S-MATRIX APPROACH TO HADRONS GAS

POK MAN LO (盧博文)

University of Wroclaw

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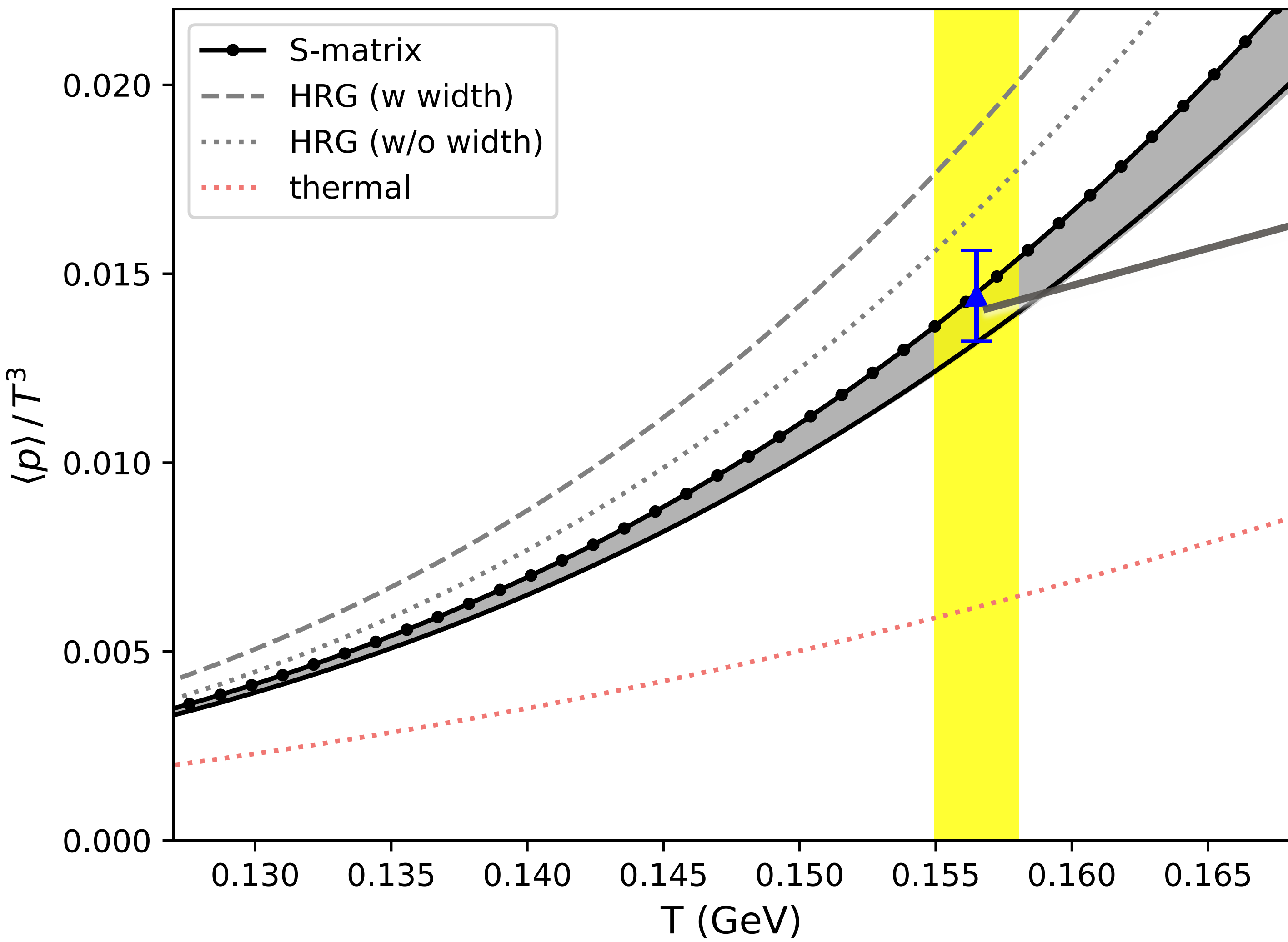
CONCLUSION



thermal model est.

ALICE proton yield

Pb-Pb @ 2.76 TeV

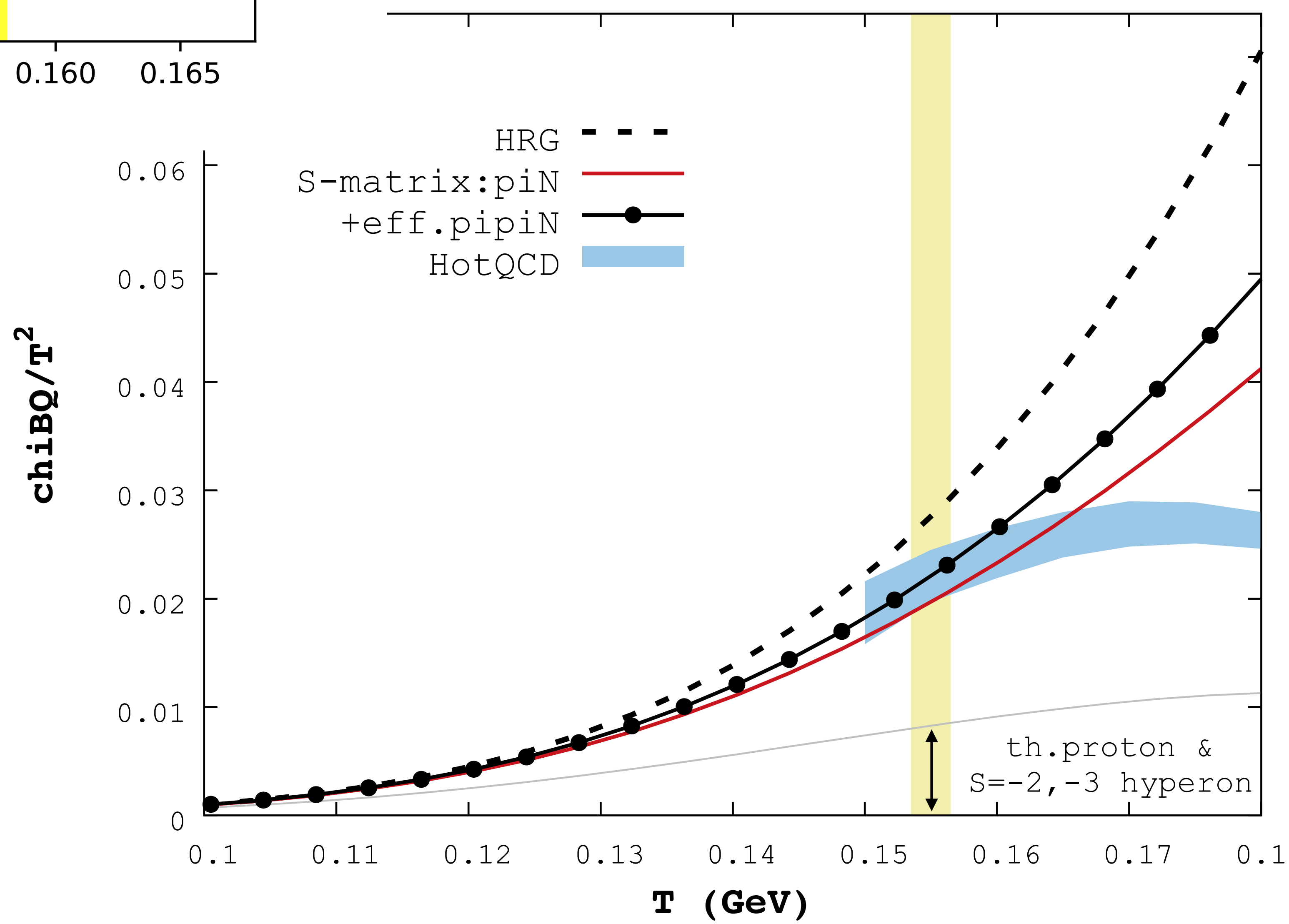
Phys. Lett. B **792**, 304 (2019)

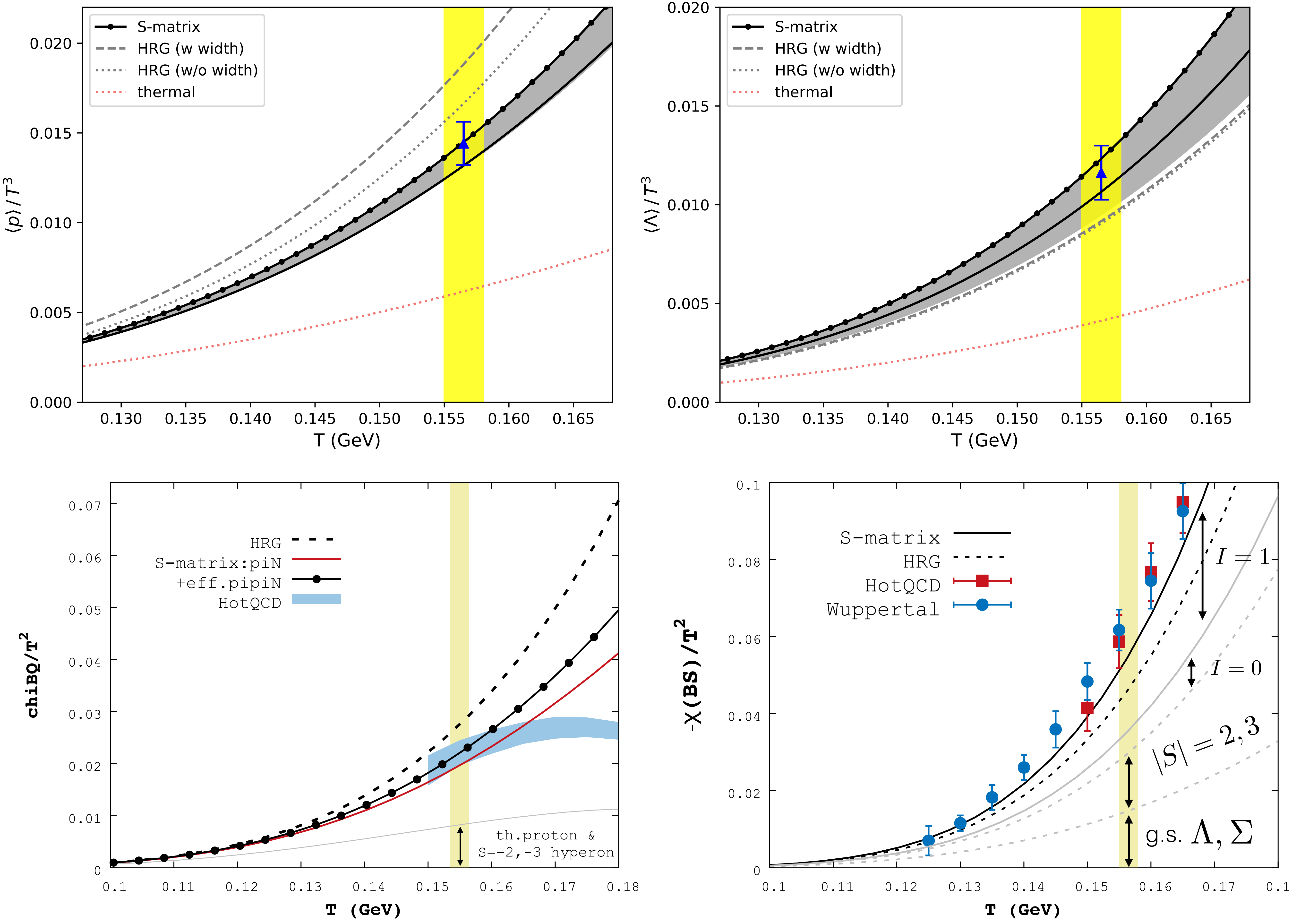
PRC **103**, 014904 (2021)

LQCD result on chiBQ

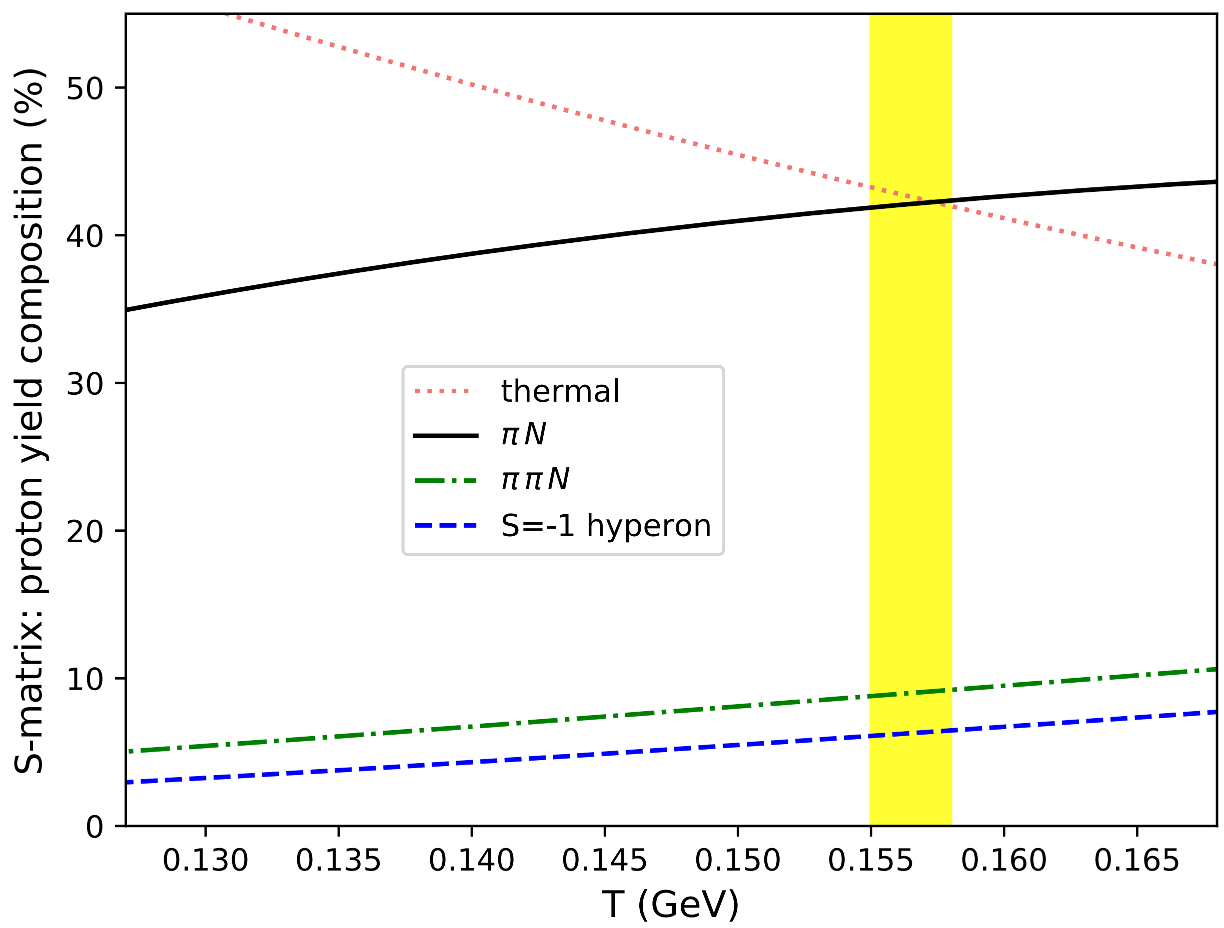
A. Bazavov, et al.,
Phys. Rev. D 86 (2012) 034509.

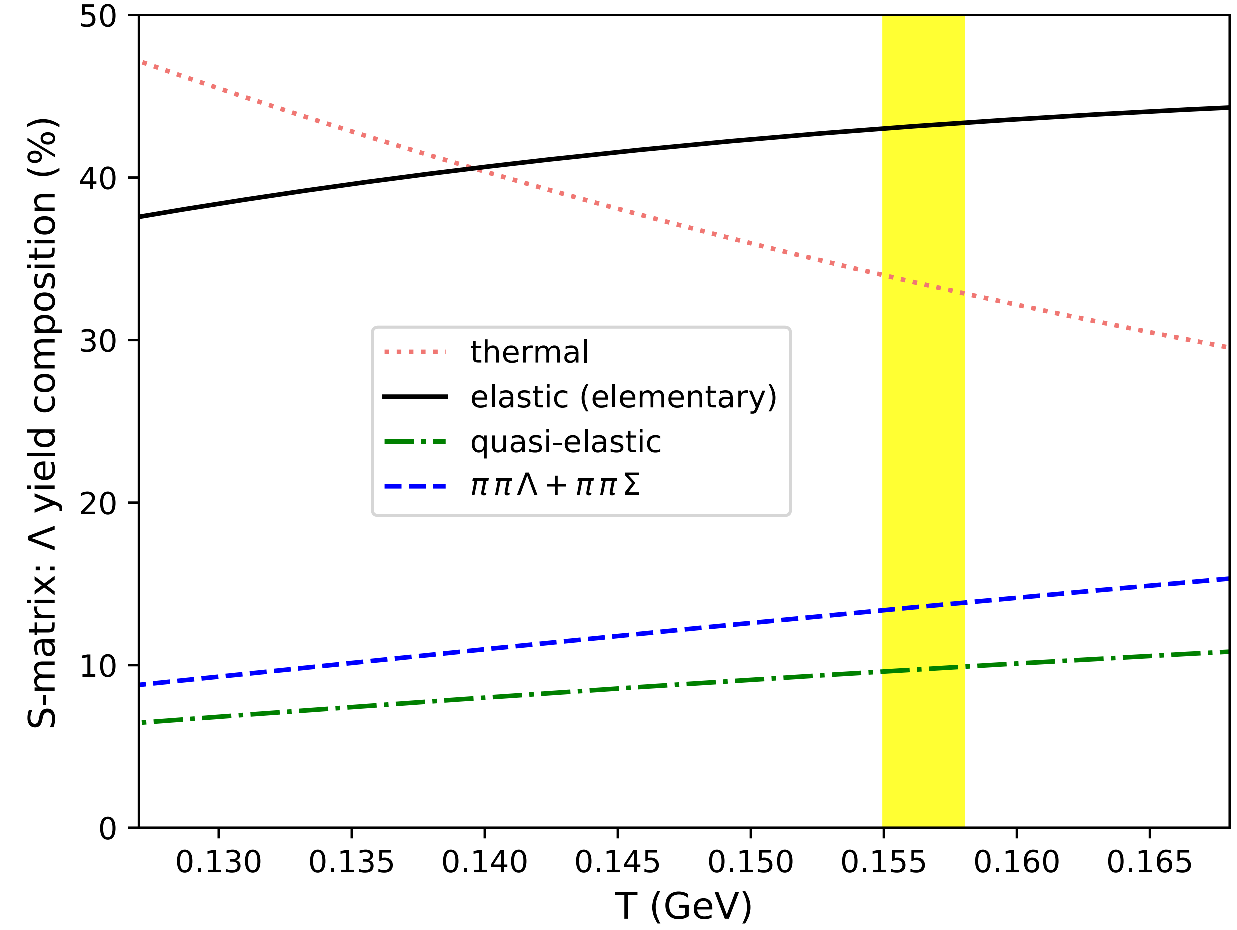
see also
Bellwied et al.
Phys. Rev. D 101, 034506 (2020)





S-matrix
HRG (w width)
HRG (w/o width)
thermal
⟨p⟩/T^3
T (GeV)
⟨Λ⟩/T^3
T (GeV)
HRG
S-matrix:piN
+eff.pipiN
HotQCD
th.proton &
S=-2,-3 hyperon
chiBQ/T^2
T (GeV)
S-matrix
HRG
HotQCD
Wuppertal
I = 1
I = 0
|S| = 2, 3
g.s. Λ, Σ
-χ(BS)/T^2
T (GeV)





SAID GWU

piN phase shifts
pipiN BGs
hyperons

JPAC

Coupled-Channel system:

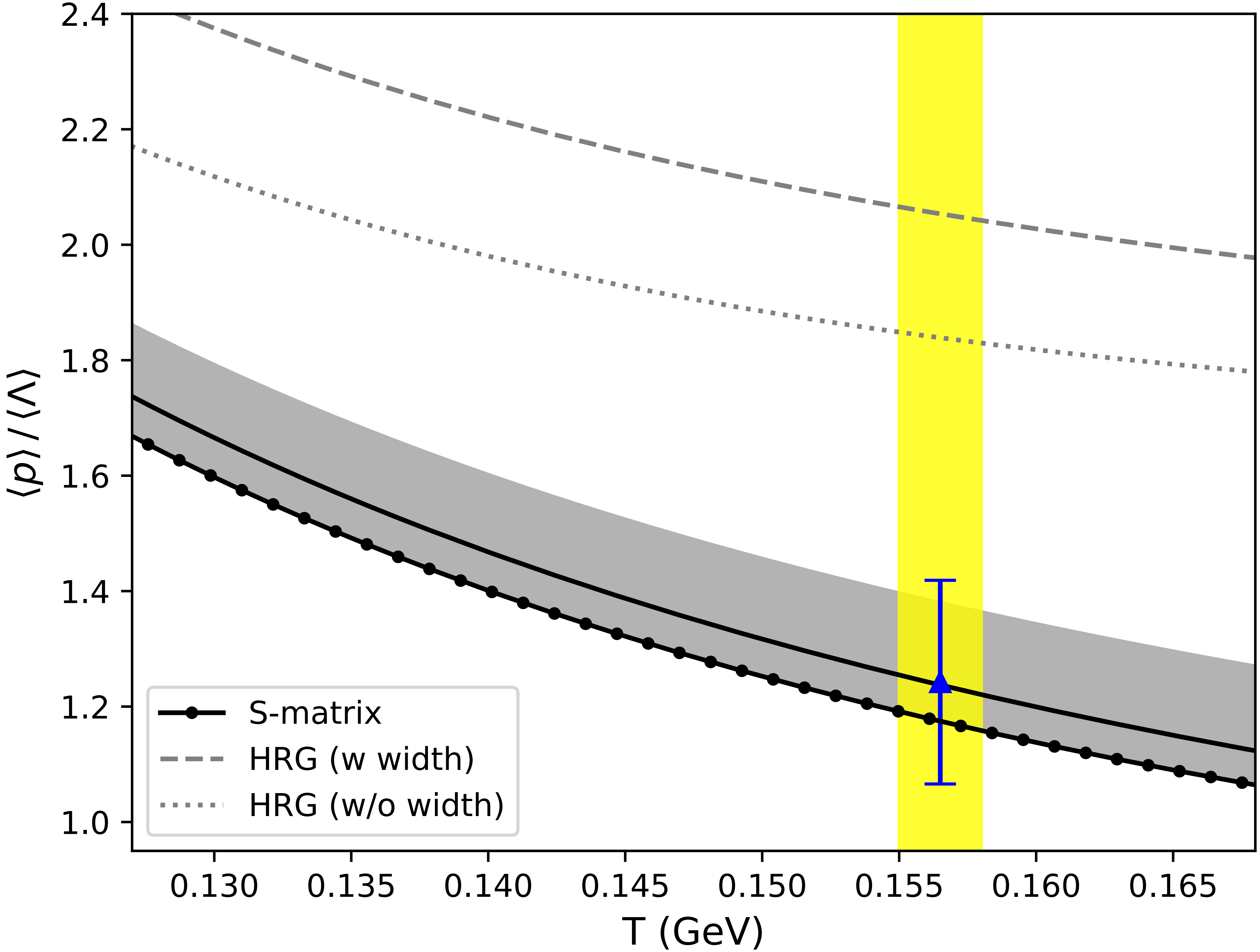
$$\bar{k}N, \pi\Lambda, \pi\Sigma, ...$$

extra hyperon states beyond PDG

unitarity BGs

consistent treatment of res and non-res. int.

ratio of yields



HADRON RESONANCE GAS
&
S-MATRIX FORMULATION

HADRON RESONANCE GAS MODEL

- Confinement

physical quantities → hadronic states representation

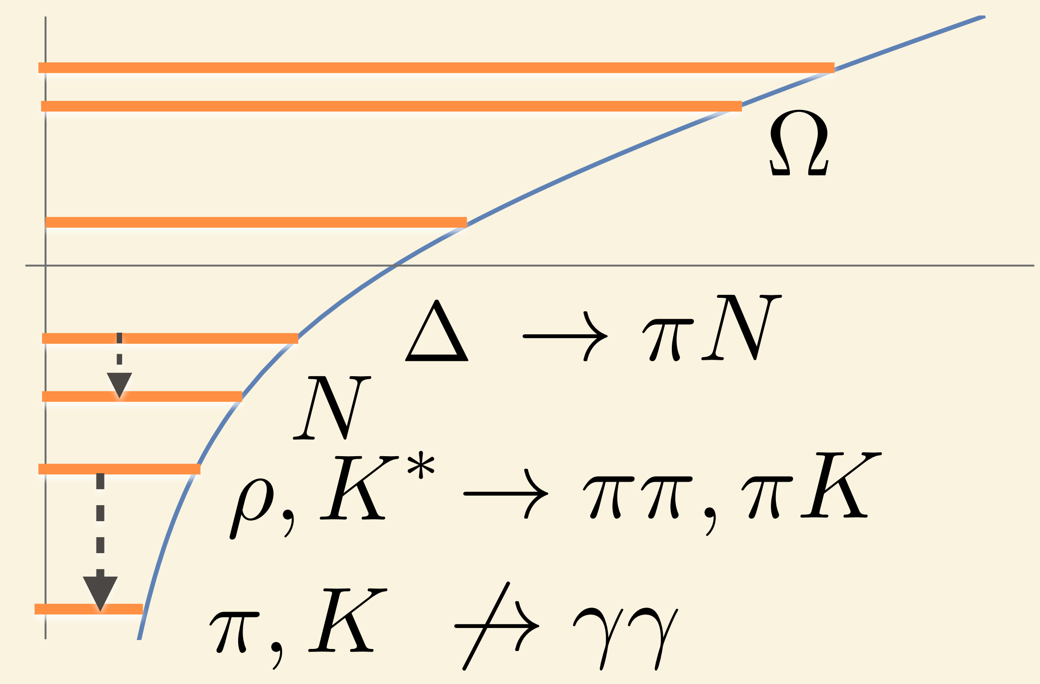
$$Z = \sum_{\alpha = B, M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

HADRON RESON
MODEL

- Confinement

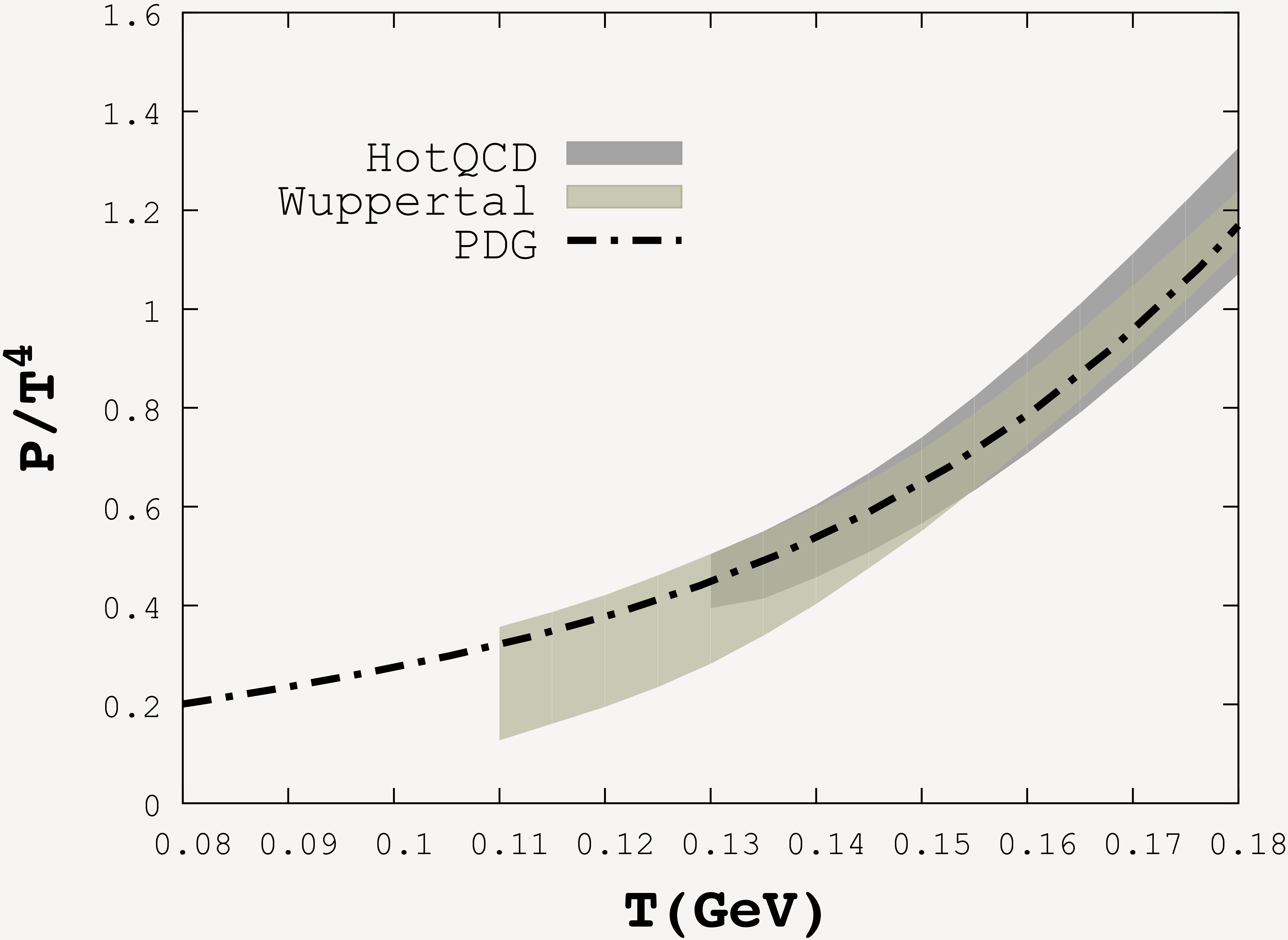
physical quantities

QCD spectrum



$$Z = \sum_{\alpha = B, M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

confinement +
spontaneous chiral symmetry breaking



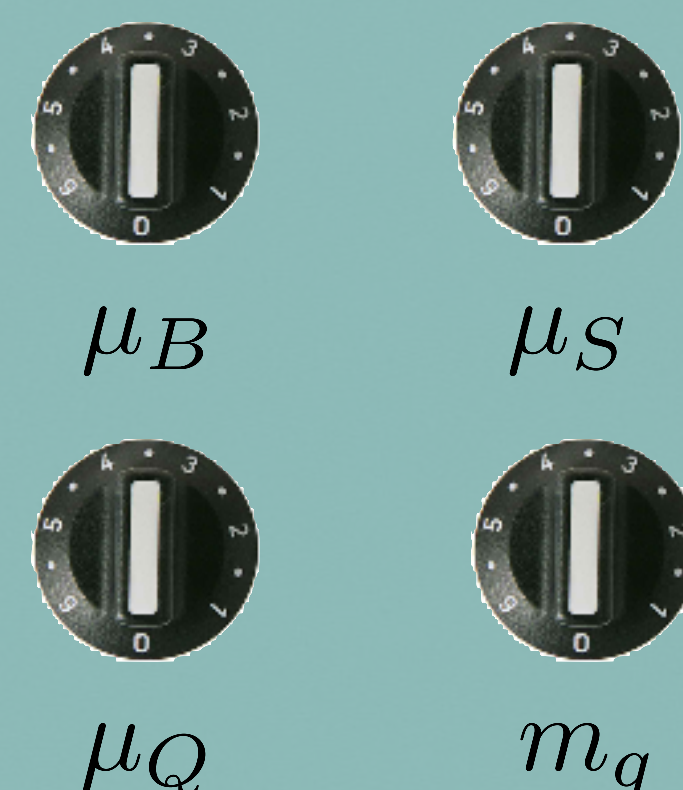
HotQCD
Wuppertal
PDG
P/T^4
T(GeV)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18

FLUCTUATIONS

- studying the system by linear response

$$\mu = \mu_B B + \mu_Q Q + \mu_S S$$

$$\chi_{B,S,...} = \frac{1}{\beta V} \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_S ...} \ln Z$$



