0\eta))\, t_{\pi^0\eta,\pi^0\eta}(M_{\rm inv}(\pi^0\eta))\right) \\ &+ \left(h_{\pi^+K^0\bar{K}^0} A\beta + h'_{\pi^+K^0\bar{K}^0} B\right) G_{K\bar{K}}(M_{\rm inv}(\pi^0\eta))\, t_{K^0\bar{K}^0,\pi^0\eta}(M_{\rm inv}(\pi^0\eta)) \\ &+ \left(h_{\pi^0K^+\bar{K}^0} A\beta + \bar{h}_{\pi^0K^+\bar{K}^0} A + \bar{h}'_{\pi^0K^+\bar{K}^0} B\gamma\right) G_{K\bar{K}}(M_{\rm inv}(\pi^+\eta))\, t_{K^+\bar{K}^0,\pi^+\eta}(M_{\rm inv}(\pi^+\eta)), \quad (15) \\ t_{D^+\to\pi^+\eta\eta} &= \frac{2}{\sqrt{2}} \left(h_{\pi^+\eta\eta} A\beta + \bar{h}_{\pi^+\eta\eta} A + h'_{\pi^+\eta\eta} B\right) \\ &\cdot \left(1 + G_{\pi\eta}(M_{\rm inv}(\pi^+\eta(1)))\, t_{\pi^+\eta,\pi^+\eta}(M_{\rm inv}(\pi^+\eta(1)))\right. \\ &+ \left. G_{\pi\eta}(M_{\rm inv}(\pi^+\eta(3)))\, t_{\pi^+\eta,\pi^+\eta}(M_{\rm inv}(\pi^+\eta(3)))\right), \end{aligned}$$

(16)

The scattering matrices and the differential width

- The Bethe-Salpeter equation

$$T = [1 - VG]^{-1} V$$



- The chiral unitary approach with the coupled channels, K^+K^-, $K^0\bar{K}^0$, $\pi^0\eta$
- The differential width:

$$\frac{d^2\Gamma}{dM_{\rm inv}(12)dM_{\rm inv}(23)} = \frac{1}{(2\pi)^3}\frac{M_{\rm inv}(12)M_{\rm inv}(23)}{8m_{D^+}^3}\,|t|^2 \qquad (17)$$

M. Artuso et al. [CLEO Collaboration], Phys. Rev. D 77, 092003 (2008)

$$\mathcal{B}(D^+ \to \pi^+\pi^0\eta) = (1.38 \pm 0.35) \times 10^{-3}. \qquad (18)$$

M. Ablikim et al. [BESIIICollaboration], Phys. Rev.D101, 052009 (2020)

$$\mathcal{B}(D^+ \to \pi^+\eta\eta) = (2.96 \pm 0.24 \pm 0.10) \times 10^{-3}, \qquad (19)$$

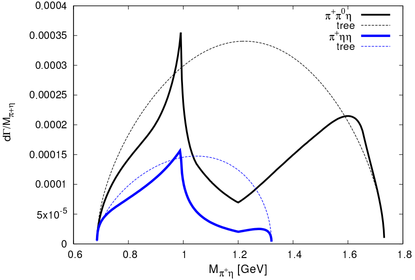
$$\mathcal{B}(D^+ \to \pi^+\pi^0\eta) = (2.23 \pm 0.15 \pm 0.10) \times 10^{-3}. \qquad (20)$$

- A, β, B, γ: $A = 1$ or -1 and find a set of three parameters β, B, γ that provide a ratio of

$$R = \mathcal{B}(D^+ \to \pi^+\eta\eta)/\mathcal{B}(D^+ \to \pi^+\pi^0\eta) = 1.33 \pm 0.16, \qquad (21)$$

Results

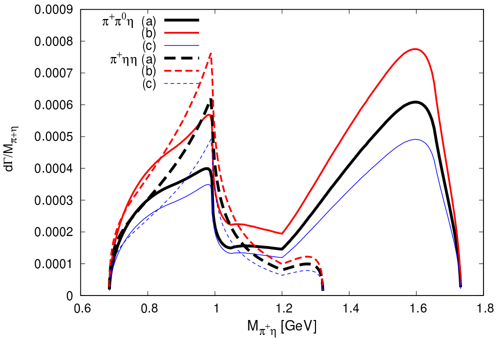
- $A = 1; \quad B \in [0.1, 0.6]; \quad \beta \in [-1, 3.0]; \quad \gamma \in [0.3, 1.5]$
- $A = 1,\ \beta = 1,\ B = \frac{1}{3},\ \gamma = 1$



$d\Gamma/dM_{\rm inv}(\pi^+\eta)$ for $D^+ \to \pi^+\pi^0\eta$ and $D^+ \to \pi^+\eta\eta$ and the phase space of $D^+ \to \pi^+\pi^0\eta$ and $D^+ \to \pi^+\eta\eta$

Results

- Differential cross sections for $D^+ \to \pi^+\pi^0\eta$ (solid lines) and $D^+ \to \pi^+\eta\eta$ (dashed lines)