FLUCTUATIONS

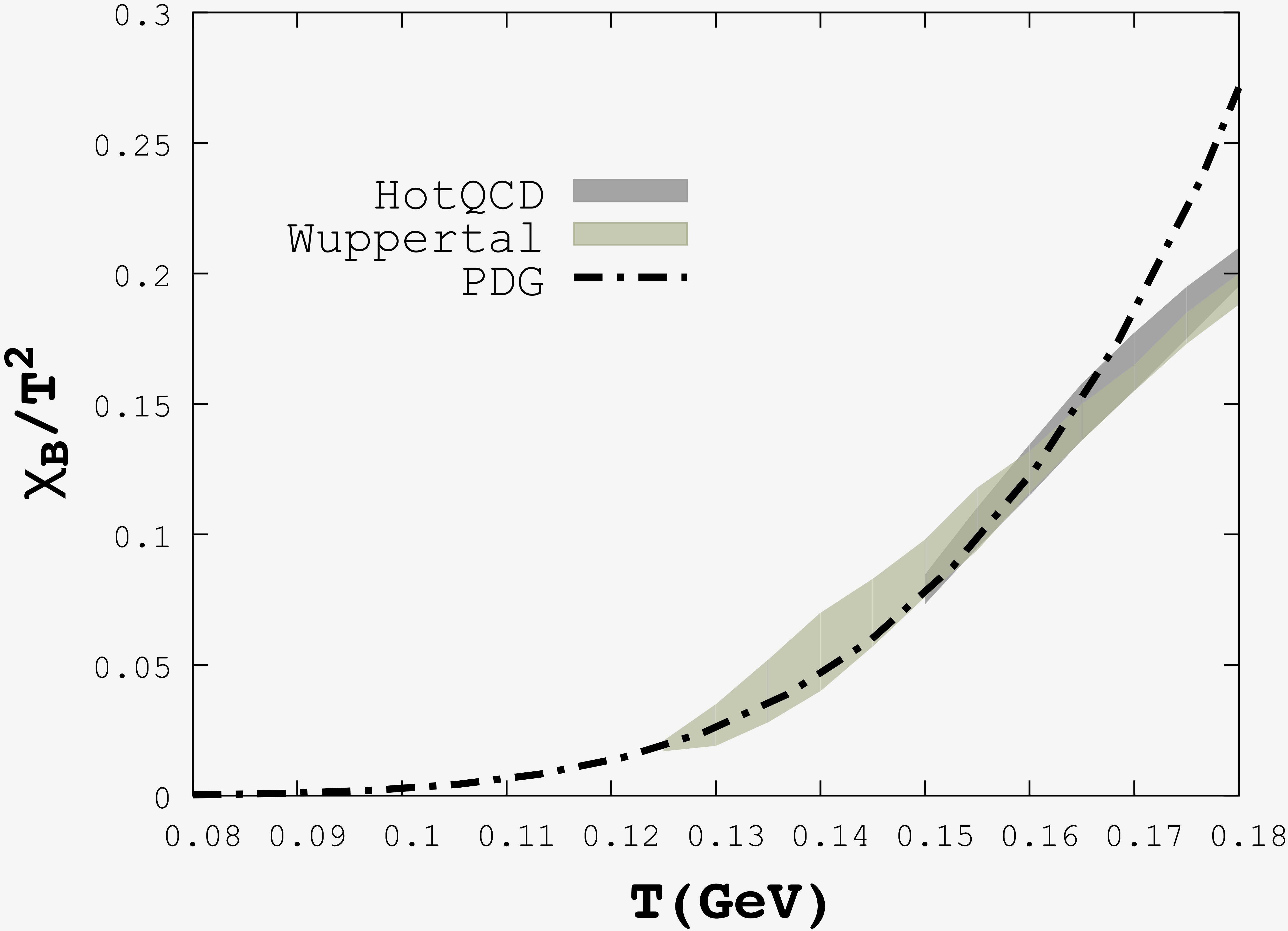
- taking derivative

$$\chi_B = \frac{\partial^2}{\partial\bar{\mu}_B \partial\bar{\mu}_B} P \qquad \textit{at the limit} \qquad \mu_B \to 0$$

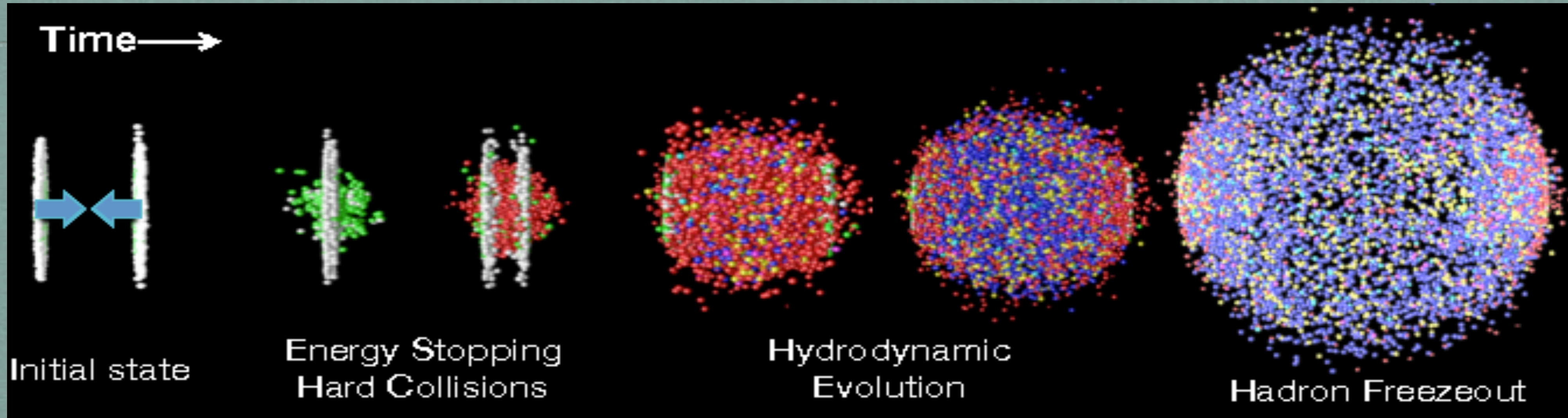
probes fluctuations

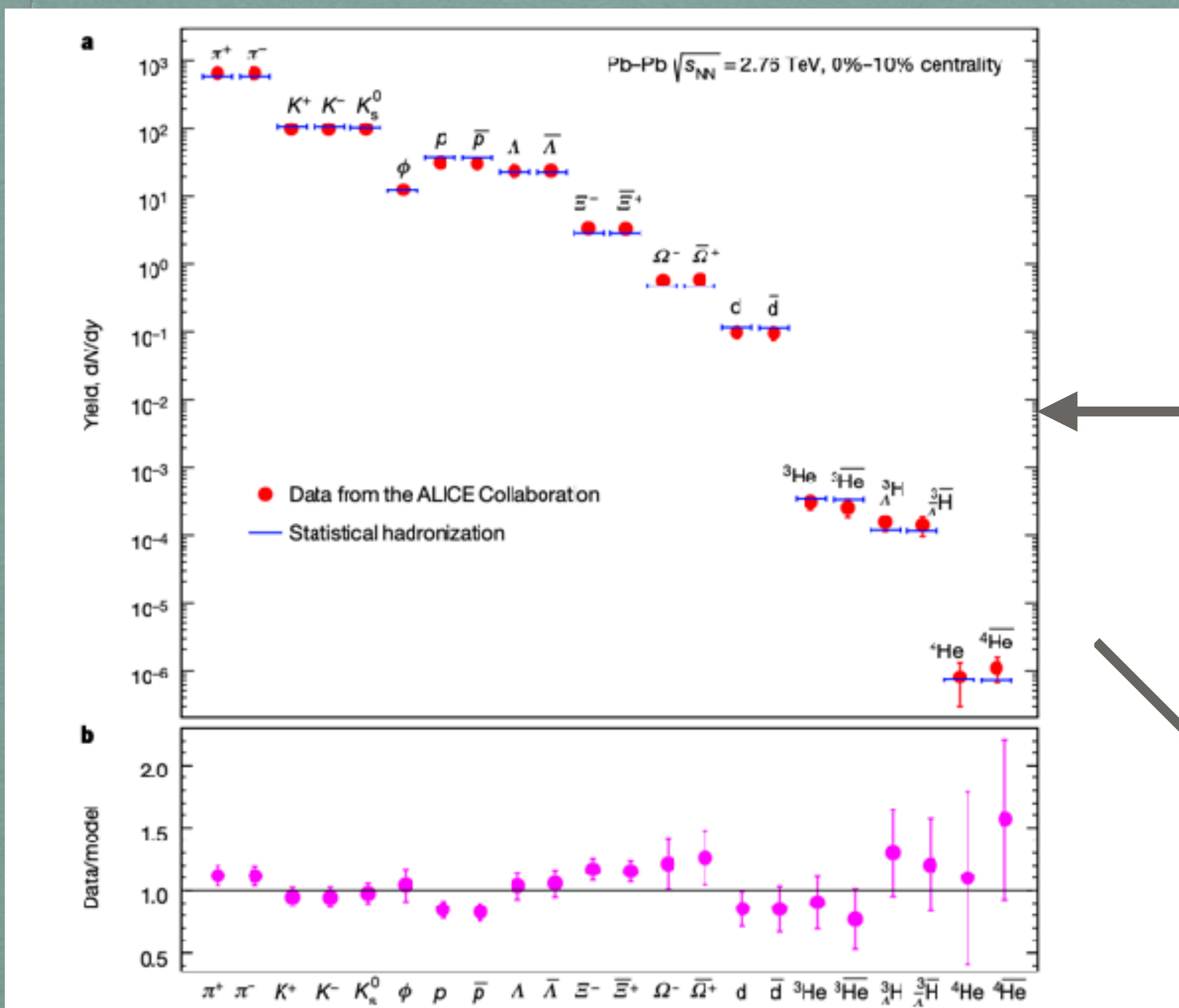
$$\chi_B = \frac{1}{\beta V} \frac{\partial^2}{\partial\bar{\mu}_B \partial\bar{\mu}_B} \ln Z$$

$$= T^2 \langle\langle \int d^4x \, \bar{\psi}(x)\gamma^0\psi(x)\bar{\psi}(0)\gamma^0\psi(0)\rangle\rangle_c$$



HotQCD
Wuppertal
PDG
χ_B/T^2
T(GeV)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18



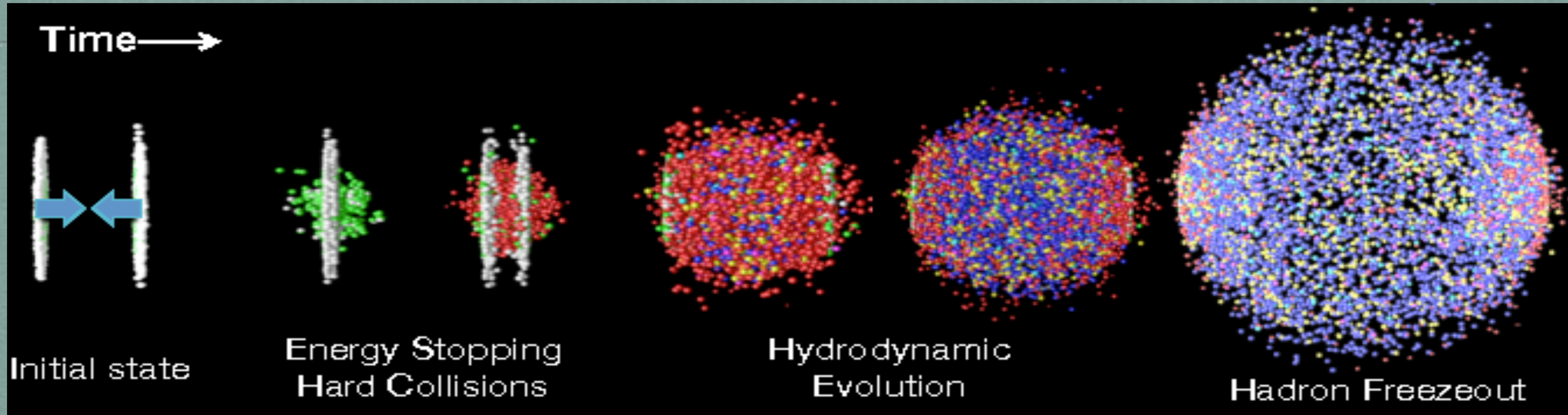


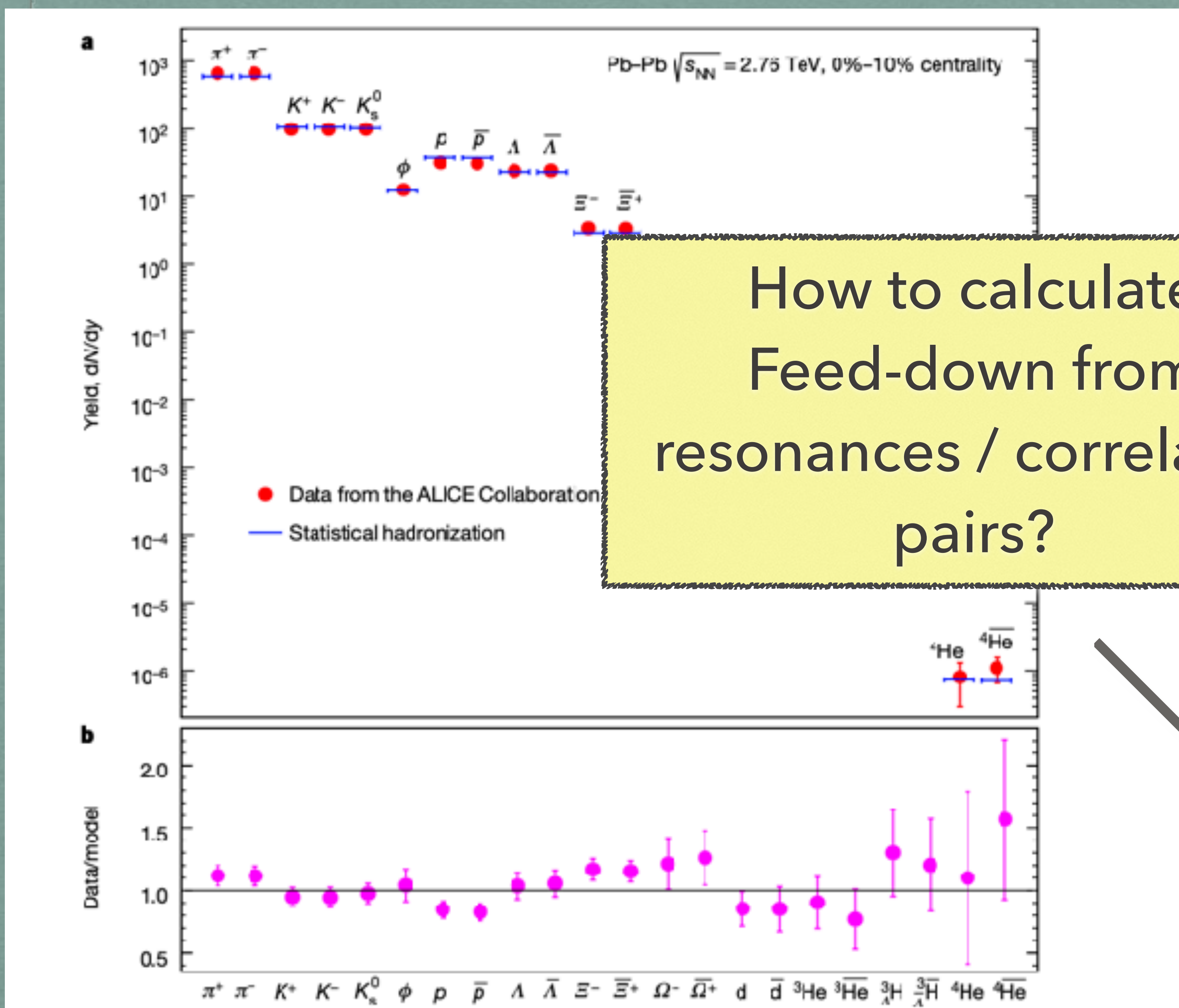
freezeout hadrons yields described by HRG

Freezeout parameters

$$T^f, \mu_B^f, \mu_S^f, \mu_Q^f, \ldots$$

A. Andronic et al 2018 Nature **561,** 321-330 (2018)





How to calculate Feed-down from resonances / correlated pairs?

freezeout hadrons yields described by HRG

Freezeout parameters

$$T^f, \mu_B^f, \mu_S^f, \mu_Q^f, \ldots$$

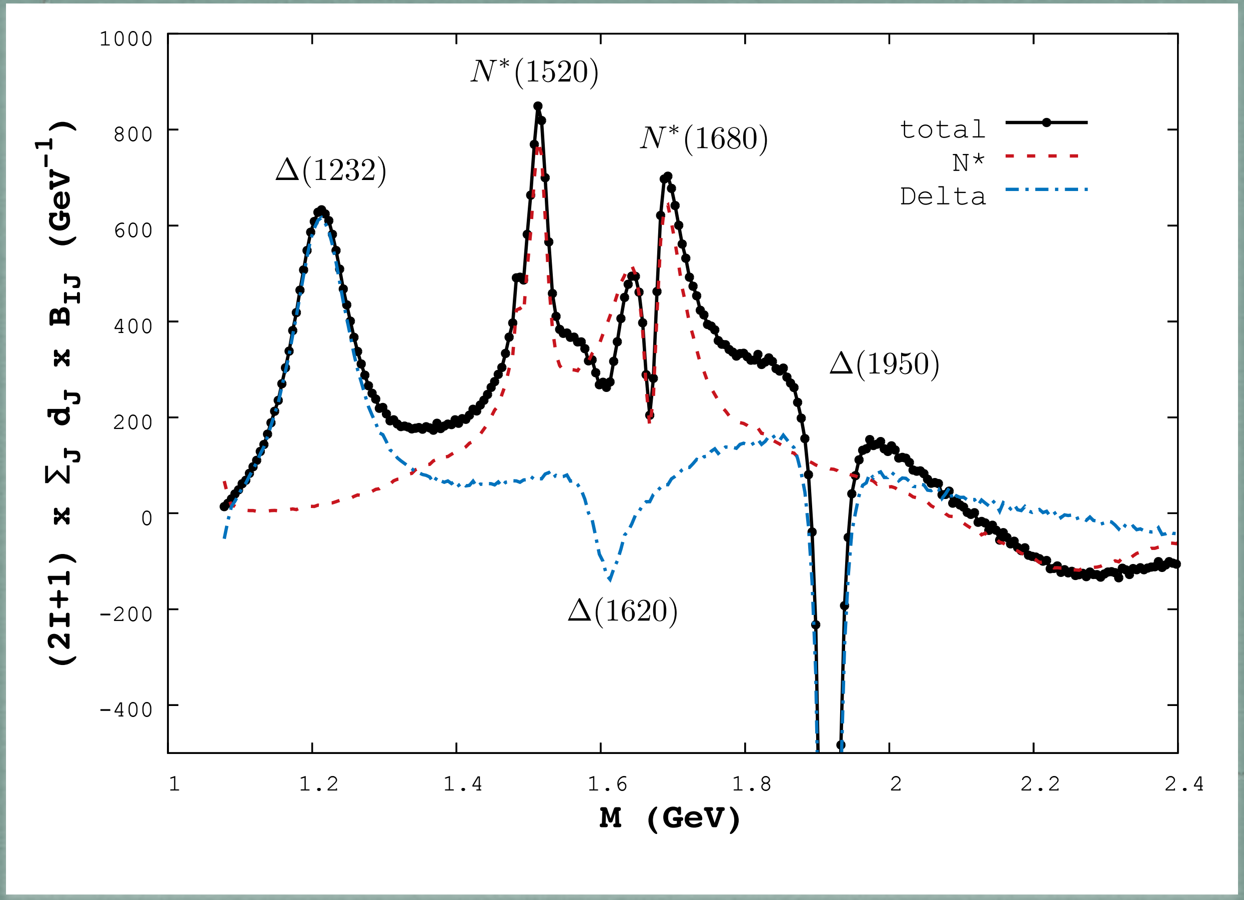
A. Andronic et al 2018 Nature **561,** 321-330 (2018)

RESONANCES / EXCITATIONS VIA SCATTERING STATES

- broad /overlapping resonances
- energy dependent branchings
- molecular states, cusps

non-resonant interactions: +/-

$$\pi N \to \Delta \to \pi N$$



scattering theory + stat. mech

S-MATRIX APPROACH TO STATISTICAL MECHANICS

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187 (1969) 345.

R. Venugopalan and M. Prakash,
Nucl. Phys. **A546**, 718 (1992).

S-MATRIX FORMULATION OF THERMODYNAMICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S_E \right\}_c$$

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187 (1969) 345.

S-MATRIX FORMULATION OF THERMODYNAMICS

thermo-statistical ***dynamical***

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QM *N-body* *QFT*

$$\mathrm{tr}\{...\} \iff \int d^3q \, \langle q| \ldots |q\rangle \longrightarrow \int (d\,k) \, \langle k_1 k_2| \, \ldots \, |k_1 k_2\rangle$$

Fock Space Expansion

$$\int (dk) \, (\cdots) \to \int \frac{d^3p_1}{(2\pi)^3} \frac{1}{2E_1} \frac{d^3p_2}{(2\pi)^3} \frac{1}{2E_2} \, (\cdots)$$

S-MATRIX FORMULATION OF THERMODYNAMICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

$$b_{\pi\pi}\xi_\pi^2 + b_{\pi K}\xi_\pi\xi_K + b_{\pi N}\xi_\pi\xi_N + b_{\pi\eta}\xi_\pi\xi_\eta + b_{K\bar{K}}\xi_K\xi_{\bar{K}} + \ldots$$

$$b_{\pi N} = 2 \times b_{\pi N}^{I=1/2} + 4 \times b_{\pi N}^{I=3/2}$$

orbital L:
S, P, D, F, etc..

R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187 (1969) 345.

S-MATRIX FORMULATION OF THERMODYNAMICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

NN effects *convergence?*

$a_S = 20\, fm$

$r \approx 0.0727$ *LHC*

$r \approx 0.36$ $T = 60\, MeV$

$r \approx 1.92$ $\mu_B = 700, 800\, MeV$

$\ldots_\pi \xi_\eta + b_{K\bar{K}} \xi_K \xi_{\bar{K}} + \ldots$

$\times\, b_{\pi N}^{I=3/2}$ *orbital L: S, P, D, F, etc..*

and H. J. Bernstein,
Rev. 187 (1969) 345.

DENSITY OF STATES

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S_E \right\}_c$$

single channel, elastic $\frac{1}{\pi} \frac{d}{dE} \delta$

N-body &
Coupled-Channel problem

multi (coupled) channel $\frac{1}{\pi} \frac{d}{dE} \mathcal{Q}$

sum over eigenphases

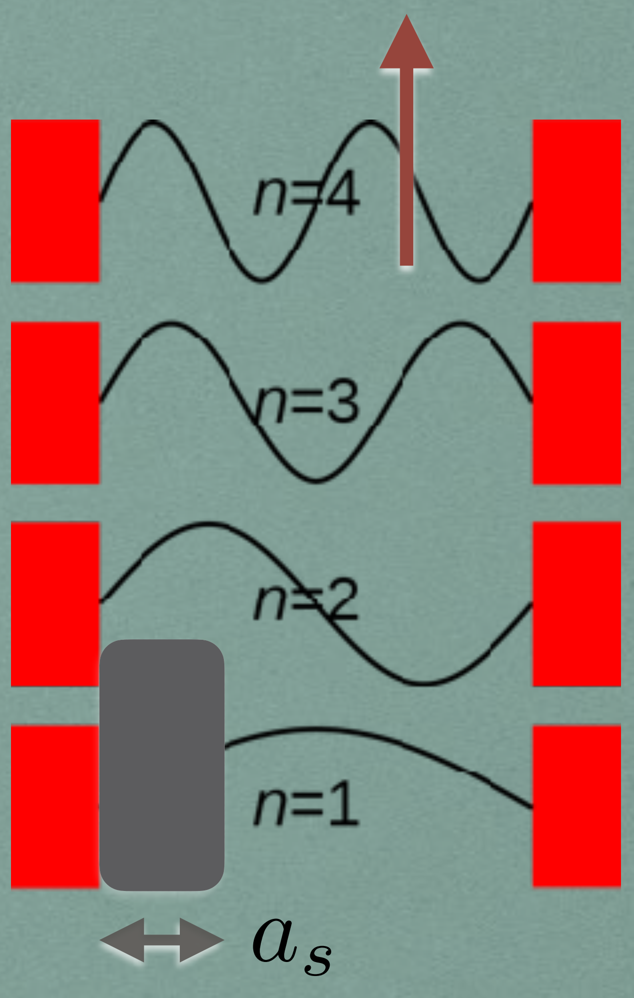
$$\mathcal{Q} = \frac{1}{2} \mathrm{Im Tr} \ln S = \sum_{\text{channels}} \lambda_i$$

PML, EPJC **77** no.8 533 (2017)

PML PRD **102**, 034038 (2020)

REPULSIONS
&
RESONANCES

PHASE SHIFT AND DENSITY OF STATES



particle in a box

$$\psi \sim \sin(k^{(0)}x) \qquad k^{(0)} = \frac{n\pi}{L}$$

in the presence of a scattering potential

$$\psi \sim \sin(kx + \delta(k))$$

density of states

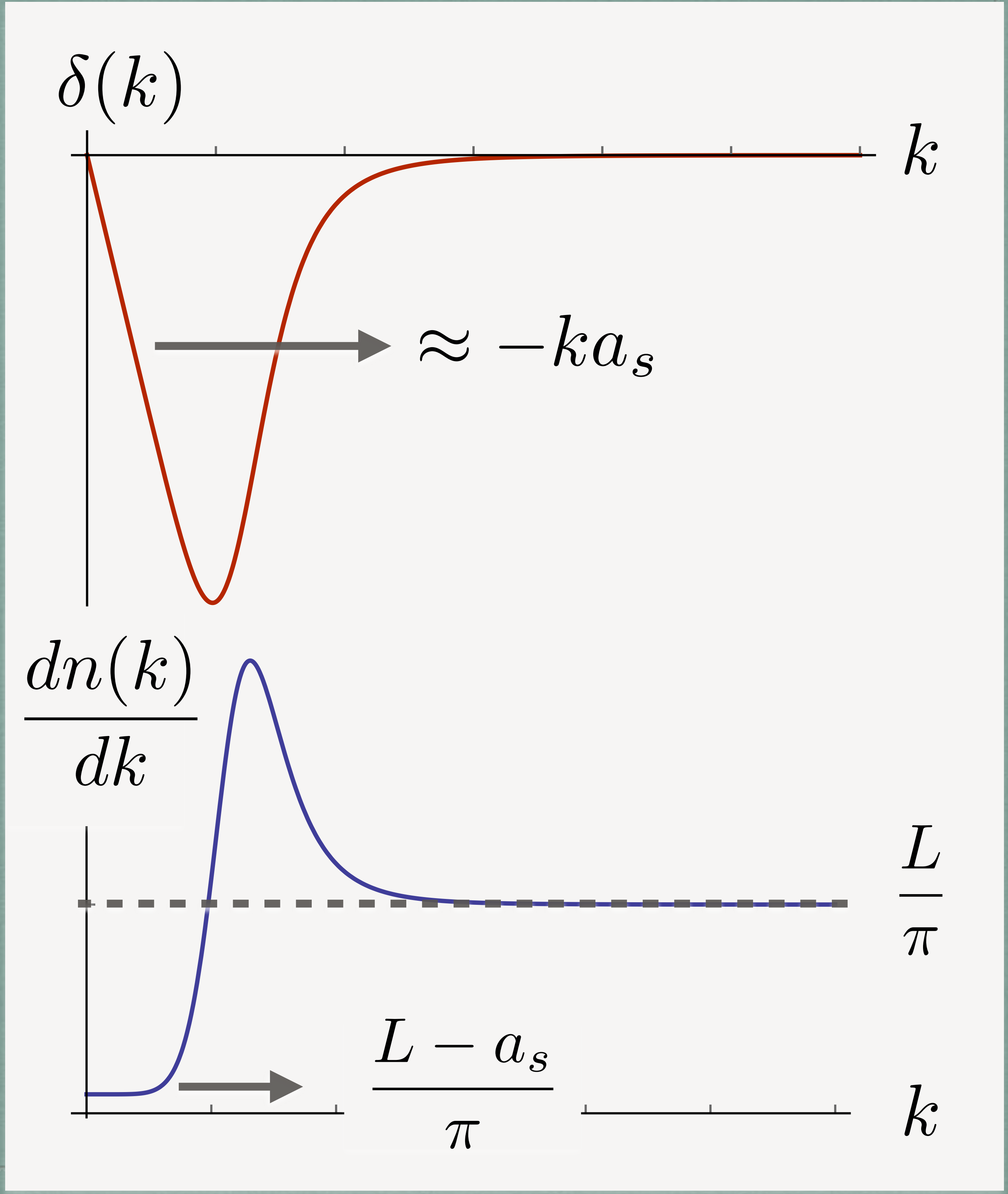
$$kL + \delta(k) = n\pi \quad \rightarrow \quad \frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

PHASE SHIFT AND DENSITY OF STATES

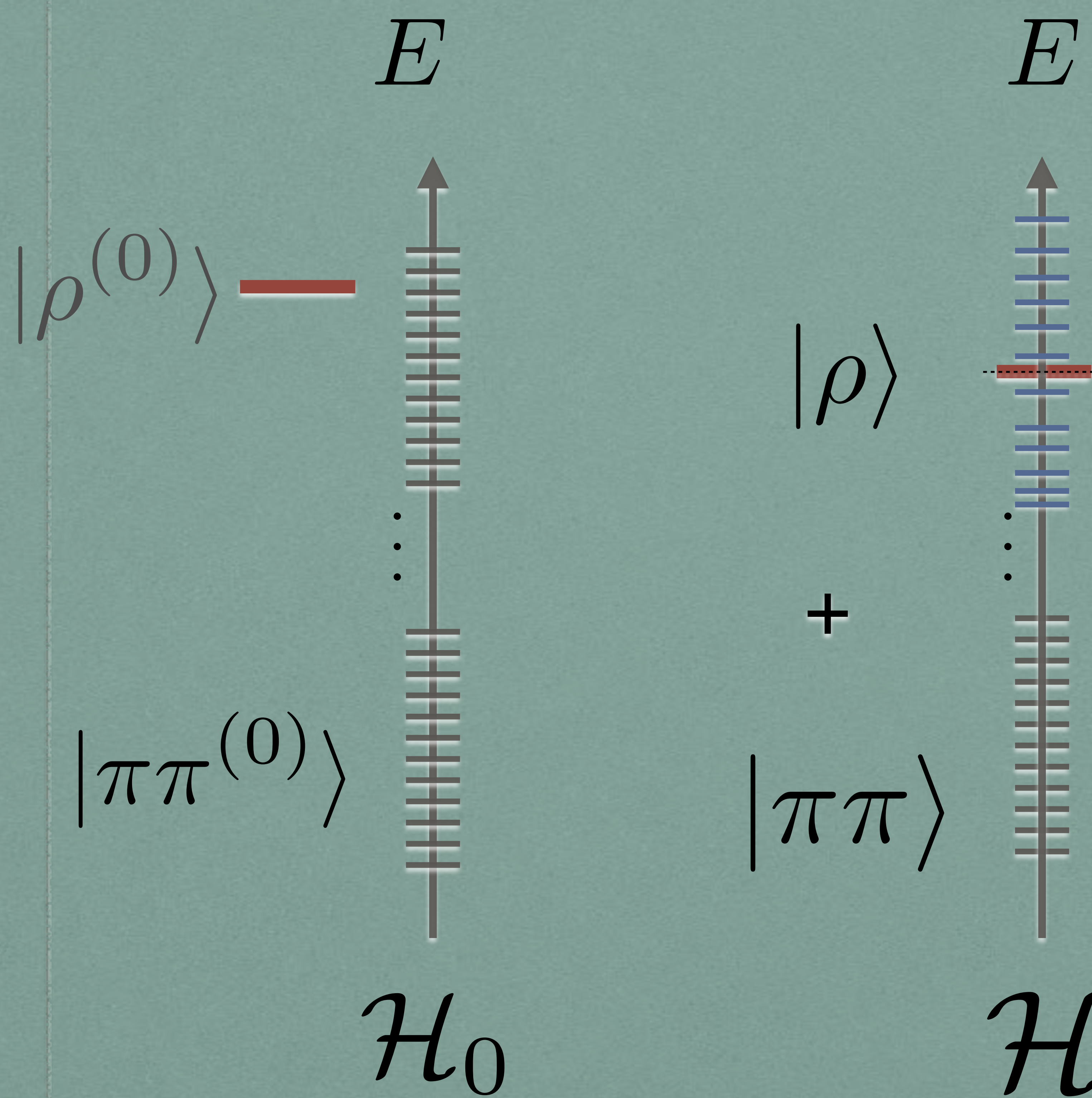
$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

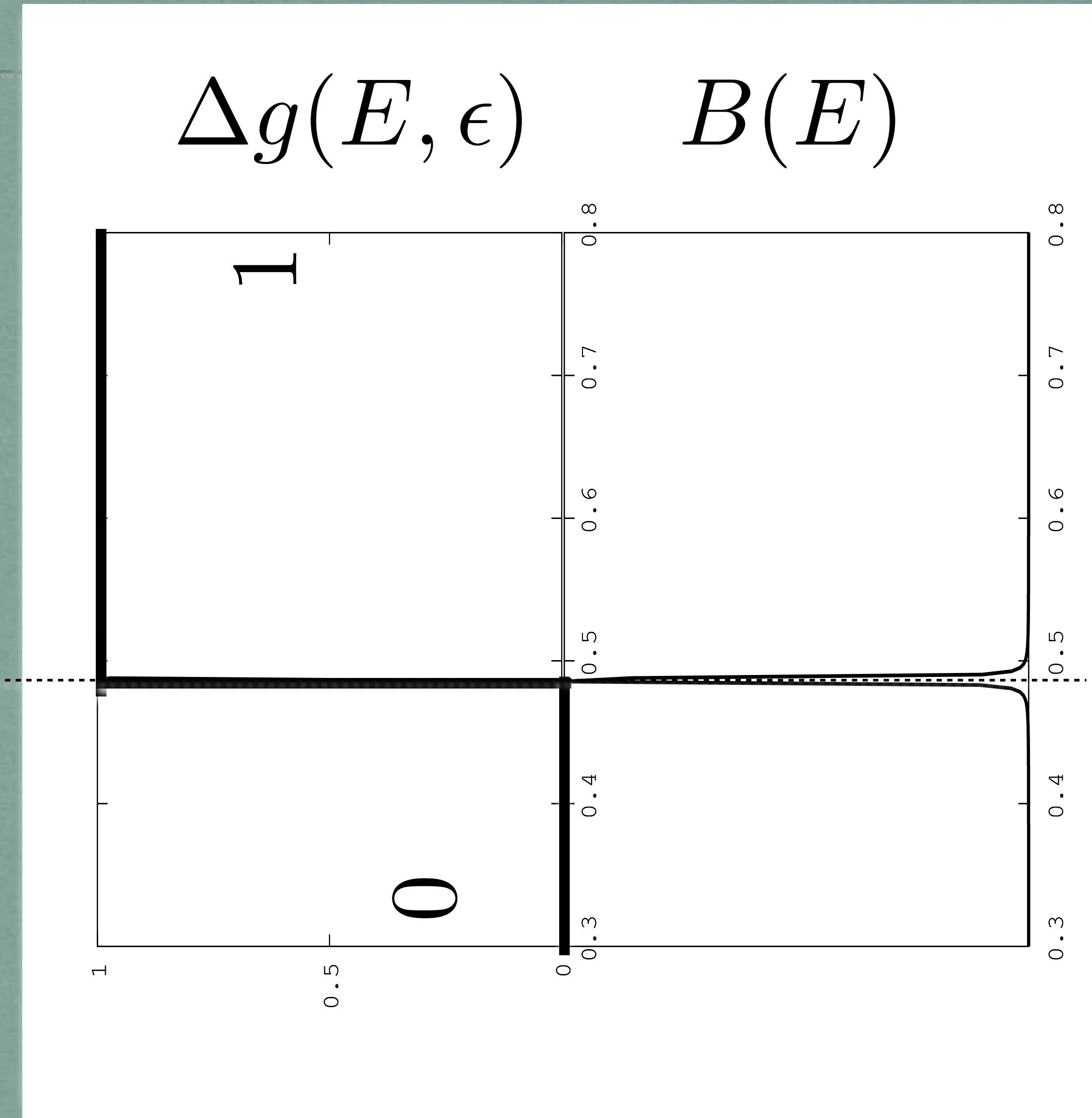
change in d.o.s. due to int.

Effect of repulsive interaction: pushing states from low k to high k



phase shift and d.o.s. (schematics)

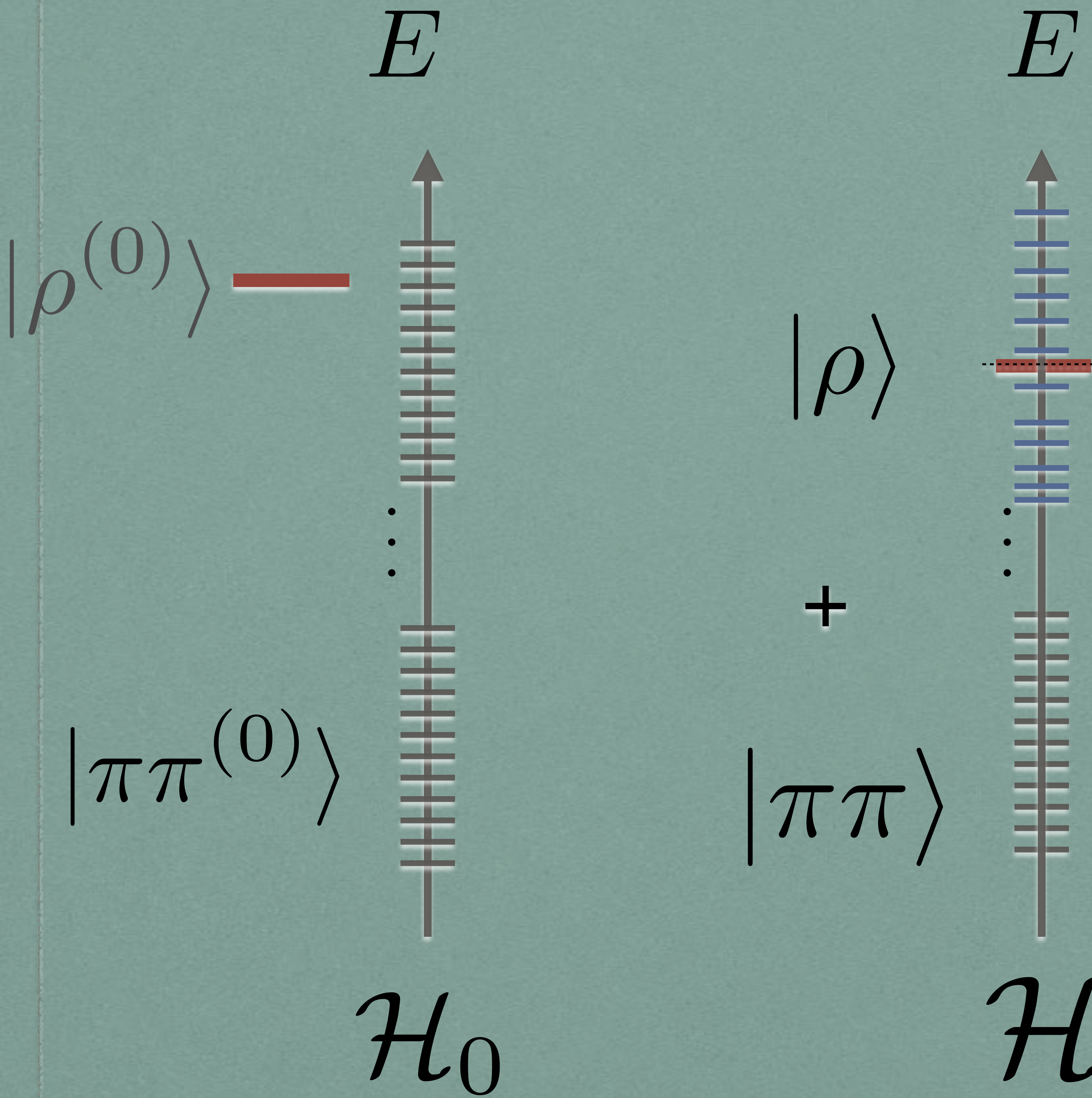


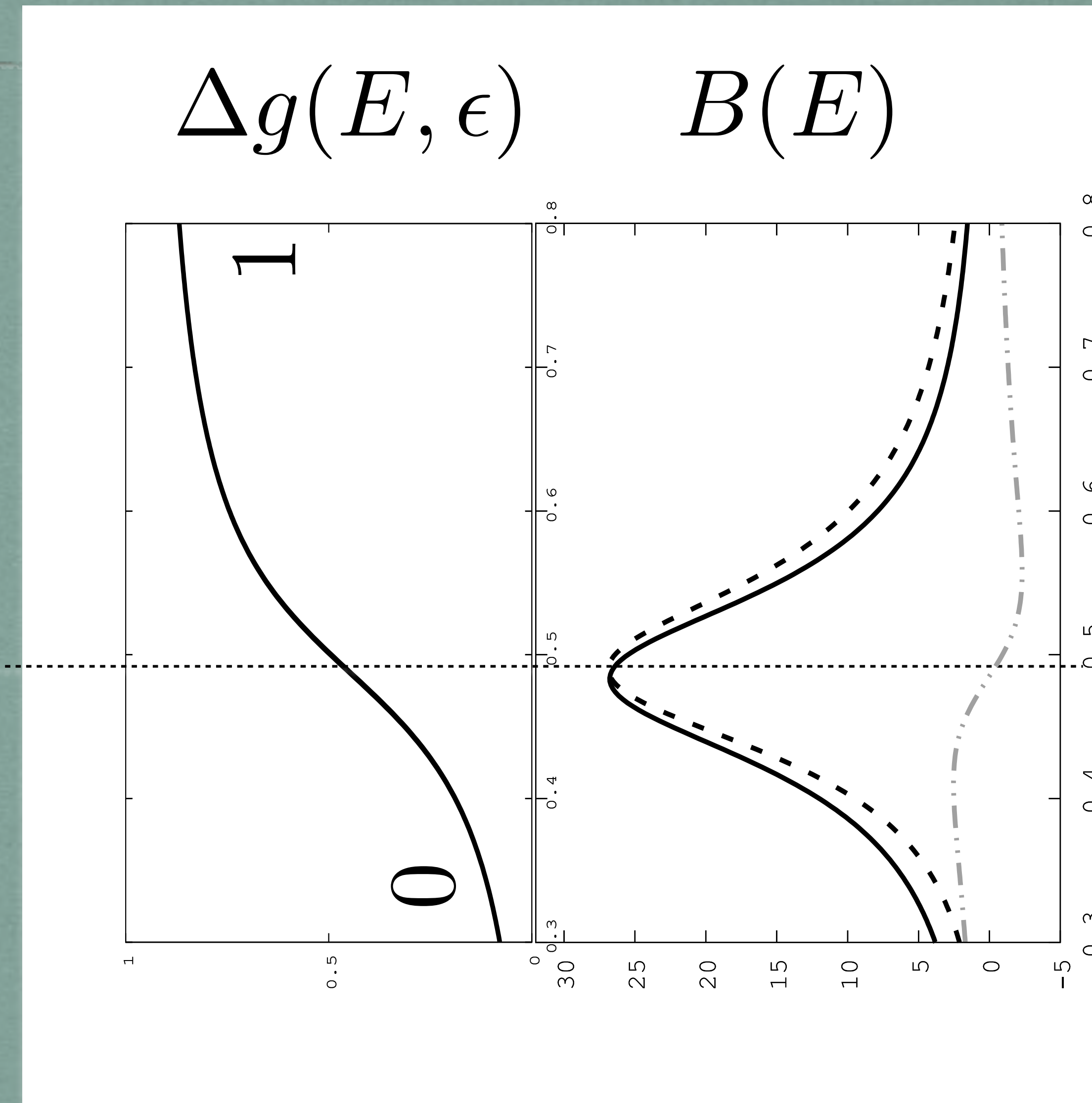


$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E,\epsilon)$$

PML, Francesco Giocosa EPJC **79** (2019)





$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

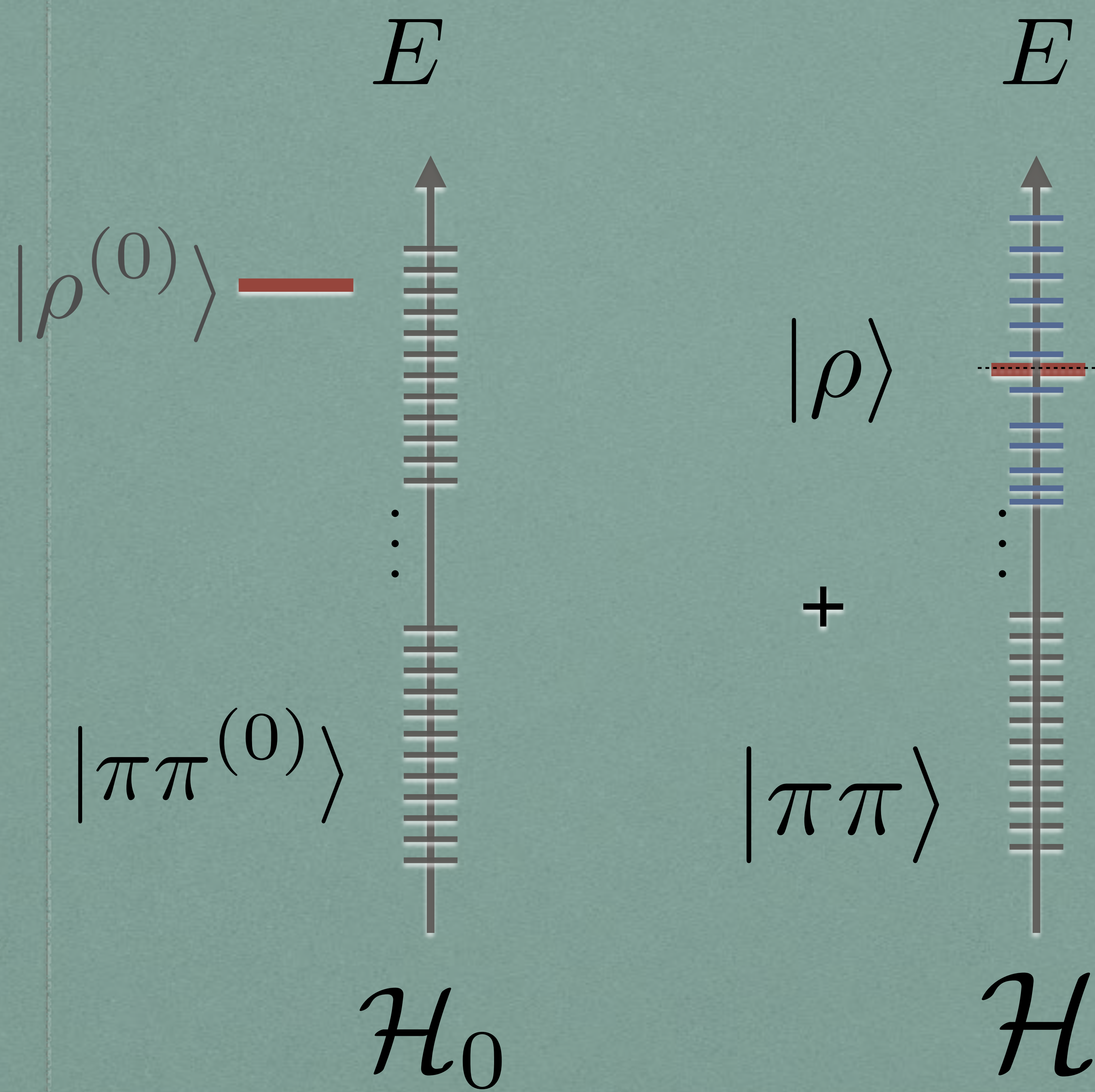
$$B(E) = 2\pi \frac{d}{dE} \Delta g(E,\epsilon)$$

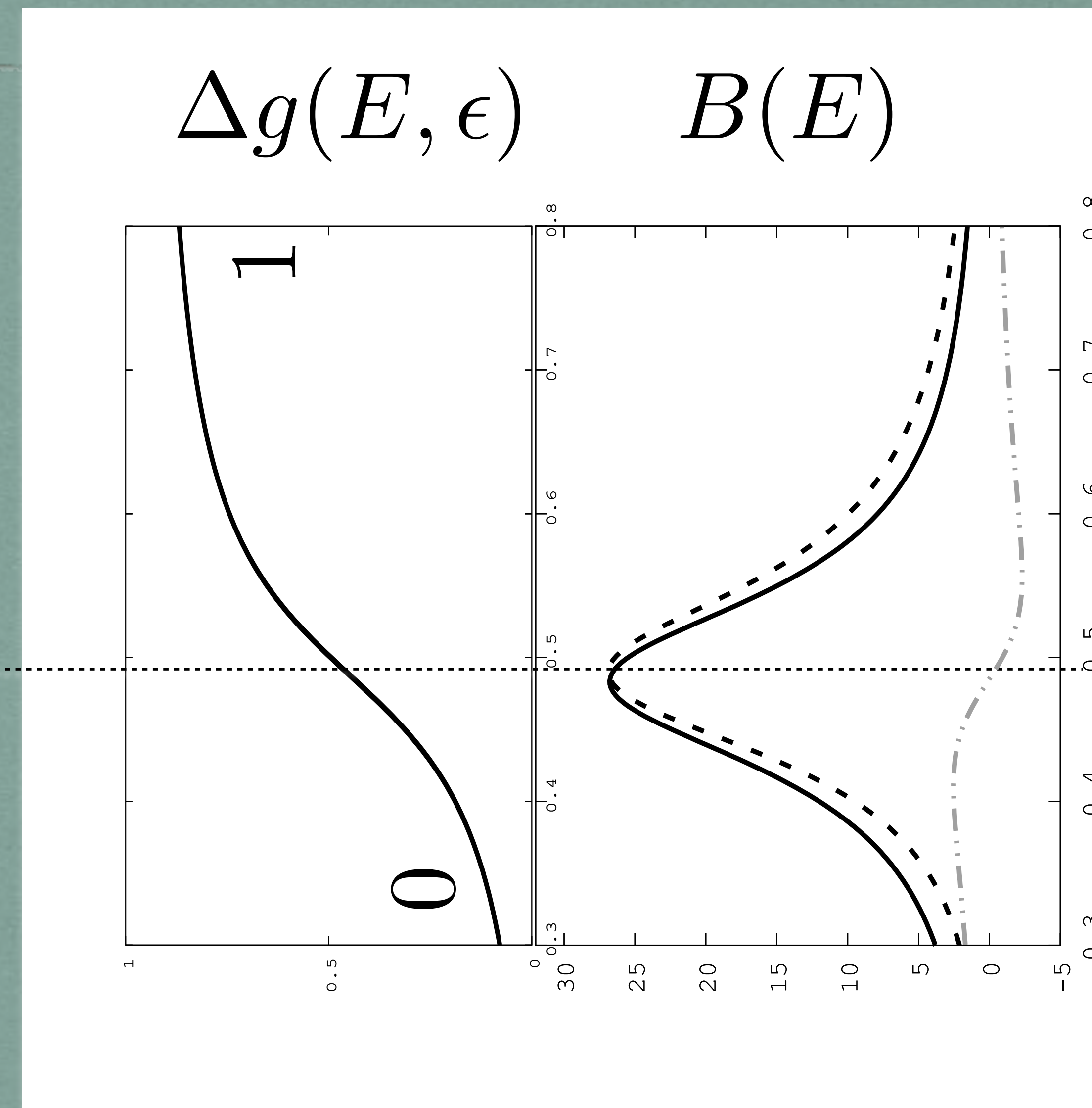
$$= A_\rho + \Delta A_{\pi\pi}$$

$\mathrm{Tr}\, e^{-\beta \mathcal{H}_0}$ vs $\mathrm{Tr}\, e^{-\beta \mathcal{H}}$

for Delta

W. Weinhold,, and B. Friman, Phys. Lett. B 433, 236 (1998).





$g(E,\epsilon) = \sum \theta_\epsilon(E - E_n)$

$\mathrm{Im}\,\Sigma$ vs $\frac{\partial}{\partial E}\Sigma$

$= A_\rho + \Delta A_{\pi\pi}$

$\mathrm{Tr}\, e^{-\beta\mathcal{H}_0}$ *vs* $\mathrm{Tr}\, e^{-\beta\mathcal{H}}$

for Delta

W. Weinhold,, and B. Friman, Phys. Lett. B 433, 236 (1998).

PHYSICS OF B

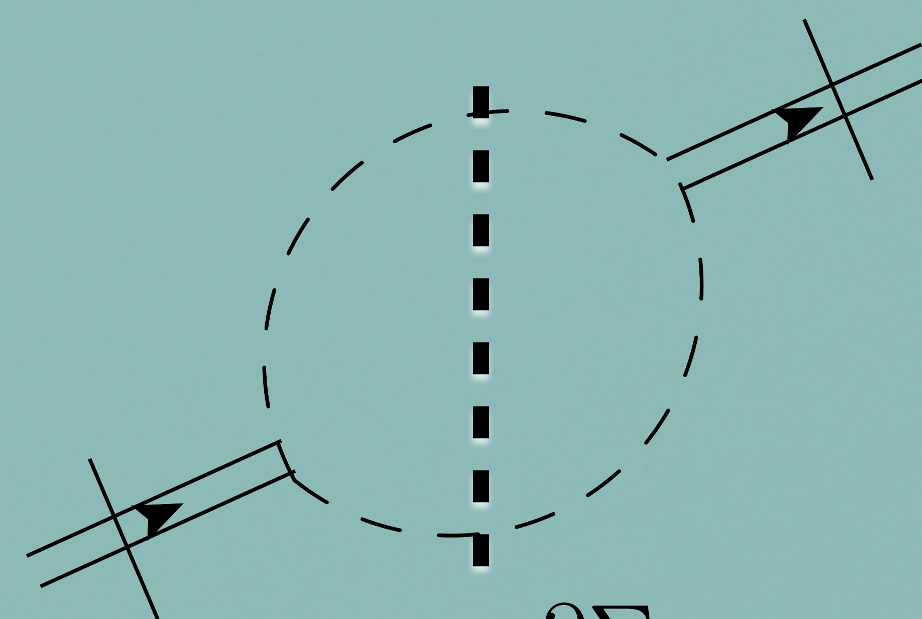
$$\delta = -\mathrm{Im}\,\mathrm{Tr}\ln G_\rho^{-1}$$

physical interpretation:

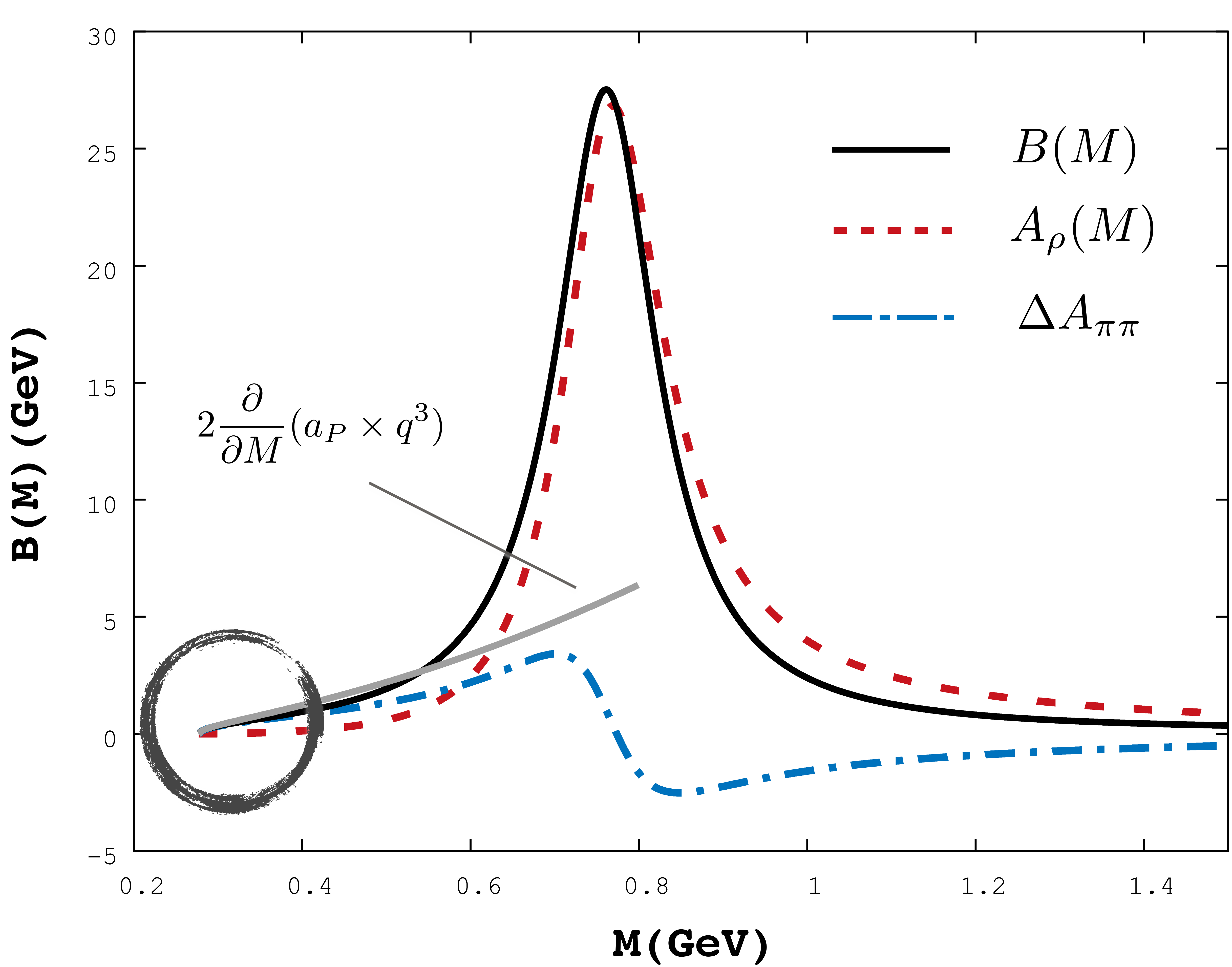
contribution from correlated pi pi pair

$$
\begin{aligned}
B &= 2\,\frac{\partial}{\partial E}\delta \\
&= -2\,\mathrm{Im}\frac{\partial}{\partial E}\ln G_\rho^{-1} \\
&= -2\,\mathrm{Im}[G_\rho](2E) + 2\,\mathrm{Im}[\frac{\partial \Sigma_\rho}{\partial E} G_\rho] \\
&= A_\rho(E) + \Delta A_{\pi\pi}
\end{aligned}
$$

$$-\frac{\partial}{\partial E}\int d\phi_E\, T_{\mathrm{re}}$$



$$\frac{\partial \Sigma_\rho}{\partial E}$$



30
25
20
15
10
5
0
-5
0.2
0.4
0.6
0.8
1
1.2
1.4
B(M) (GeV)
M(GeV)
$B(M)$
$A_\rho(M)$
$\Delta A_{\pi\pi}$
$2\frac{\partial}{\partial M}(a_P \times q^3)$

PROTON PUZZLE

A. Andronic, P. Braun-Munzinger, B. Friman, PML,
K. Redlich, J. Stachel
Phys. Lett. B **792**, 304 (2019)

proton yields

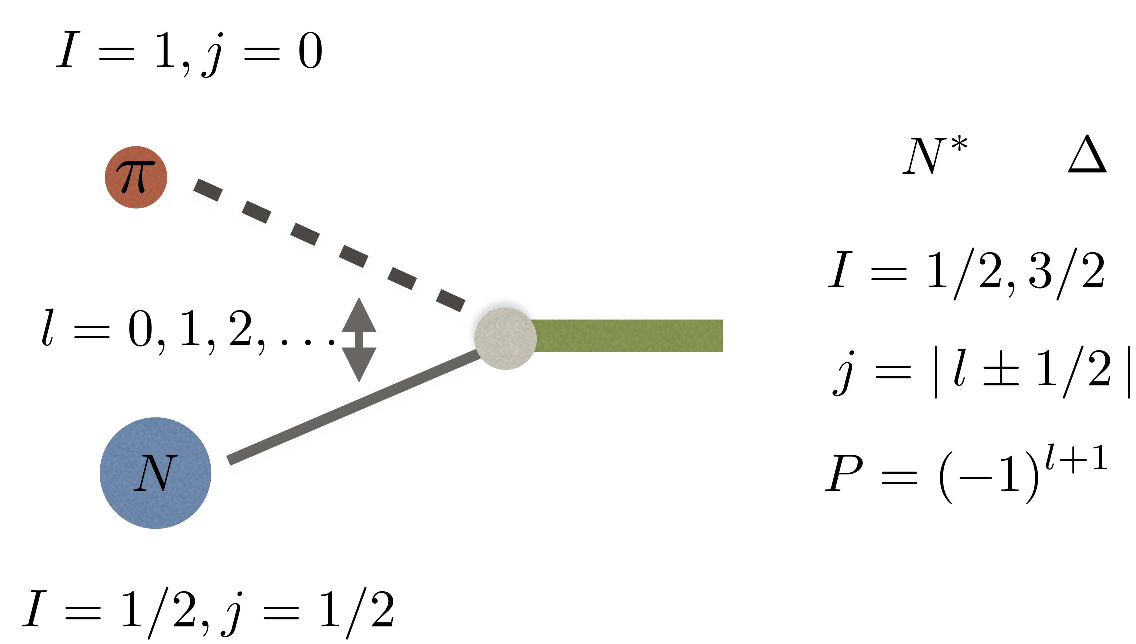
$$\langle p \rangle = \langle p \rangle_{th.} + \langle p \rangle_{N^*} + \langle p \rangle_{\Delta} + \langle p \rangle_{hyp.} + \ldots$$

$$\langle p \rangle_{N^*} = \frac{2}{3}\langle N^*_{Q=0} \rangle + \frac{1}{3}\langle N^*_{Q=1} \rangle \approx \frac{1}{2}\langle N^* \rangle$$

$$\langle p \rangle_{\Delta} = \langle \Delta_{Q=2} \rangle + \frac{2}{3}\langle \Delta_{Q=1} \rangle + \frac{1}{3}\langle \Delta_{Q=0} \rangle \approx \frac{1}{2}\langle \Delta \rangle$$

isospin symmetric @ LHC

$$n_a(T) = \int_{m_{th}}^{\infty} \frac{dM}{2\pi} B_a(M)\, n^{(0)}(T, M)$$



empirical phase shifts

resonant and non-resonant int.

BQ CORRELATION

$$\chi_{BQ}/T^2 = \sum_{I_z;B} d_J \, \underline{B\, Q} \times \int_{m_{\rm th}}^{\infty} \frac{d\sqrt{s}}{2\pi} B_{I,J}(s)$$

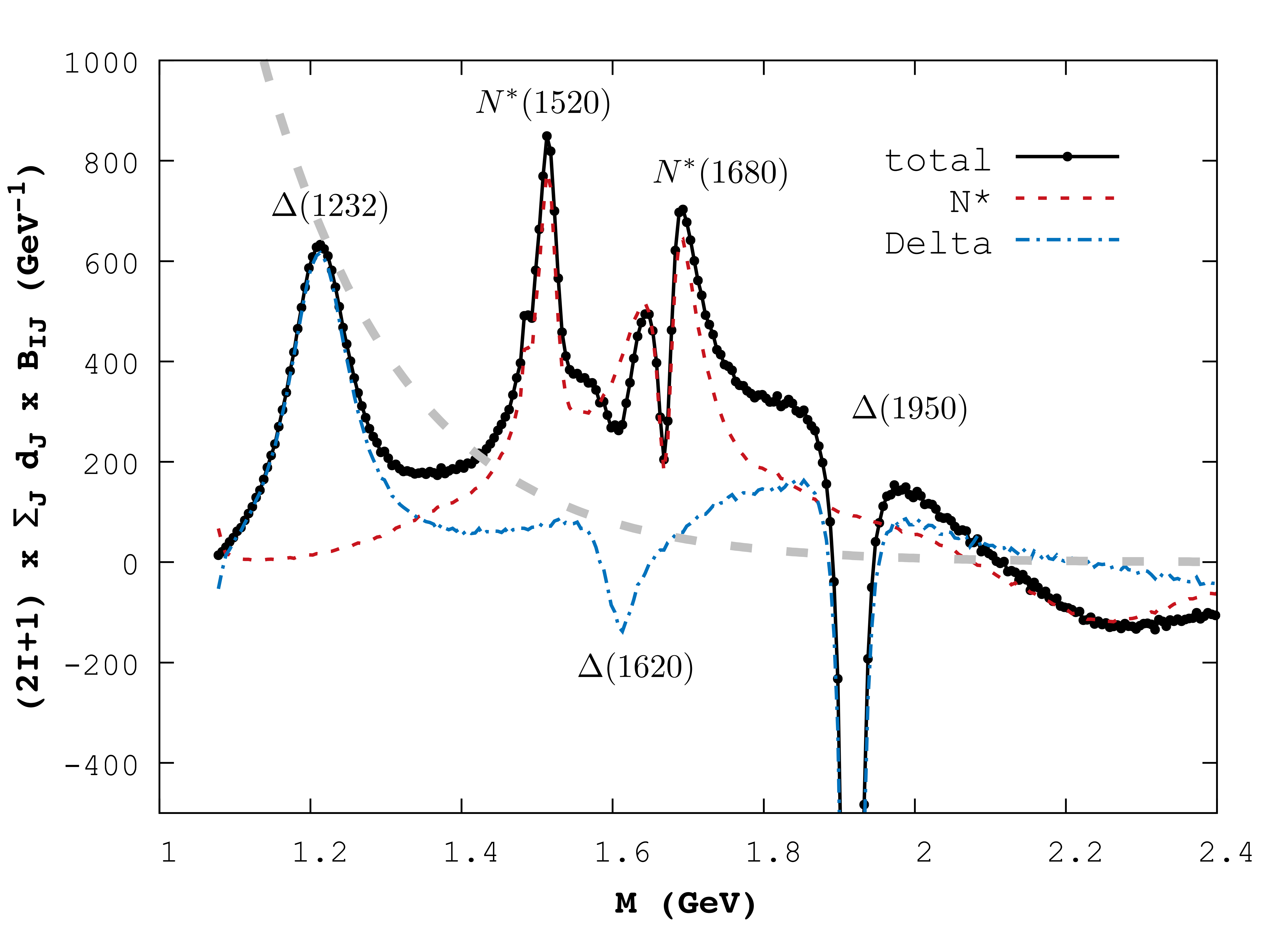
$$\times \frac{1}{T^3} \int \frac{d^3p}{(2\pi)^3} \frac{e^{\beta\sqrt{p^2+s})}}{(e^{\beta\sqrt{p^2+s}}+1)^2}$$

- (in)famous resonances:
  N*: 1535 (S11), 1440 (P11), 1520 (D13)
  Δ: 1232 (P33), 1620 (S31)