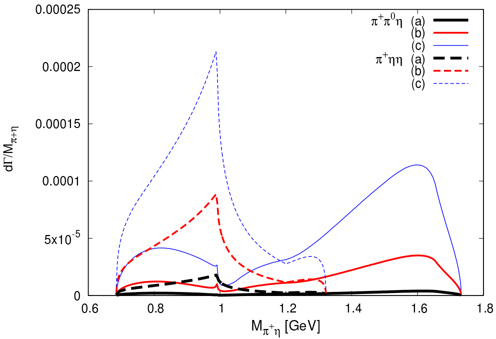


A=1

$$
\begin{aligned}
(a)\quad & \beta = 3.0, \quad B = 0.15, \quad \gamma = 0.33, \quad R = 0.46, \\
(b)\quad & \beta = 3.0, \quad B = 0.55, \quad \gamma = 0.33, \quad R = 0.44, \\
(c)\quad & \beta = 2.6, \quad B = 0.15, \quad \gamma = 0.33, \quad R = 0.45.
\end{aligned} \tag{22}
$$

Results

- Differential cross sections for $D^+ \to \pi^+\pi^0\eta$ (solid lines) and $D^+ \to \pi^+\eta\eta$ (dashed lines)

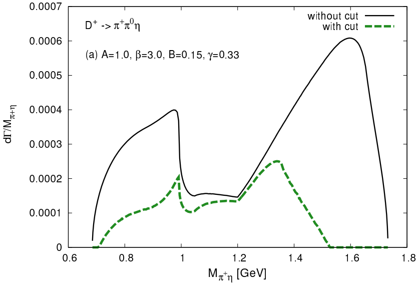


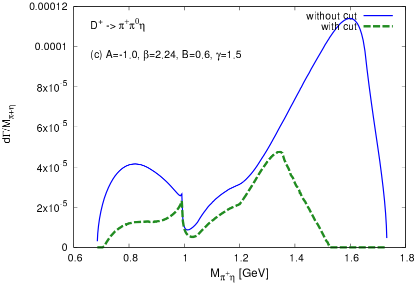
A=-1

$$\begin{aligned} (a)\quad &\beta = 0.72, \quad B = 0.6, \quad \gamma = 1.21, \quad R = 2.22, \\ (b)\quad &\beta = 1.48, \quad B = 0.6, \quad \gamma = 1.50, \quad R = 1.35, \\ (c)\quad &\beta = 2.24, \quad B = 0.6, \quad \gamma = 1.50, \quad R = 1.00. \end{aligned} \tag{23}$$

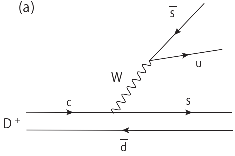
Results for $D^+ \to \pi^+\pi^0\eta$ with and without the cut $M_{\rm inv}(\pi^+\pi^0) > 1$ GeV

- to remove the $\rho^+\eta$ contribution (Fig. (b)) $\Rightarrow$ a cut $M_{\rm inv}(\pi^+\pi^0) > 1$ GeV

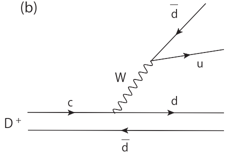




(a)

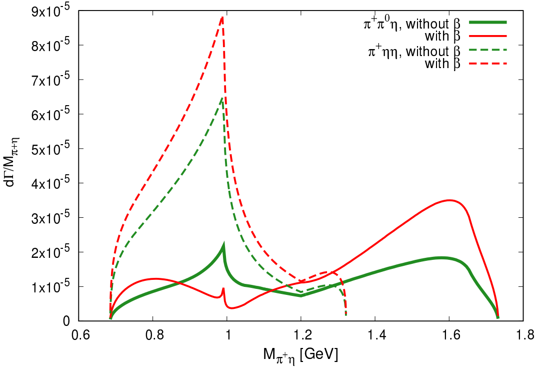


(b)



Results

- Mass distributions with and without β term.
- $A = -1$, $B = -0.68$. $\gamma = -1.5$ for which $R = 1.35$
- $A = -1$, $\beta = 1.48$, $B = 0.6$, $\gamma = 1.50$, $R = 1.35$



Summary and Conclusion

- We have studied the $D^+ \to \pi^+\eta\eta$ and $D^+ \to \pi^+\pi^0\eta$ reactions
- Both reactions are single Cabibbo suppressed
- Both the internal and external emission mechanisms are possible
- In all cases the $a_0(980)$ signal was visible in the $\pi^+\eta$ invariant mass distributions
- More neat in the $D^+ \to \pi^+\eta\eta$ reaction
- There are still uncertainties in the theory
- When the actual mass distributions are measured
  $\Rightarrow$ provide information on the reaction mechanism
  $\Rightarrow$ more assertive conclusions on the role played by the $a_0(980)$ resonance in these reactions

THANK YOU FOR YOUR ATTENTION