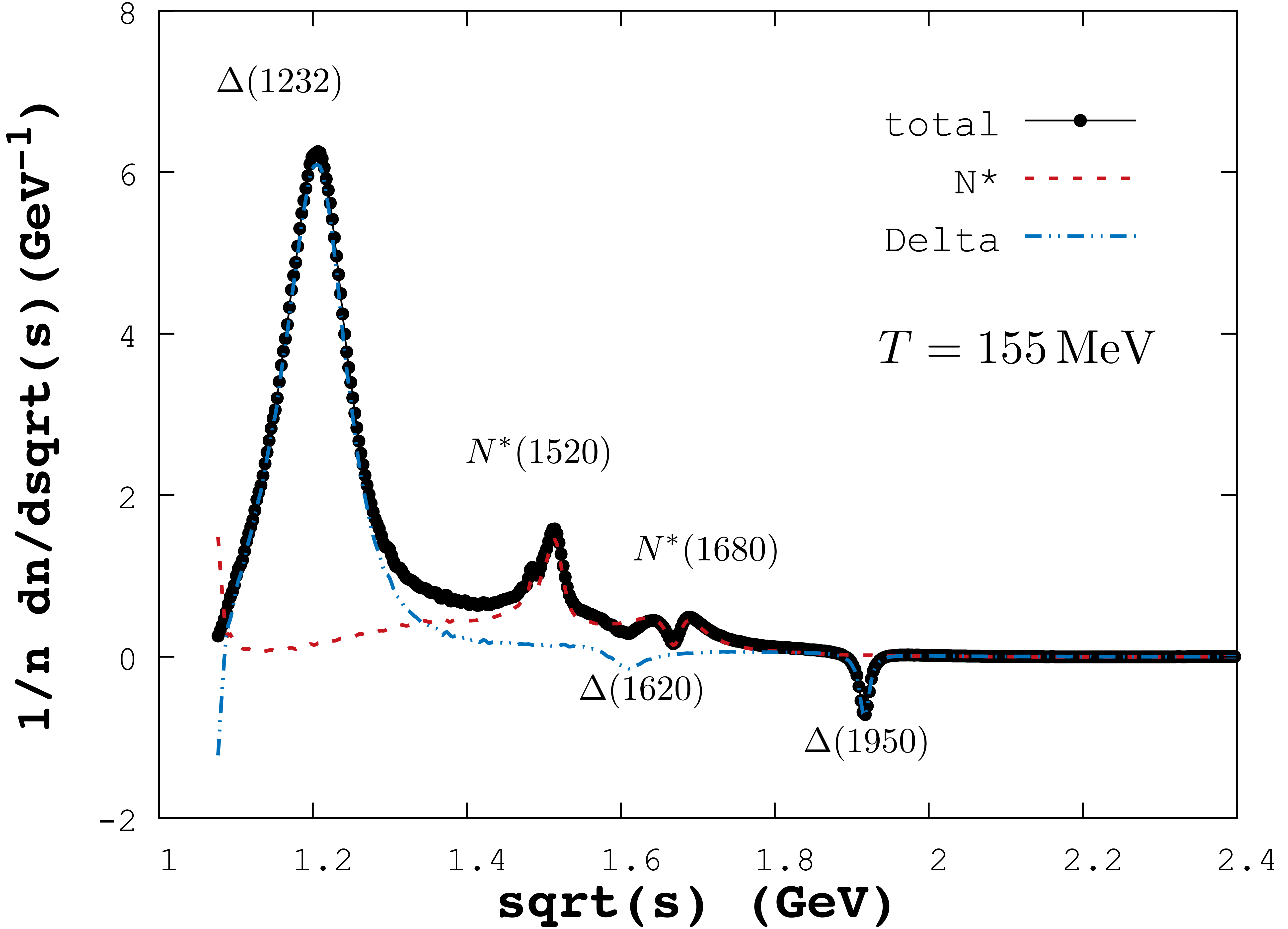
- robust for probing protons
  S = -1 hyperons are excluded

$$Q = I_z + \frac{1}{2}(B+S)$$

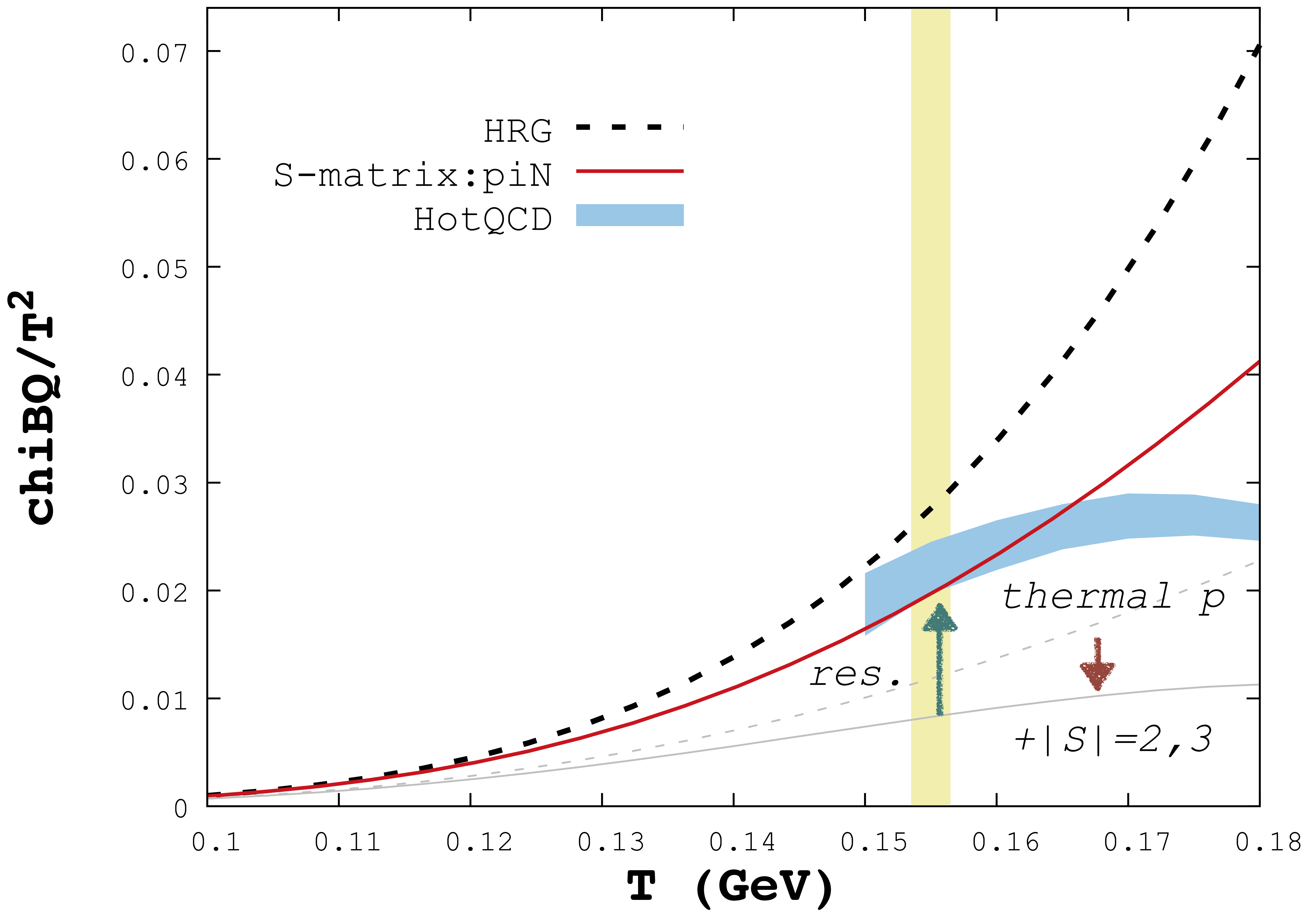
$$\chi_{BB} = 2\chi_{BQ} + |\chi_{BS}|$$



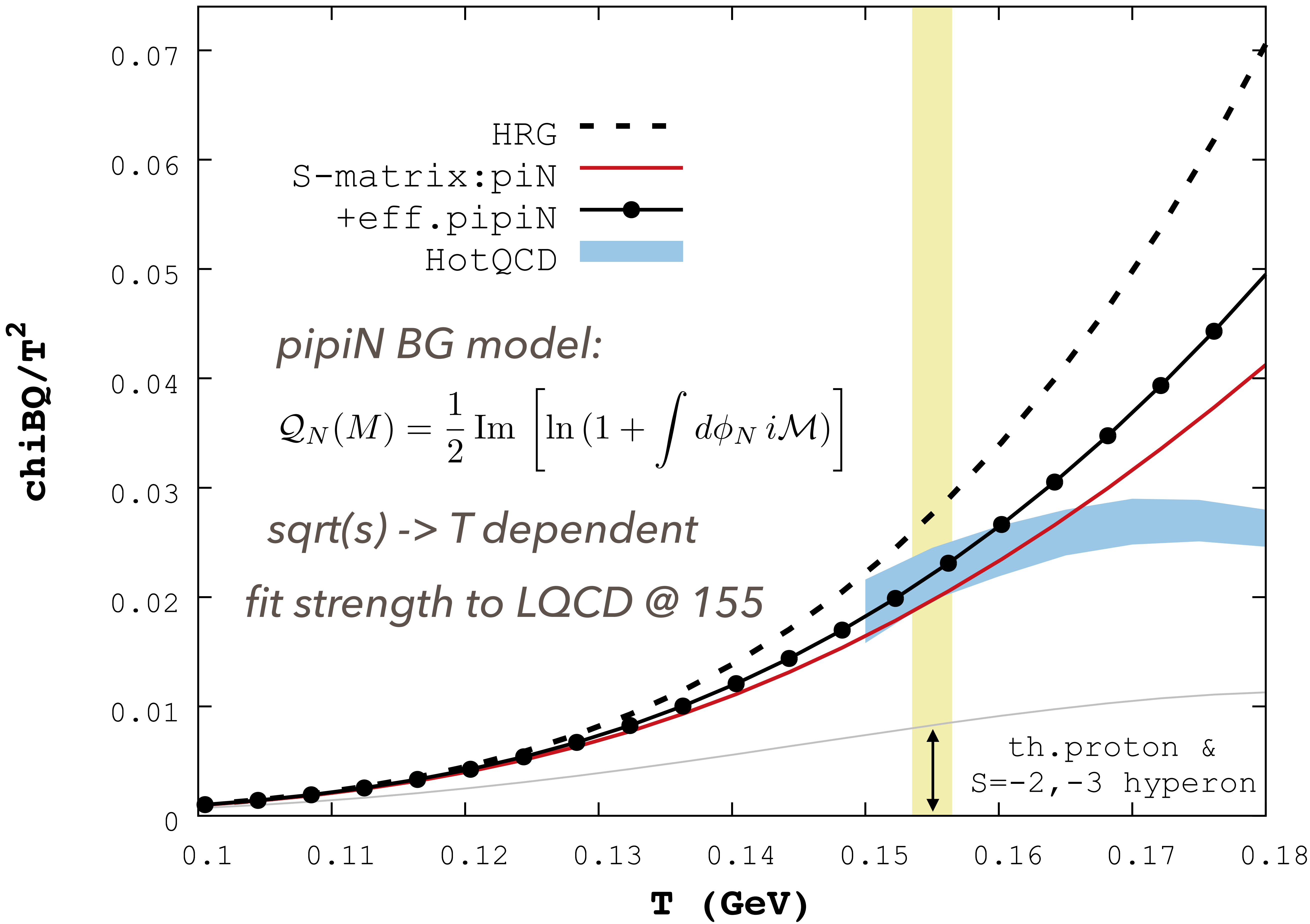
1000
800
600
400
200
0
-200
-400
1
1.2
1.4
1.6
1.8
2
2.2
2.4
M (GeV)
(2I+1) x $\sum_J$ d_J x B_{IJ} (GeV^{-1})
$N^*(1520)$
$N^*(1680)$
$\Delta(1232)$
$\Delta(1950)$
$\Delta(1620)$
total
N*
Delta



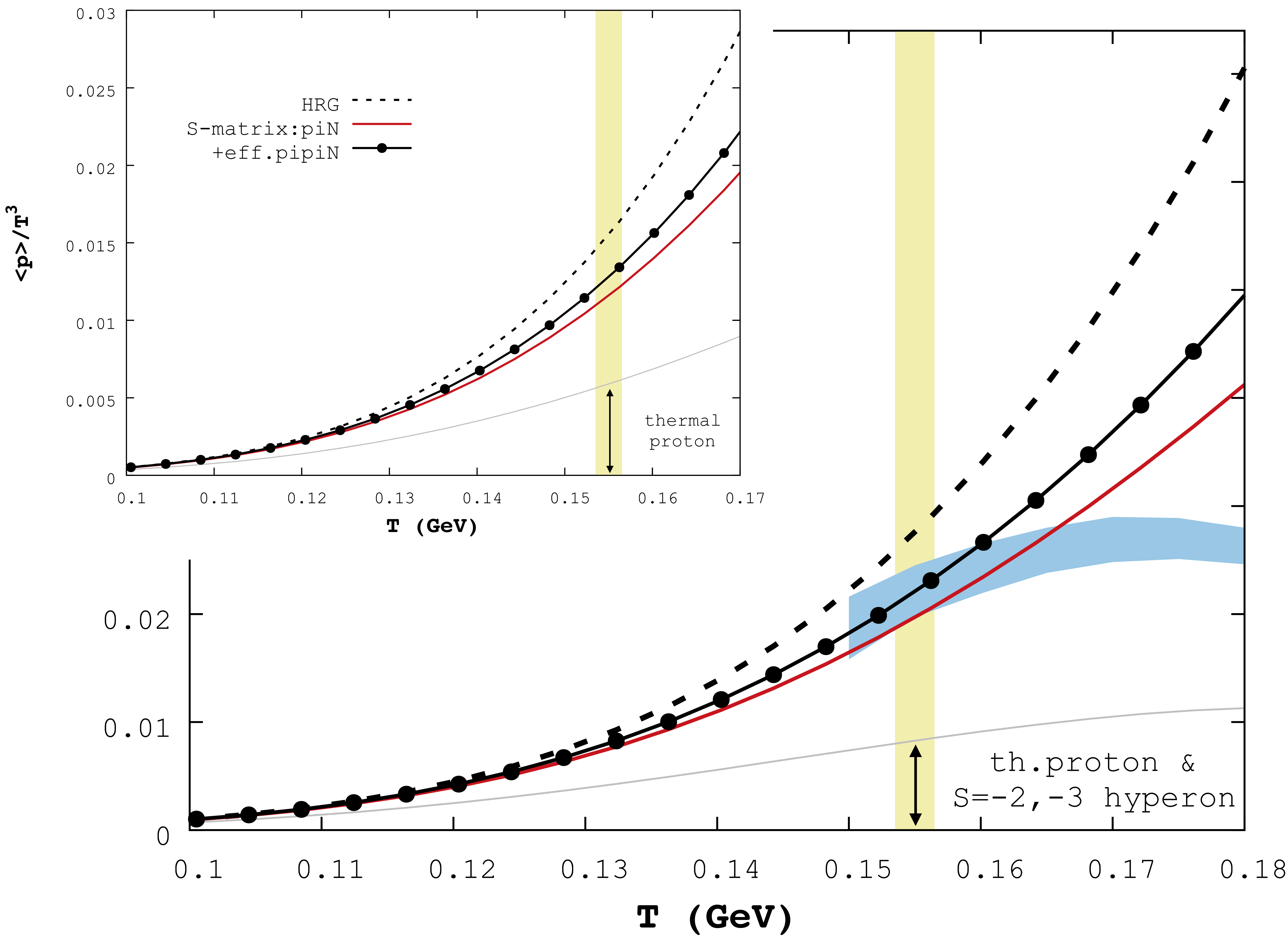
$\Delta(1232)$
$N^*(1520)$
$N^*(1680)$
$\Delta(1620)$
$\Delta(1950)$
total
N*
Delta
$T = 155\,\mathrm{MeV}$
1/n dn/dsqrt(s) (GeV^{-1})
sqrt(s) (GeV)
8
6
4
2
0
-2
1
1.2
1.4
1.6
1.8
2
2.2
2.4



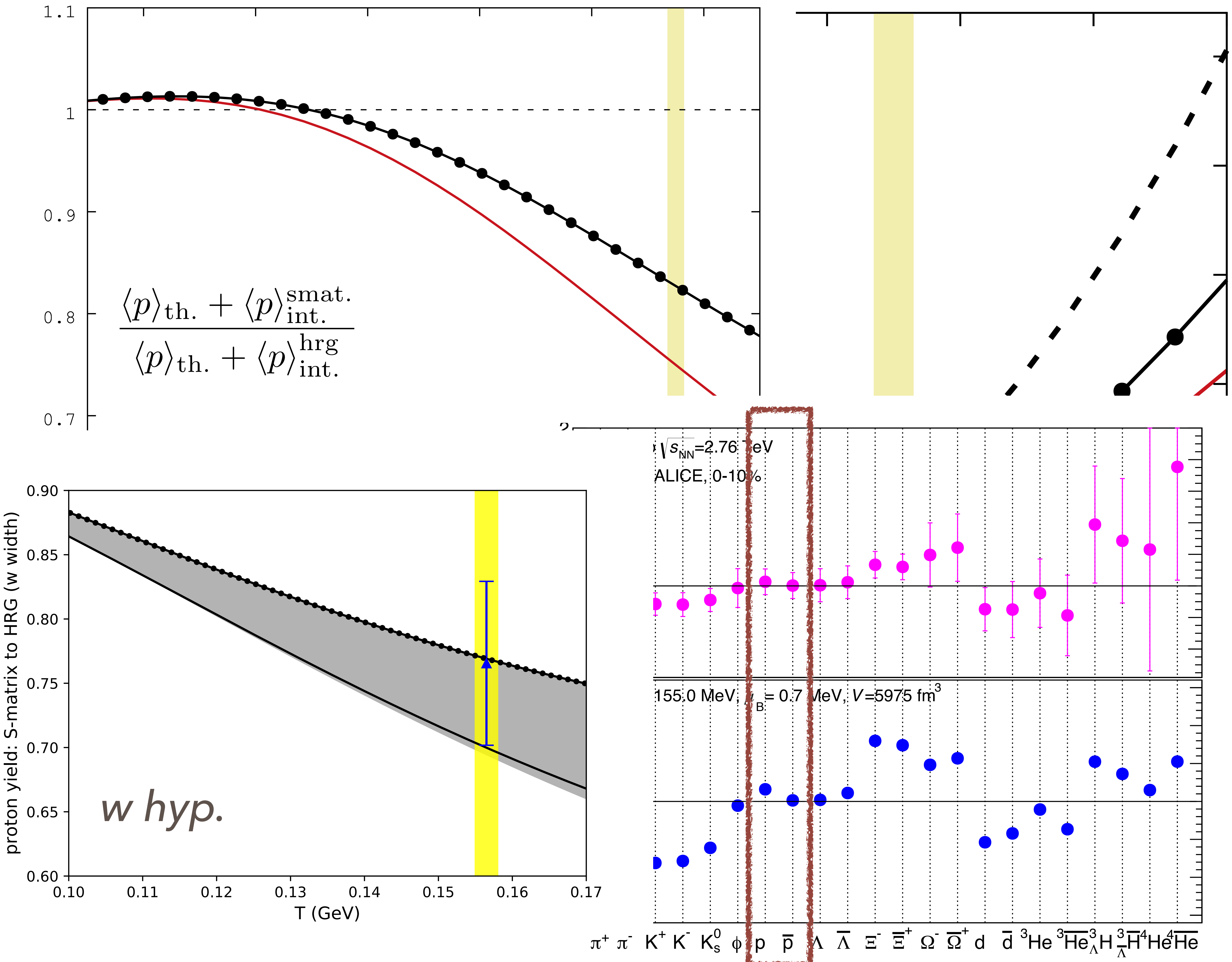
HRG
S-matrix:piN
HotQCD
thermal p
res.
+|S|=2,3
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
chiBQ/T^2
T (GeV)



chiBQ/T^2
T (GeV)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
HRG
S-matrix:piN
+eff.pipiN
HotQCD
pipiN BG model:
$\mathcal{Q}_N(M) = \frac{1}{2}\,\mathrm{Im}\left[\ln\left(1 + \int d\phi_N\, i\mathcal{M}\right)\right]$
sqrt(s) -> T dependent
fit strength to LQCD @ 155
th.proton &
S=-2,-3 hyperon

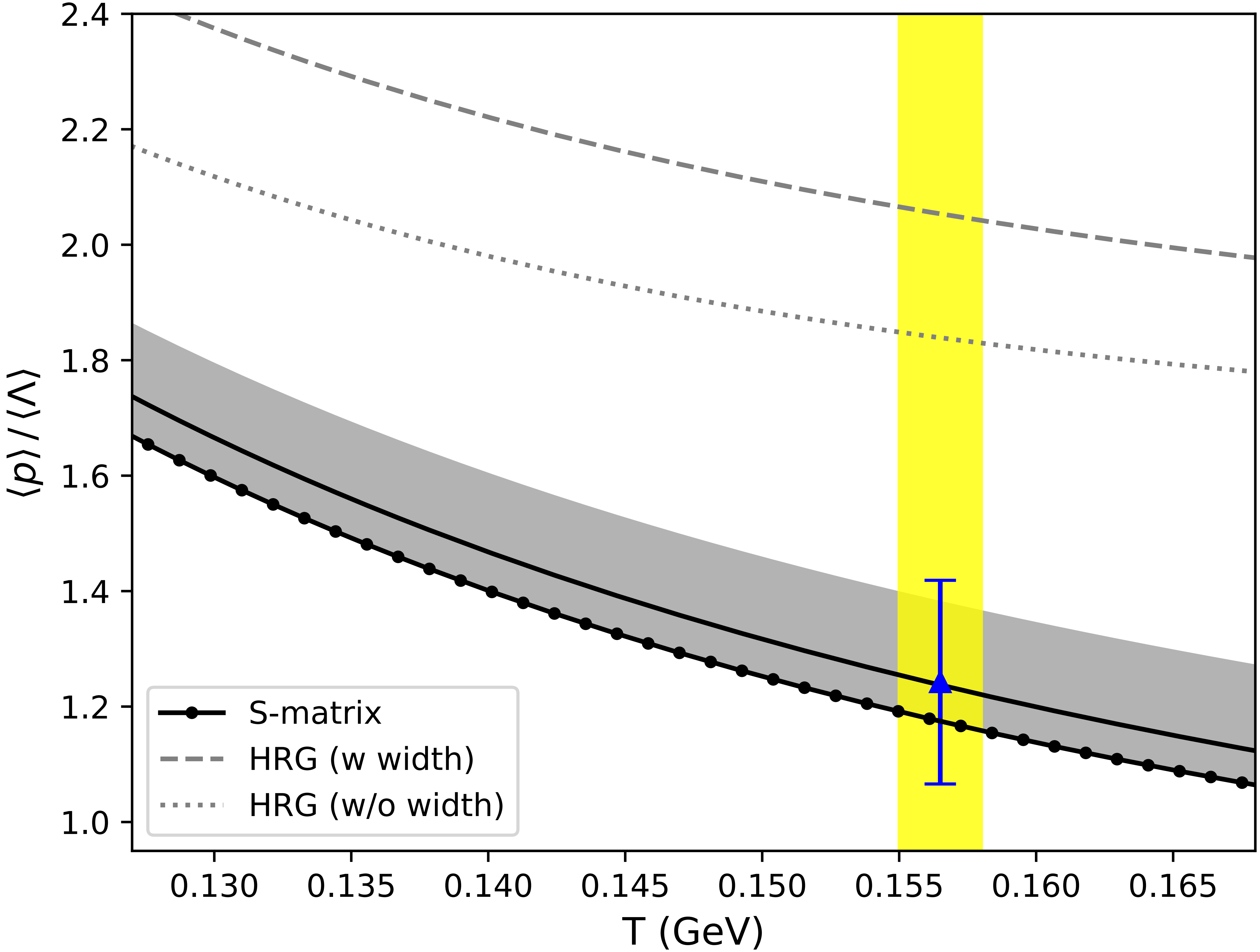


HRG
S-matrix:piN
+eff.pipiN
thermal
proton
<p>/T^3
T (GeV)
th.proton &
S=-2,-3 hyperon



$$\frac{\langle p\rangle_{\rm th.} + \langle p\rangle^{\rm smat.}_{\rm int.}}{\langle p\rangle_{\rm th.} + \langle p\rangle^{\rm hrg}_{\rm int.}}$$
proton yield: S-matrix to HRG (w width)
T (GeV)
w hyp.
ALICE, 0-10%
155.0 MeV
π+ π- K+ K- K0s φ p p̄ Λ Λ̄ Ξ- Ξ̄+ Ω- Ω̄+ d d̄ 3He 3He̅ 3ΛH 3Λ̄H̄ 4He 4He̅

ratio of yields



DOS OF COUPLED CHANNEL SYSTEM

C. Fernandez-Ramirez, PML, and P. Petreczky,
PRC **98**, 044910 (2018)

COUPLED-CHANNEL PROBLEM

$$\{\gamma_1, \gamma_2, m_{\text{res}}\} \longleftrightarrow \{\delta_1, \delta_2, \eta\}$$

$$S = \begin{pmatrix} \eta\, e^{2\,i\,\delta_I} & i\sqrt{1-\eta^2}\, e^{i\,(\delta_I+\delta_{II})} \\ i\sqrt{1-\eta^2}\, e^{i\,(\delta_I+\delta_{II})} & \eta\, e^{2\,i\,\delta_{II}} \end{pmatrix}$$

$$\begin{aligned} \mathcal{Q}(M) &\equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln\ \mathrm{S}) \\ &= \frac{1}{2}\,\mathrm{Im}\,(\ln\ \det\,[\mathrm{S}]) \\ &= \delta_I + \delta_{II}. \end{aligned}$$

$a_0(980)$ *system*

$$\pi\eta \to \begin{pmatrix} \pi\eta \\ K\bar{K} \end{pmatrix} \to \pi\eta$$

$$K\bar{K} \to \begin{pmatrix} \pi\eta \\ K\bar{K} \end{pmatrix} \to K\bar{K}$$

WIGNER, EISENBUD, SMITH, …

$$S \to U^\dagger S_d U$$

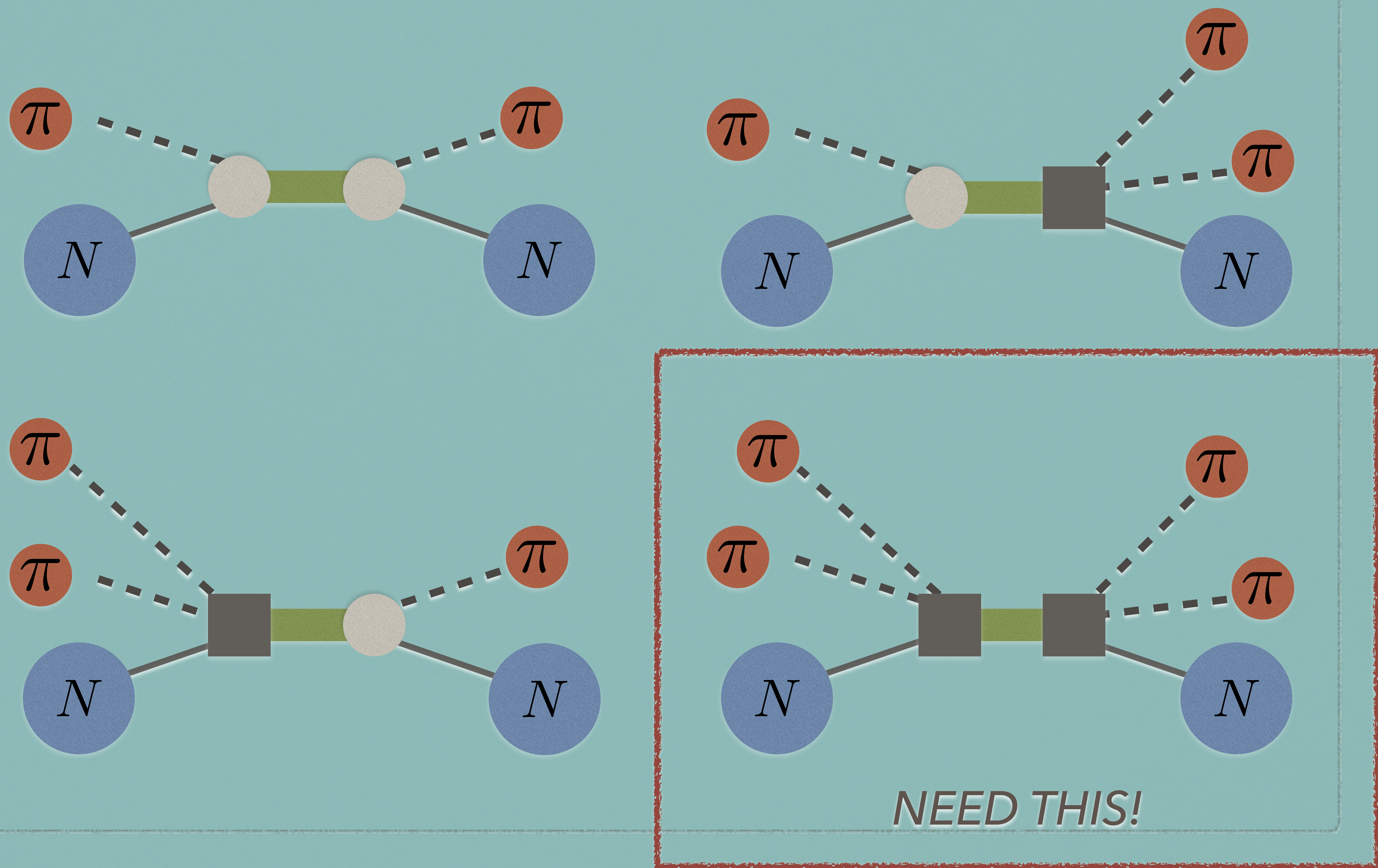
$$S_d = \begin{pmatrix} e^{2i\delta_{\rm res}(s)} & 0 \\ 0 & 1 \end{pmatrix},$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

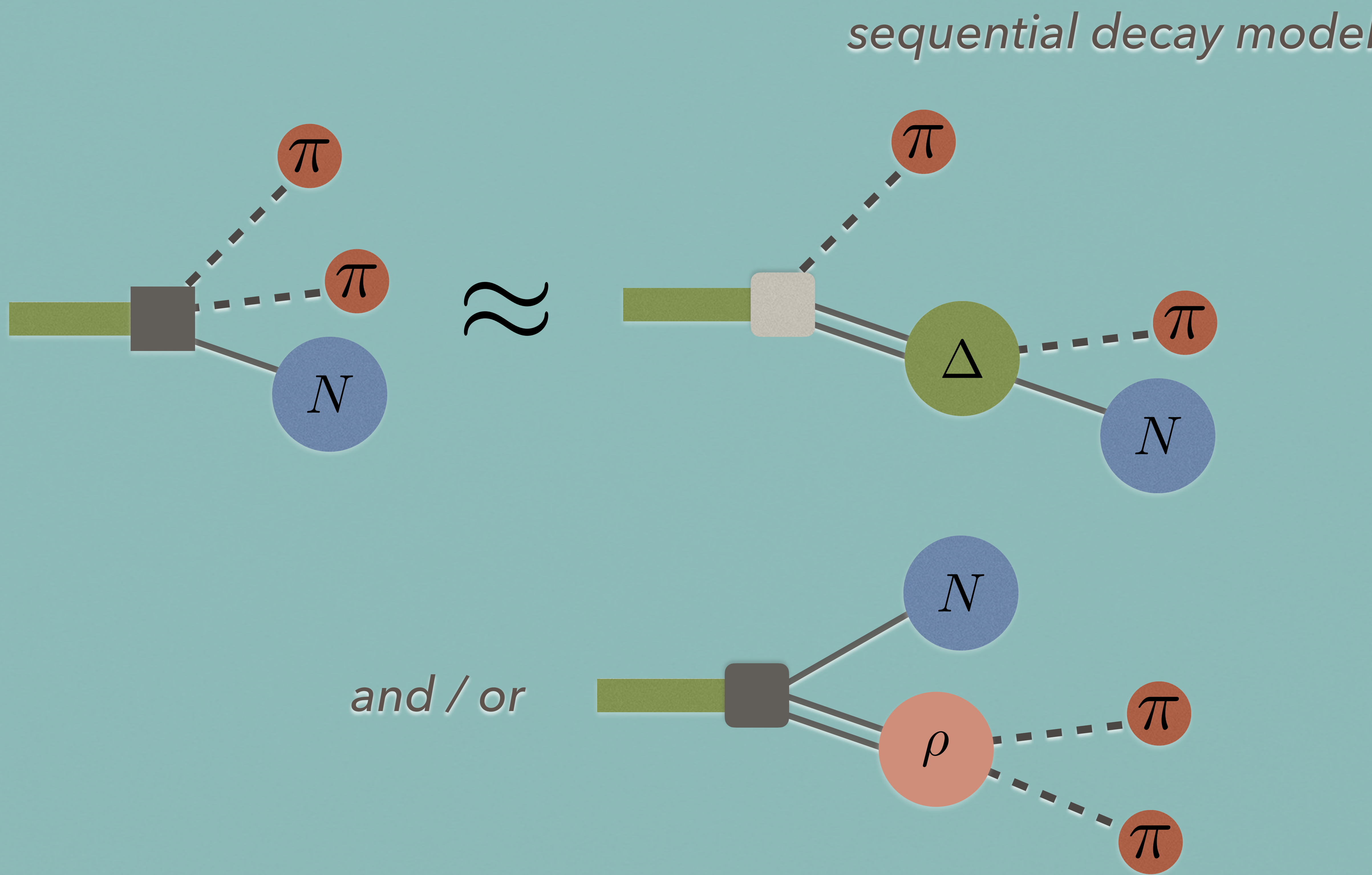
$$\mathrm{BR}_a = \cos^2\theta = \frac{g_a^2\,\phi_a}{g_a^2\,\phi_a + g_b^2\,\phi_b},$$

$$\mathrm{BR}_b = \sin^2\theta = \frac{g_b^2\,\phi_b}{g_a^2\,\phi_a + g_b^2\,\phi_b}.$$

ISOBAR MODEL



ISOBAR MODEL



S = -1 HYPERONS
COUPLED CHANNEL SYSTEM

JPAC, PRD **93**, 034029 (2016)

C. Fernandez-Ramirez, PML, and P. Petreczky,
PRC **98**, 044910 (2018)

J. Cleymans, PML, K. Redlich, and N. Sharma
PRC **103**, 014904 (2021)

PHASE SHIFT FROM PWA

```
------------------------------------------------------------------------------------

Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

------------------------------------------------------------------------------------
------------------------------------------------------------------------------------

Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)
Igor V. Danilkin (Jefferson Lab)
Vincent Mathieu (Indiana University)
Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)
This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosphy. If your
parameters do not make sense, the output will not make sense either.
2 - You can use, share and modify this code under your own responsability.
3 - This code is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
4 - No PhD students or postdocs were severely damaged during the development
of this project.

----------------------------------------------------------------------
```

- 1 $\to \bar{K}N$,
- 2 $\to \pi\Sigma$,
- 3 $\to \pi\Lambda$,
- 4 $\to \eta\Lambda$,
- 5 $\to \eta\Sigma$,
- 6 $\to \bar{K}_1 N$,
- 7 $\to [\bar{K}_3 N]_-$,
- 8 $\to [\bar{K}_3 N]_+$,
- 9 $\to [\pi\Sigma^*]_-$,
- 10 $\to [\pi\Sigma^*]_+$,
- 11 $\to [\bar{K}\Delta]_-$,
- 12 $\to [\bar{K}\Delta]_+$,
- 13 $\to [\pi\Lambda(1520)]_-$,
- 14 $\to [\pi\Lambda(1520)]_+$,
- 15 $\to \pi\pi\Lambda$,
- 16 $\to \pi\pi\Sigma$.

elastic scatterings (elementary)

quasi elastic scatterings

elastic scatterings (elementary)

STRANGENESS CONTENT IN A HADRON GAS

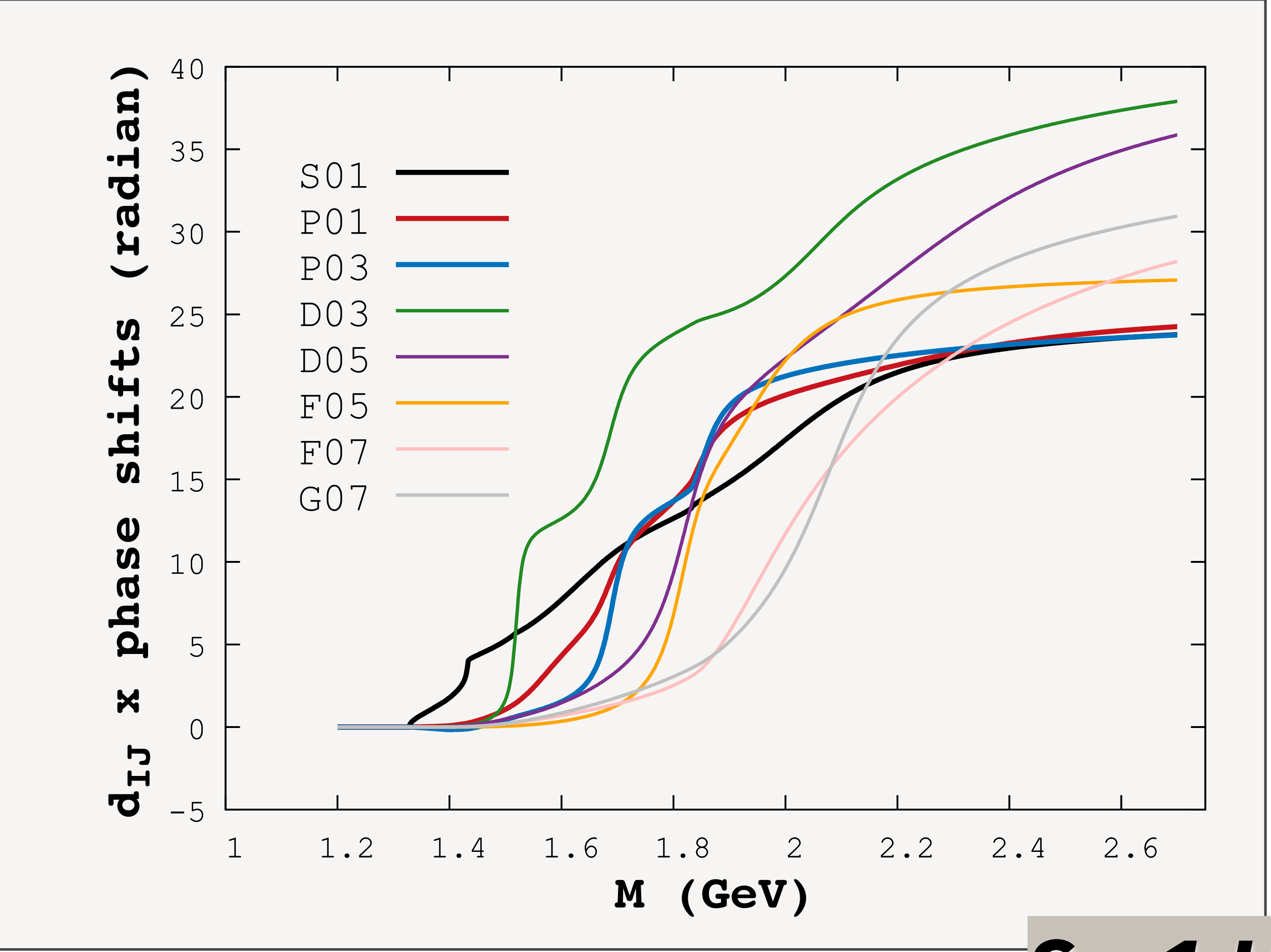
- K-N system requires a coupled channel analysis

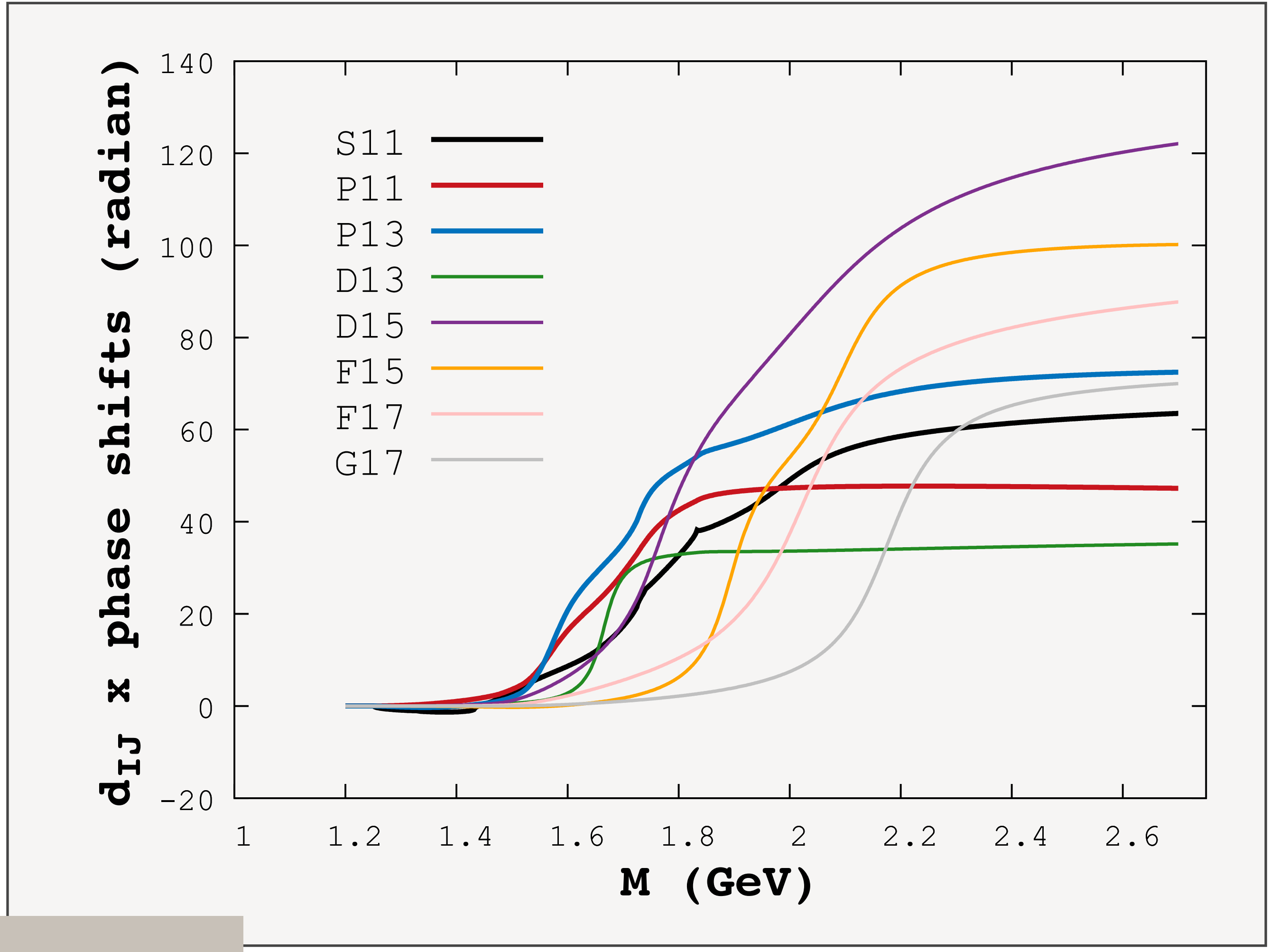
$$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle, \ldots$$

16 basis states

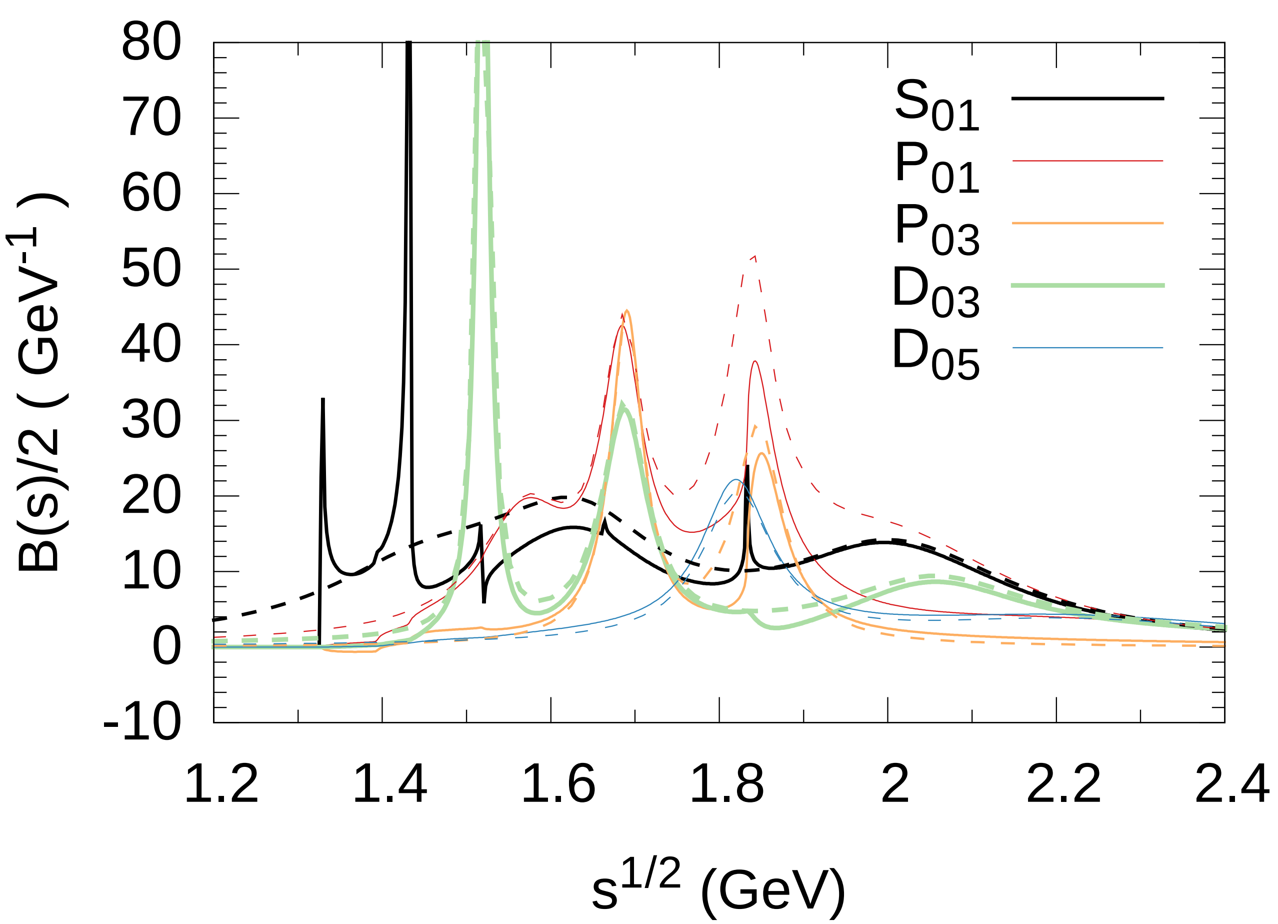
$$\mathcal{Q}(M) \equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln\ \mathrm{S}) = \frac{1}{2}\,\mathrm{Im}\,(\ln\ \det\,[\mathrm{S}]) = \delta_{\bar{K}N} + \delta_{\pi\Sigma} + \delta_{\pi\Lambda} + \ldots$$

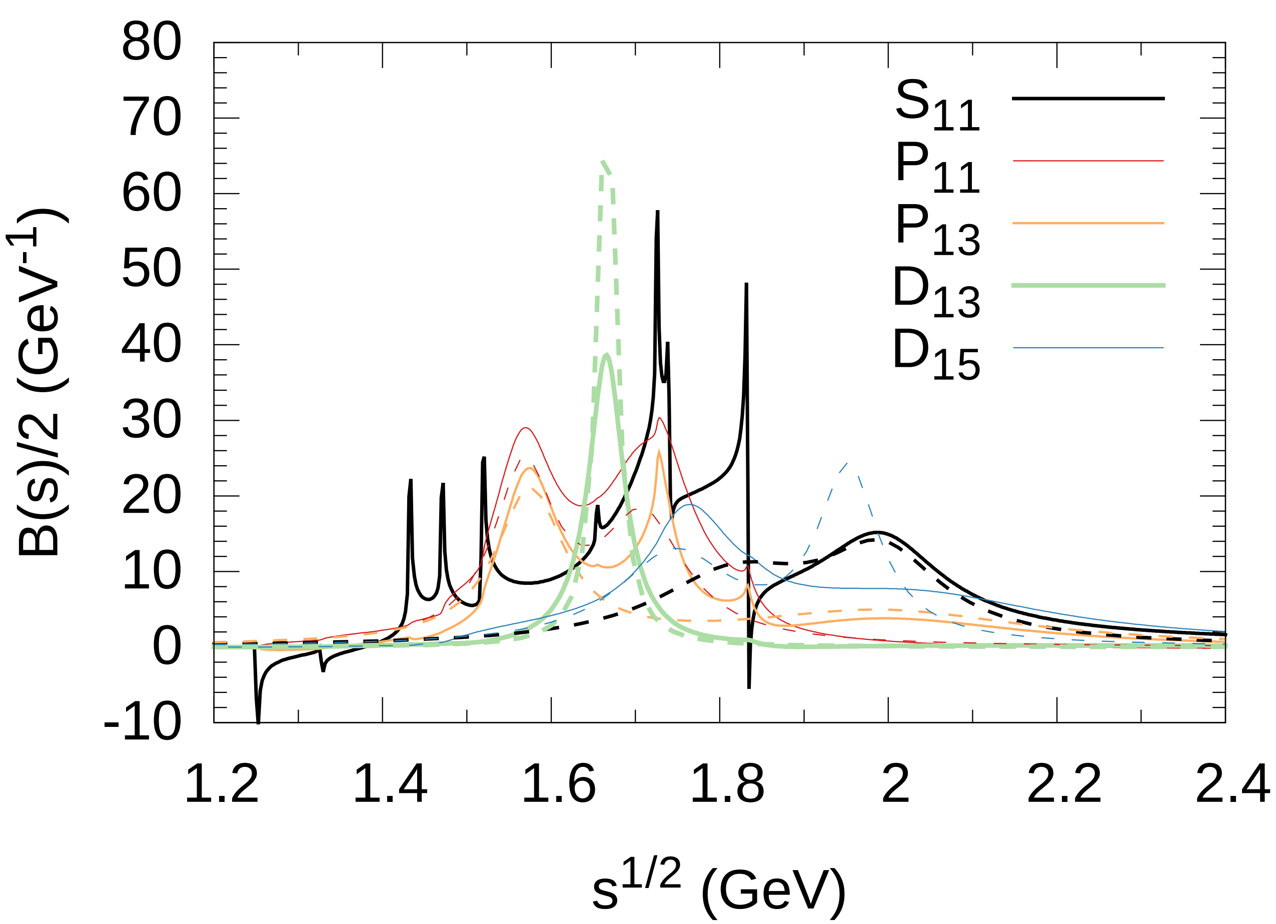
recipe to extract eigenphases from PWA

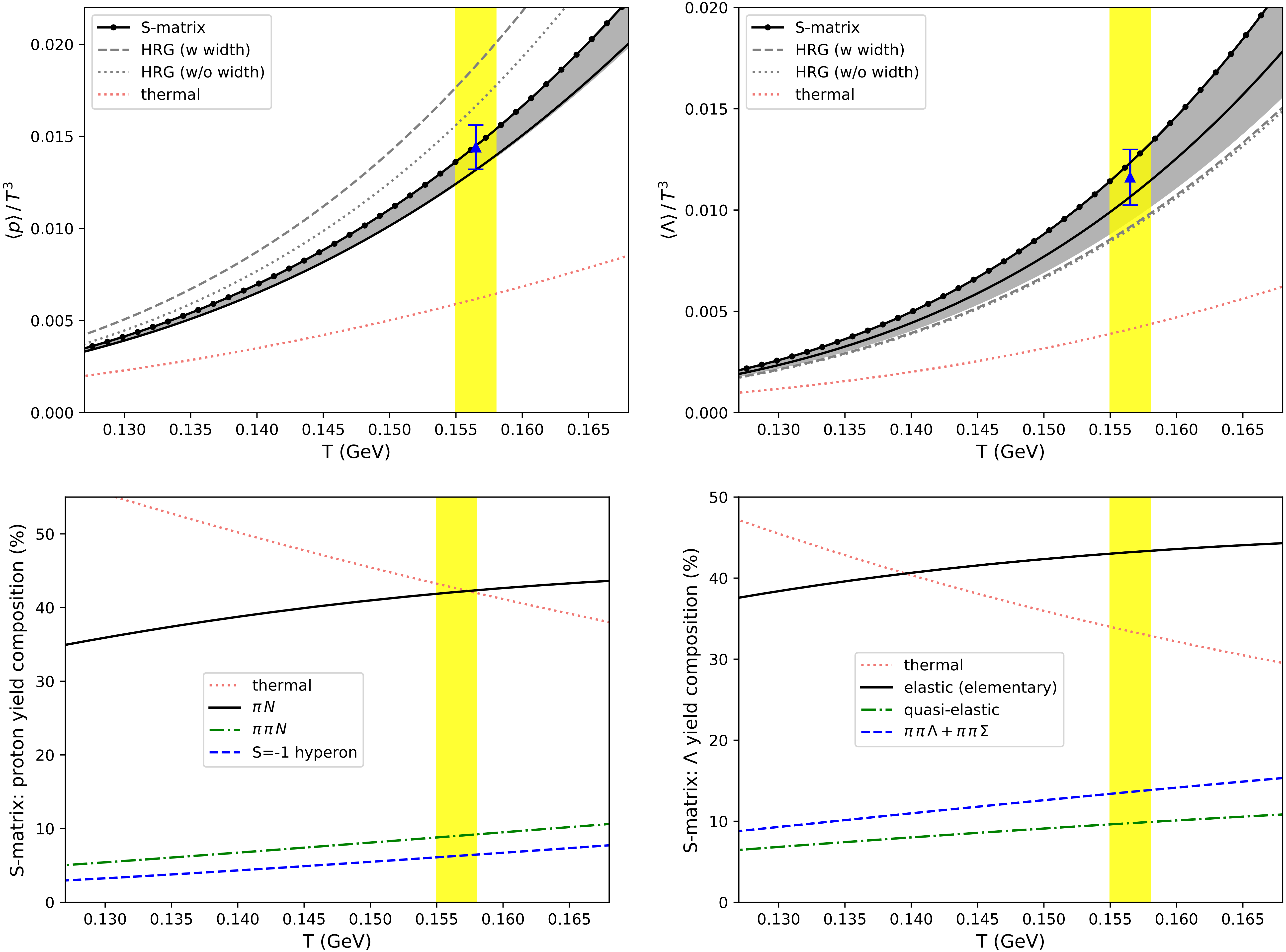

dIJ x phase shifts (radian)
M (GeV)
S01
P01
P03
D03
D05
F05
F07
G07


dIJ x phase shifts (radian)
M (GeV)
S11
P11
P13
D13
D15
F15
F17
G17

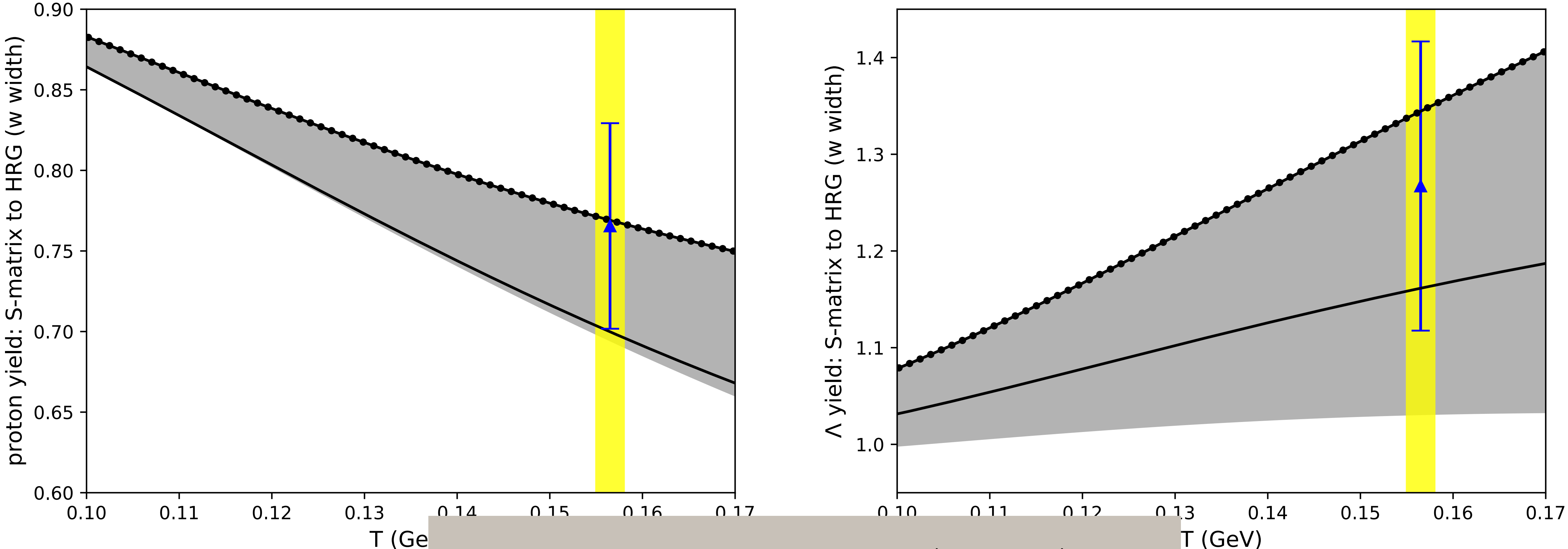
S=-1 Hyperon


B(s)/2 (GeV-1)
s1/2 (GeV)
S01
P01
P03
D03
D05

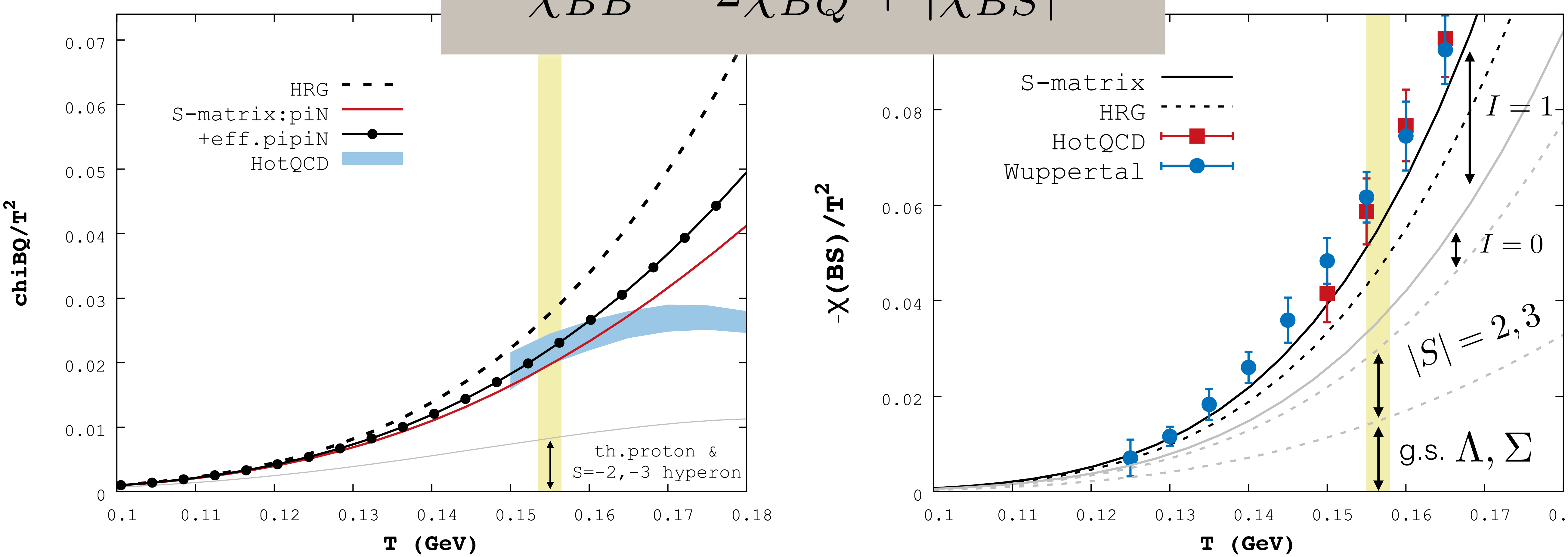

B(s)/2 (GeV-1)
s1/2 (GeV)
S11
P11
P13
D13
D15



S-matrix
HRG (w width)
HRG (w/o width)
thermal
⟨p⟩ / T^3
T (GeV)
⟨Λ⟩ / T^3
S-matrix: proton yield composition (%)
πN
ππN
S=-1 hyperon
S-matrix: Λ yield composition (%)
elastic (elementary)
quasi-elastic
ππΛ + ππΣ



$$\chi_{BB} = 2\chi_{BQ} + |\chi_{BS}|$$



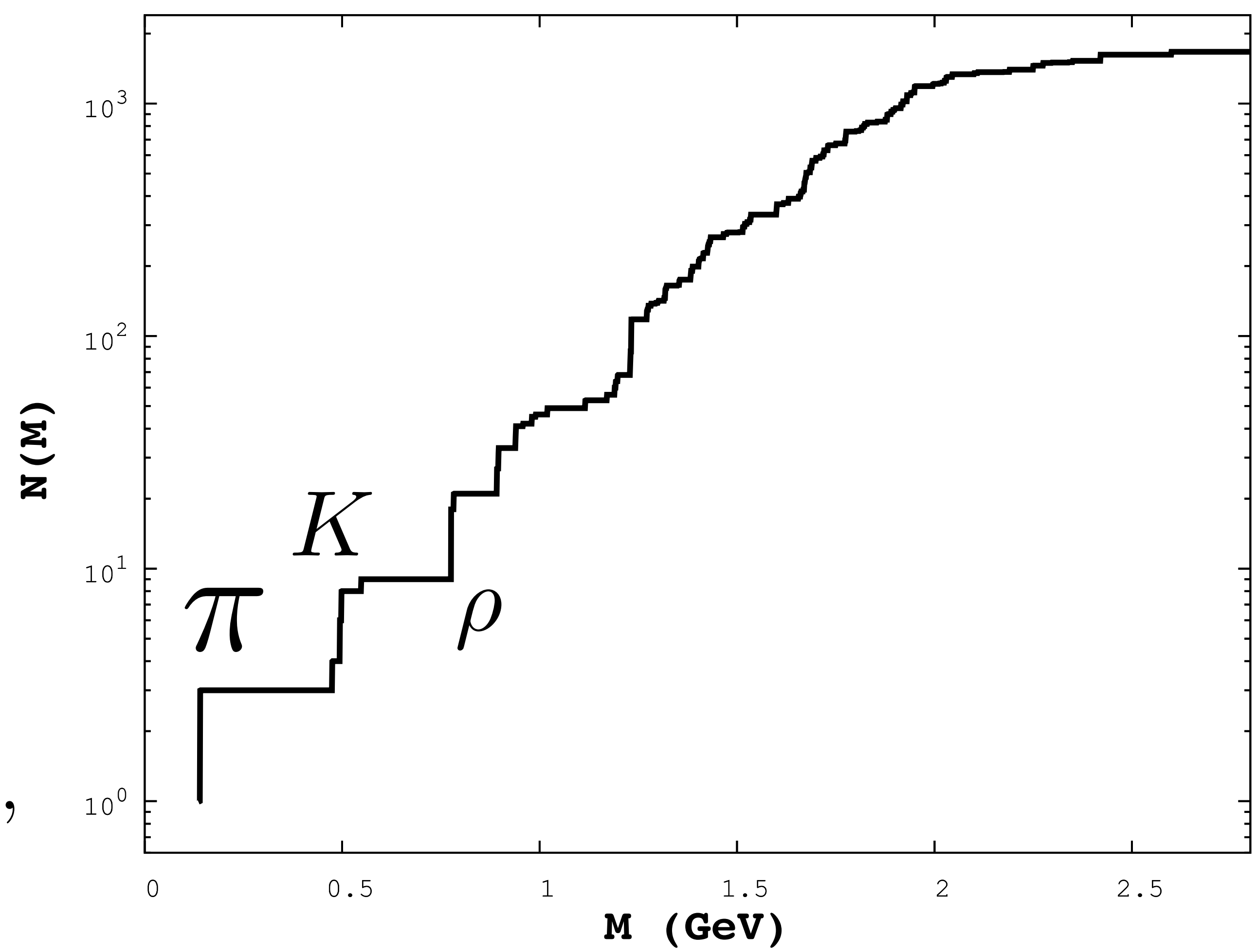
POLES AND ROOTS

HRG AS AN S-MATRIX SCHEME

$$\det S(E) = \prod_{\{\text{res}\}} \frac{z^{\star}_{\text{res}} - E}{z_{\text{res}} - E}, \qquad z_{\text{res}} \approx m_{\text{res}} - i\,0^{+}.$$

$$\mathcal{Q}(M) \equiv \frac{1}{2}\,\text{Im}\,(\text{tr}\ln\,\text{S})$$

$$\mathcal{Q}_{\text{HRG}}(E) = \sum_{\text{res}} d_{IJ} \times \pi\,\theta(E - m_{\text{res}}),$$

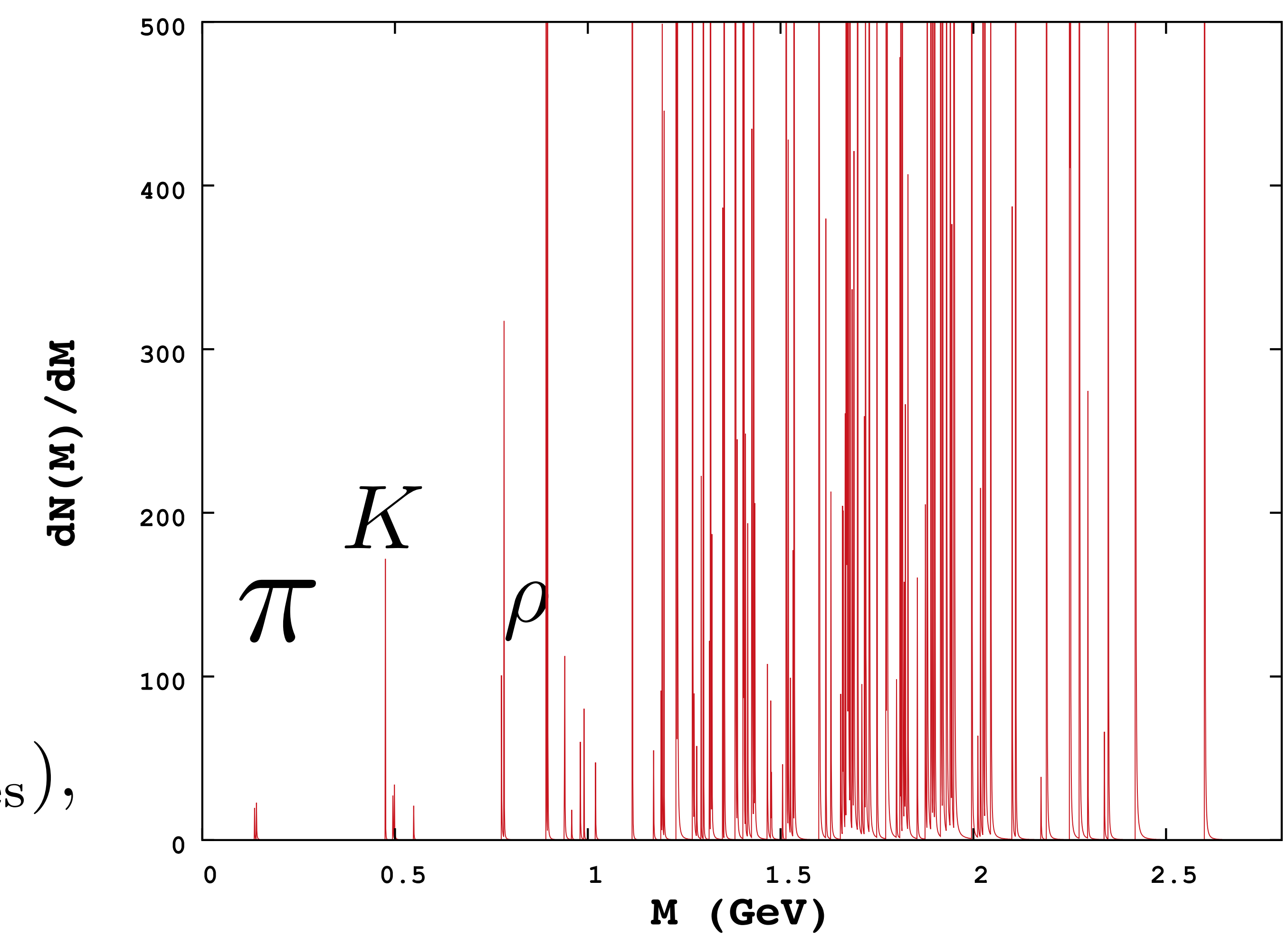


HRG AS AN S-MATRIX SCHEME

$$\det S(E) = \prod_{\{\text{res}\}} \frac{z^{\star}_{\text{res}} - E}{z_{\text{res}} - E}, \qquad z_{\text{res}} \approx m_{\text{res}} - i\,0^{+}.$$

$$\mathcal{Q}(M) \equiv \frac{1}{2}\,\text{Im}\,(\text{tr}\ln\,S)$$

$$\frac{\partial}{\partial E}$$



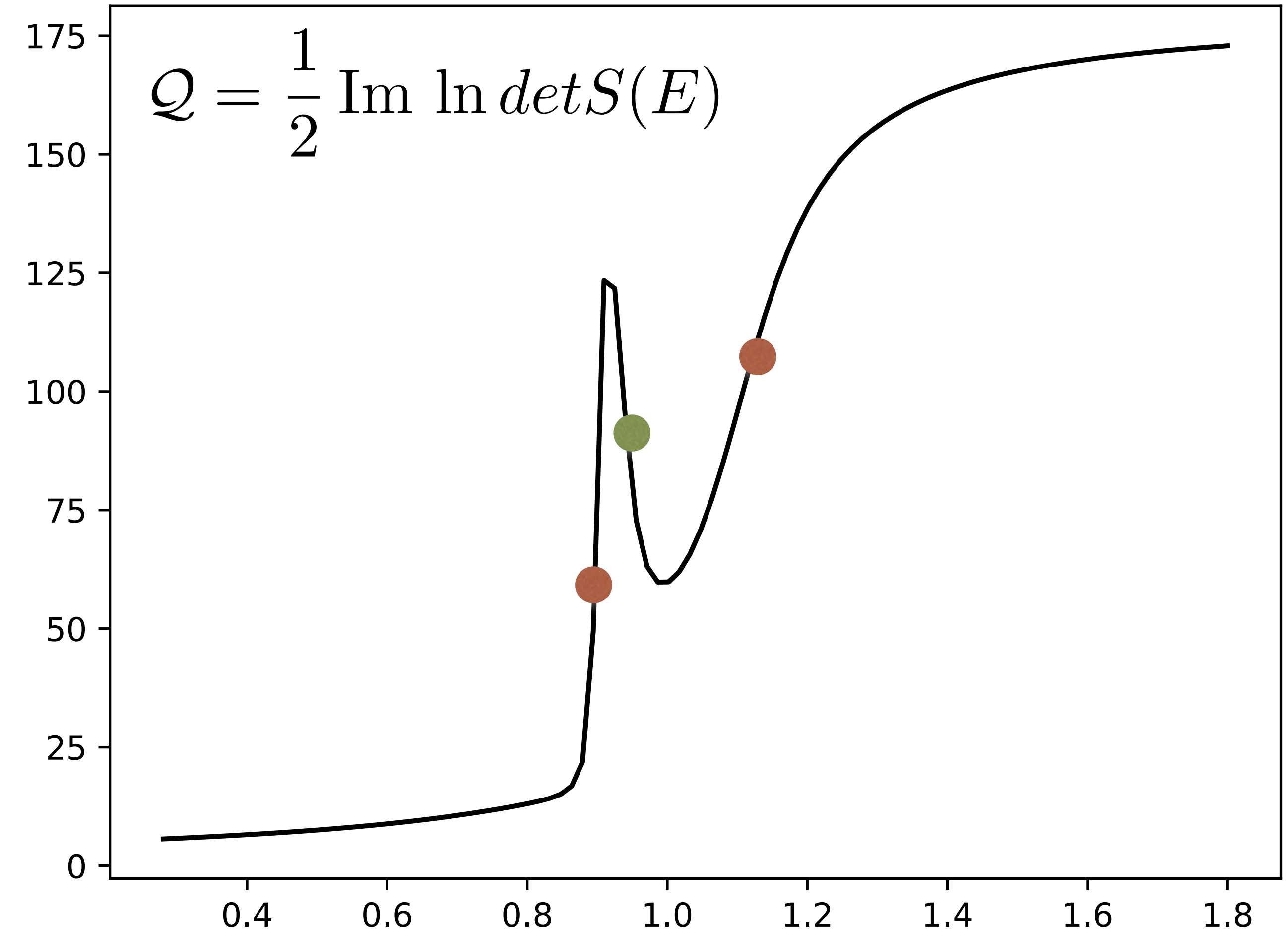
$$\mathcal{Q}_{\text{HRG}}(E) = \sum_{\text{res}} d_{IJ} \times \pi\,\theta(E - m_{\text{res}}),$$

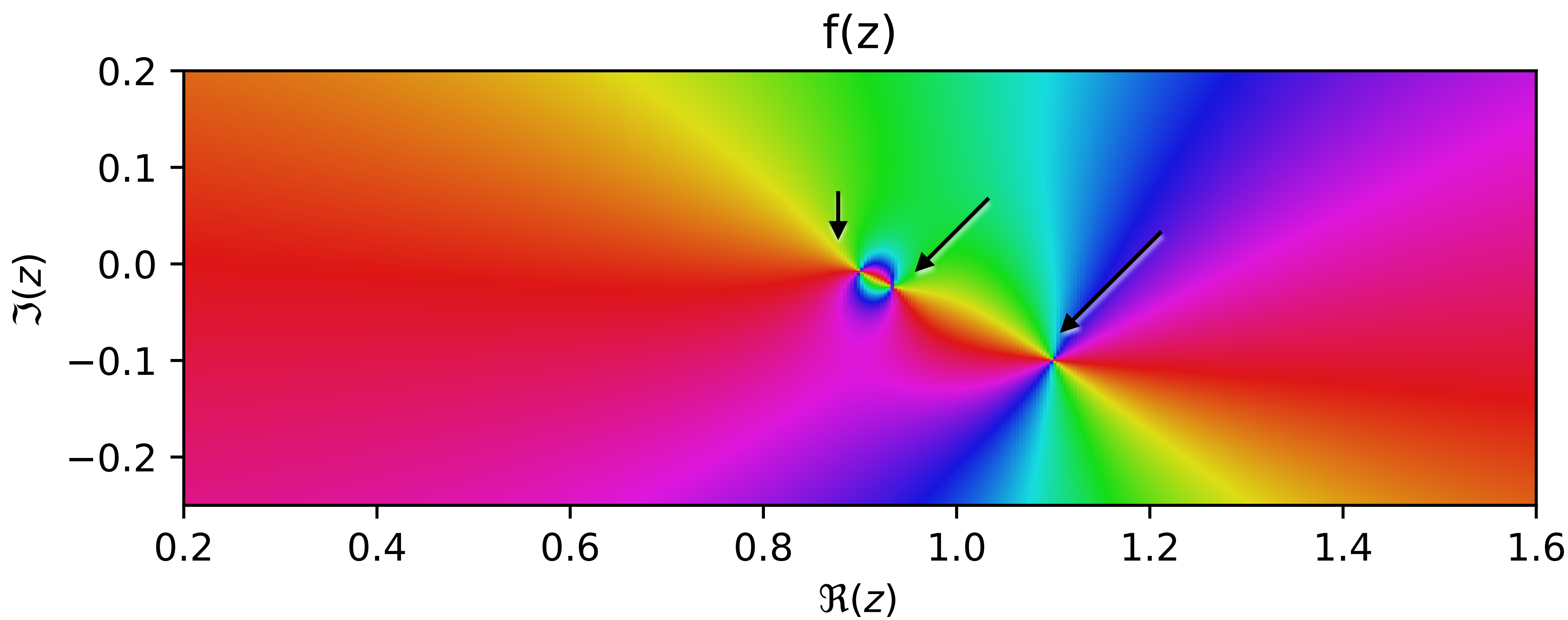
ROOTS IN S-M

2 pole + 1 root

$$detS \propto \frac{g_1}{E - p_1} + \frac{g_2}{E - p_2}$$

$$detS \propto \frac{g_1}{E - p_1} \times \frac{g_2}{E - p_2}$$





DYNAMICAL GENERATION OF BS / RESONANCES

- dynamical generation of bound states / resonances:
  f(980) close to K Kbar threshold
  f(500) dynamically generated

- coupling of open channels: pipi, kkbar
  with a $|q\bar{q}\rangle$ state

Locher, Markushin, Zheng, EPJC 4 (1998)
Kaminski, Lesniak, Loiseau, EPJC 9 (1999)

PML PRD **102**, 034038 (2020)

what you give $\neq$ *what you get* *1 in 5 out!*

$$\frac{1}{E - \mathcal{H}_0} = \begin{array}{r} |\pi\pi\rangle \\ |K\bar{K}\rangle \\ |R^0\rangle \ (|q\bar{q}\rangle) \end{array} \begin{bmatrix} \Pi_{\pi\pi}(E) & & \\ & \Pi_{K\bar{K}}(E) & \\ & & \frac{1}{E - m^0_{res}} \end{bmatrix}$$

$$V_{int} = \begin{bmatrix} g_{\pi\pi} & g_{\pi K} & g_{\pi R} \\ g_{\pi K} & g_{KK} & g_{KR} \\ g_{\pi R} & g_{KR} & \end{bmatrix}$$

$$G = G_0 + G_0 V_{int} G$$

From Hamiltonian to Scattering Matrix

$$\frac{1}{E - \mathcal{H}_0 \pm i\delta}$$

$$\begin{aligned}
\tilde{S} &= \left(I - G^0_- V\right)\left(I + G^0_+ T\right) \\
&= I - G^0_- V + G^0_+ T - G^0_- V G^0_+ T \\
&= I - G^0_- V + G^0_+ V + G^0_+ V G^0_+ T - G^0_- V G^0_+ T \\
&= I + (G^0_+ - G^0_-) V + (G^0_+ - G^0_-) V G^0_+ T \\
&= I + (G^0_+ - G^0_-) T \\
&\to I + 2\, i \, \mathrm{Im}\,(G^0_+) \times T.
\end{aligned}$$

on-shell limit

TESTING THE ROBUSTNESS

$$Q(E) = \frac{1}{2}\mathrm{Im}\mathrm{Tr}\{\ln S_E\}$$

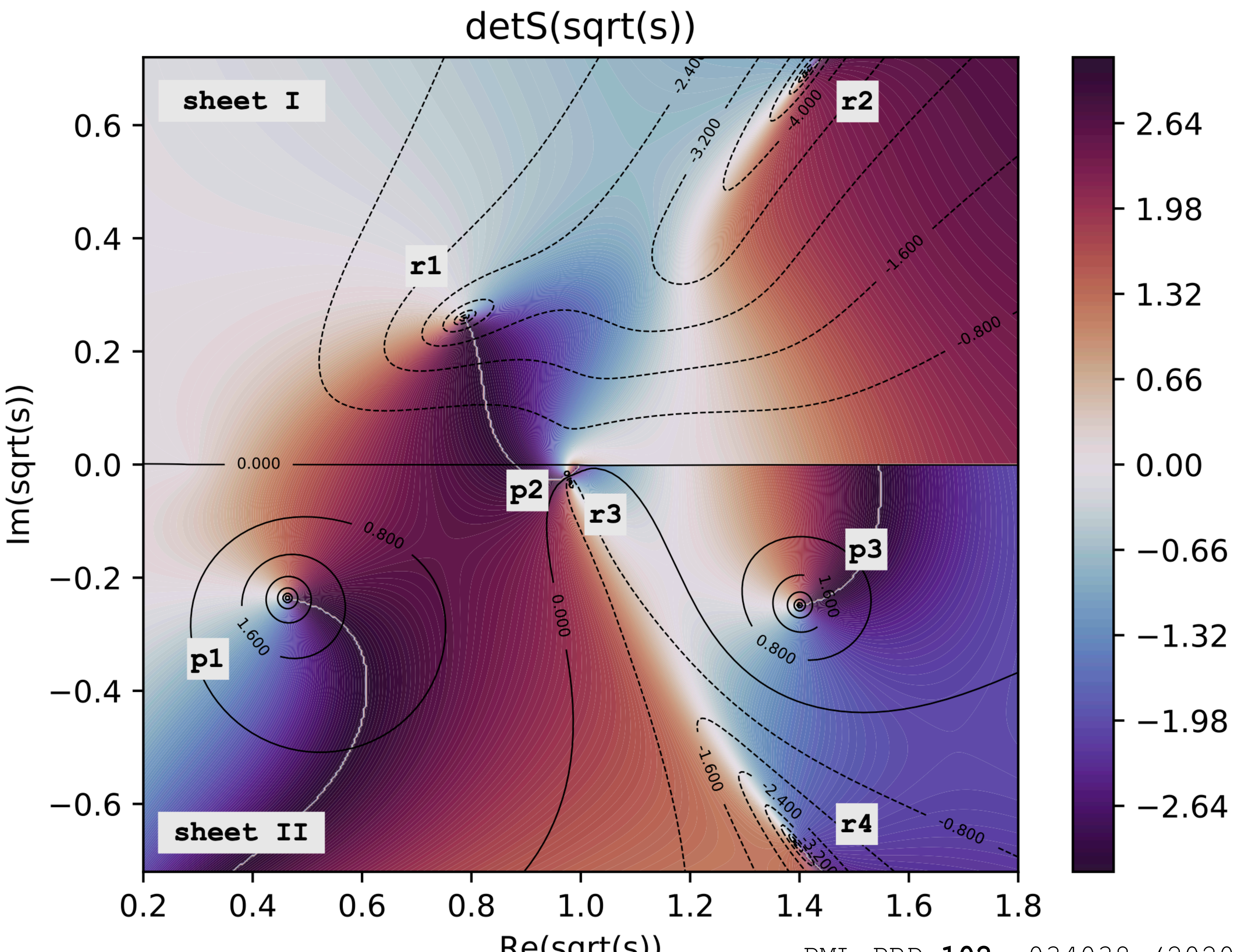
Getting Effective DOS on REAL Energy

effective DOS

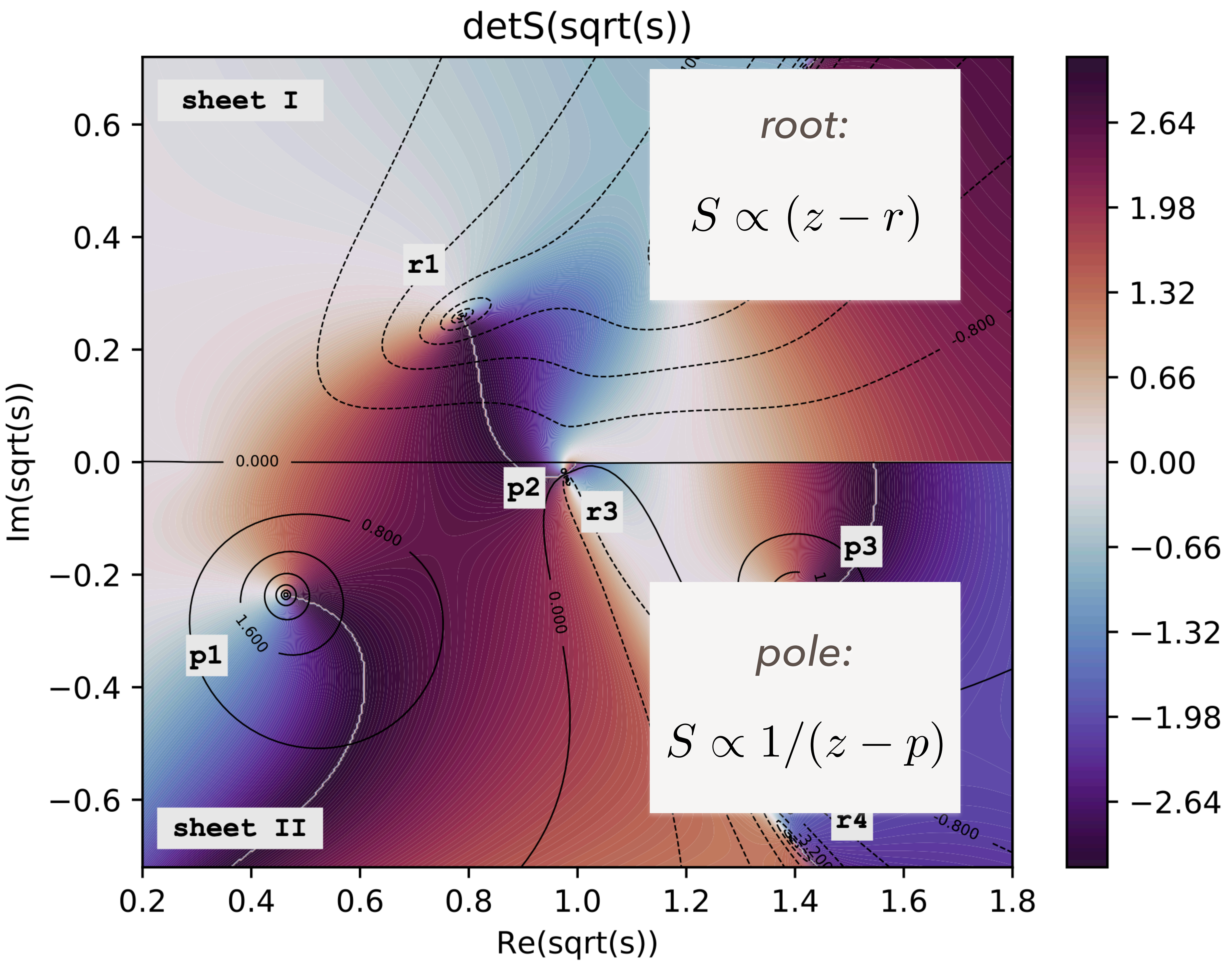
$$B = 2\frac{d}{dE}Q$$

what is being counted?

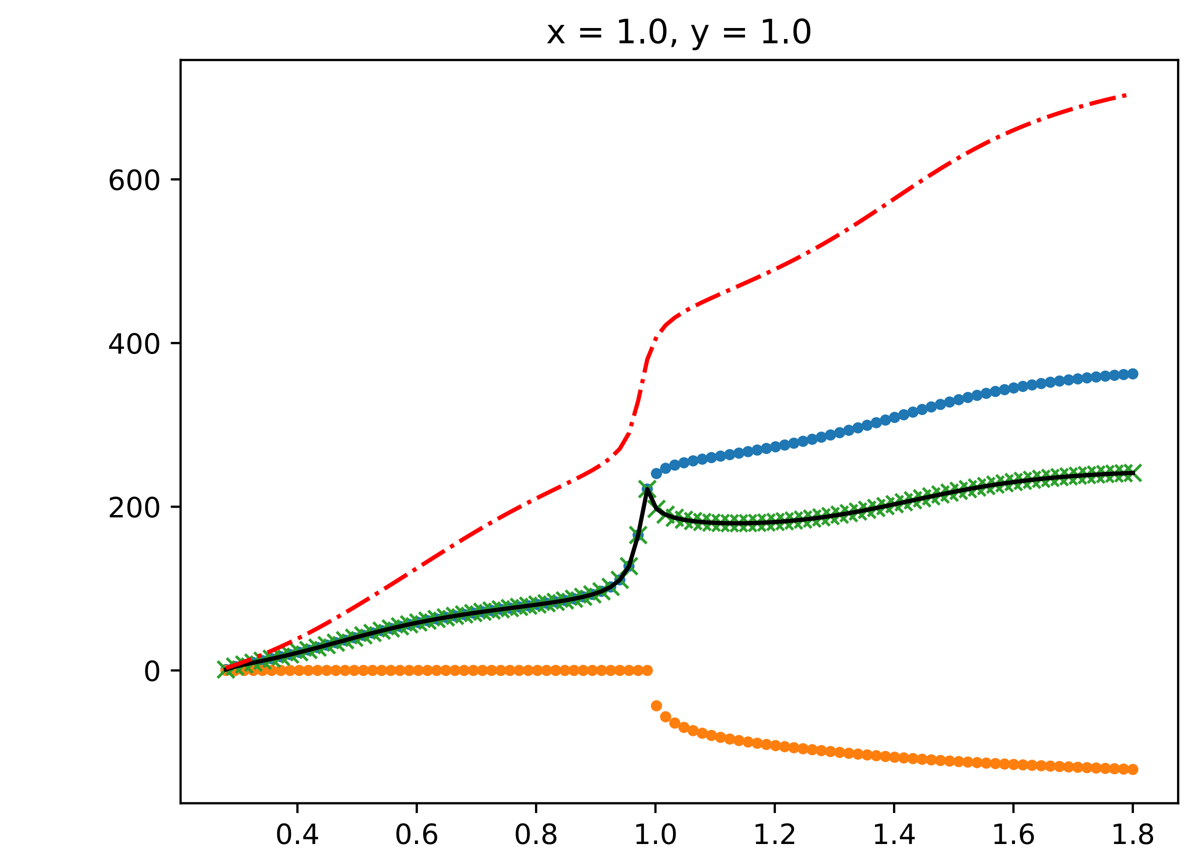
can it handle dynamically generated states?



PML PRD **102**, 034038 (2020)



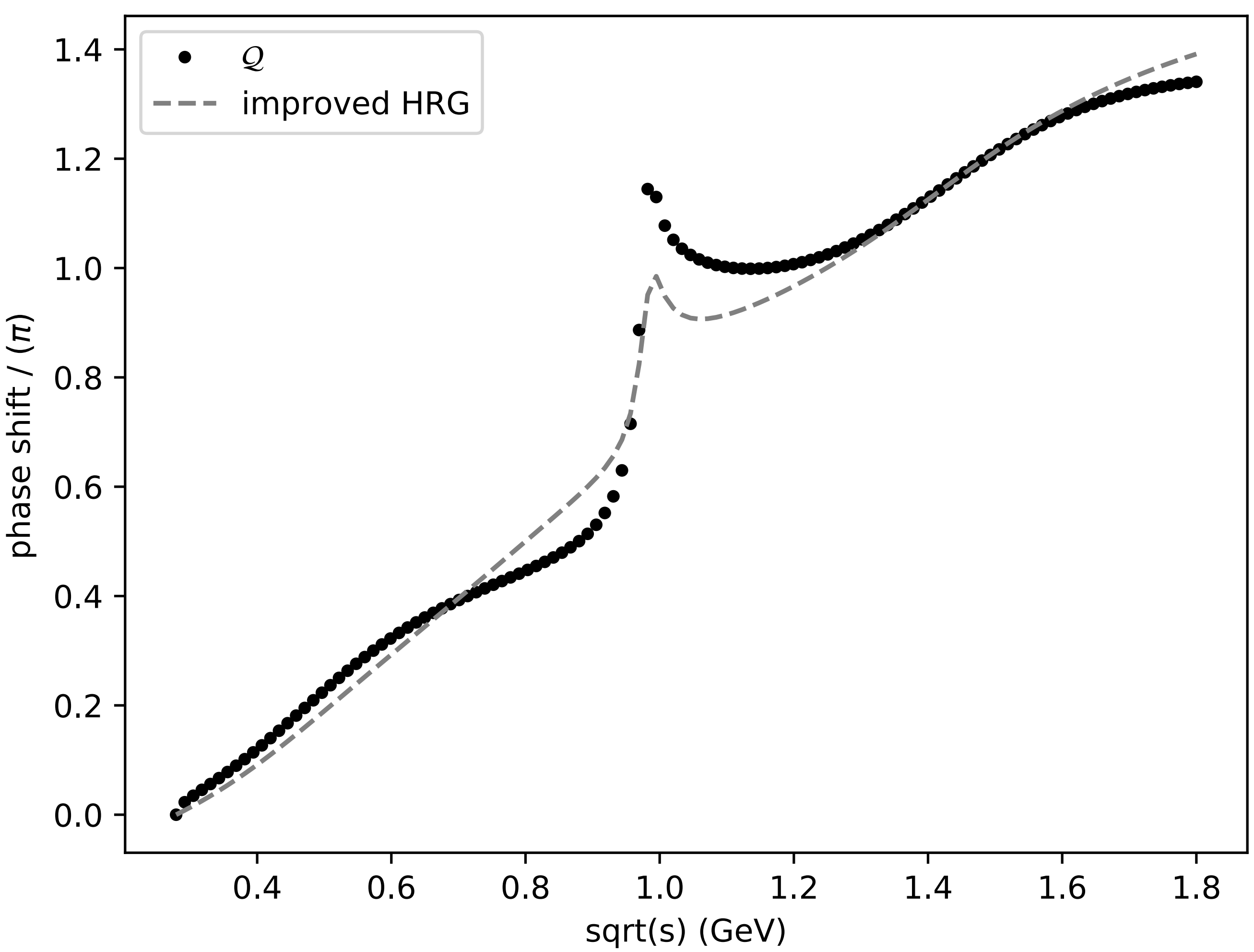
detS(sqrt(s))
sheet I
sheet II
root:
$S \propto (z - r)$
pole:
$S \propto 1/(z - p)$
r1
r3
r4
p1
p2
p3
0.000
0.800
1.600
-0.800
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.64
1.98
1.32
0.66
0.00
−0.66
−1.32
−1.98
−2.64



ef. [54, 55]

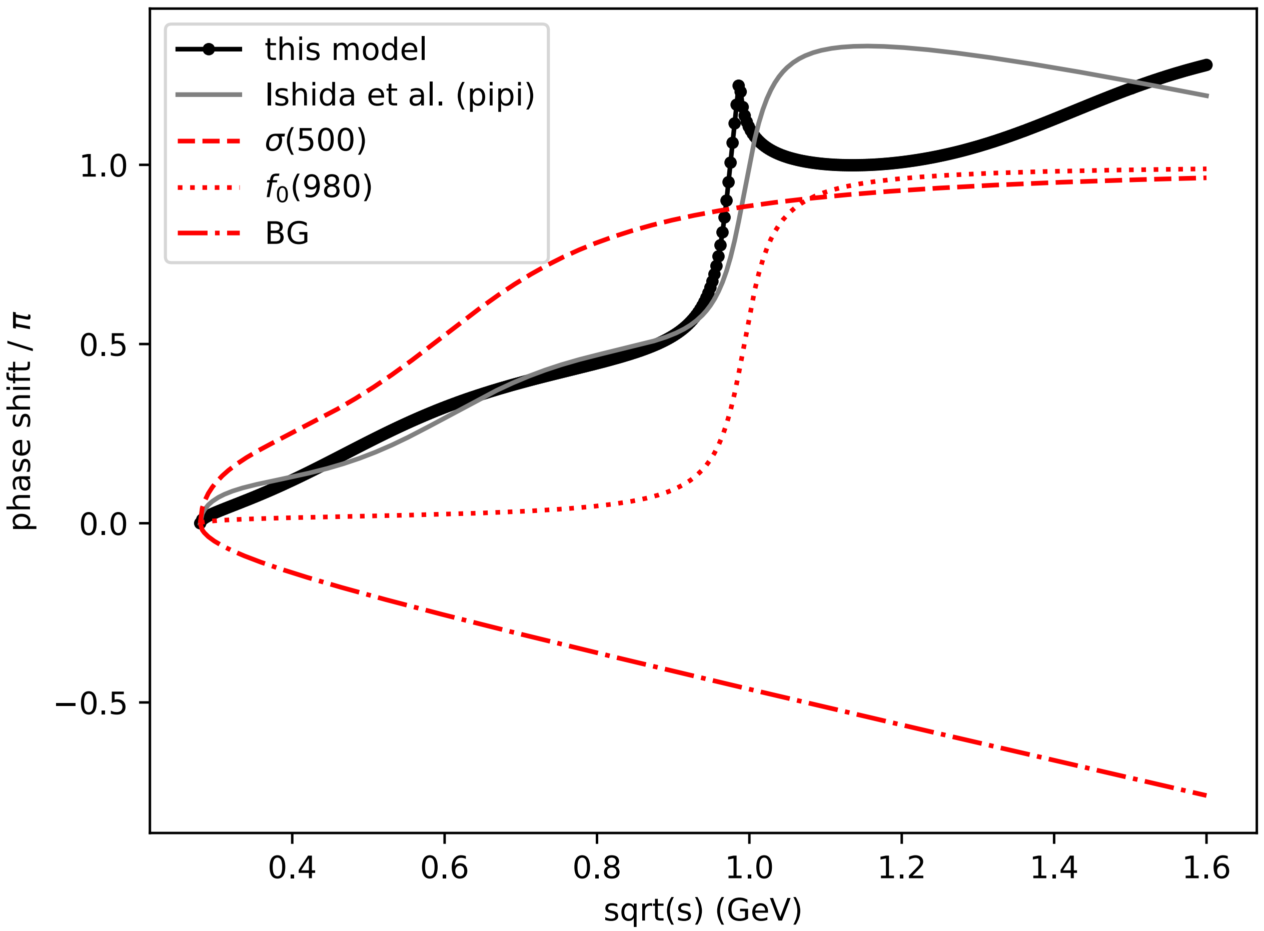
	Re $\sqrt{s}$	Im $\sqrt{s}$	sheet
p1	0.4637	-0.2357	II
p2	0.975	-0.0164	II
p3	1.401	-0.249	II
p4	0.6654	-0.2263	III
p5	1.4176	-0.2640	III
r1	0.787	+0.259	I
r2	1.410	+0.691	I
r3	0.981	-0.032	II
r4	1.393	-0.669	II
r5	0.918	+0.248	IV

II. Location of resonance poles (p_i) and roots (r_i) d in the model.



repulsive corrections in HRG-like scheme: via roots

M (GeV)



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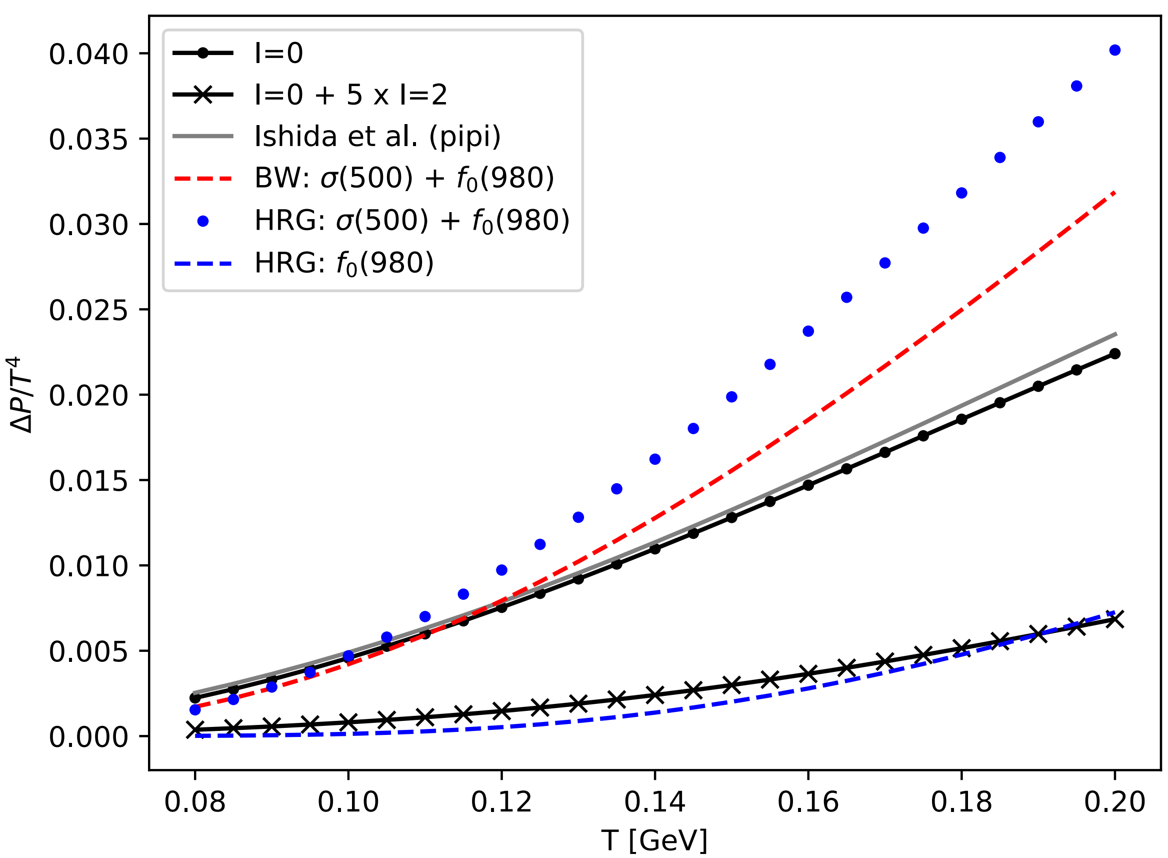
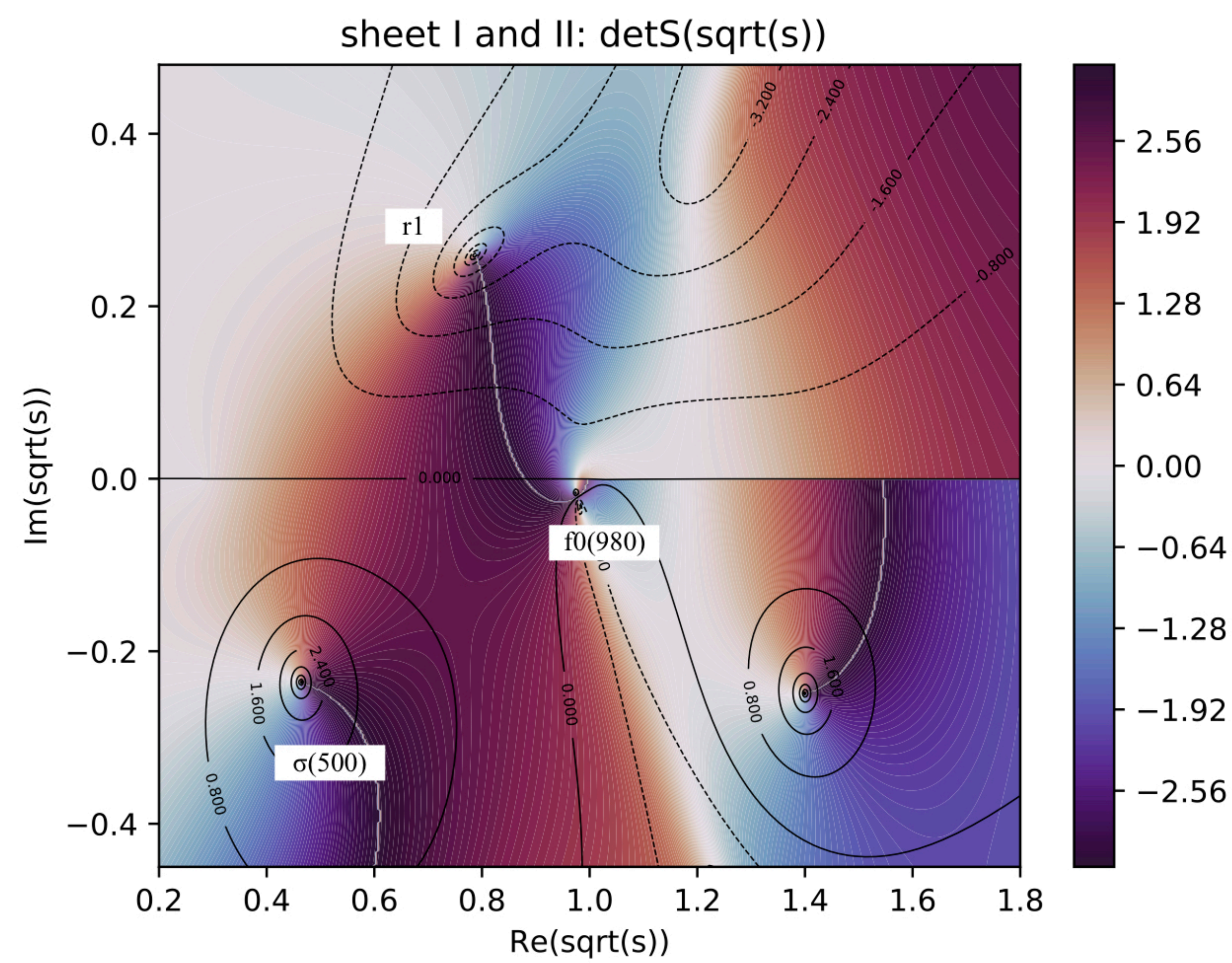
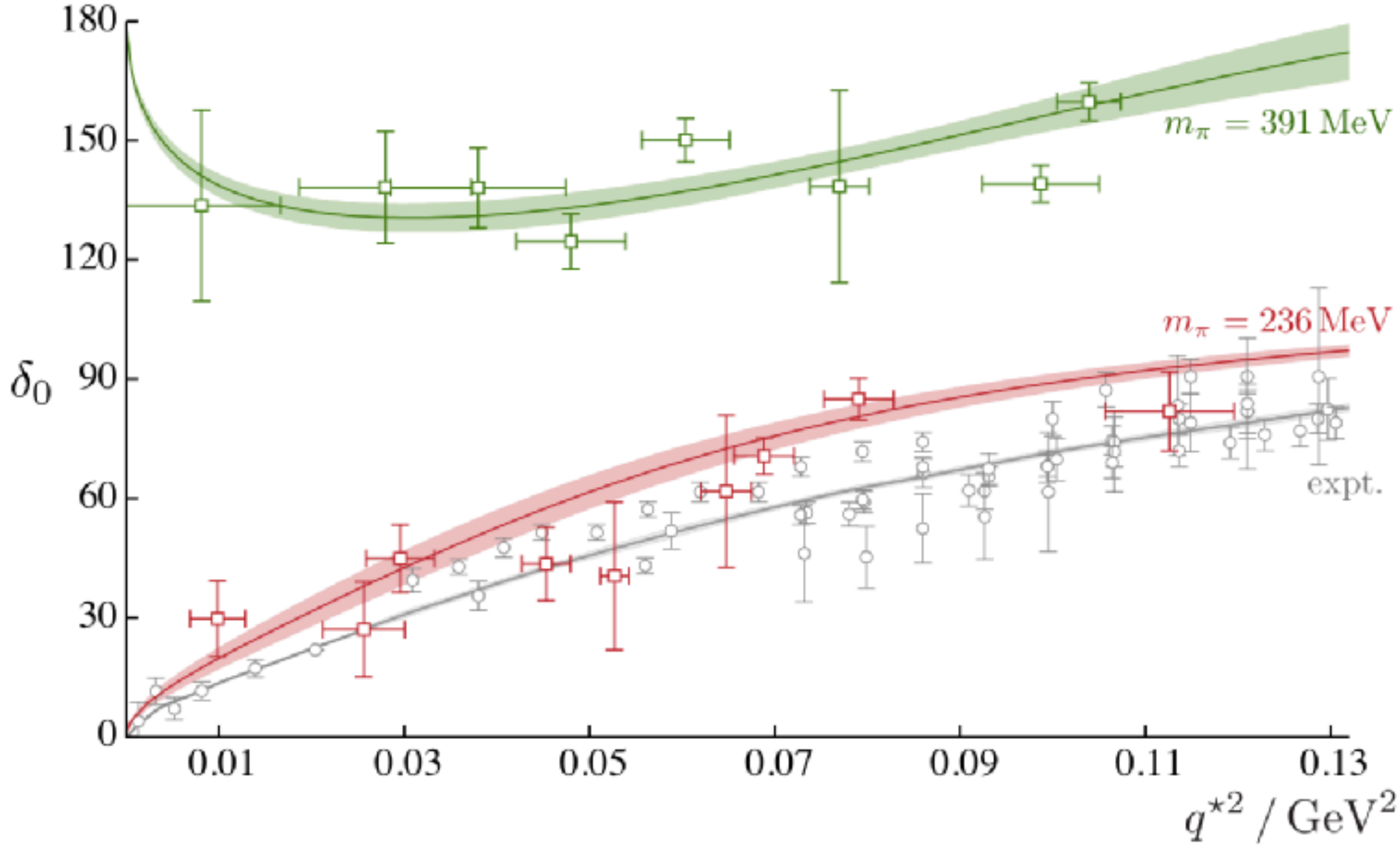




LATTICE COMPUTATIONS ON PHASE SHIFT

deuteron physics?



R. A. Briceno, J. J. Dudek and R. D. Young, arXiv:1706.06223 [hep-lat].

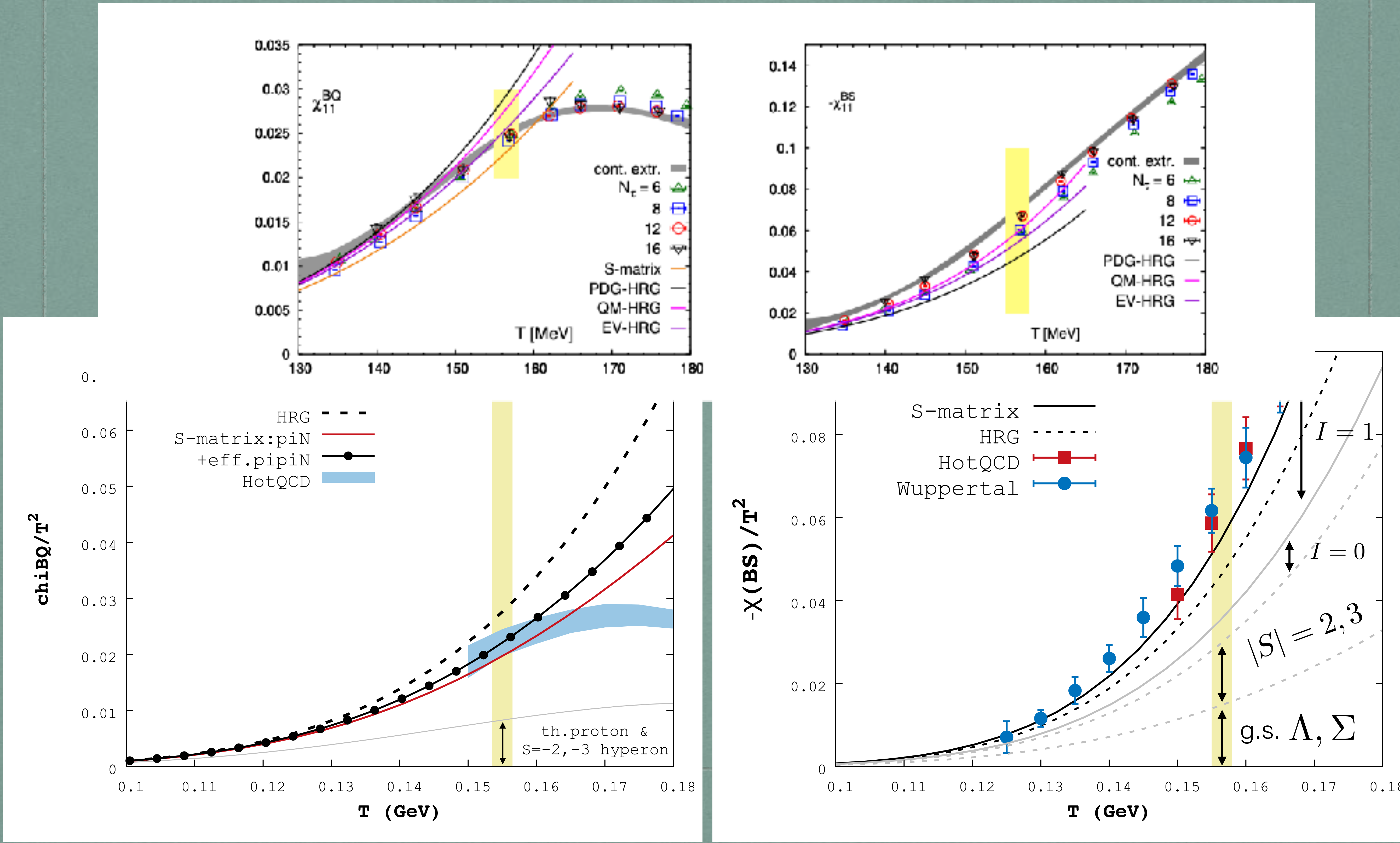
$$\frac{\partial}{\partial g} \ln Z$$ *is smooth?*

TO DO
&
I OWE YOU('S)

TO DO...

Updated LQCD results

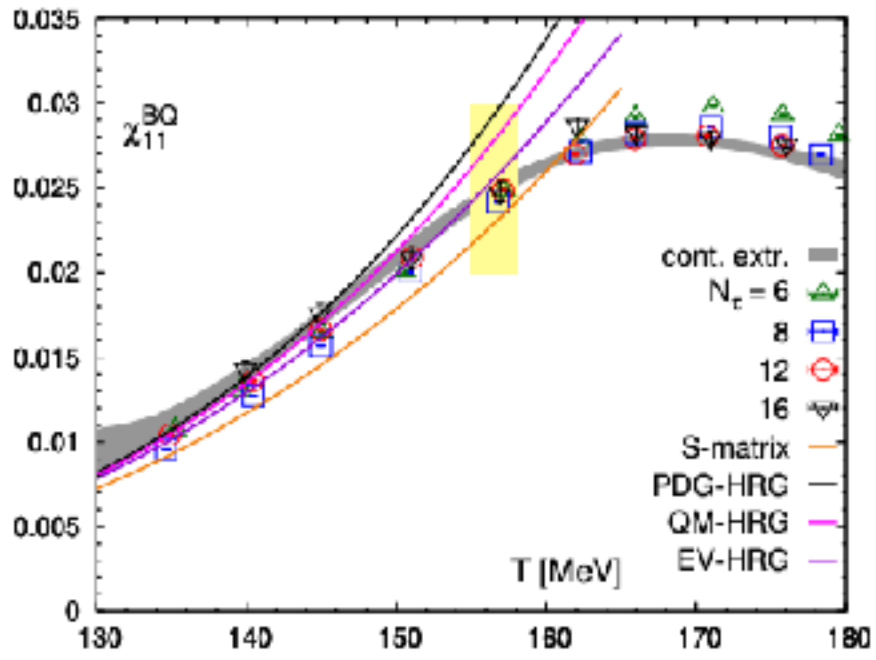
Goswami et al. 2011.02812

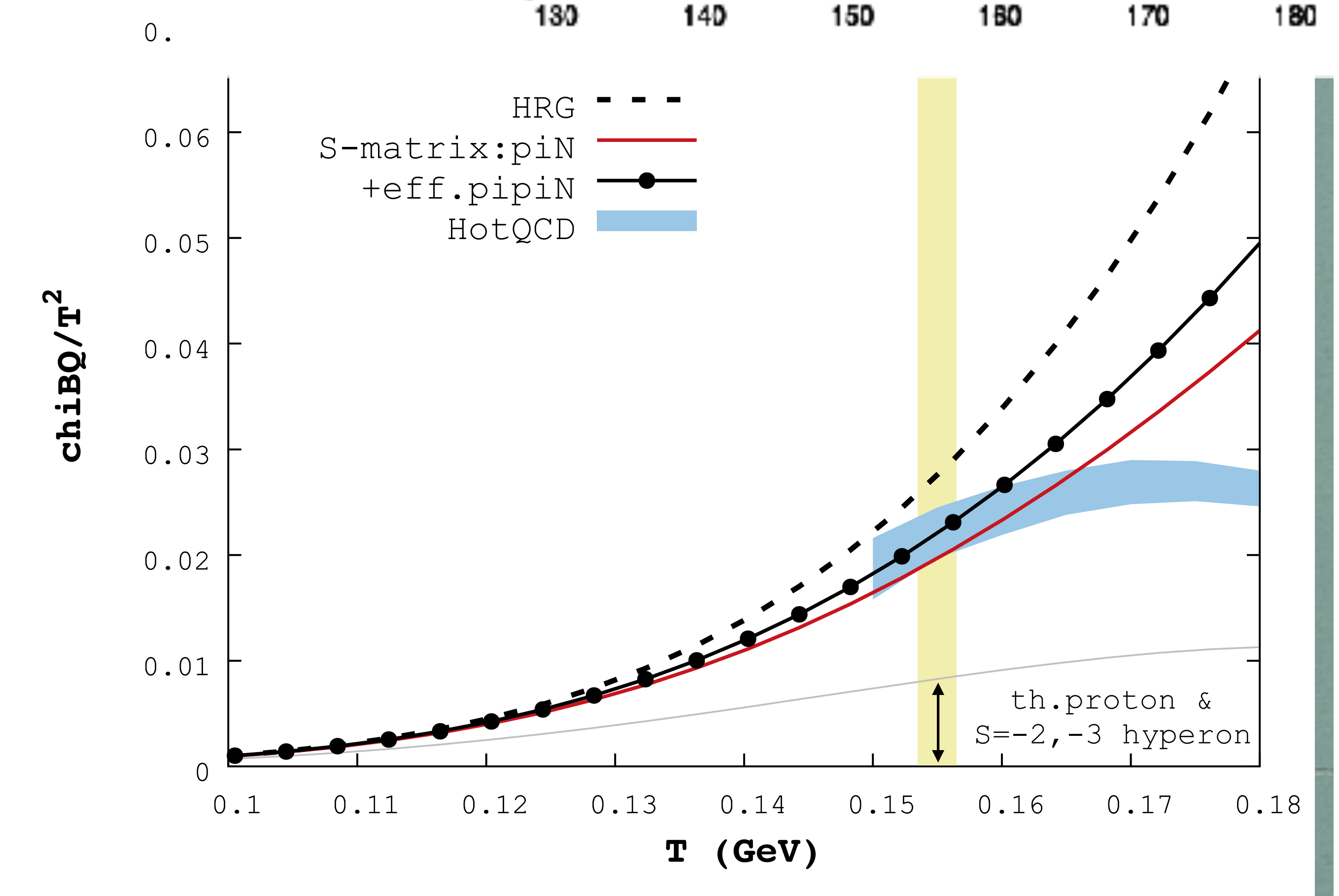


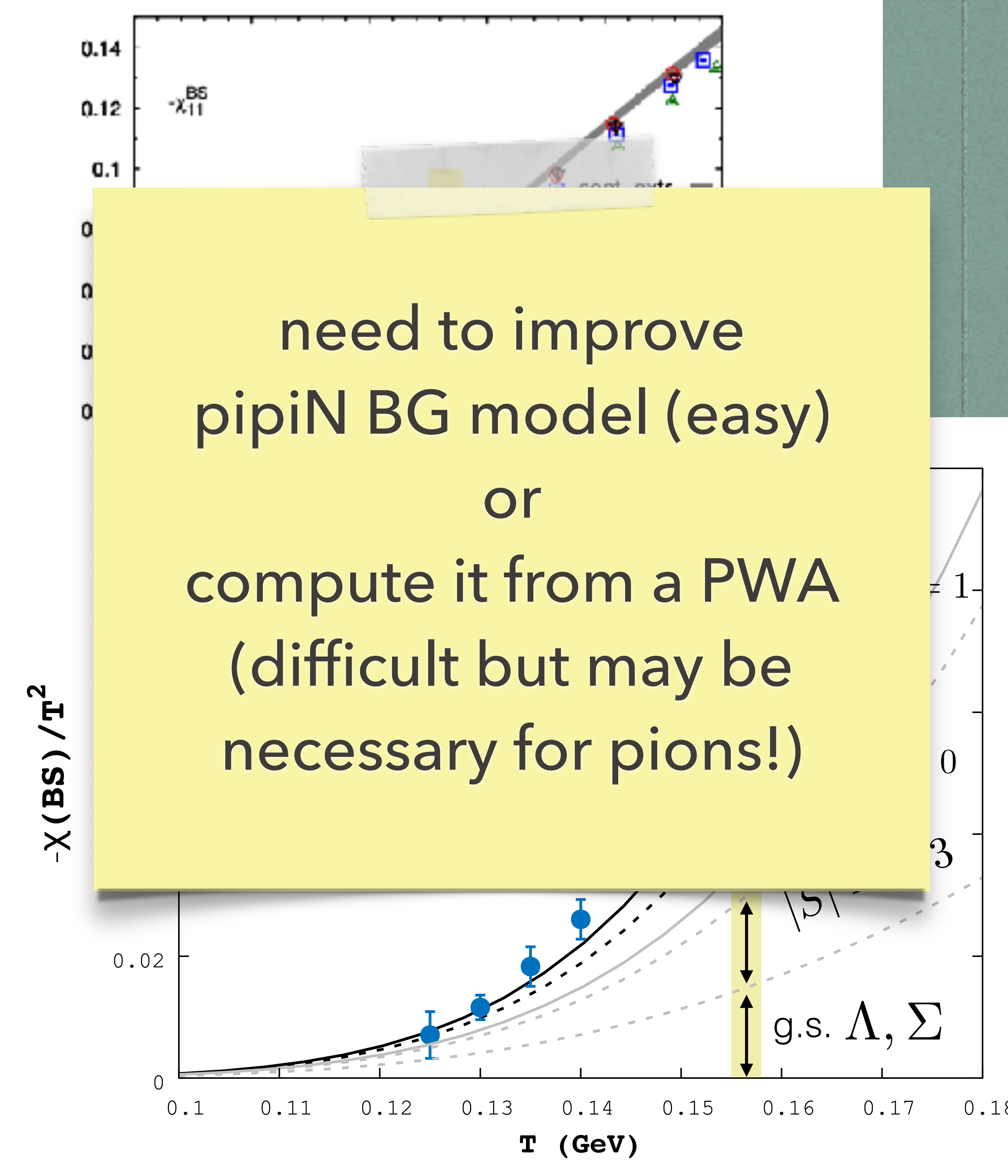
TO DO...

Updated LQCD results

Goswami et al. 2011.02812







need to improve
pipiN BG model (easy)
or
compute it from a PWA
(difficult but may be
necessary for pions!)

TO DO…

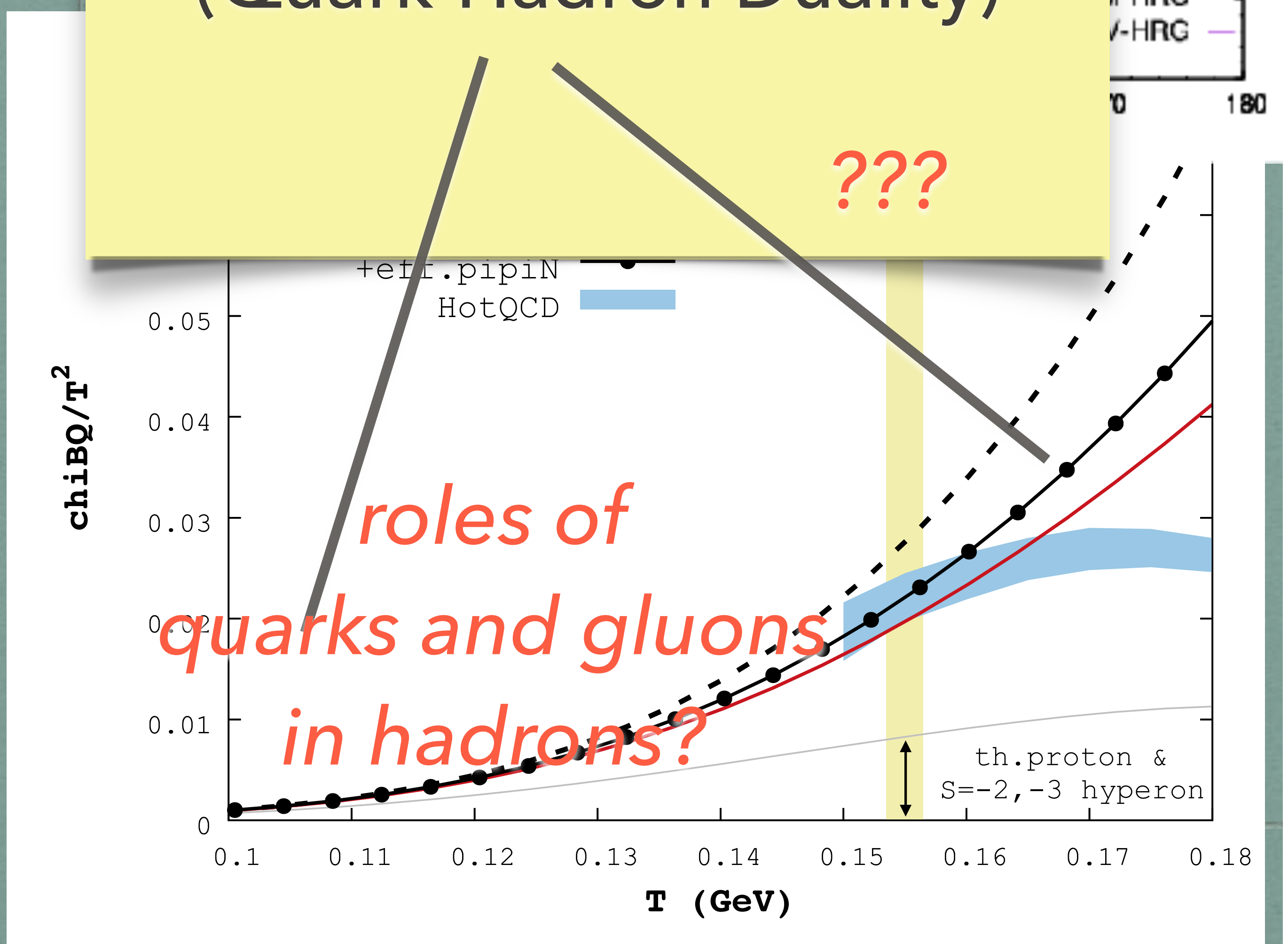
Updated LQCD results

Goswami et al. 2011.02812

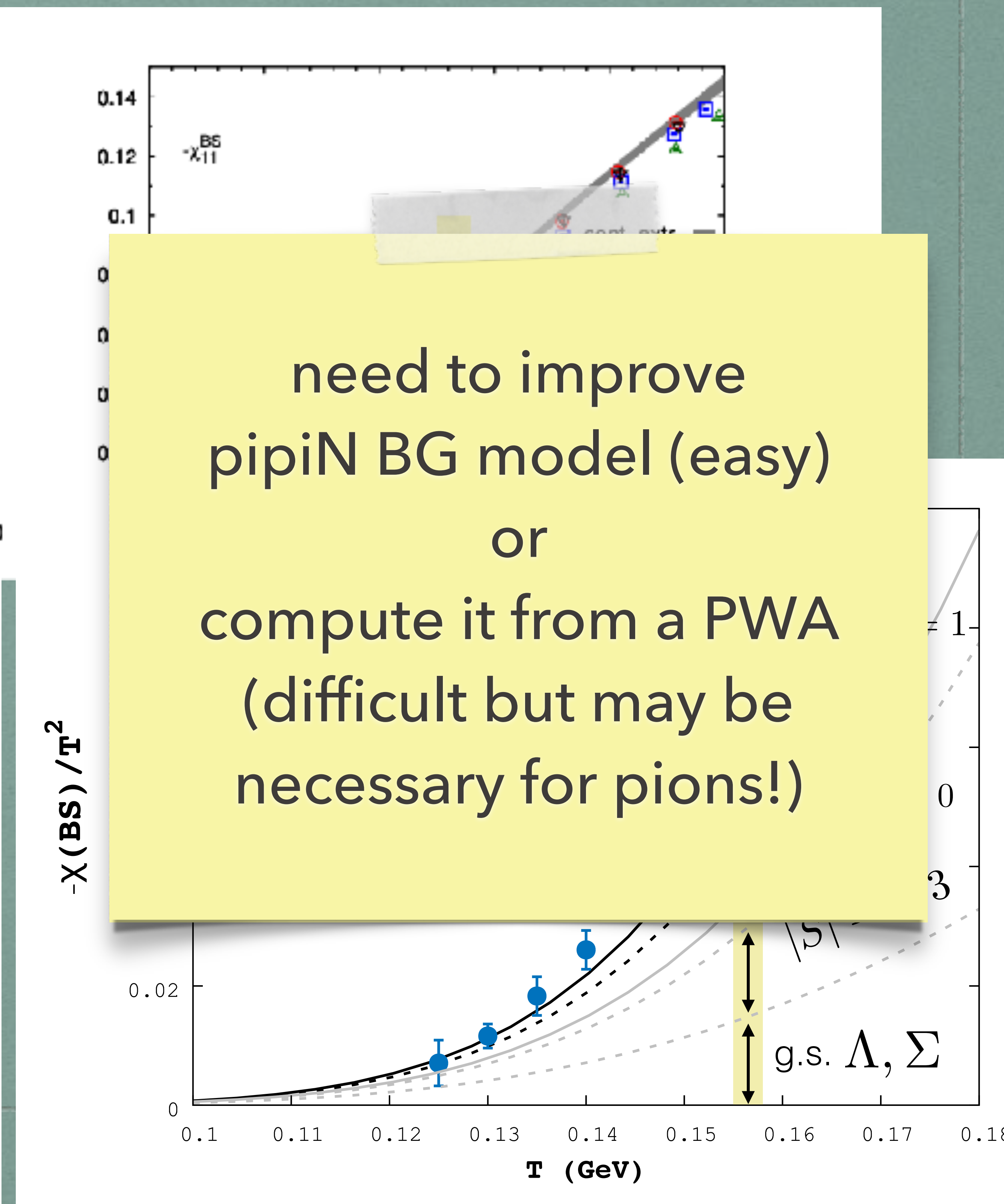
amplitudes should be independent of DoFs used to calculate them (Quark-Hadron Duality)

???

need to improve pipiN BG model (easy) or compute it from a PWA (difficult but may be necessary for pions!)



roles of quarks and gluons in hadrons?



NO SERIOUS MESON SPECTROSCOPY
WITHOUT SCATTERING*

George Rupp

CFIF, Instituto Superior Técnico, Universidade de Lisboa, 1049-001, Portugal

Eef van Beveren

CFC, Departamento de Física, Universidade de Coimbra, 3004-516, Portugal

Susana Coito

Institute of Modern Physics, CAS, Lanzhou 730000, China

(Received January 25, 2015)

CONTINUUM

$$Z = \sum_{\alpha=B,M} \langle\alpha|e^{-\beta H}|\alpha\rangle$$

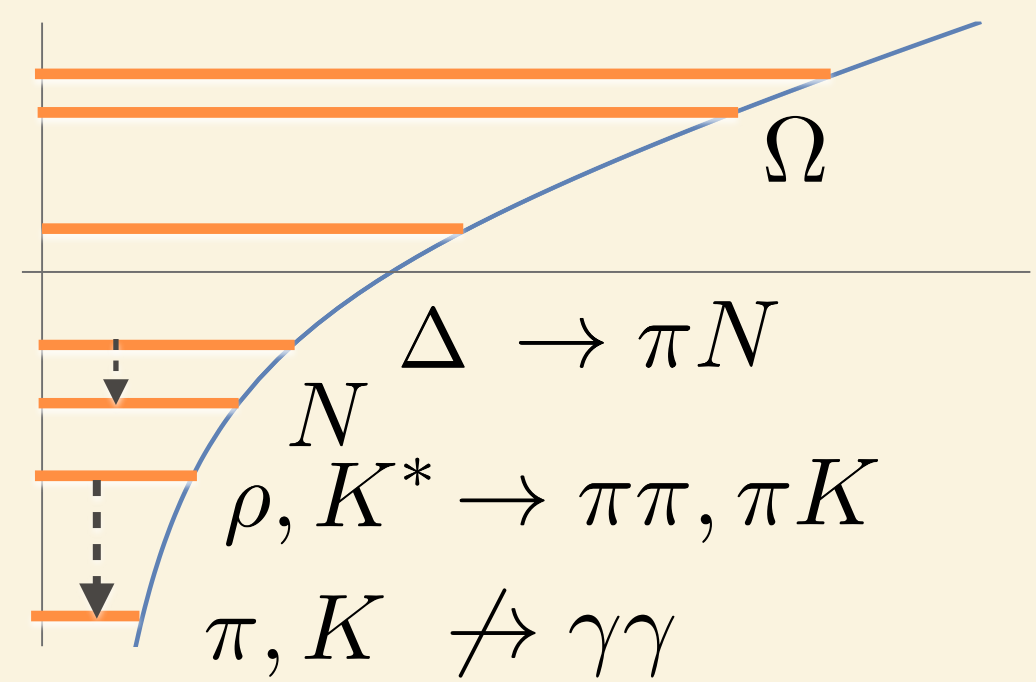
meson loops effects:

shift in hadron masses

S-matrix issues:

how resonances are expressed by the scattering states?
(and by quarks and gluons?)

QCD spectrum



DENSE (R) MEDIUM

CONVERGENCE?

$$\Delta P = T\xi_N^2 \int \frac{d^3P}{(2\pi)^3}\frac{d\epsilon}{2\pi} e^{-\beta(\frac{P^2}{2m_{\rm tot}}+\epsilon)} \times 2\frac{\partial}{\partial\epsilon}(-qa_S)$$

$$= -a_S \frac{2\pi}{m_{\rm red}}\xi_N^2 n_N^2$$

$$P^{(0)} = n_N T \xi_N \qquad n_N = (\lambda_T^{-1})^3 = (\frac{m_N T}{2\pi})^{3/2}$$

$$r = \frac{\Delta P}{P^{(0)}} = -2 \times (a_S/\lambda_T) \times \xi_N$$

$$= -2a_S\,(\frac{m_N T}{2\pi})^{1/2}\, e^{(\mu - m_N)/T}$$

CONVERGENCE?

$$\Delta P = T\xi_N^2 \int \frac{d^3P}{(2\pi)^3}\frac{d\epsilon}{2\pi} e^{-\beta(\frac{P^2}{2m_{\rm tot}}+\epsilon)} \times 2\frac{\partial}{\partial\epsilon}(-q a_S)$$

$$= -a_S \frac{2\pi}{m_{\rm red}}\xi_N^2 n_N^2$$

$$P^{(0)} = n_N T \xi_N$$

$$r = \frac{\Delta P}{P^{(0)}} = -2 \times (a_S/\lambda$$

$$= -2a_S \left(\frac{m_N T}{2\pi}\right)^{1/2} e^{(\mu - m_N)/T}$$

$a_S = 20\, fm$

$r \approx 0.0727$ LHC

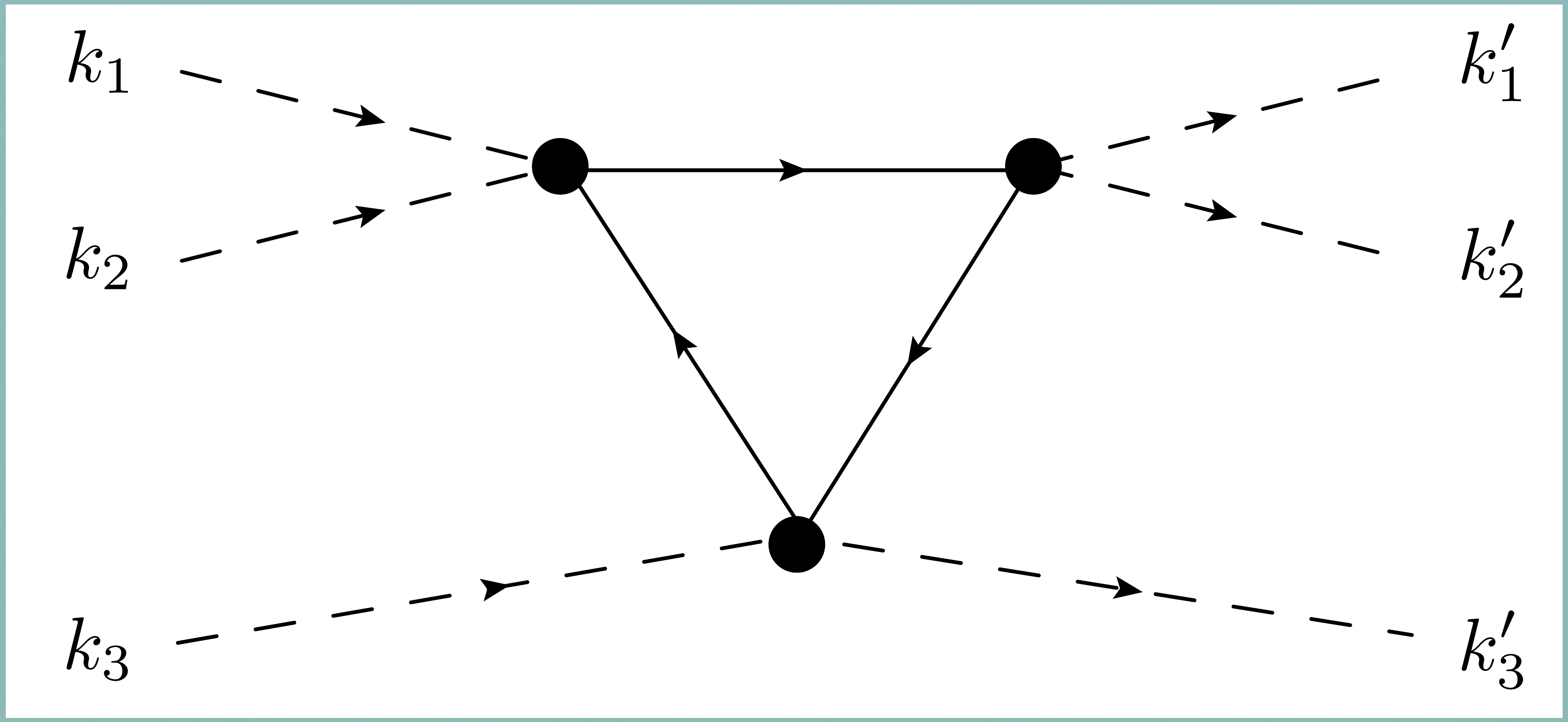
$r \approx 0.36$ $T = 60\, MeV$

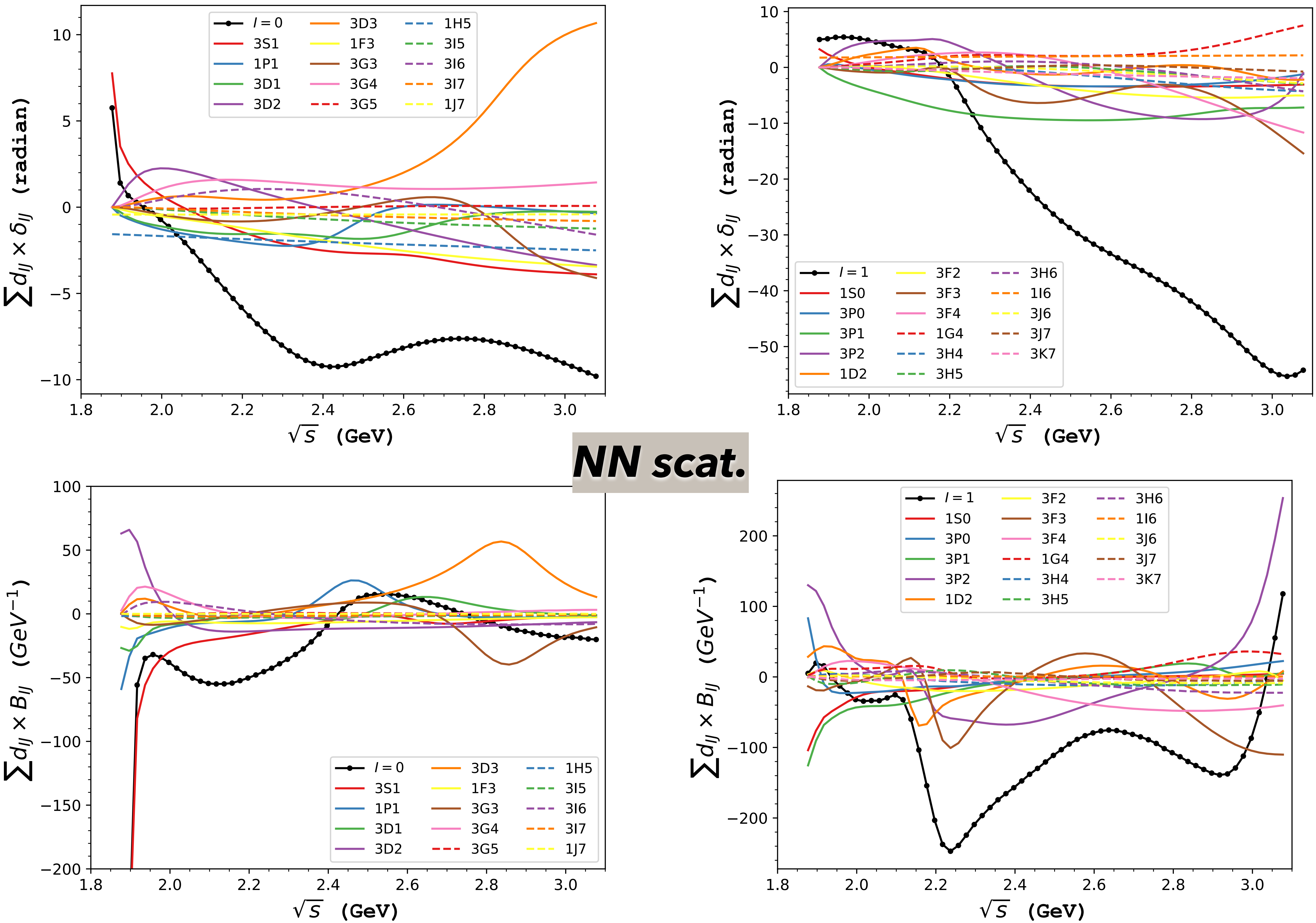
$r \approx 1.92$ $\mu_B = 700, 800\, MeV$

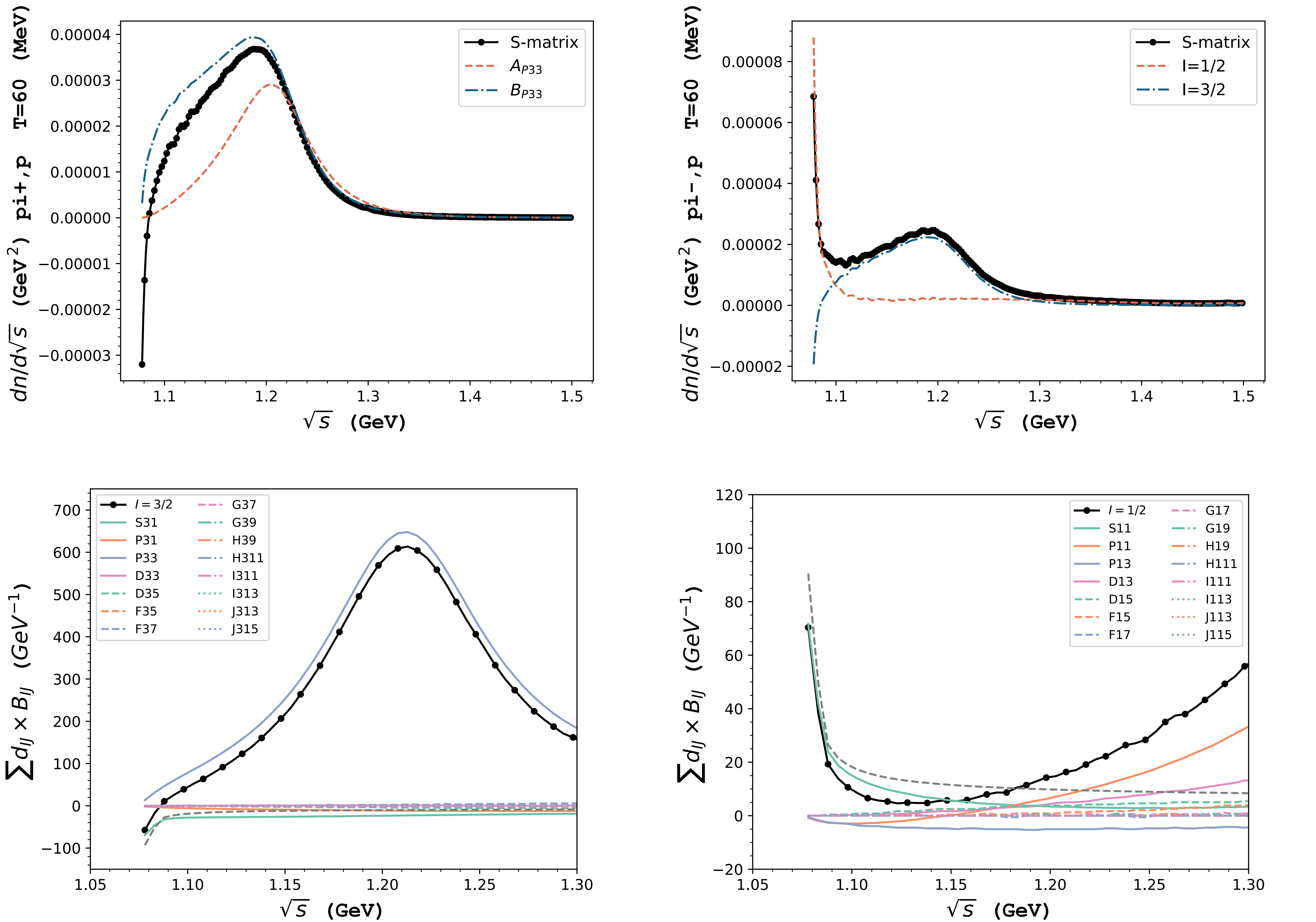
TRIANGLE DIAGRAM

- 3-body diagram

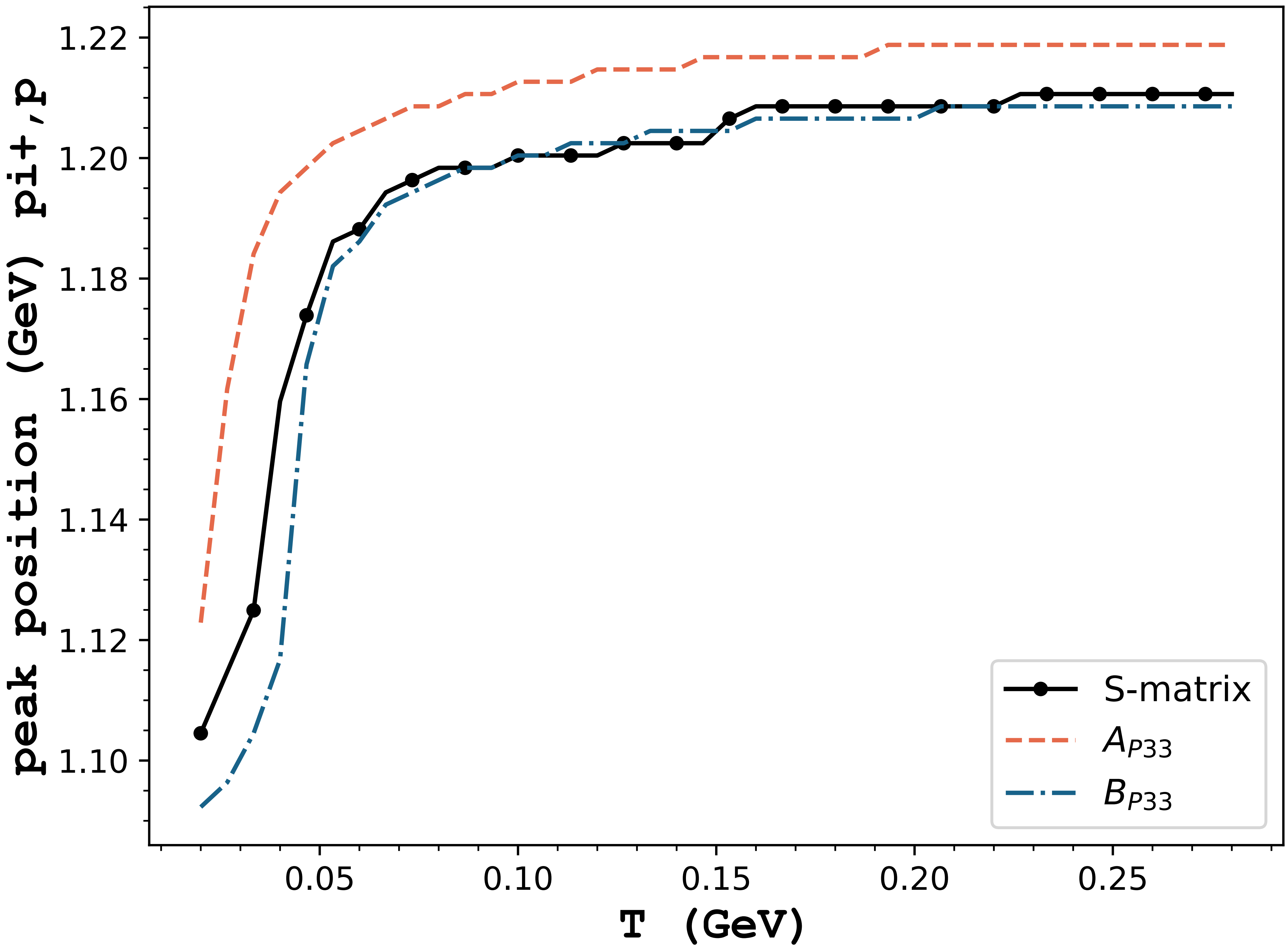
$$b_3 \propto a_S^3$$




NN scat.
$\sum d_{IJ} \times \delta_{IJ}$ (radian)
$\sqrt{s}$ (GeV)
$I = 0$
3S1
1P1
3D1
3D2
3D3
1F3
3G3
3G4
3G5
1H5
3I5
3I6
3I7
1J7
$I = 1$
1S0
3P0
3P1
3P2
1D2
3F2
3F3
3F4
1G4
3H4
3H5
3H6
1I6
3J6
3J7
3K7
$\sum d_{IJ} \times B_{IJ}$ (GeV^{-1})
1.8
2.0
2.2
2.4
2.6
2.8
3.0
10
5
0
−5
−10
−20
−30
−40
−50
100
50
−50
−100
−150
−200
200



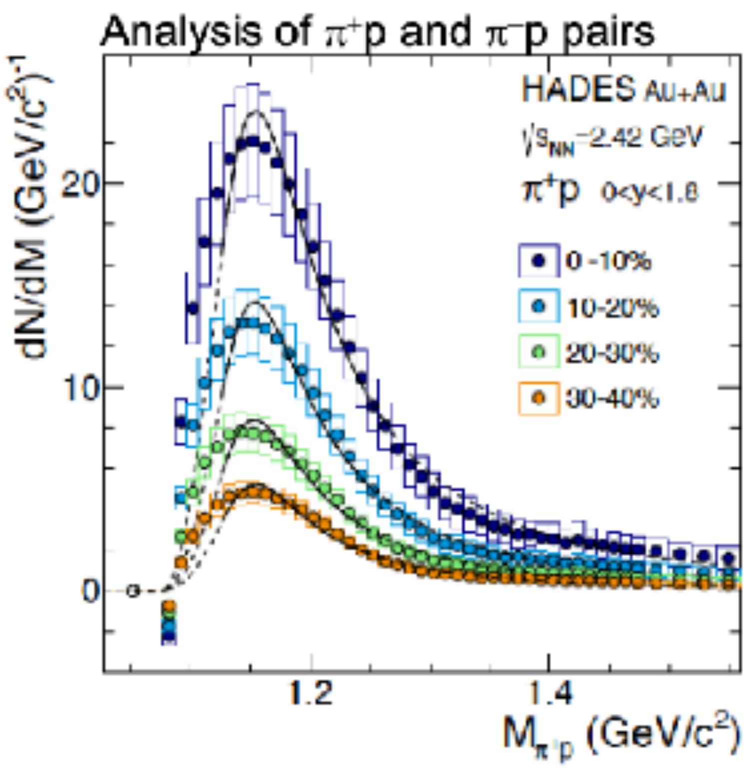
S-matrix
A_{P33}
B_{P33}
$dn/d\sqrt{s}$ (GeV^2) pi+,p T=60 (MeV)
$\sqrt{s}$ (GeV)
S-matrix
I=1/2
I=3/2
$dn/d\sqrt{s}$ (GeV^2) pi-,p T=60 (MeV)
$\sqrt{s}$ (GeV)
$I = 3/2$
S31
P31
P33
D33
D35
F35
F37
G37
G39
H39
H311
I311
I313
J313
J315
$\sum d_{IJ} \times B_{IJ}$ (GeV^{-1})
$\sqrt{s}$ (GeV)
$I = 1/2$
S11
P11
P13
D13
D15
F15
F17
G17
G19
H19
H111
I111
I113
J113
J115
$\sum d_{IJ} \times B_{IJ}$ (GeV^{-1})
$\sqrt{s}$ (GeV)



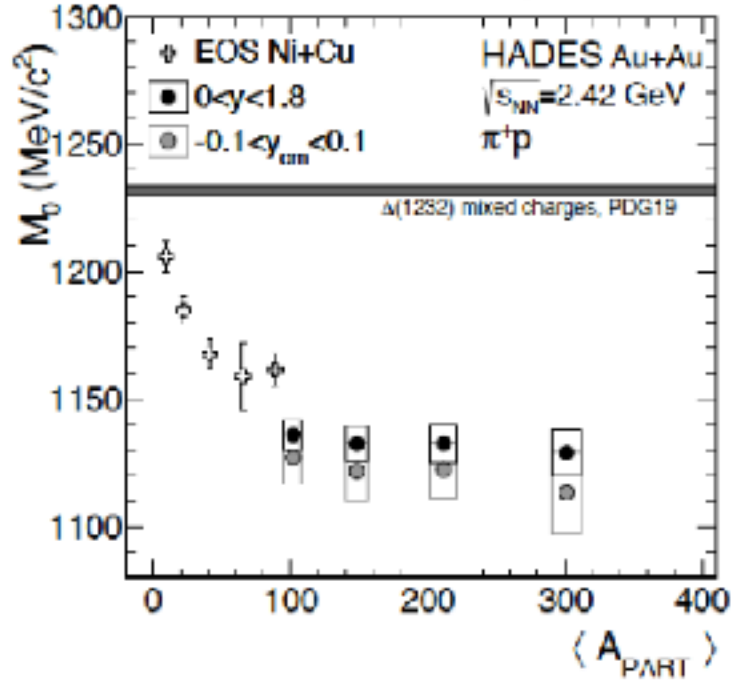
peak position (GeV) pi+,p
1.22
1.20
1.18
1.16
1.14
1.12
1.10
0.05
0.10
0.15
0.20
0.25
T (GeV)
S-matrix
A_{P33}
B_{P33}

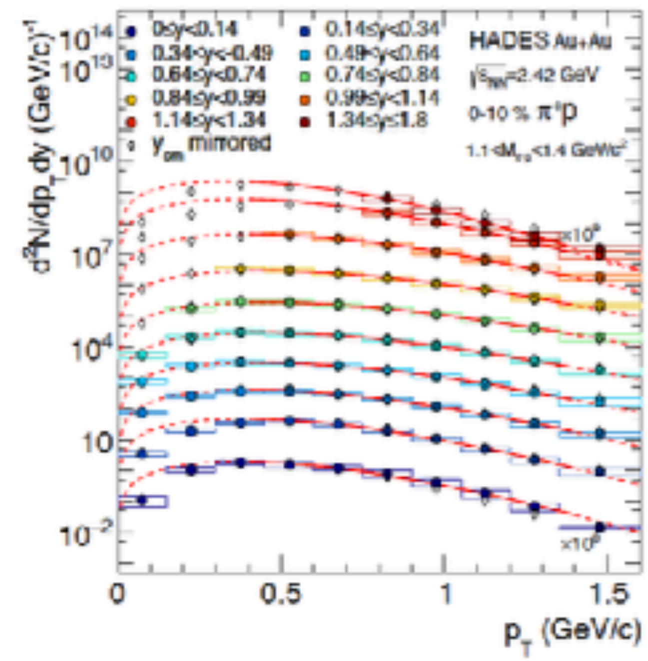
Correlatd Pion-Proton Pair Emission

HADES Collaboration, MS. submitted



$$\sigma_{BW}(M_{\pi p}) = \frac{q^3}{q^3+\mu^3}\frac{\sigma_0}{1+4[(M_{\pi p}-M_0)/\Gamma_0]^2}$$





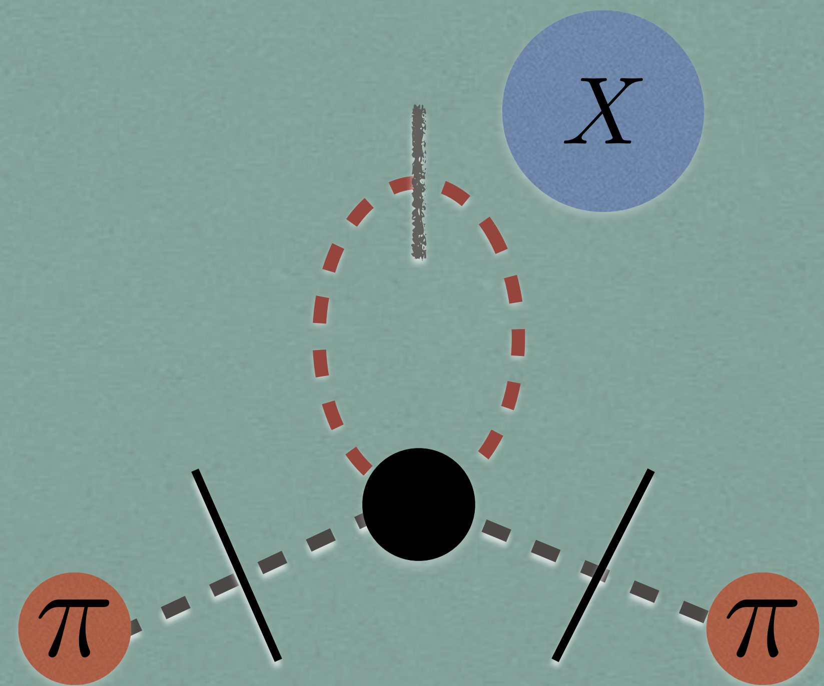
- High statistics allows multi-differential analysis
- Understanding of "kinematical" mass shift with *S*-matrix formalism
- Comparison to microscopic transport

UrQMD, T. Reichert et al., 2004.10539 [nucl-th]; Astron.Nachr. 340 (2019) 9-10, 1018-1022

P.M. Lo, EPJC 77 (2017) no.8, 533
R. Dashen *et al.*, Phys. Rev. 187 (1969) 345
R.Venugopalan *et al.*, NPA 546 (1992) 718
W. Weinhold, B. Friman, PLB 433 (1998) 236

IN-MEDIUM EFFECTS FROM S-MATRIX

$$\Sigma_\pi =$$



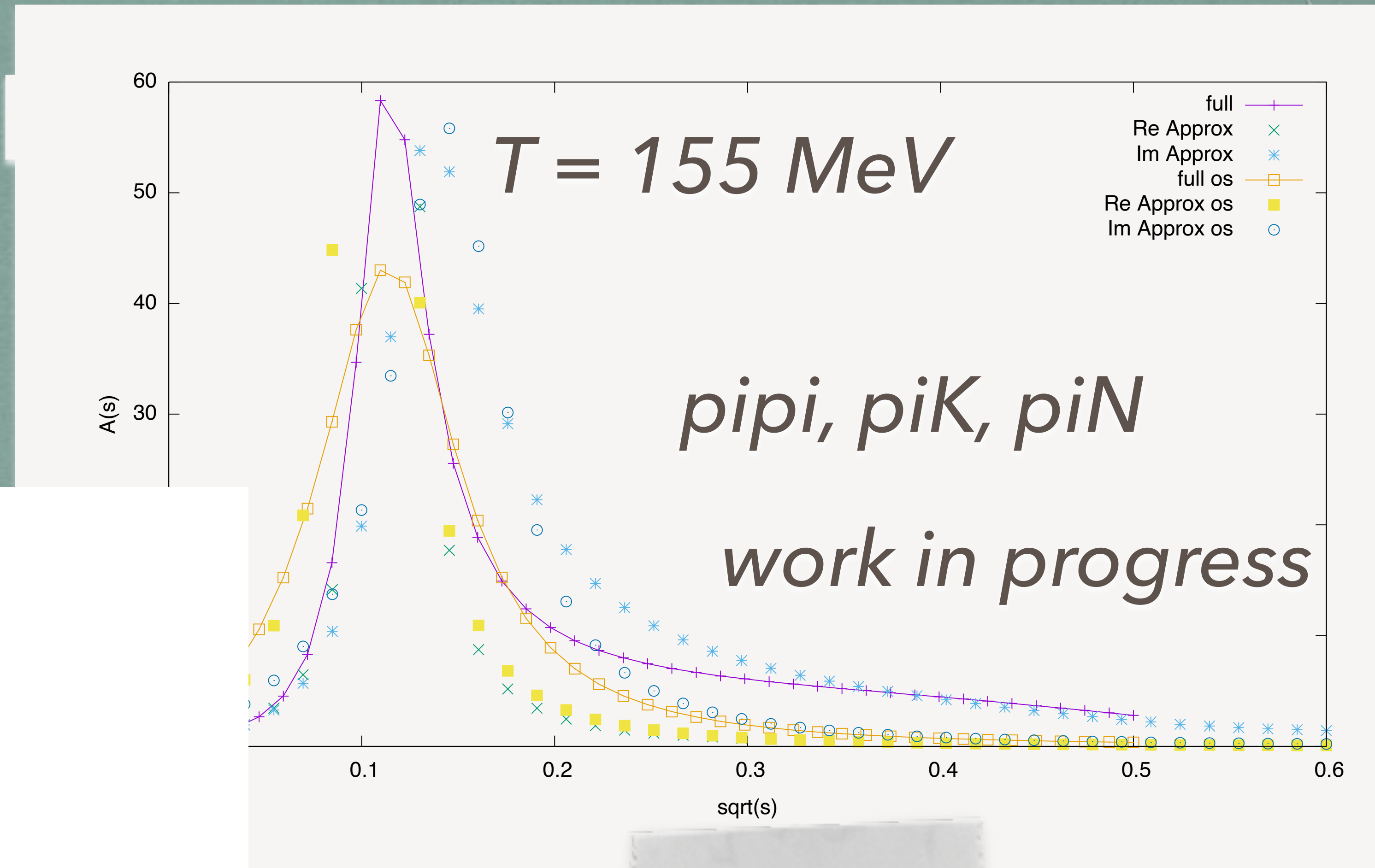
$$\propto \int \frac{d^3q}{\omega_q} n_X T_{\pi X}(s)$$

forward amplitude

A. Schenk NPB 363(1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)

IN-MEDIUM EF
S-MATRIX



COLLECTIVE INTERACTION OF MESONS IN HOT HADRONIC MATTER

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Abstract: We calculate momentum-dependent optical potentials (both real and imaginary parts) for mesons propagating in hot hadronic matter. For sufficiently dilute matter the results obtained are essentially model-independent, and based directly on experimental information about hadronic scattering amplitudes. Even at the lowest (break-up) temperatures available in high-energy hadronic collisions, $T_f = 120$–150 MeV, these potentials can significantly modify spectra of the outgoing hadron, leading to a peak at small p_t. Other observable consequences of these phenomena are changes of the line shape of ρ-, ω- and ϕ-mesons, which can be observed in the dilepton mode. For such mesons, decaying inside the matter, we estimate both the expected shift and broadening of the peaks.

1. Introduction

One of the main goals of experiments with high-energy nuclear collisions is to produce and to study superdense matter, in particular, trying to reach conditions at which the phase transitions into the so-called "quark-gluon plasma" phase [1-6]) can take place.

However, one may approach the problem from another side, looking at the QCD phase transition from the low-temperature end. In this paper we study properties of hot hadronic matter in which volumes per particle are a few cubic fermi, so that it certainly is made out of individual hadrons. So far, physics of such matter has not been studied much, and we should start this paper with its general motivations.

First of all, one has to mention a simple practical consideration: *such matter is*

+ coupled-channel aspects

T

045015 (1998)

THANK YOU

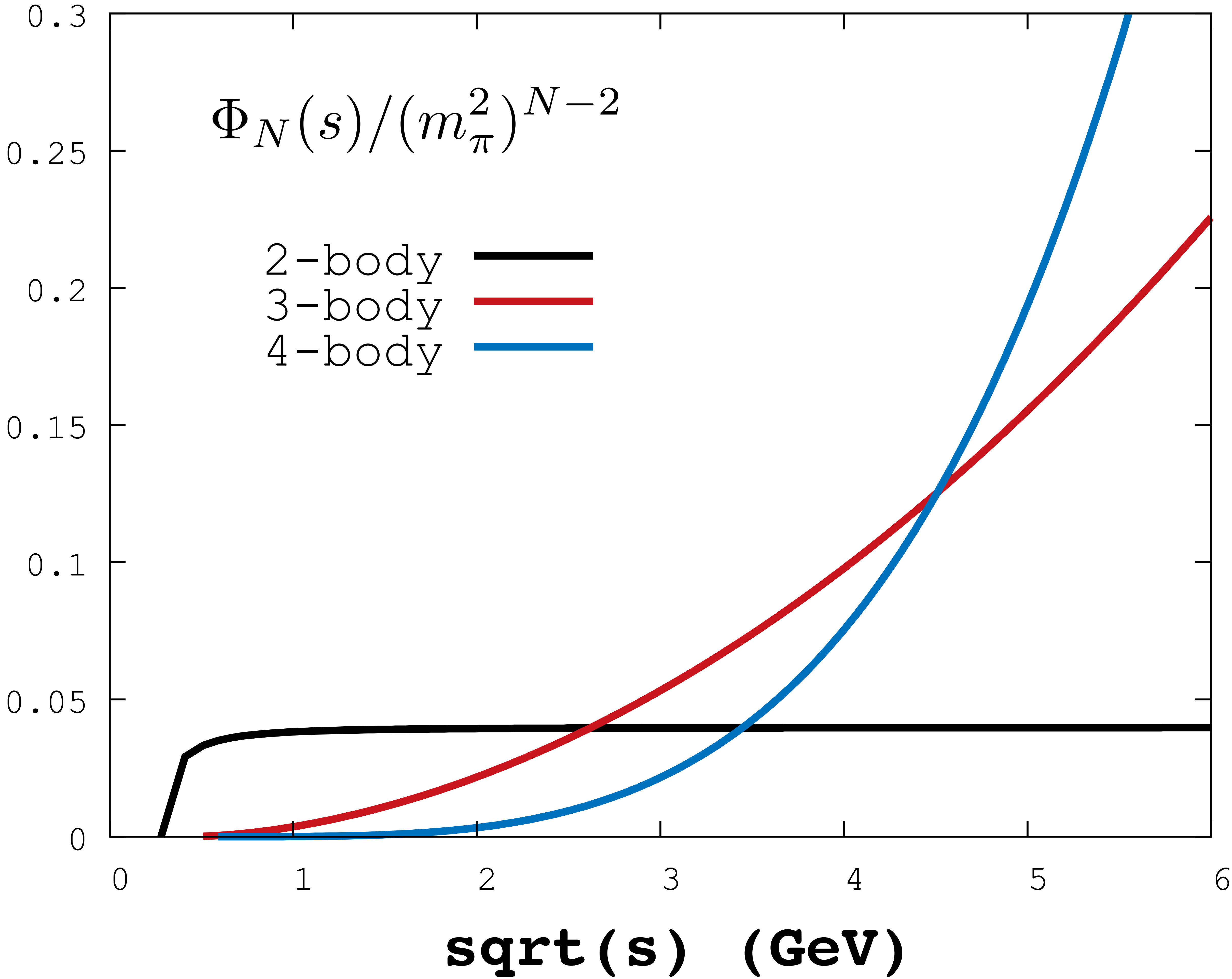
T-MATRIX REPRESENTATION

$$\frac{1}{4\,i}\,\mathrm{tr}\,\left[S^{-1}\,\frac{\overleftrightarrow{\partial}}{\partial E}\,S\right]_c \longleftrightarrow \frac{\partial \delta_E}{\partial E}$$

$$\frac{1}{4}\,\frac{\partial}{\partial E}\,\mathrm{tr}\,\left[T + T^\dagger\right]_c \longleftrightarrow (1 - 2\sin^2\delta_E) \times \frac{\partial \delta_E}{\partial E}$$

$$\frac{1}{4\,i}\,\mathrm{tr}\,\left(T^\dagger\,\frac{\overleftrightarrow{\partial}}{\partial E}\,T\right)_c \longleftrightarrow 2\sin^2\delta_E \times \frac{\partial \delta_E}{\partial E}.$$

Landau Lifshitz classification



0.3
0.25
0.2
0.15
0.1
0.05
0
$\Phi_N(s)/(m_\pi^2)^{N-2}$
2-body
3-body
4-body
0
1
2
3
4
5
6
sqrt(s) (GeV)